# Linear SVM: objective

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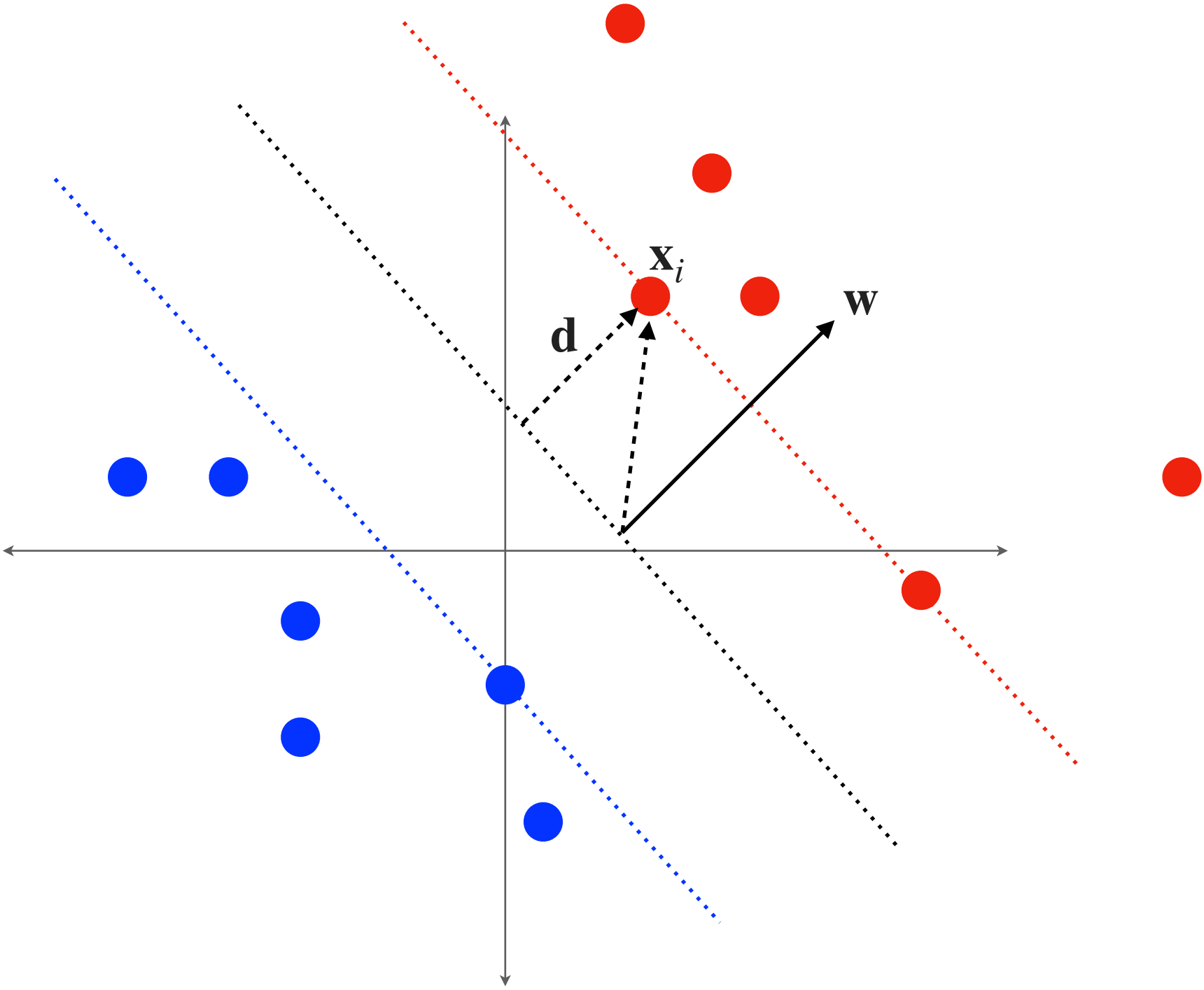
$\mathbf{x}_i$
d
w

starting objective: $$\gamma_{\mathbf{w},b} = \min_{\mathbf{x}_i \in \mathbf{X}} \frac{|\mathbf{w}^T \mathbf{x}_i + b|}{\|\mathbf{w}\|_2}$$

What's the problem with this?

$\mathbf{w}, b$ are unconstrained!

constraint: $\forall i \; y_i(\mathbf{w}^T\mathbf{x}_i + b) \geq 0$

new objective: $\max_{\mathbf{w},b} \frac{1}{\|\mathbf{w}\|_2} \left[ \min_{\mathbf{x}\in\mathbf{X}} |\mathbf{w}^T\mathbf{x} + b| \right]$ s.t. $y_i(\mathbf{w}^T\mathbf{x}_i + b) \geq 0$

new problem: scale of $\mathbf{w}, b$ unconstrained; sol's not unique

new constraint: $\min_{\mathbf{x} \in \mathbf{X}} | \mathbf{w}^T \mathbf{x}_i + b | = 1$

new objective: $\min_{\mathbf{w},b} \|\mathbf{w}\|_2^2$ s.t. $\forall i \; y_i\left(\mathbf{w}^T\mathbf{x}_i + b\right) \geq 1$

# Linear SVM: objective

starting objective: $\gamma_{\mathbf{w},b} = \min_{\mathbf{x}_i \in \mathbf{X}} \frac{|\mathbf{w}^T\mathbf{x}_i + b|}{\|\mathbf{w}\|_2}$

What's the problem with this? $\mathbf{w}, b$ are unconstrained!

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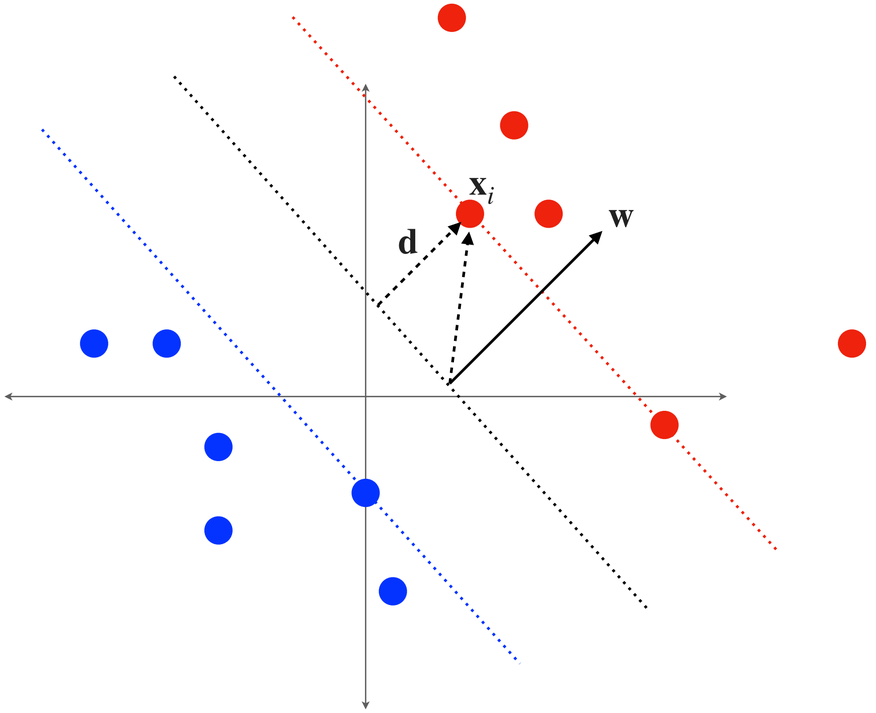

# The primal SVM objective is a quadratic program

primal form of objective: