- Somewhat counter intuitively, in high dimensional space, volume gets concentrated at the boundaries. In the world of decision boundaries, this means that presence of a linearly separating hyperplane increases with the number of dimensions, and in fact is guaranteed to exist at … great news!?!?!

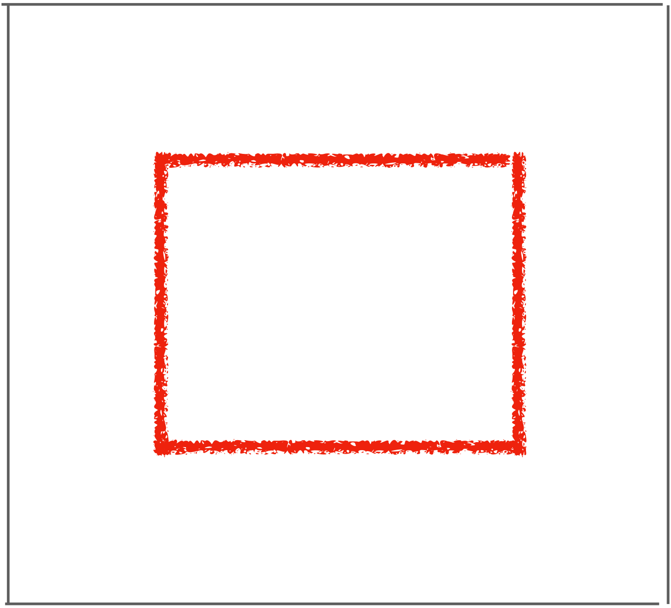
Curse of dimensionality

7

intuition: given a hypercube $\in \mathbb{R}^N$ with sides of length d, what fraction of space is occupied by the inner hypercube with sides of length $d/2$?

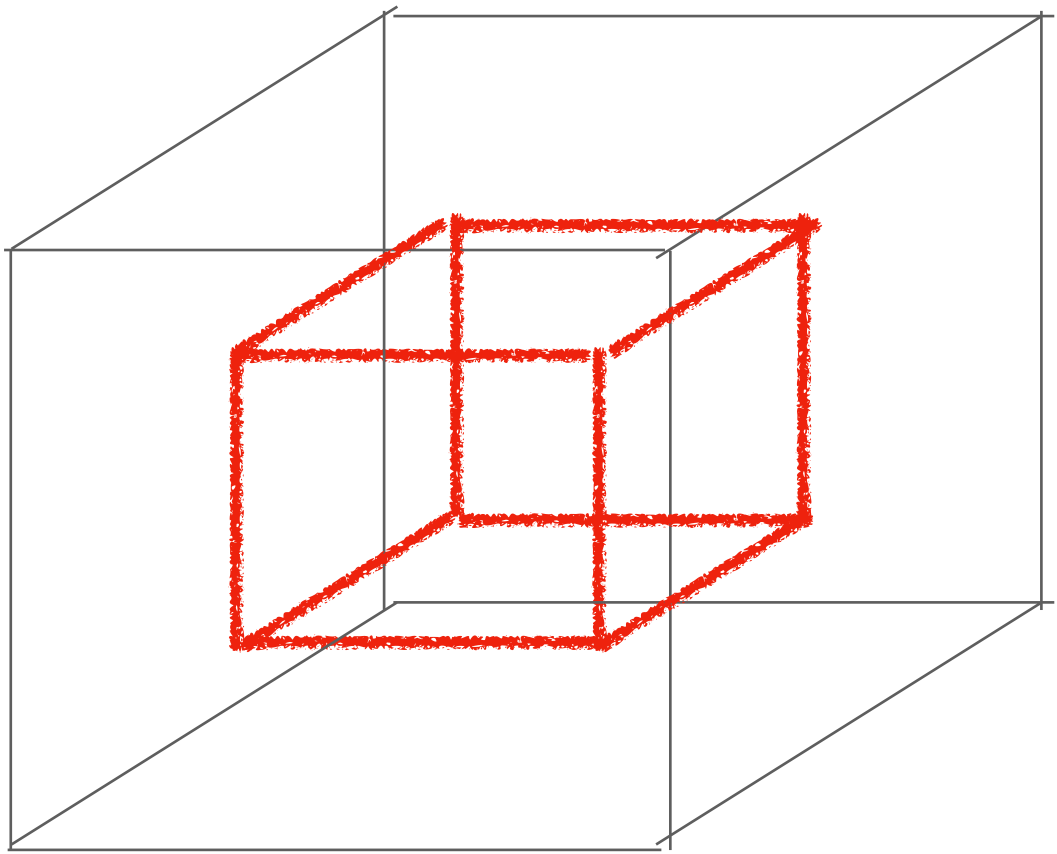
$$N = 2$$

$$\frac{v}{v_0} = \frac{1}{4}$$



$$N = 3$$

$$\frac{v}{v_0} = \frac{1}{8}$$



$$N = 4 \qquad\qquad N = 5$$

$$\frac{v}{v_0} = \frac{1}{16} \qquad\qquad \frac{v}{v_0} = \frac{1}{32}$$

$$N \to \infty$$

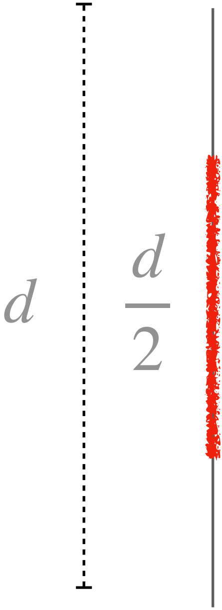
$$N = 1000$$

$$\cdots\cdots\cdot \quad \frac{v}{v_0} \approx \frac{1}{1 \times 10^{-301}} \quad \cdots\cdots\cdot \quad \frac{v}{v_0} = \frac{1}{2^N}$$

$\longrightarrow$ the "curse"

$$N = 1$$

$$\frac{v}{v_0} = \frac{1}{2}$$



Curse of dimensionality

- Somewhat counter intuitively, in high dimensional space, volume gets concentrated at the boundaries. In the world of decision boundaries, this means that presence of a linearly separating hyperplane increases with the number of dimensions, and in fact is guaranteed to exist at $N \to \infty$ ... great news!?!?!

intuition: given a hypercube $\in \mathbb{R}^N$ with sides of length d, what fraction of space is occupied by the inner hypercube with sides of length $d/2$?

$N = 1$ $\quad \frac{v}{v_0} = \frac{1}{2}$

$N = 2$ $\quad \frac{v}{v_0} = \frac{1}{4}$

$N = 3$ $\quad \frac{v}{v_0} = \frac{1}{8}$

$N = 4$ $\quad \frac{v}{v_0} = \frac{1}{16}$

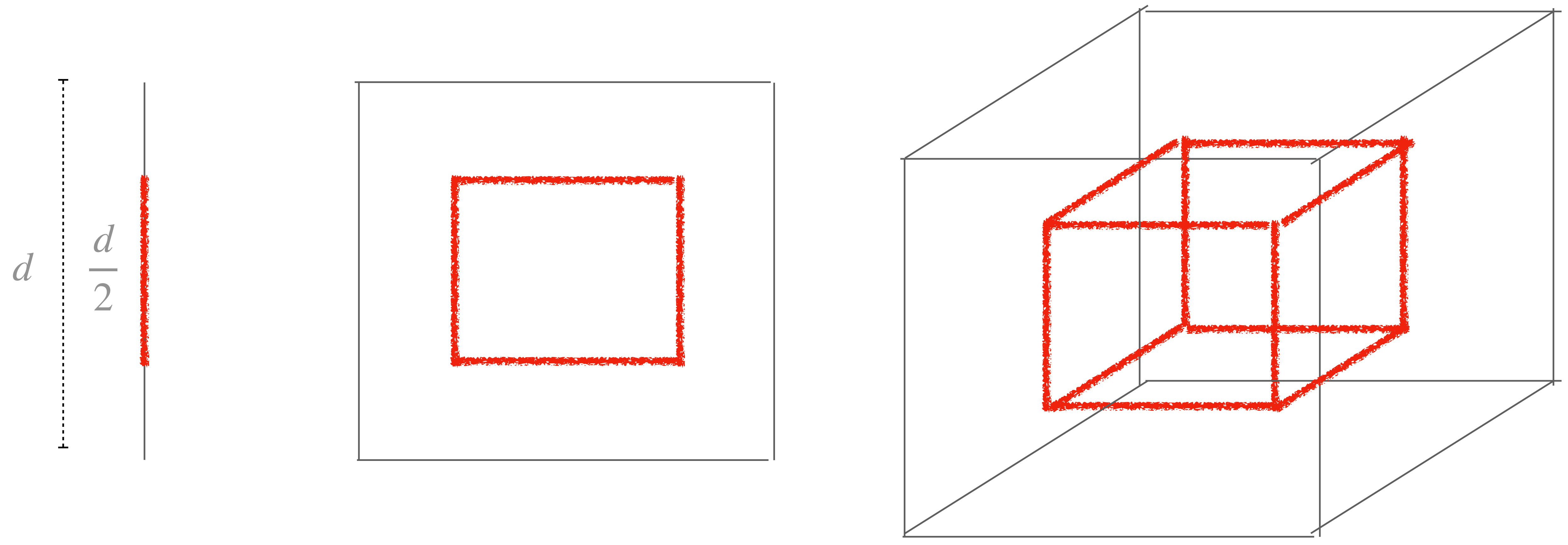
$N = 5$ $\quad \frac{v}{v_0} = \frac{1}{32}$

. . . .

$N = 1000$ $\quad \frac{v}{v_0} \approx \frac{1}{1 \times 10^{-301}}$

. . . .

$\frac{v}{v_0} = \frac{1}{2^N}$



the "curse"

Georgetown University

The Perceptron