Expected value and covariance functions

- Expectation: $\mathbb{E}_{x\sim P}[f(x)] = \sum_x P(x)f(x)$

- Variance: $Var(f(x)) = \mathbb{E}_x\left[\left(f(x) - \mathbb{E}[f(x)]\right)^2\right]$

- Covariance: $Cov(f_1(x), f_x(x)) = \mathbb{E}\left[\left(f_1(x) - \mathbb{E}[f_1(x)]\right) \cdot \left(f_2(x) - \mathbb{E}[f_2(x)]\right)\right]$

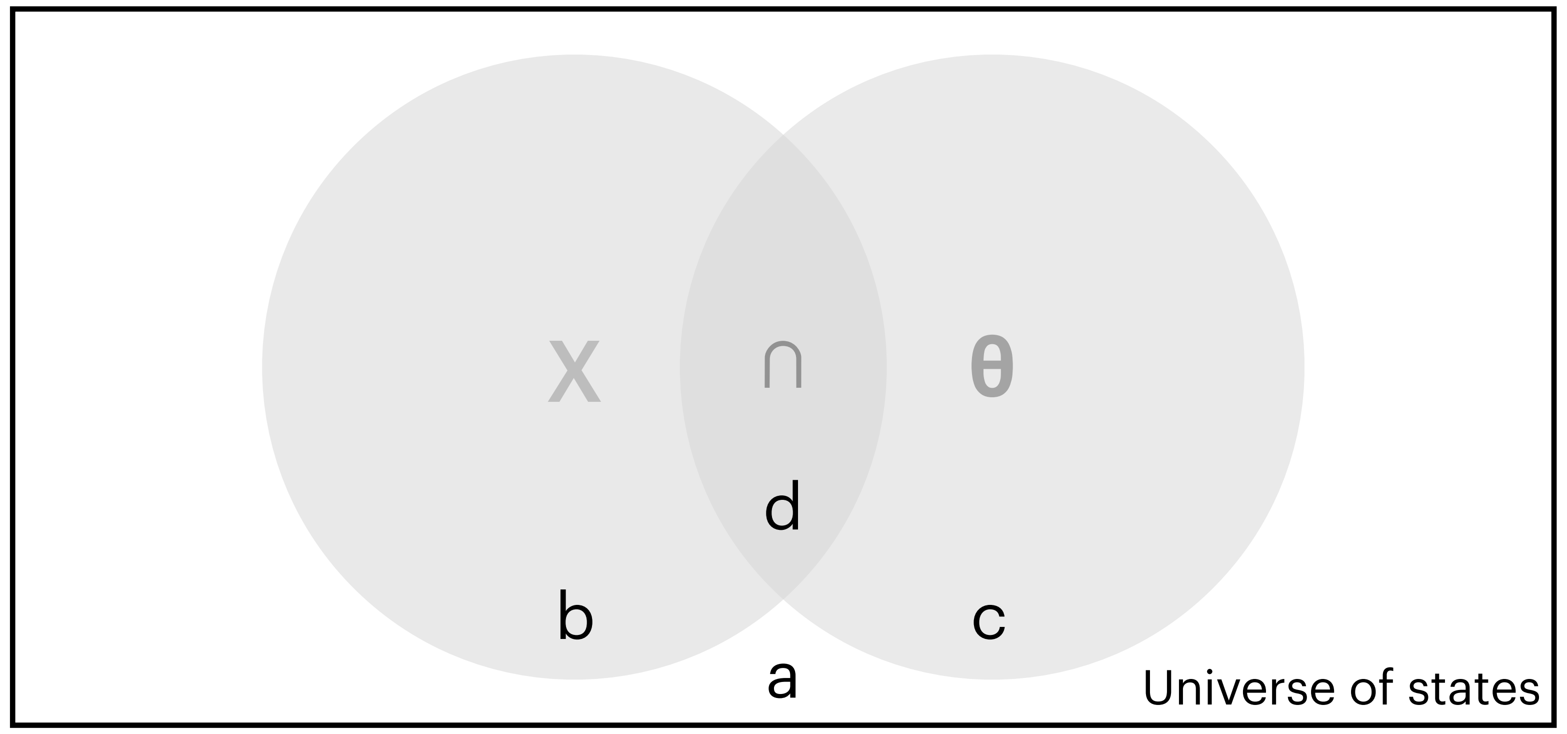
- Covariance of random vector, ***x***:

$$\begin{aligned} Cov(\mathbf{x}, \mathbf{x}) &= \mathbb{E}\left[(\mathbf{x} - \mathbb{E}[\mathbf{x}])(\mathbf{x} - \mathbb{E}[\mathbf{x}])^T\right] \\ &= \mathbb{E}\left[\mathbf{x}\mathbf{x}^T - \mathbb{E}[\mathbf{x}]\mathbb{E}[\mathbf{x}]^T\right] \end{aligned}$$

Bayes' Rule

- Intuitive statement: $P(x|\theta)P(\theta) = P(\theta|x)P(x)$

  - Proof $\dfrac{d}{\cancel{c}} \cdot \dfrac{\cancel{c}}{a} = \dfrac{d}{\cancel{b}} \cdot \dfrac{\cancel{b}}{a} = \dfrac{d}{a}$



- Bayes' Rule follows by deduction: $P(\theta|x) = \dfrac{P(x|\theta)P(\theta)}{P(x)}$ $\quad posterior = \dfrac{likelihood \times prior}{evidence}$

- Prescription for how to update a model given new evidence (i.e., new data)