Vector and tensor norms

- Measure of the size, or magnitude of a vector or tensor

- Definition: The following criteria qualify $f(\cdot)$ as a norm:
  - Positive definite: $f(\mathbf{x}) = 0 \quad iff \quad \mathbf{x} = \mathbf{0}$
  - Triangle inequality: $f(\mathbf{x}+\mathbf{y}) \leq f(\mathbf{x}) + f(\mathbf{y})$
  - Homogeneity: $\forall \alpha \in \mathbb{R} : f(\alpha \mathbf{x}) = |\alpha| f(\mathbf{x})$

- The ones we care about are:

  - Lp norm: $\|\mathbf{x}\|_p = \left(\sum_i |x_i|^p \right)^{1/p}$

  - Frobenius norm: $\|\mathbf{A}\|_F = \sqrt{\sum_{i,j} A_{i,j}^2}$

Distance metrics

- The two distance metrics used most often in machine learning are the Manhattan (L1) and Euclidean (L2) distances, which can be defined using the Lp norm:

  - Manhattan: $$\|\mathbf{x_1} - \mathbf{x_2}\|_1 = \sum_i |x_i^{(1)} - x_i^{(2)}|$$

  - Euclidean: $$\|\mathbf{x_1} - \mathbf{x_2}\|_2 = \sqrt{\sum_i (x_i^{(1)} - x_i^{(2)})^2}$$