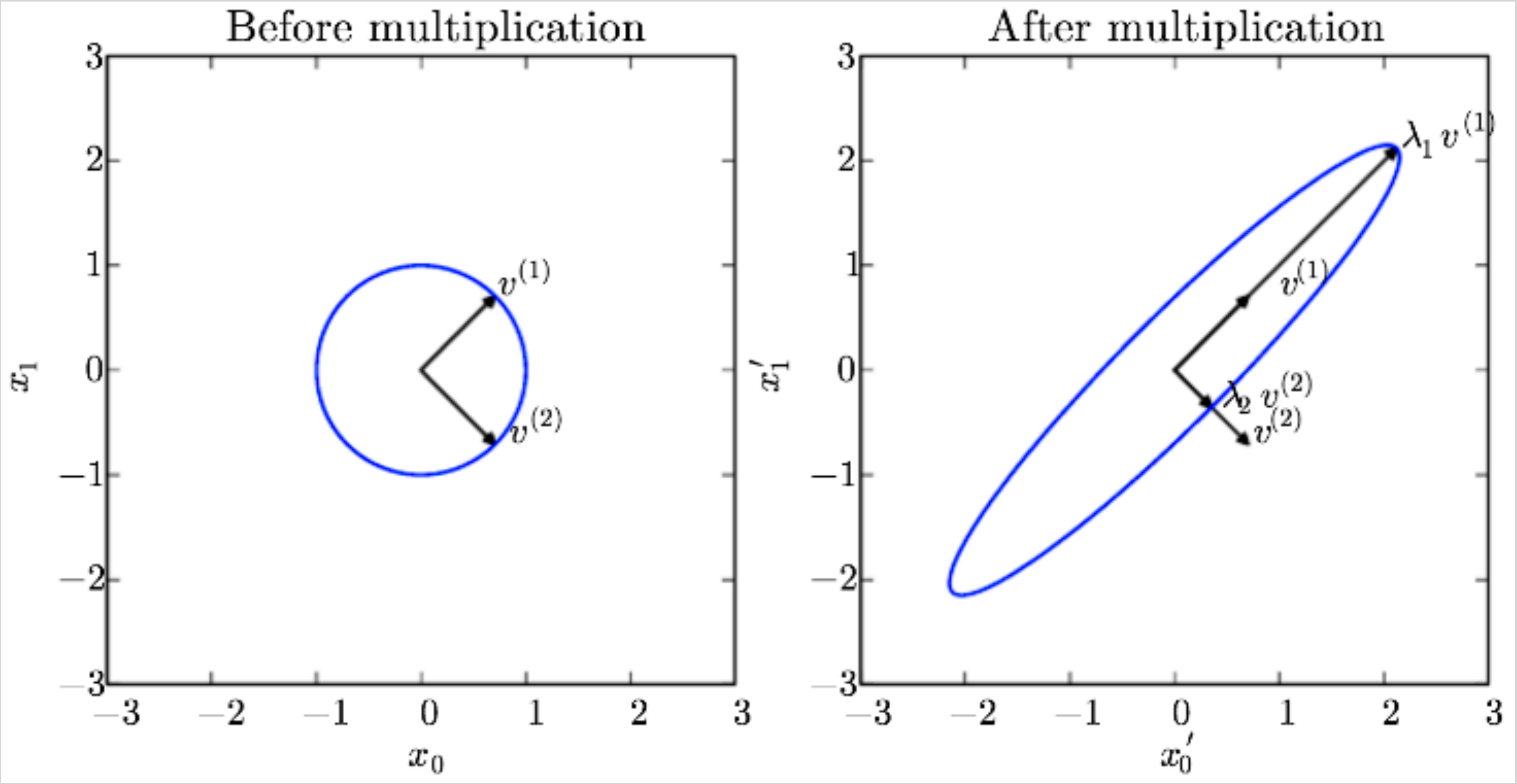
Eigendecomposition

- Definition: $\mathbf{A}\mathbf{v} = \lambda \mathbf{v}$
- Decomposition: $\mathbf{A} = \mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^T$ *where* $\mathbf{Q} = \begin{bmatrix} v_1^{(1)} & \dots & v_1^{(N)} \\ \vdots & \vdots & \vdots \\ v_N^{(1)} & \dots & v_N^{(N)} \end{bmatrix}$ *and* $\boldsymbol{\Lambda} = \begin{bmatrix} \lambda_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_N \end{bmatrix}$
- Applies only to square matrices



Goodfellow et al., 2016, *Deep Learning*

Singular value decomposition

- What if our matrix is not square? SVD is a widely used factorization method in this case.

- Definition: $\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T \quad where \quad \mathbf{A} \in \mathbb{R}^{M\times N}, \quad \mathbf{U} \in \mathbb{R}^{M\times M}, \quad \boldsymbol{\Sigma} \in \mathbb{R}^{M\times N}, \quad \mathbf{V} \in \mathbb{R}^{N\times N}$

$$\mathbf{A} = \begin{bmatrix} u_1^{(1)} & \dots & u_1^{(M)} \\ \vdots & \vdots & \vdots \\ u_M^{(1)} & \dots & u_M^{(M)} \end{bmatrix} \begin{bmatrix} \sigma_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_N \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1^{(1)} & \dots & v_N^{(1)} \\ \vdots & \vdots & \vdots \\ v_1^{(N)} & \dots & v_N^{(N)} \end{bmatrix}$$

* *M>N case*

- Numerically stable relative to eigendecomposition, widely used for compression, ps