

# MATH70098 Ethics in Data Science and AI

## Exercise Sheet - Multi-objective optimisation and Pareto Fronts

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### Exercise sheet: Multi-objective optimisation and Pareto Fronts

The questions on this sheet are designed to let you test your own understanding of the course content on multi-objective optimisation and Pareto fronts. Some questions will test basic notions, while others will encourage you to think more deeply about some of the concepts introduced this week.

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#### Q1: Wiki contributions

The Wikipedia article on multi-objective optimisation provides a brief but comprehensive introduction to the topic. The “Examples of applications” section of this page lists application areas for multi-objective optimisation including economics, finance and optimal design. Along with each example application is a short one or two paragraph explanation of how multi-objective optimisation can be used in that area. Ethical AI is not currently on this list.

Write one to two paragraphs, in language suitable for this format, describing how multi-objective optimisation may be applied in the context of ethical AI.

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#### Q2: Formal definitions of a Pareto-optimal and a dominated solution

Formally define and explain in plain language what is meant by a dominated and a Pareto-optimal solution to the multi-objective optimisation problem:

$$\max_{x=(x_1, \dots, x_n) \in X} g(f_1(x), f_2(x), \dots, f_k(x)) \text{ s.t. } x \in \mathcal{X}.$$

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#### Q3: A concrete example

Consider the size classifiers given in the table below.

	classifier	k_anon	equalised_odds	error_rate	false_positive_rate
1	A	2	FALSE	0.11	0.51
2	B	4	TRUE	0.13	0.22
3	C	3	TRUE	0.05	0.61
4	D	5	FALSE	0.20	0.17
5	E	3	TRUE	0.13	0.86
6	F	2	TRUE	0.08	0.64

- (i) Consider the four pairs of objectives given below. Visualise the bi-objective performance of the six classifiers for each of these pairs. Identify which classifiers are Pareto-optimal and which are dominated in each case.
- (a) false positive rate vs error rate,
  - (b) k-anonymity vs equalised odds,
  - (c) k-anonymity vs error rate,
  - (d) Equalised odds vs error rate.
- (ii) When considering all four objectives at once, which (if any) of the classifiers are Pareto optimal?
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**Q4: Scalarisation**

- (a) Define in your own words what it means to scalarise a multi-objective optimisation problem.
- (b) Linearisation and epsilon-constraint are two methods of linearisation. Explain in plain language how each method works.
- (c) Give linearised and epsilon-constrained formulations for the multi-objective optimisation in Question 2.