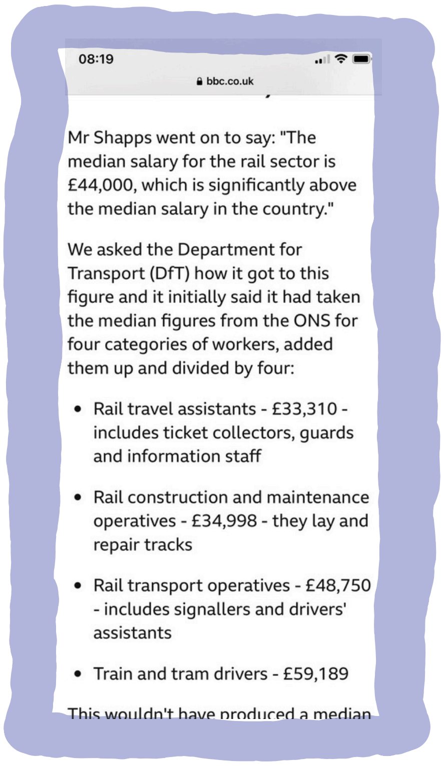
Ethics Live Session: Week 8.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Mr Shapps went on to say: "The median salary for the rail sector is £44,000, which is significantly above the median salary in the country."

We asked the Department for Transport (DfT) how it got to this figure and it initially said it had taken the median figures from the ONS for four categories of workers, added them up and divided by four:

- Rail travel assistants - £33,310 - includes ticket collectors, guards and information staff
- Rail construction and maintenance operatives - £34,998 - they lay and repair tracks
- Rail transport operatives - £48,750 - includes signallers and drivers' assistants
- Train and tram drivers - £59,189

This wouldn't have produced a median

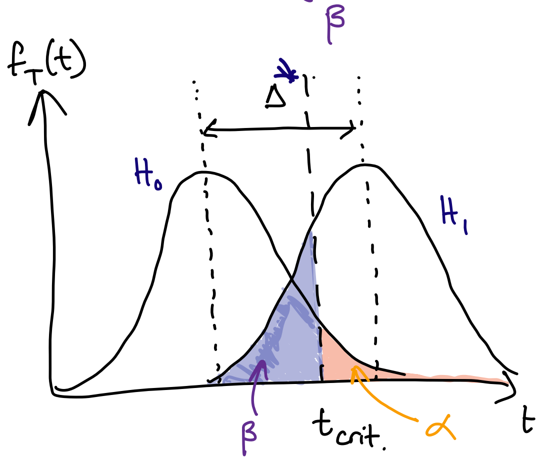
Opening discussion:

How does this link to what we have been looking at?

Review of Error types

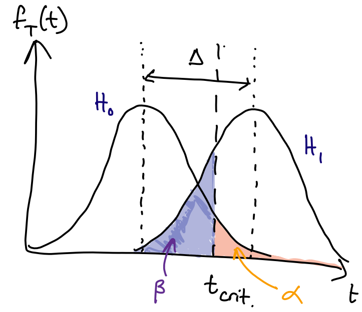
	fail to reject H_0	reject H_0
H_0 true	✓	type 1 error (α)
H_0 false	type 2 error (β)	✓

- Within a hypothesis test we have a null & alternative hypothesis.
- Construct a test statistic T and it's sampling dist$^{\underline{n}}$ under the null.
- Reject H_0 if t is very unlikely under H_0.



Power of a test

$$\text{Power} = 1-\beta$$
$$= Pr(\text{reject } H_0 \mid H_0 \text{ false})$$
$$= \text{ability to detect real difference.}$$



Will depend on:

- Δ - true effect size — Separation
- n - Sample size — Concentration
- α - Size of test ← ROC curves
- Choice of test statistic — dist^n
- dist^n of observations — dist^n
- nature of alternative hypothesis — Critical value

Sample Size Calculations

AIM

Want to pick n such that for given α, Δ, T we get at least power $1-\beta$.

1. Establish dist^n of T under H_0 and H_1.
2. Agree on minimum relevant difference Δ.
3. $1-\beta = pr(\text{reject } H_0 \mid H_1 \text{ true})$
   $= pr(T > t_{crit} \mid H_1 \text{ true})$
   $\gtrsim Pr(T > t_{crit} \mid \text{worst } H_1)$
   $= h(n, \theta)$
4. Solve $h(n,\theta) = 1-\beta$ for n.

EXAMPLE

Drug for blood pressure

$X_1, \dots, X_n$ Change in BP following medication

Hypotheses : $H_0: \mu_x = 0 \qquad H_1: \mu_x < 0$

* for simplicity, suppose BP variance is known & unchanged by medication. * (one sample Z-test)

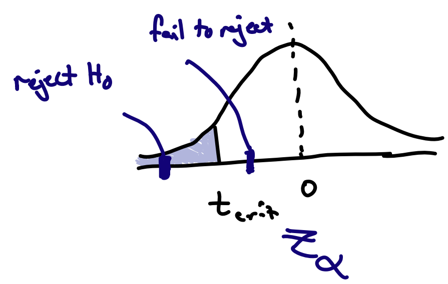
TEST STATISTIC :

Estimating μ_x by $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ we have

$$H_0: \bar{X} \overset{\text{approx}}{\sim} N\left(0, \sigma_x^2/n\right),$$

$$H_1: \bar{X} \overset{\text{approx}}{\sim} N\left(\Delta, \sigma_x^2/n\right). \quad \text{where } \Delta < 0$$

Test statistic $T = \dfrac{\bar{X} - 0}{\sigma_x/\sqrt{n}} \sim N(0,1)$ under null.



Power Calculation

$$\text{Power} = 1 - \beta$$

$$= \Pr\left(\underline{\text{Reject } H_0} \mid H_1 \text{ true}\right)$$

$$= \Pr\left(\underline{\frac{\bar{X}\sqrt{n}}{\sigma_x}} < \underline{Z_\alpha} \;\middle|\; H_1 \text{ true} \right)$$

$$= \Pr\left(\bar{X} < \frac{\Phi^{-1}(\alpha)\,\sigma_x}{\sqrt{n}} \;\middle|\; \underline{H_1 \text{ true}} \right)$$

$$\geq \Pr\left(\bar{X} < \frac{\Phi^{-1}(\alpha)\,\sigma_x}{\sqrt{n}} \;\middle|\; \bar{X} \sim N\left(\Delta^{*}, \sigma_x^2/n\right) \right)$$

$$= \Pr\left(Z < \frac{\left[\frac{\Phi^{-1}(\alpha)\,\sigma_x}{\sqrt{n}} - \Delta \right]}{\sigma_x/\sqrt{n}} \right)$$

$$= \Pr\left(Z < \Phi^{-1}(\alpha) - \frac{\Delta\sqrt{n}}{\sigma_x} \right)$$

$$h(n, \theta) = \Phi\left(\Phi^{-1}(\alpha) - \frac{\Delta\sqrt{n}}{\sigma_x} \right) = 1 - \beta.$$

Rearrange for n — note $\Phi^{-1}(1-\beta) = -\Phi^{-1}(\beta)$

$$n \geq \left[\frac{\sigma_x}{\Delta} \left(\Phi^{-1}(\beta) + \Phi^{-1}(\alpha) \right) \right]^2$$

↳ Subs for $\alpha, \beta, \Delta, \sigma_x$.

$\boxed{\text{FIN}}$

CHALLENGES

- Properly defining H_0 and H_1
- Agreeing on Δ
- Establishing dist^n of test statistic
- Inverting $h(n, \theta) = 1-\beta$.
  - ↳ 2 sided tests have absolute values
  - ↳ σ_x^2 unknown $\Rightarrow$ t-test $\Rightarrow$ df depend on n.
  - ↳ 2 sample tests: dealing with group size
  - ↳ Estimating something other than a mean.