Imperial College London

MSc Machine Learning & Data Science

Applicable Mathematics

Ethics of Data Science – Part II

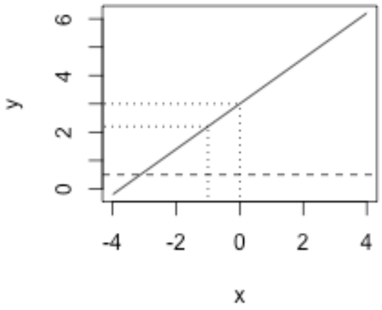
Measuring feature effects in classical models: logistic regression

Dr. Chris Anagnostopoulos, Hon. Senior Lecturer

Logistic regression

$$y = \beta_0 + \beta_1 X$$

”A unit increase in X will result in an increase in y by β_1 units”



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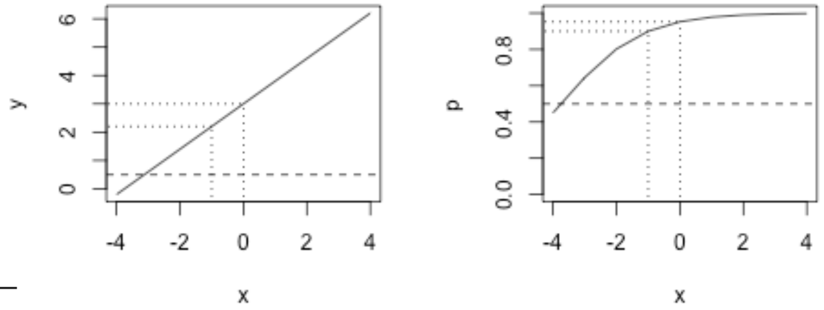
$$\text{logit}(y) = \beta_0 + \beta_1 X, \text{ where } \text{logit} p = \log \frac{p}{1-p}$$

”A unit increase in X will result in an increase in logit(p) by β_1 units”



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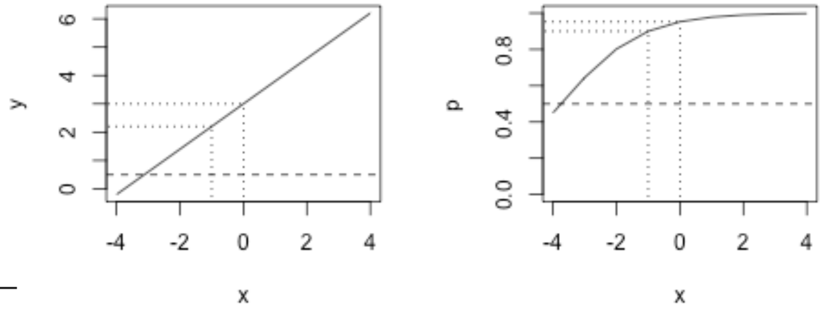

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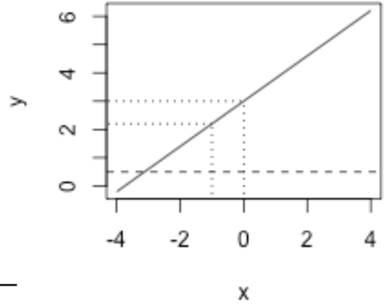

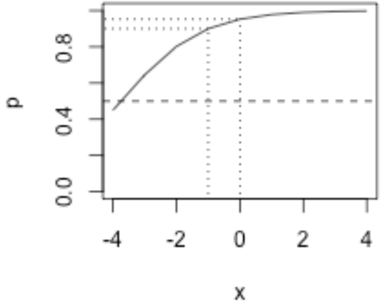

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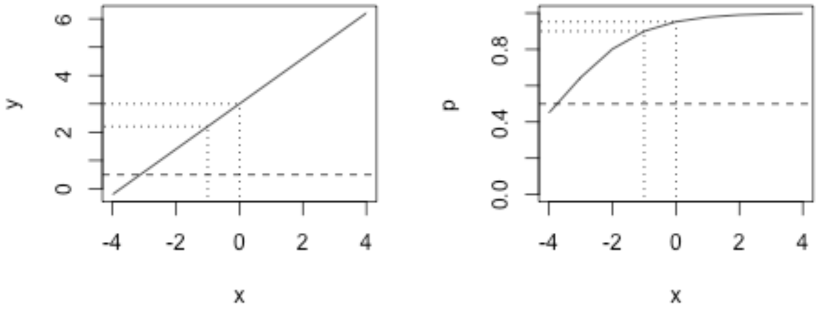

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If β_1=0.8, then exp(0.8) = 2.2 times higher odds. If β_1=0, same odds.

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Logistic regression – non-linear case (e.g., GAM)

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increase from X=-1 to X=0

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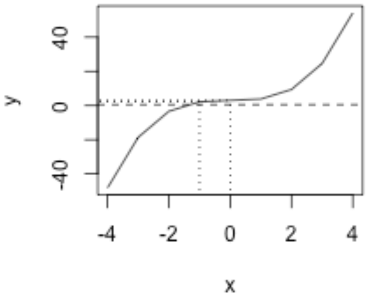

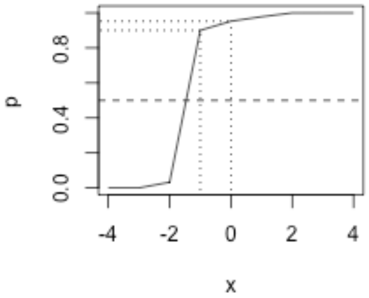

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Summary and afterthoughts

- Both linear and logistic regression are “linear” in a sense that allows us to describe the effect of changing one variable on the response as a multiple of that change, holding other things constant.
- Additivity is a separate consideration, which we will better understand when we discuss interactions over the next few days.

Some afterthoughts:

- Can we offer as precise statements using PDPs or other such XAI tools for general machine learning techniques?
- Can/should classification be treated as logistic regression, under a log-odds interpretation? Is that clarity necessary?