Optimization Modeling

Disusun oleh:

Tim dosen Pemodelan dan Simulasi S1 Informatika

APA ITU OPTIMISASI

Optimization → Optimal → Terbaik

- Mengoptimasi suatu hal berarti membuat hal tersebut menjadi yang paling baik.
- “Terbaik” atau “optimal” bisa memiliki beberapa kriteria.
- Pemain bola terbaik adalah yang larinya tercepat dan paling minimum salah operan bola.
- Misal rencana wisata terbaik untuk seseorang itu bisa jadi yang harganya “paling terjangkau”, “jarak terekat” , dan “jumlah wahana/spot wisata terbanyak”

→ **Mathematical optimization**

MATHEMATICAL OPTIMIZATION IN THE “REAL WORLD”

- Merupakan cabang terapan matematika yang digunakan di banyak bidang berbeda.

enginerring

Manufactur

Transportasi

Penjadwalan

Ekonomi

Finance

Dan masih banyak lagi

OPTIMIZATION COMPONENT

Goal: Finding the value of decision variables that maximize/minimize the objective function while satisfying the given constraints

- Decision Variable
- Objective Function : the function being minimized/maximized
- Constrains (specific requirement/ limitation that satisfied by the solution (decision variables))

OPTIMIZATION NOTATION

Goal: Finding the value of decision variables that maximize/minimize the objective function while satisfying the given constraints

Objective function, $\boldsymbol{f}(\boldsymbol{x})$, which is the output needed to be maximum or minimum.

Decision variables, ex. $\boldsymbol{x_1}, \boldsymbol{x_2}$, $\mathrm{x}_3, \cdots$ which are the input things to be controlled.

Constraints, which are equations that place limits on how big or small some variables can get. Equality constraints are usually noted $h_n(x)$ and inequality constraints are noted $g_n(x)$.

OPTIMIZATION EXAMPLE

A football coach is planning practices for his running backs.

- His main goal is to maximize running yards – this will become his objective function.
- He can make his athletes spend practice time in the weight room; running sprints; or practicing ball protection. The amount of time spent on each is a variable.
- However, there are limits to the total amount of time he has. Also, if he completely sacrifices ball protection he may see running yards go up, but also fumbles, so he may place an upper limit on the amount of fumbles he considers acceptable. These are constraints.

Note that the variables influence the objective function and the constraints place limits on the domain of the variables.

OPTIMIZATION MATH. NOTATION

example

$$\min_{x_1, x_2 \in \mathbb{Z}} f(x_1, x_2)$$

Subject to:

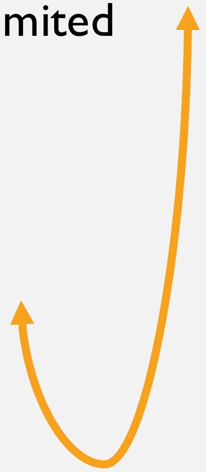
$$g_1:\ x_1 + x_2 > 100,$$
$$g_2:\ x_1 \geq 2x_2$$

The problem is to minimize objective function $f(x_1, x_1)$ over the integer number of x_1 and x_2 that satisfying constraint g_1 and g_2.

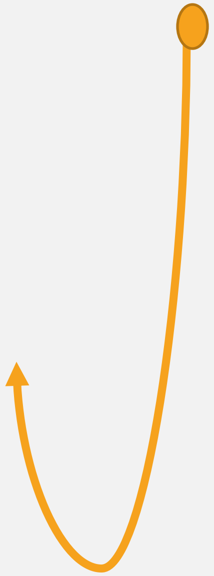
TYPE OF OPTIMIZATION PROBLEMS

Without constraints

Solution is unlimited



With constraints



Solution is limited

TYPE OF OPTIMIZATION PROBLEMS

Single variable

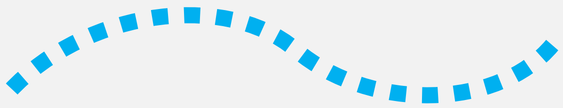
$$\{x\}$$

Multi-variables

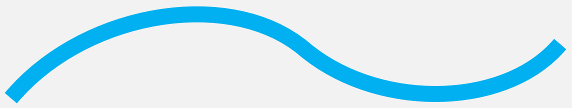
$$\{x_1, x_2, \cdots, x_n\}$$

TYPE OF OPTIMIZATION PROBLEMS

Discrete



Continuous



TYPE OF OPTIMIZATION PROBLEMS

Static

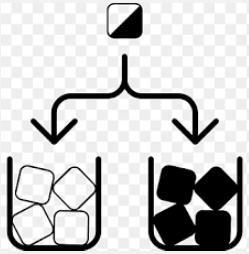
Do not change over time

Dynamic

continual adjustments must be made as changes occur

TYPE OF OPTIMIZATION PROBLEMS

Deterministic



specific causes produce specific effects

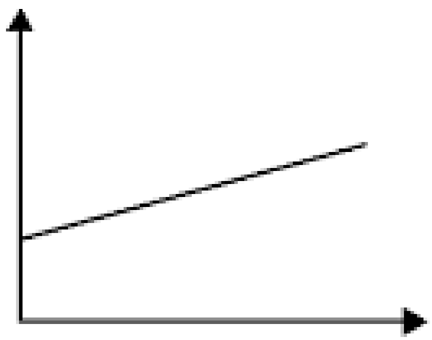
Stocastic

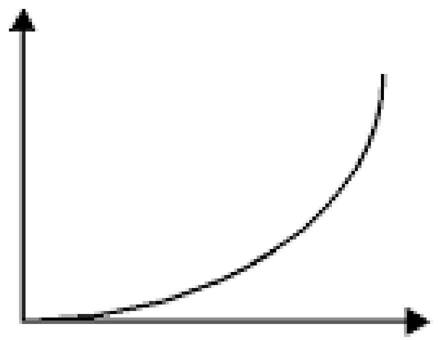
Involve randomness/probability

TYPE OF OPTIMIZATION PROBLEMS

Linear

Non-linear





WHY OPTIMIZATION IS IMPORTANT

- Better than "guess-and-check" methods
- Less expensive than building and testing

SOLVING OPTIMIZATION PROBLEM

- Draw a picture, if possible
- Decide what quantity f is to be maximized or optimized.
- Assign letters to other quantities that may vary, which are assigned as variables of f.
- Determine the "objective equation" that expresses f as a function of the variables in step 3.
- Find the "constraint equation" that relates the variables to each other and to any constants that are given in the problem
- Use the constraints equation to simplify the objective function in such way that f become a function of only one variable. Determine the domain of the function
- Sketch the graph of the function obtained from step-6 and use tis graph to solve the optimization problem.

QUESTION FOR GROUP DISCUSSION

Group the following into what might be maximized, minimized, or cannot be optimized. If it might be an optimization problem, explain what are the variables and the objective, also the restriction if exists any.

1. When choosing a new phone and plan, you might consider: minutes of talk time per month; how much is charged for overages; whether extra minutes roll over; amount of data allowed; cost per month; amount of storage/memory; how many phones are available; brands/types of available phones; cost of the phone; amount of energy used; time it takes to download apps or music; whether or not you get signal in your home.
2. A student must create a poster project for a class. Managing contents layout for the poster.
3. A shipping company must deliver packages to customers.
4. A grocery store must decide how to organize the store layout.

Review: Optimization Problem Component

$$\min_{x_1, x_2 \in \mathbb{Z}}/\max f(x_1, x_2)$$

Subject to:

$$g_1\colon\ x_1 + x_2 > 100,$$
$$g_2\colon\ x_1 \geq 2x_2$$

The problem is to minimize objective function $f(x_1, x_1)$ over the integer number of x_1 and x_2 that satisfying constraint g_1 and g_2.

Outline

- Optimization model in decision making
- How to model optimization problem
- Linear Programming (LP)
- Technique to approach solution LP
- Group Discussion
- Some Application LP

The role of Mathematical model in Operation decision making

- Advances in business and engineering research and computer technology have expanded managers' use of mathematical models. A model represents the essential features of an object, system, or problem without unimportant details.
- The models in this supplement have the important aspects represented in mathematical form using variables, parameters, and functions.
- Analyzing and manipulating the model gives insight into how the real system behaves under various conditions. From this we determine the best system design or action to take.
- Mathematical models are cheaper, faster, and safer than constructing and manipulating real systems.

Example: The Farmer John Problem

Farmer Jones decides to supplement his income by baking and selling two types of cakes, chocolate and vanilla. Each chocolate cake sold gives a profit of $3, and the profit on each vanilla cake sold is $4. Each chocolate cake uses 4 eggs and 4 pounds of flour, while each vanilla cake uses 2 eggs and 6 pounds of flour. If Farmer Jones has only 32 eggs and 48 pounds of flour available, how many of each type of cake should be baked in order to maximize Farmer Jones's profit? (For now, assume all cakes baked are sold, and fractional cakes are OK.)

Let C be a variable that represents the number of chocolate cakes Farmer Jones bakes.

Let V be a variable that represents the number of vanilla cakes Farmer Jones bakes

Can you write down the allowed values for C and V with the profit?

C	V	Profit	Residu
1	7	1x3+7x4=31 ($)	Eggs:14 ; floor: 2 pounds
2	6	2x3+6x4=30($)	Eggs: 12; floor: 4 pounds
3	?	?	?
⋮	⋮	⋮	⋮

Finding all possibilities are exhausted, then we should expect something smarter!

Example: The Farmer John Problem

Can we make generalization of the problem, means:

- Can we describe the profit in terms of C and V ?
- Can we describe the set of all possible values for C and V ?

In the other words

can we describe Farmer Jones's problem as an optimization model?

Let's write an Optimization model for Farmer Jones's model!

Should be something like this →

$$\min_{x_1, x_2 \in \mathbb{Z}} / \max f(x_1, x_2)$$

Subject to:

$$g_1:\ x_1 + x_2 > 100,$$
$$g_2:\ x_1 \geq 2x_2$$

Example: The Farmer John Problem

Profit(C,V) = $3 \cdot C + 4 \cdot V$ (in dollars)

Total eggs should be $\leq$ 32 or in math expression: $4 \cdot C + 2 \cdot V \leq 32$

Total used flour should be $\leq$ 48 pounds or in math expression: $4 \cdot C + 6 \cdot V \leq 48$

Thus, math expression for problem above where we want to determine the optimal (profit maximum) of C and V satisfying the allowed amount of eggs and flour, can be written as:

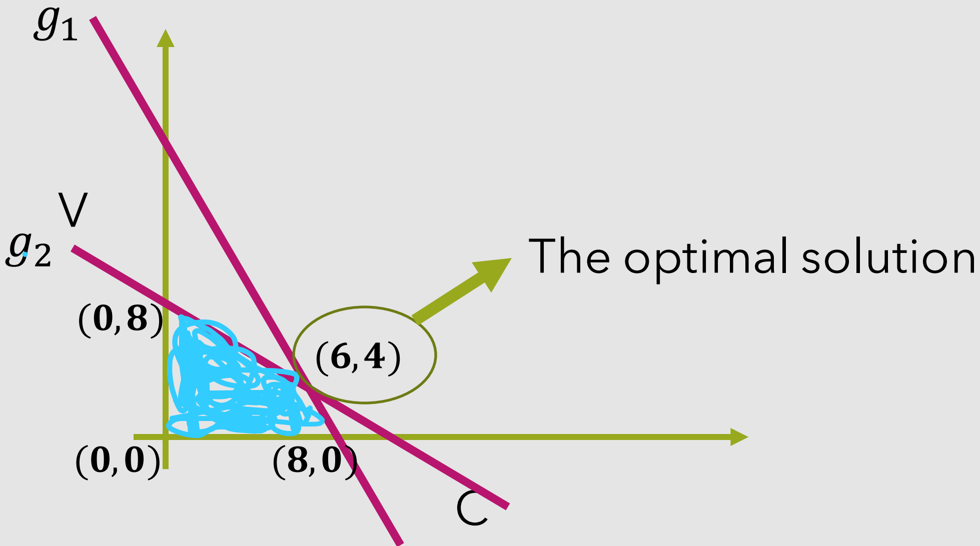
$$\max_{C,V \in \mathbb{Z}^+} profit(C,V) = 3 \cdot C + 4 \cdot V$$ →Linear objective function.

Subject to:

$$g_1: 4 \cdot C + 2 \cdot V \leq 32,$$
$$g_2: 4 \cdot C + 6 \cdot V \leq 48$$
$$g_3: C \geq 0$$
$$g_4: V \geq 0$$

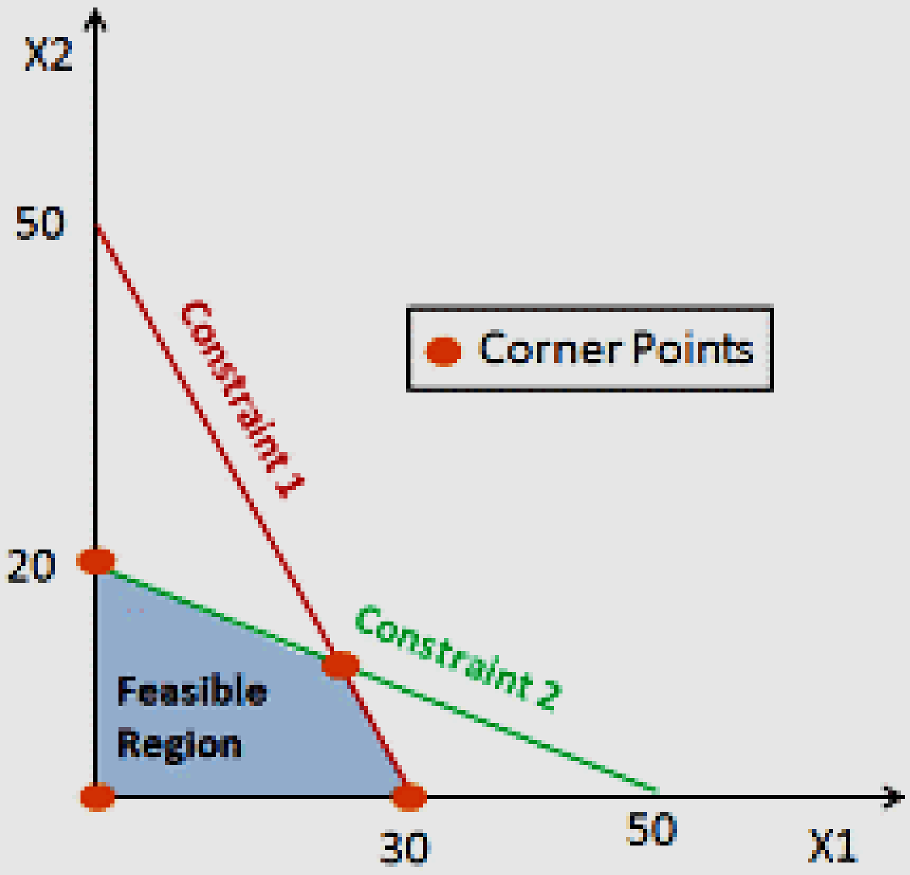
→Linear constraints

Remember in high school. Geometry of the problem



Solutions and Values of Optimization Models

- **A feasible solution** to an optimization model is a choice of values for the decision variables that satisfies all constraints
- **The feasible region** of an optimization model is the collection of all feasible solutions to the model
- **The value of a feasible solution** is its objective function value
- **An optimal solution** to an optimization model is a feasible solution whose value is as good as the value of all other feasible solutions



Classification of Optimization Model

Based on characteristic of decision variables	**Continuous (lies in a real valued interval) and integer (specified interval of integers)**
Base on Objective function	**Linear and nonlinear.**
Based on constraints	**Constraint and non-constraint**

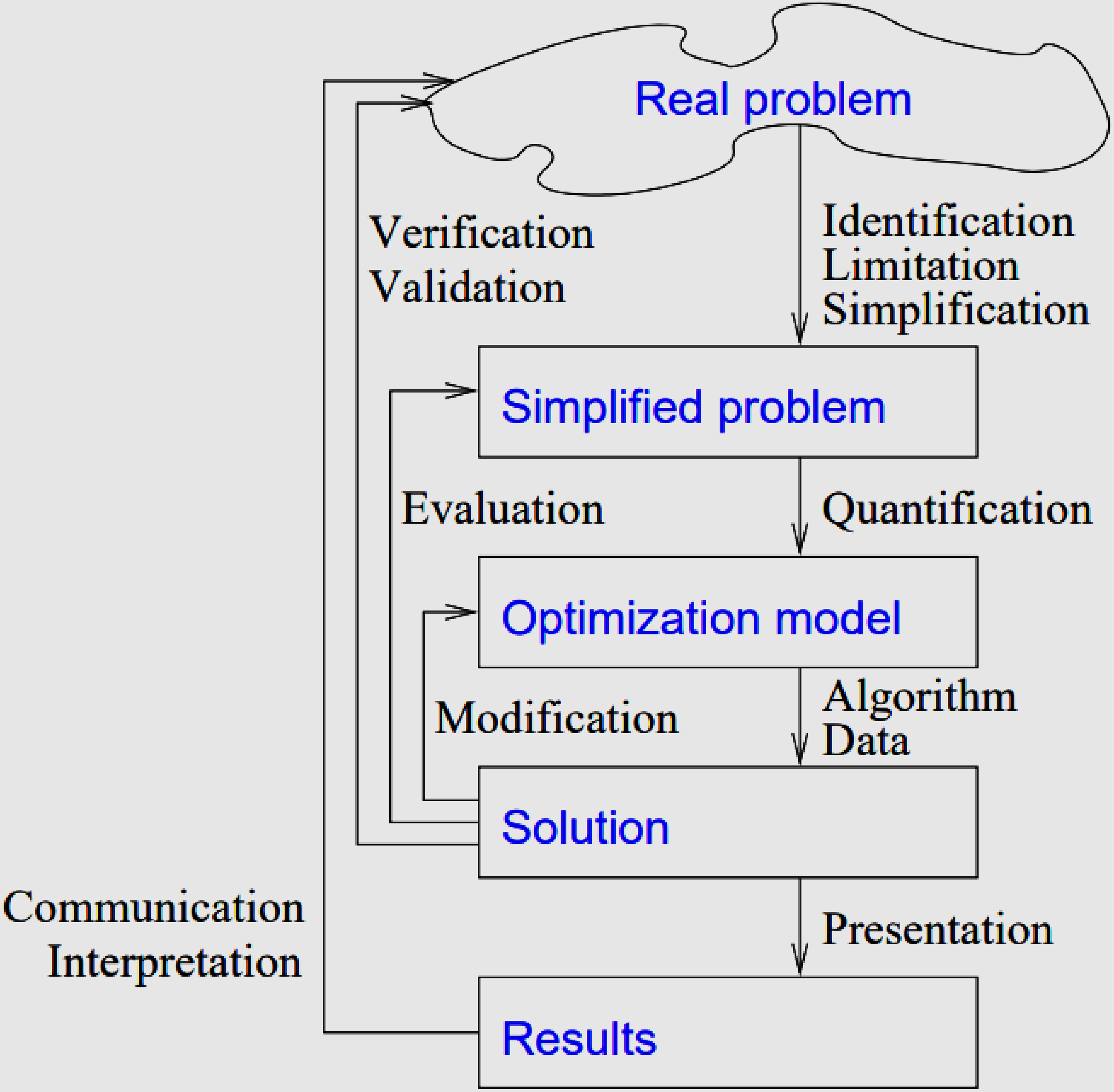
Constraint can be linear and non-linear. Are these function linear or non-linear?

- $f(x_1, x_2, x_3) = 9x_1 - 17x_3$
- $f(x_1, x_2, x_3) = \frac{5}{x_1} + 3x_2 - 6x_3$
- $f(x_1, x_2, x_3) = \frac{x_1 - x_2}{x_2 + x_3}$
- $f(x_1, x_2, x_3) = x_1 x_2 + 3x_3$

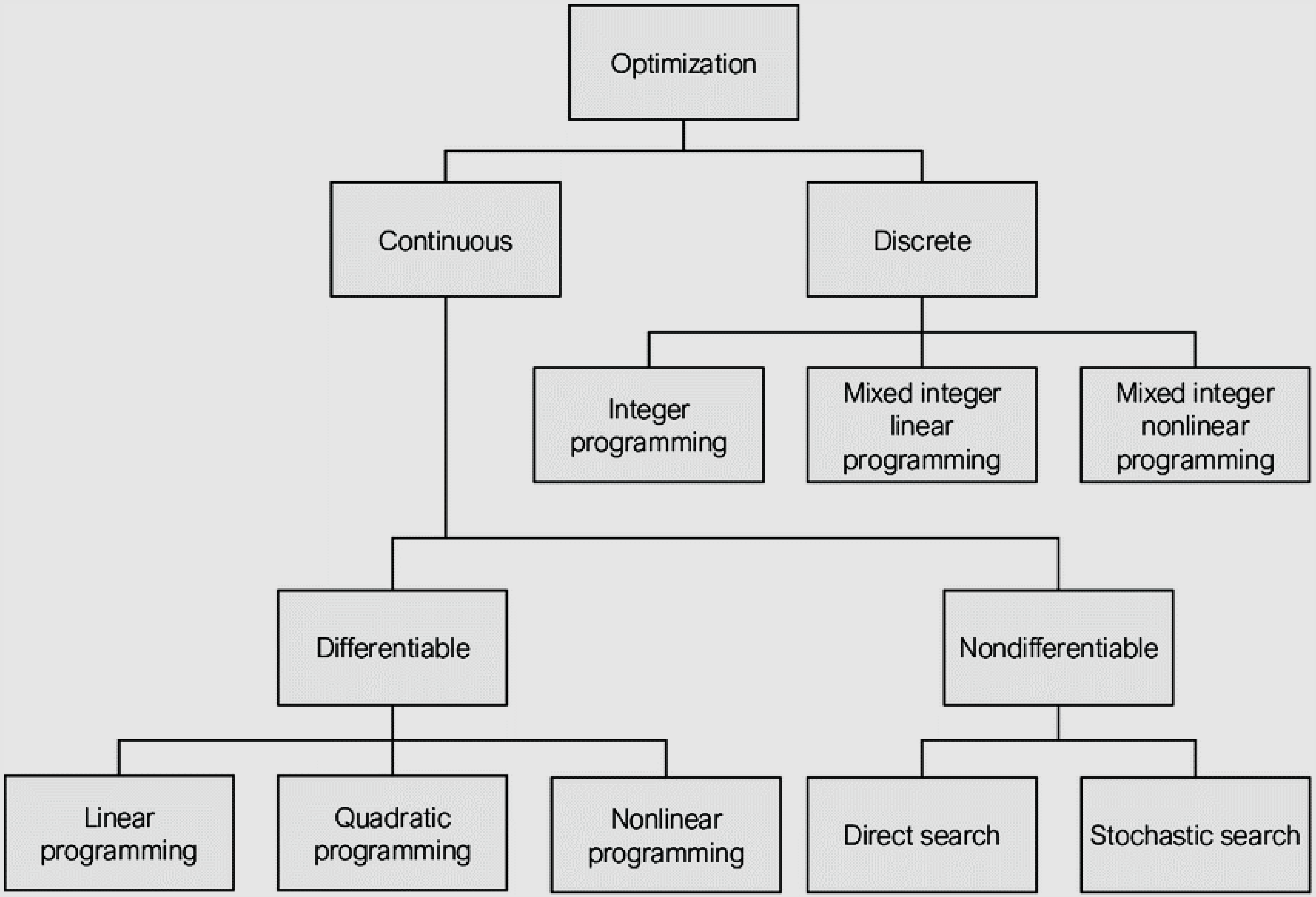
Constraint can be written in the form:

$$g(x_1, \ldots, x_n) \left\{ \begin{array}{c} \leq \\ = \\ \geq \end{array} \right\} b$$

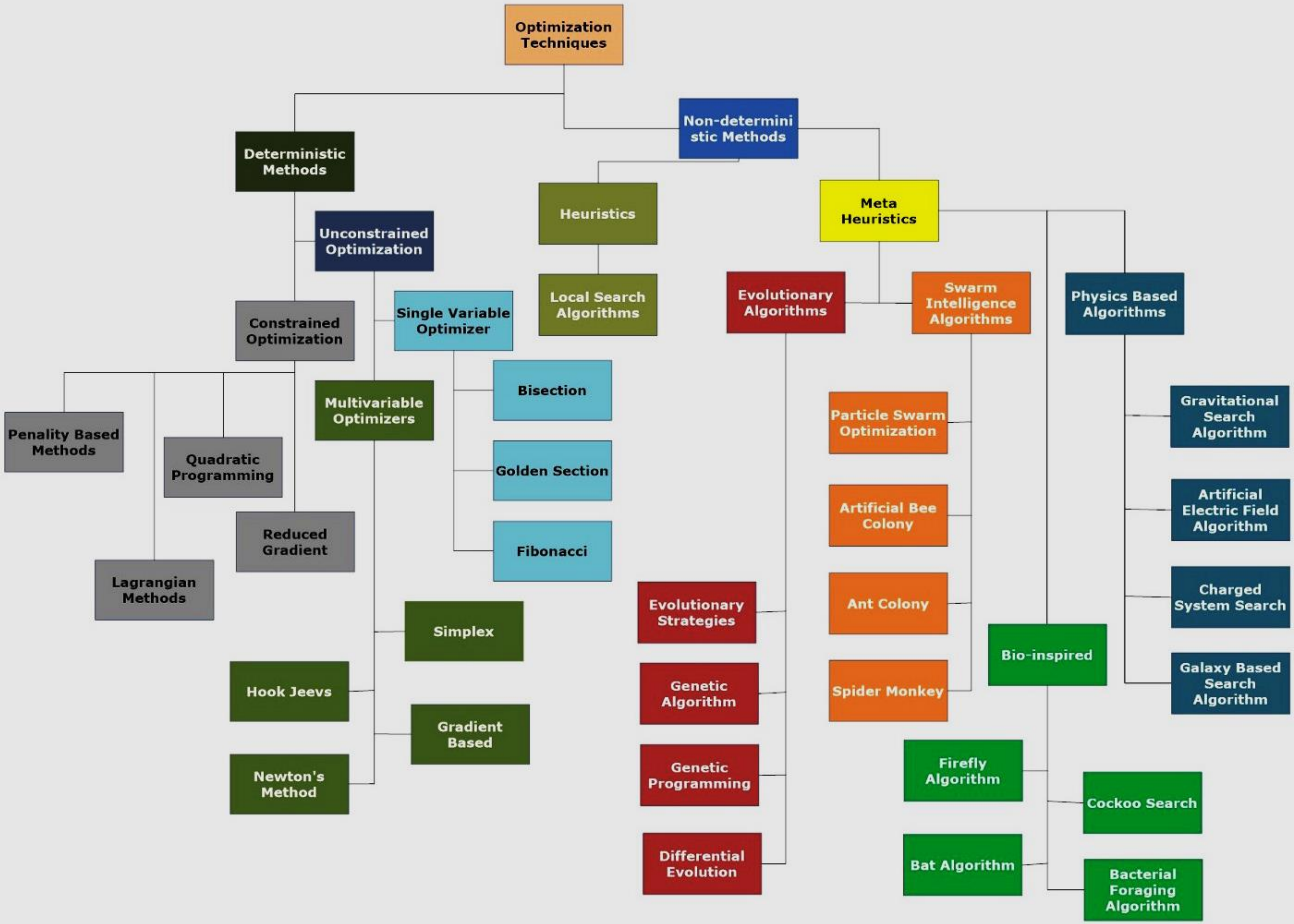
The Process of Optimization



Classification of Optimization Problem



Optimization Technique Characteristics



Linear Programming

An optimization problem is a linear program (LP) if :

- The decision variables are continuous,
- The objective function is linear, and
- The constraints are linear.

Identify, are these optimization models linear program?

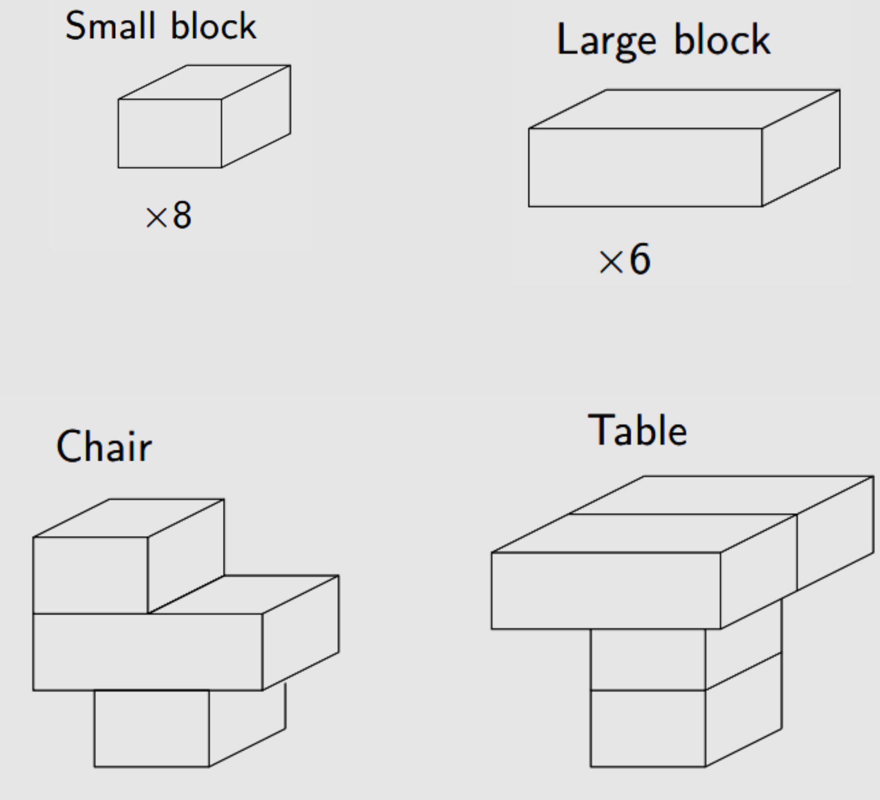
$$\begin{aligned} \max \quad & 3z_1 + 14z_2 + 7z_3 \\ \text{s.t.} \quad & 10z_1 + 5z_2 \le 25 - 18z_3 \\ & z_1 \ge 0, z_2 \ge 0, z_3 \ge 0 \end{aligned}$$

$$\begin{aligned} \min \quad & 3w_1 + 14w_2 - w_3 \\ \text{s.t.} \quad & 3w_1 + w_2 \le 1 \\ & w_1 w_2 w_3 = 1 \\ & w_1 + 2w_2 + w_3 = 10 \\ & w_1 \ge 0, w_3 \ge 0 \\ & w_1 \text{ integer} \end{aligned}$$

Farmer Jones's model

Linear Programming modeling: example

Suppose that we plan to produce chairs and tables from this following available blocks



1. A chair is assembled from one large and two small blocks
2. A table is assembled from two blocks of each size
3. Only 6 large and 8 small blocks are available
4. A table is sold at a revenue of 1.6 millions IDR
5. A chair is sold at a revenue of 1 million IDR
6. Assume that all items produced can be sold and determine an optimal production plan

What is the maximum revenue that possible to earn?

Linear Programming modeling: example

Suppose that we plan to produce chairs and tables from this following available blocks

1. A chair is assembled from **one large and two small blocks**
2. A table is assembled from **two blocks of each size**
3. Only **6 large and 8 small blocks are available**
4. A table is sold at a revenue of **1.6 millions IDR**
5. A chair is sold at a revenue of **1 million IDR**
6. Assume that all items produced can be sold and determine an optimal production plan

$$\max_{x,y\in\mathbb{Z}^+} R(x,y) = 1.6x + y$$

Subject to:

$$g_1: x + 2y \leq 6,$$
$$g_2: 2x + 2y \leq 8$$
$$g_3: x, y \geq 0$$

Let x and y are respectively be the number of chairs and table that can be made.

Total revenue must be a function of x and y:

$$R(x,y) = 1.6 \cdot x + 1 \cdot y \text{ in millions IDR}$$

The available materials are **6 large and 8 small blocks.**

g_1: $x + 2y \leq 6$ (the available large blocks)

g_2: $2x + 2y \leq 8$ (the available small blocks)

$g_3: x, y \geq 0$

Group Discussion Problem-1

The Healthy Pet Food Company manufactures two types of dog food: Meaties and Yummies. Each package of Meaties contains 2 pounds of cereal and 3 pounds of meat; each package of Yummies contains 3 pounds of cereal and 1.5 pounds of meat. Healthy believes it can sell as much of each dog food as it can make. Meaties sell for $2.80 per package and Yummies sell for $2.00 per package. Healthy's production is limited in several ways. First, Healthy can buy only up to 400,000 pounds of cereal each month at $0.20 per pound. It can buy only up to 300,000 pounds of meat per month at $0.50 per pound. In addition, a special piece of machinery is required to make Meaties, and this machine has a capacity of 90,000 packages per month. The variable cost of blending and packing the dog food is $0.25 per package for Meaties and $0.20 per package for Yummies. This information is given in Table B-1.

Table B-1 Healthy Pet Food Data

	Meaties	Yummies
Sales price per package	$2.80	$2.00
Raw materials per package		
Cereal	2.0 lb.	3.0 lb.
Meat	3.0 lb.	1.5 lb.
Variable cost—blending and packing	$0.25 package	$0.20 package
Resources		
Production capacity for Meaties	90,000 packages per month	
Cereal available per month	400,000 lb.	
Meat available per month	300,000 lb.	

Suppose you are the manager of the Dog Food Division of the Healthy Pet Food Company. Your salary is based on division profit, so you try to maximize its profit. How should you operate the division to maximize its profit and your salary?

Group Discussion Problem-2

International Wool Company operates a large farm on which sheep are raised. The farm manager determined that for the sheep to grow in the desired fashion, they need at least minimum amounts of four nutrients (the nutrients are nontoxic so the sheep can consume more than the minimum without harm). The manager is considering three different grains to feed the sheep. Table B-2 lists the number of units of each nutrient in each pound of grain, the minimum daily requirements of each nutrient for each sheep, and the cost of each grain. The manager believes that as long as a sheep receives the minimum daily amount of each nutrient, it will be healthy and produce a standard amount of wool. The manager wants to raise the sheep at minimum cost.

Table B-2 International Wool Data

		Grain 1	Grain 2	Grain 3	Minimum Daily Requirement (units)
Nutrient	A	20	30	70	110
Nutrient	B	10	10	0	18
Nutrient	C	50	30	0	90
Nutrient	D	6	2.5	10	14
Cost (¢/lb)		41	36	96	

Group Discussion Problem-3

Solar Oil Company is a gasoline refiner and wholesaler. It sells two products to gas stations: regular and premium gasoline. It makes these two final products by blending together four raw gasolines and some chemical additives (the amount and cost of the additives per barrel are assumed to be independent of the mixture). Each gasoline has an octane rating that reflects its energy content. Table B-3 lists the octane, purchase price per barrel, and availability at that price per day. This table also gives the required minimum octane for each final gasoline, the net selling price per barrel (removing the cost of the additives), and the expected daily demand for gas at that price. Solar Oil can sell all the gas it produces up to that amount.

The blending of gasoline is approximately a linear operation in terms of volume and octane. If x barrels of 80 octane gasoline are blended with y barrels of 90 octane gasoline, this produces $x + y$ barrels of gasoline with an octane of $(80x + 90y)/(x + y)$. There is no significant volume gain or loss, and octane of the mixture is a weighted average of the octanes of the inputs.

Table B-3 Solar Oil Data

		Octane	Cost ($/b)	Available daily
Raw gasolines	1	86	17.00	20,000
	2	88	18.00	15,000
	3	92	20.50	15,000
	4	96	23.00	10,000
		Octane	**Price ($/b)**	**Maximum daily demand**
Products	Regular	89	19.50	35,000
	Premium	93	22.00	23,000

Group Discussion Problem-4

Basel Tool and Die Company (BTD) makes large industrial pipe wrenches in one of its factories. The marketing department estimates demand for this product during the next 6 months to be:

Month	**Demand**
January	370
February	430
March	380
April	450
May	520
June	440

With the current labor force, BTD believes it can make approximately 420 pipe wrenches per month at a cost of $40 per wrench using regular-time production. An additional 80 wrenches per month can be made using overtime production at a cost per wrench of $45. Wrenches can be made in advance and held in inventory for later shipment at a cost of $3 per month per wrench. The monthly demand for wrenches must be satisfied every month. At the end of December (beginning of January) BTD has 10 wrenches in inventory. BTD wants to plan its production, including overtime, and inventory for the next 6 months so as to maximize profit. Assuming the revenue for these wrenches is fixed, the production manager can maximize profit by minimizing the total costs incurred in producing and delivering the wrenches.

Solution:

The quantities that the decision maker controls are (1) the number of wrenches to make each month using regular-time production, (2) the number of wrenches to make each month using overtime production, and indirectly (3) the number of wrenches to keep in inventory at the end of each month. We define our decision variables as follows (to keep a clear time convention, we assume that wrenches are made during a month; at the end of the month, wrenches are shipped to customers; any wrench not shipped incurs a holding cost for that month):

Linear Programming solution approach

1. Graphical Approach
2. Simplex Algorithm

Graphical Approach

Steps for Graphical Method

Step 1 Formulate the LPP

Step 2 Construct a graph and plot the constraint lines

Step 3 Determine the valid side of each constraint line

Step 4 Identify the feasible solution region

Step 5 Find the optimum points

Step 6 Calculate the co-ordinates of optimum points

Step 7 Evaluate the objective function at optimum points to get the required maximum/minimum value of the objective function

Try to solve the problems we discussed using graphical approach!

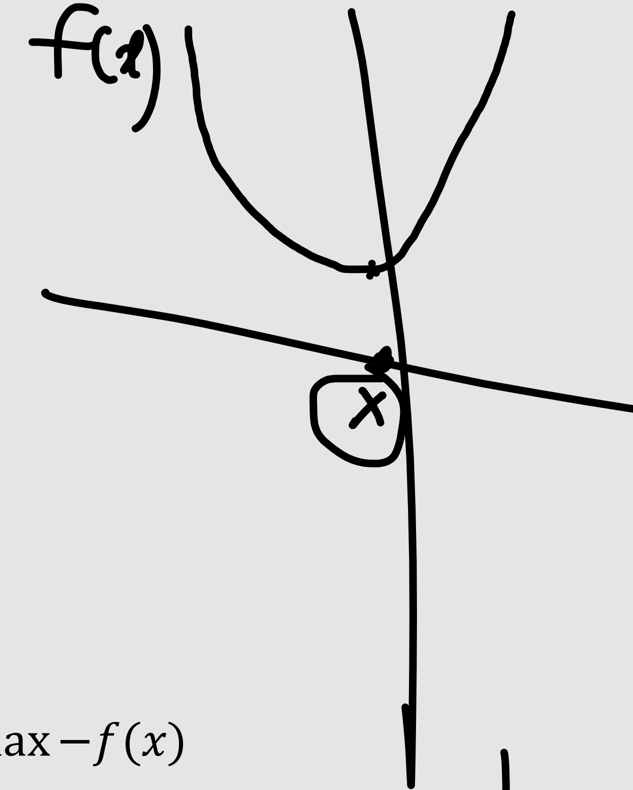
Simplex Approach

- The Simplex method is an approach to solving linear programming models by hand using slack variables, tableaus, and pivot variables as a means to finding the optimal solution of an optimization problem.
- A linear program is a method of achieving the best outcome given a maximum or minimum equation with linear constraints.
- Most linear programs can be solved using an online solver such as MatLab or python library (scipy.optimize → linprog), but the Simplex method is a technique for solving linear programs by hand.

Steps of using Simplex method to solve Linear programming model:

1. Standard form
2. Introducing slack variables
3. Creating the tableau
4. Pivot variables
5. Creating a new tableau
6. Checking for optimality
7. Identify optimal values

Simplex Approach (Example)



Problem example:

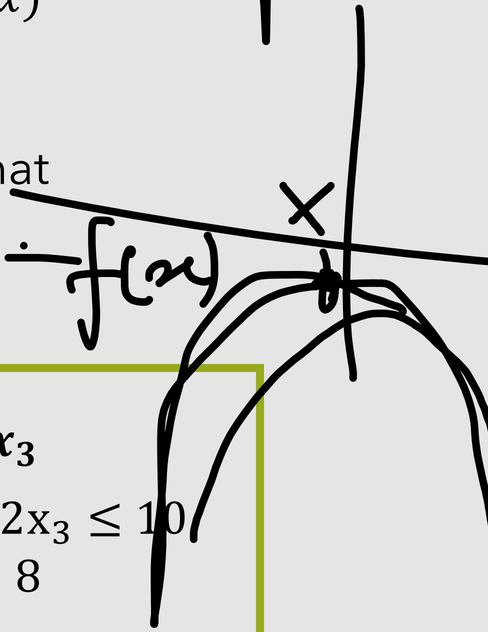
$$
\begin{aligned}
Minimize &: -z = -8x_1 - 10x_2 - 7x_3 \\
s.t. &: x_1 + 3x_2 + 2x_3 \leq 10 \\
& -x_1 - 5x_2 - x_3 \geq -8 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

Step-1: **Standard form**

(1) Must be a maximization problem, problem of $\min f(x)$ equilavent to $\max -f(x)$

(2) All linear constraints must be in **a less-than-or-equal-to** inequality,
Similar to point-1, multiply both side of greaten-that-or-equal-to with -1, so that it changes the form to a less-than-or-equal-to.

(3) All variables are non-negative.



Therefore, for problem example the problem change to**:** $\mathbf{\max z = 8x_1 + 10x_2 + 7x_3}$

Subject to: $x_1 + 3x_2 + 2x_3 \leq 10$

$x_1 + 5x_2 + x_3 \leq 8$

$x_1, x_2, x_3 \geq 0$

Simplex Approach (Example)

Standard form of the problem:

$$\max z = 8x_1 + 10x_2 + 7x_3$$

Subject to: $x_1 + 3x_2 + 2x_3 \leq 10$

$$x_1 + 5x_2 + x_3 \leq 8$$

$$x_1, x_2, x_3 \geq 0$$

Step-2: **Introducing slack variables (in the constraint)**

$$x_1 + 3x_2 + 2x_3 + \mathbf{s_1} = 10$$

$$x_1 + 5x_2 + x_3 + \boldsymbol{s_2} = 8$$

$$x_1, x_2, x_3, s_1, s_2 \geq 0$$

.

s_1 and s_2 are called slack variables, and the value are always positive cause they are completing the less-than-or-equal-to inequality to be equality. Slack variables are artificial we need to add for the computation.

Simplex Approach (Example)

the problem with slack variables: $\max z = \mathbf{8x_1 + 10x_2 + 7x_3}$

Subject to: $x_1 + 3x_2 + 2x_3 + \mathbf{s_1} = 10$

$x_1 + 5x_2 + x_3 + \boldsymbol{s_2} = 8$

$x_1, x_2, x_3, s_1, s_2 \geq 0$

Step-3: Setting up the Tableau

	x1	x2	x3	s1	s2	z	b
g_1:	1	3	2	1	0	0	10
g_2:	1	5	1	0	1	0	8
objective function (sign z should be positive.)	-8	-10	-7	0	0	1	0

Simplex Approach (Example)

Step-4: Check Optimality

	x1	x2	x3	s1	s2	z	b
g_1:	1	3	2	1	0	0	10
g_2:	1	5	1	0	1	0	8
objective function (sign z should be positive.)	-8	-10	-7	0	0	1	0

- To check optimality using the tableau, all values in the last row must contain values greater than or equal to zero.
- If a value is less than zero, it means that variable has not reached its optimal value.
- As seen in the previous tableau, three negative values exists in the bottom row indicating that this solution is not optimal.
- If a tableau is not optimal, the next step is to identify the pivot variable to base a new tableau on, as described in Step 5.

Simplex Approach (Example)

Step 5: Identify Pivot Variable

x1	x2	x3	s1	s2	z	b	Indicator
1	3	2	1	0	0	10	10/3
1	5	1	0	1	0	8	8/5
-8	-10	-7	0	0	1	0	

Smallest Value

The pivot variable is used in row operations to identify which variable will become the unit value and is a key factor in the conversion of the unit value. The pivot variable can be identified by looking at the bottom row of the tableau and the indicator.

In the example shown below, -10 is the smallest negative in the last row. This will designate the x_2 column to contain the pivot variable. Solving for the indicator gives us a value of $\frac{10}{3}$ for the first constraint, and a value of $\frac{8}{5}$ for the second constraint. Due to $\frac{8}{5}$ being the smallest non-negative indicator, the pivot value will be in the second row and have a value of 5.

Simplex Approach (Example)

Step 6: Create the New Tableau

x1	x2	x3	s1	s2	z	b
1/5	1	1/5	0	1/5	0	8/5

Pivot row

Elementary row operation, as like in MK Matriks dan Ruang Vector.

x1	x2	x3	s1	s2	z	b
2/5	0	7/5	1	-3/5	0	26/5
1/5	1	1/5	0	1/5	0	8/5
-6	0	-5	0	2	1	16

New pivot row

Simplex Approach (Example)

Step 7: Check Optimality

- Optimality will need to be checked after each new tableau to see if a new pivot variable needs to be identified.
- **A solution is considered optimal if all values in the bottom row are greater than or equal to zero.**
- If all values are greater than or equal to zero, the solution is considered optimal and Steps 8 through 11 can be ignored.
- If negative values exist, the solution is still not optimal and a new pivot point will need to be determined which is demonstrated in Step 8.

Simplex Approach (Example)

Step 8: Identify New Pivot Variable

- If the solution has been identified as not optimal, a new pivot variable will need to be determined.
- The pivot variable was introduced in Step 5 and is used in row operations to identify which variable will become the unit value and is a key factor in the conversion of the unit value.
- The pivot variable can be identified by the intersection of the row with the smallest non-negative indicator and the smallest negative value in the bottom row.

x1	x2	x3	s1	s2	z	b	Indicator
2/5	0	7/5	1	−3/5	0	26/5	(26/5) / (2/5) = 13
1/5	1	1	0	1/5	0	8/5	(8/5) / (1/5) = 8
-6	0	-5	0	2	1	0	

Smallest Value

Simplex Approach (Example)

Step 9: Create New Tableau

x1	x2	x3	s1	s2	z	b
1	5	1	0	1	0	8

x1	x2	x3	s1	s2	z	b
0	-2	1	1	-1	0	2
1	5	1	0	1	0	8
0	30	1	0	8	1	64

Step 10: Check Optimality

All in the last row are non-negative, then optimal has been achieved.

Step 11: Identify Optimal Values

x_1	= 8	s_1	= 2
x_2	= 0	s_2	= 0
x_3	= 0	z	= 64

Practice!

Some Application in linear programming

1. Product Mix: Find the best mix of product to produce
2. Shipping: Find the optimal shipping assignments
3. Stock Control: Determine the optimal mix of product to hold in inventory
4. Supplier Selection: Find the optimal combination of suppliers to minimize unwanted inventory
5. Plants and warehouse: Determine optimal location of a plant or warehouse
6. Stock Cutting: Find the cutting pattern that minimize the amount of scrap
7. Production: Find the minimum-cost production schedule
8. Staffing: Find the optimal staffing level
9. Blends: Find the optimal proportions of various ingredients used to make products.
10. Shifts: Determine the minimum cost assignment of worker to shift.
11. Vehicles: Assign vehicles to products or customers.
12. Routing: Find the optimal routing of a service or product through several sequential processes.

Solving linear problem using Scipy package.

https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.linprog.html

Please try to solve the problems we discussed before using that package!

Python practice:
https://colab.research.google.com/drive/1ohEtbUmlk1PodnJxXDTBFSD2laqYUbXZ?usp=sharing

References

1. https://blogs.epfl.ch/extrema/documents/Maison%2020.05.10.pdf
2. https://www.usna.edu/Users/math/dphillip/sa305.s15/barkley/lessons/02/02.pdf
3. https://www.uky.edu/~dsianita/300/online/LP.pdf