Sum of three numbers from unformly distributed set equals to zero

I'm reading Sedgewick's "Algorithms" and completely stuck at one exercise. It is formulated like that:

> Develop an appropriate mathematical model describing the number of triples of N random int values that sum to 0, where the values are uniformly distributed between –M and M, where M is not small

I wrote a program to calculate such triplets. It iterates through all possible **distinct triplets** in array **A** of N numbers. **A** may have repeating numbers, but these numbers are form uniform random generator.

Example:

> **A** = [7, -3, -4, 0] gives 4 distinct triplets: **{7, -3, -4}**, {7, -3, 0}, {7, -4, 0}, {-3, -4, 0}. We have only one triplet (first) that sums to 0.

I already calculated the number of 3-samples, it's sampling without replacement and without order: N! / 3! (N - 3)!, but I have no idea how to formulate quantity of triplets that sum to zero.

I want a model and mathematical basis, to **calculate average quantity of such triplets among all 3-samples from N**.

(combinatorics) (uniform-distribution)

edited Mar 3 '14 at 13:55

asked Feb 10 '14 at 20:38
oroboros
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3-samples and triples are not the same thing, in most definitions - a triple is an ordered sequence of three numbers. Also, nothing said that the values in the triple have to be distinct. – Thomas Andrews Feb 10 '14 at 20:43

"Triplets" in my case are just samples of three numbers. And, yes, values may not be distinct. – oroboros Feb 10 '14 at 20:50

So is there a distinction between the triplet $\langle -1,-1,2\rangle$ and $\langle -1,2,-1\rangle$? – Thomas Andrews Feb 10 '14 at 20:54

It does not depend on values (which are just *uniformly distributed*). Set **A** may have repeating numbers. Your two triplets should consist of different elements from **A**, e.g. {a[0], a[1], a[2]} and {a[0], a[1], a[3]} are distinct. But {a[0], a[1], a[2]} and {a[1], a[2], a[0]} are not considered in this task. – oroboros Feb 10 '14 at 21:00

"triples of N random int values$\ldots$where the values are uniformly distributed between $-M$ and $M\ldots$. To me, if say N is 4 and M is large, then one of these things is something like $(\{1,1,1,-3\},\{1,2,3,-6\},\{2,2,1,-5\})$s that right? It looks like there is confusion between what role N has and what the role of 3 is. – alex.jordan Feb 10 '14 at 21:28

@alex.jordan N is the size of set, we should select only 3 distinct elements at a time. If N = 4, we have set like [34,76,-10,3]. From set we can select triples: ({34,76,-10}, {34,76,3}, {34,-10,3}, {76,-10,3}). – oroboros Feb 10 '14 at 22:19

1 Answer

We are counting the number of pairs (A,t) where $A=(A_1,\ldots,A_N)$ is a sequence of N numbers in the range $[-M,M]$ and t is a subset of $\{1,\ldots,N\}$ of size 3 such that $\sum_{i\in t}A_i=0$.

We first pick a triple (this can be done in $\binom{N}{3}$ ways), then pick three values from $[-M,M]$ that sum to 0 (this can be done in $3M^2+3M+1$). Finally we pick the values of the remaining $N-3$ positions (can be done in $(2M+1)^{N-3}$ ways).

The total is thus $\binom{N}{3}(3M^2+3M+1)(2M+1)^{N-3}$, and the average number of t's per A is therefore

$$\frac{\binom{N}{3}(3M^2+3M+1)(2M+1)^{N-3}}{(2M+1)^N} = \frac{\binom{N}{3}(3M^2+3M+1)}{(2M+1)^3} \sim \frac{1}{16}\frac{N^3}{M}.$$

EDIT: I fixed the count of triples from $[-M, M]$ with sum 0, and calculated the average number of t's, which is what the OP asked about (rather than the probability that a random triple of a random int vector will have sum zero).

edited Mar 4 '14 at 18:53 answered Mar 3 '14 at 14:09

Eric
370 1 6

Is number of triplets = N * 3/16M ? Or I should multiply "total" and "fraction" instead? – oroboros Mar 4 '14 at 8:50

Based on my program statistics quantity of zero triplets = N^3 / 16M. Looks like, there must be error in your answer (or in my understanding of it). – oroboros Mar 4 '14 at 9:31

1 You are right - I made a mistake in the calculation (see edit for details). – Eric Mar 4 '14 at 18:54