Financial Econometrics

Homework assigment

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Problem 1

MA(1) process:

$$R_t = \epsilon_t + 0.25\epsilon_{t-1}$$

ϵ_t is white noise therefore $\epsilon_t \sim N(0, \delta_\epsilon)$
$E[\epsilon_t] = 0$, $Var(\epsilon_t) = \delta_\epsilon^2$, $\delta_\epsilon = 0.01$ and $E[\epsilon_t, \epsilon_s] = 0 \ \forall s \neq t$

1.

lag-1 autocorrelation:

$$E[R_t] = E[\epsilon_t + 0.25\epsilon_{t-1}] = 0$$

$$\begin{aligned} Var(R_t) &= Var(\epsilon_t + 0.25\epsilon_{t-1}) = Var(\epsilon_t) + 0.25^2 Var(\epsilon_{t-1}) \\ &= 0.01^2(1 + 0.25^2) = 0.00010625 \end{aligned}$$

$$\begin{aligned} Cov(R_t, R_{t-1}) &= E[(R_t - E(R_t))(R_{t-1} - E(R_{t-1}))] = E[R_t R_{t-1}] \\ &= E[(\epsilon_t + 0.25\epsilon_{t-1})(\epsilon_{t-1} + 0.25\epsilon_{t-2})] \\ &= E[\epsilon_t \epsilon_{t-1}] + 0.25E[\epsilon_t \epsilon_{t-2}] + 0.25E[\epsilon_{t-1}\epsilon_{t-1}] + 0.25^2 E[\epsilon_t \epsilon_{t-2}] \\ &= 0.25E[\epsilon_t^2] = 0.25 * 0.01^2 = 0.000025 \end{aligned}$$

$$corr(R_t, R_{t-1}) = \frac{cov((R_t, R_{t-1}))}{\sqrt{Var(R_t)}\sqrt{Var(R_{t-1})}} = \frac{0.025}{0.10625} = 0.2353$$

lag-2 autocorrelation:

$$\begin{aligned} Cov(R_t, R_{t-2}) &= E[(R_t - E(R_t))(R_{t-2} - E(R_{t-2}))] = E[R_t R_{t-2}] \\ &= E[(\epsilon_t + 0.25\epsilon_{t-1})(\epsilon_{t-2} + 0.25\epsilon_{t-3})] \\ &= E[\epsilon_t \epsilon_{t-2}] + 0.25E[\epsilon_t \epsilon_{t-3}] + 0.25E[\epsilon_{t-1}\epsilon_{t-2}] + 0.25^2 E[\epsilon_t \epsilon_{t-2}] \\ &= 0 \end{aligned}$$

$$corr(R_t, R_{t-2}) = \frac{cov((R_t, R_{t-2}))}{\sqrt{Var(R_t)}\sqrt{Var(R_{t-2})}} = 0$$

2.

1-month ahead forecast:

$$\begin{aligned} \hat{R}_{100}(1) &= E[\epsilon_{101}|\epsilon_{100}, \epsilon_{99}, ...\epsilon_0] = E[\epsilon_{101} + 0.25\epsilon_{100}|\epsilon_{100} = 0.01] \\ &= 0.25 * 0.01 = 0.0025 \end{aligned}$$

Forecast error: $e_{100}(1) = \epsilon_{101}$

$$Var(e_{100}(1)) = \sigma_\epsilon^2 = 0.0001$$

2-month ahead forecast:

$\hat{R}_{100}(2) = E[\epsilon_{102}|\epsilon_{100}, \epsilon_{99}, ...\epsilon_0] = E[\epsilon_{102} + 0.25\epsilon_{101}|\epsilon_{100} = 0.01] = 0$

Forecast error:$e_{100}(2) = \epsilon_{102} + 0.25\epsilon_{101}$

$$Var(e_{100}(2)) = (1 + 0.25^2)\sigma_\epsilon^2 = 0.00010625$$

3.

standard error of forecast:

$$se(1) = \sqrt{Var(e_{100}(1))} = 0.01$$
$$se(2) = \sqrt{Var(e_{100}(2))} = 0.0103$$

Problem 2

AR(1) process:

$$r_t = 0.01 + 0.5r_{t-1} + \epsilon_t$$

where $\epsilon_t \sim N(0, 0.03)$

1.

$$\begin{aligned} E[r_t] &= E[0.01 + 0.5r_{t-1} + \epsilon_t] \\ &= 0.01 + 0.5E[0.01 + 0.5r_{t-2} + \epsilon_{t-1}] \\ &= 0.01\frac{1}{1-0.5} = 0.02 \end{aligned}$$

$$Var(r_t) = \frac{\sigma_\epsilon^2}{1-0.5^2} = \frac{0.009}{0.75} = 0.0012$$

2.

$r_{100} = -0.01$ and $r_{99} = 0.02$

1-month forecast:

$$\begin{aligned} \hat{r}_{100}(1) &= E[r_{101}|r_{100}, r_{99}...r_0] = E[0.01 + 0.5r_{100} + \epsilon_{101}|r_{100} = -0.01] \\ &= 0.01 + 0.5 * (-0.01) = 0.005 \end{aligned}$$

Forecast error: $e_{100}(1) = \epsilon_{100}$

$$Var(e_{100}(1)) = \sigma_\epsilon^2 = 0.03$$

2-months forecast:

$$\begin{aligned}
\hat{r}_{100}(2) &= E[r_{102}|r_{100}, r_{99}...r_0] = E[0.01 + 0.5r_{101} + \epsilon_{101}|r_{100} = -0.01] \\
&= E[0.01 + 0.5(0.01 + 0.5r_{100} + \epsilon_{101}) + \epsilon_{102}|r_{100} = -0.01] \\
&= 0.01 + 0.5(0.01 + 0.5 * (-0.01)) = 0.0125
\end{aligned}$$

Forecast error : $e_{100}(2) = 0.5\epsilon_{101} + \epsilon_{102}$

$$Var(e_{100}(2)) = (0.5^2 + 1)\sigma_\epsilon^2 = 0.0375$$

3.

$$se(1) = \sqrt{Var(e_{100}(1))} = 0.1732$$
$$se(1) = \sqrt{Var(e_{100}(2))} = 0.1936$$