Financial Econometrics

Homework assigment

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Problem 1

$$R_t = \epsilon_t + 0.25\epsilon_{t-1}$$

ϵ_t is white noise therfore $\epsilon_t \sim N(0, \delta_\epsilon)$ and $E[\epsilon_t] = 0$, $Var(\epsilon_t) = \delta_\epsilon^2$ and $E[\epsilon_t, \epsilon_s] = 0 \ \forall s \neq t$ where $\delta_\epsilon = 0.01$

1.
lag-1 autocorrelation:

$$E[R_t] = E[\epsilon_t + 0.25\epsilon_{t-1}] = 0$$

$$Var(R_t) = Var(\epsilon_t + 0.25\epsilon_{t-1}) = Var(\epsilon_t) + 0.25^2 Var(\epsilon_{t-1}) = 0.01^2(1 + 0.25^2) = 0.00010625$$

$$\begin{aligned}
&Cov(R_t, R_{t-1}) \\
&\quad = E[(R_t - E(R_t))(R_{t-1} - E(R_{t-1}))] = E[R_t R_{t-1}] \\
&\quad = E[(\epsilon_t + 0.25\epsilon_{t-1})(\epsilon_{t-1} + 0.25\epsilon_{t-2})] \\
&\quad = E[\epsilon_t\epsilon_{t-1}] + 0.25E[\epsilon_t\epsilon_{t-2}] + 0.25E[\epsilon_{t-1}\epsilon_{t-1}] + 0.25^2 E[\epsilon_t\epsilon_{t-2}] \\
&\quad = 0.25E[\epsilon_t^2] = 0.25 * 0.01^2 = 0.000025
\end{aligned}$$

$$corr(R_t, R_{t-1}) = \frac{cov((R_t, R_{t-1}))}{\sqrt{Var(R_t)}\sqrt{Var(R_{t-1})}} = \frac{0.025}{1.0625} = 0.2353$$

lag-2 autocorrelation:

$$\begin{aligned}
&Cov(R_t, R_{t-2}) \\
&\quad = E[(R_t - E(R_t))(R_{t-2} - E(R_{t-2}))] = E[R_t R_{t-2}] \\
&\quad = E[(\epsilon_t + 0.25\epsilon_{t-1})(\epsilon_{t-2} + 0.25\epsilon_{t-3})] \\
&\quad = E[\epsilon_t\epsilon_{t-2}] + 0.25E[\epsilon_t\epsilon_{t-3}] + 0.25E[\epsilon_{t-1}\epsilon_{t-2}] + 0.25^2 E[\epsilon_t\epsilon_{t-2}] \\
&\quad = 0
\end{aligned}$$

$$corr(R_t, R_{t-2}) = \frac{cov((R_t, R_{t-2}))}{\sqrt{Var(R_t)}\sqrt{Var(R_{t-2})}} = 0$$

2.

1-month ahead forecast:

$$\hat{R}_{100}(1) = E[\epsilon_{101}|\epsilon_{100}, \epsilon_{99}, ...\epsilon_0] = E[\epsilon_{101} + 0.25\epsilon_{100}|\epsilon_{100} = 0.01] = 0.25 * 0.01 = 0.0025$$

Forecast error: $e_{100}(1) = \epsilon_{101}$

$$Var(e_{100}(1)) = \sigma_\epsilon^2 = 0.0001$$

2-month ahead forecast:

$$\hat{R}_{100}(2) = E[\epsilon_{102}|\epsilon_{100}, \epsilon_{99}, ...\epsilon_0] = E[\epsilon_{102} + 0.25\epsilon_{101}|\epsilon_{100} = 0.01] = 0$$

Forecast error:$e_{100}(2) = \epsilon_{102} + 0.25\epsilon 101$

$$Var(e_{100}(2)) = (1 + 0.25^2)\sigma_\epsilon^2 = 0.00010625$$

3.

standard error of forecast

$$se(1) = \sqrt{Var(e_{100}(1))} = 0.01$$
$$se(2) = \sqrt{Var(e_{100}(2))} = 0.0103$$

Problem 2

$AR(1) r_t = 0.01 + 0.5r_{t-1} + \epsilon_t$, where $\epsilon_t \sim N(0, 0.03)$

1.

$$\begin{aligned} E[r_t] &= E[0.01 + 0.5r_{t-1} + \epsilon_t] \\ &= 0.01 + 0.5E[0.01 + 0.5r_{t-2} + \epsilon_{t-1}] \\ &= 0.01\frac{1}{1 - 0.5} = 0.05 \end{aligned}$$

$$Var(r_t) = \frac{\sigma_\epsilon^2}{1 - 0.5^2} = \frac{0.009}{0.75} = 0.0012$$

2.

$r_{100} = -0.01$ and $r_{99} = 0.02$

1-month forecast:

$$\hat{r_{100}}(1) = E[r_{101}|r_{100}, r_{99}...r_0] = E[0.01+0.5r_{t-1}+\epsilon_t|r_{100} = -0.01] = 0.01+0.5*(-0.01) = 0.005$$