Machine Learning for Maths - Session 4

1. Vector Notation

- A vector x in d-dimensions is represented as a column:

  x = [x1, x2, ..., xd]

2. Norms

- L2 Norm (Euclidean): ||x|| = sqrt(x1 + x2 + ... + xd)
- L1 Norm (Manhattan): ||x|| = |x1| + |x2| + ... + |xd|

3. Dot Product

- x y = xy = x1*y1 + x2*y2 + ... + xd*yd

4. Angle Between Vectors

- cos() = (xy) / (||x|| * ||y||)
- Helps understand directional similarity
- < 90 positive dot product same side
- > 90 negative dot product opposite side

5. Equation of Line and Halfspace

- Line: wx + w = 0
- Defines the decision boundary
- Halfspaces:
  * wx + w > 0 positive halfspace
  * wx + w < 0 negative halfspace

6. Distance from Origin to Line

- d = |w| / ||w||

7. Distance from Point to Line

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- d = |wp + w| / ||w||
- Sign indicates which side of the hyperplane the point lies on

8. Distance Between Two Hyperplanes

- d = |w - w| / ||w||
- Valid only when w = w = w

9. Loss and Gain Functions (SVM Intuition)

- Loss function tells how good the structure is
- Lower the loss, better the separation
- Gain function:

  Gain(f(x)) = [(wxi + w) / ||w|| * yi] over i = 1 to n

- Loss = -Gain(f(x))
- Good line: High sum of signed distances (SOD) large margin

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Revision :

① Vector : $\bar{x} : \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix}$

② Norm : $\|x\|$

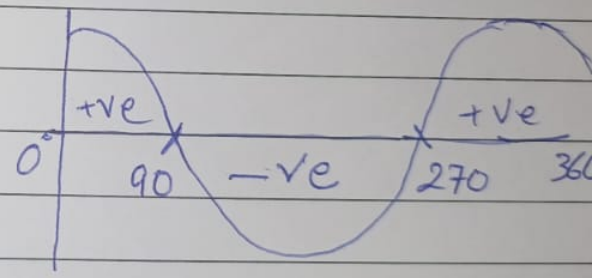
∠2 norm : $\sqrt{x_1^2 + x_2^2 + \dots x_d^2}$

∠1 norm : $|x_1| + |x_2| + |x_3| \dots$

③ Dot product : $\bar{x} \cdot \bar{y} = x^T y = x_1 y_1 + x_2 y_2 + \dots x_d y_d$

④ Angle b/w vectors

$$\cos\theta = \frac{\bar{x}^T y}{\|x\| \|y\|}$$

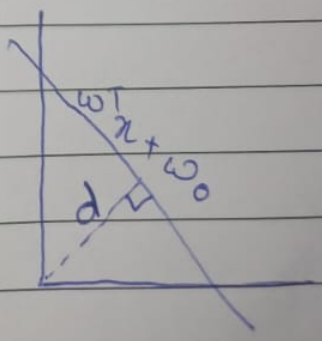


⑤ Eq^n of line & Halfspace

$$w^T x + w_0 = 0$$

⑥ Distance b/w origin & Line

$$d = \frac{w_0}{\|w\|}$$



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Loss function : tells us how good the structure is.

- Lower the loss better the function
- A Good Line will have SOD as high as possible

$$\text{Gain func} = \left(\sum_{i=1}^{n} \frac{w^T x_i + w_0}{\|w\|} \right) y_i$$

$$\text{loss func} = -\text{Gain } f(x)$$

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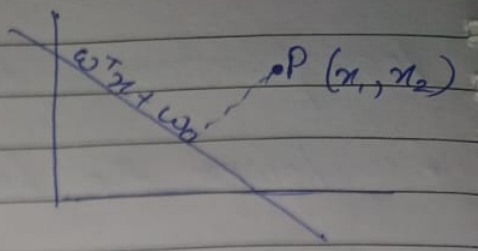
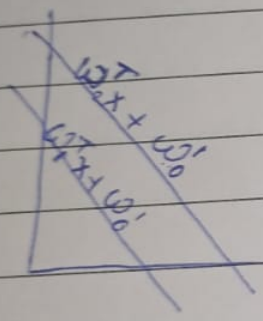
⑦ Dist b/w point & line:

$$d = \frac{w^T p + w_0}{\|w\|}$$

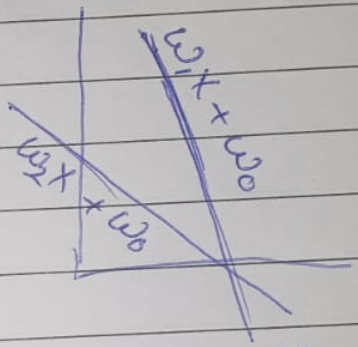


If $w^T p + w_0$ +ve, direction of point, +ve halfspace
-ve, opposite of point, -ve halfspace

Distance b/w 2 Hyperplane



$w_1 = w_2 = w^T$
$d \neq 0$



If $w_1 \neq w_2$
$d \doteq 0$

~~Hyperplanes~~ $d_{plane} = \left| \frac{w_0^2 - w_0^1}{\|w\|} \right|$