The Puzzling Labor Market Sorting Pattern in Expanding and Contracting Firms*

Andreas Gulyas†

Abstract

Using German matched employer-employee data I document a disconnect between the cross-sectional sorting pattern and firm-level workforce skill adjustments over time. Whereas larger firms match with higher-skilled workers in the cross section, low-skilled workers become more valuable to expanding firms as shown by: (i) expanding firms downgrade average workforce skills, (ii) expanding firms hire worse workers and separate from better workers, (iii) lower-skilled workers see larger wage gains in expanding firms. These patterns reverse for contracting firms. I develop a multi-worker firm model with search frictions, firm dynamics, and worker-firm complementarities and multi-dimensional sorting that replicates these patterns, as well as many other salient features of firm dynamics. Using the model I show that not accounting for workforce skill adjustments overstates the typical firm-level labor supply elasticity estimates by up to 12 percent. My model further predicts stronger effects of dismissal taxes compared to a model without sorting.

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†University of Mannheim; andreas.gulyas@uni-mannheim.de

1 Introduction

One of the central roles of the labor market is to allocate workers and jobs across firms. A well-documented labor market outcome is positive sorting between firms and workers, i.e., more productive and larger firms match with better workers in the cross section. Standard sorting models, such as Becker (1973), predict that firms upgrade worker skills as they expand in response to positive productivity shocks, and downgrade worker skills as they contract. Yet, there is surprisingly little evidence on how worker composition changes with firm growth in the literature.

I use German social security data and show that these predictions are not borne out in the data. While larger firms match with better workers in the cross section, I find that low-skilled workers become more valuable to expanding firms, in contradiction with the predictions of standard labor market sorting models.

First, I show that expanding firms downgrade the skill intensity of their workforce, whereas contracting firms increase the skill intensity of their workforce. The negative relationship between firm growth and changes in worker skills is a robust and salient feature of firm dynamics and holds for different worker skill measures, over the entire life cycle of the firm, over different time horizons, and different econometric specifications.

Second, zooming in on worker flows, I show that expanding firms replace better workers with worse workers, i.e., the average hire has fewer skills than the average separating worker, whereas the opposite is true for contracting firms. This happens because expanding firms hire worse workers, while contracting firms hire better workers. A decomposition shows that both hires and separations contribute to the negative relationship between firm growth and changes in workforce skills. I present additional evidence that changes in the occupation structure and skill shortages for expanding firms cannot explain these features.

Third, in expanding firms, low-skilled workers see higher wage gains compared to high-skilled workers, whereas in contracting establishments it is the higher-skilled workers who fare comparatively better. In summary, despite the fact that larger, more successful firms match with better workers in the cross section, firm dynamics paint the opposite picture: low-skilled workers become more valuable to firms that become more successful and expand.

To shed light on the driving forces behind the adjustment patterns of workforce skills over time, I develop a tractable multi-worker firm model with search frictions, on-the-job search, firm dynamics originating from idiosyncratic firm-level shocks and sorting between firm and worker types. In my model, workers and firms are heterogeneous in their productive capacity and complementarities in production induce sorting in equilibrium, as in Becker (1973). As in Shimer and Smith (2000), search frictions impede the reallocation of workers across firms, so equilibrium sorting is imperfect. The bargaining setup is similar to Postel-Vinay and Robin (2002) and Postel-Vinay and Turon (2010).

To explain the disconnect between the cross-sectional positive sorting and the firm level adjustments over time, I depart from most of the sorting literature in two dimensions. First, I assume that firms face idiosyncratic productivity shocks.[1] Given that a majority of firms change size every quarter (Davis, Faberman, and Haltiwanger, 2006, 2012), it is highly implausible to assume fixed firm types even over short time periods. Second, I relax the assumption of one-dimensional firm productivity. I assume that firms are characterized by a persistent and a transitory productivity component. A priori, there is no reason to assume why these two firm productivity components should interact the same way with worker skills. Investments in modern capital equipment, which tend to be persistent, are typically thought to be complementary to worker skills (e.g., Krusell et al., 2000). But, for example, a good management team, which temporarily makes the firm more productive, or investments in new artificial intelligence (AI) technology, which depreciate quickly, could substitute for worker skills. Key to explaining the disconnect between the cross-sectional positive sorting and firm level adjustments over time is that the persistent and transitory firm components are allowed to interact differently with worker skills.

The key identification idea to recover the complementarities in production is to use the empirically documented adjustment patterns as identifying moments. In my setup, more productive firms operate at a larger scale.[2] Because firms face convex job creation costs and a linear job destruction probability, firms with positive productivity shocks expand in size, whereas firms with negative ones shrink. This allows me to map changes in unobserved productivity to observable changes in firm size in the German social security dataset. In addition, firms adjust the skill composition of their workforce in response to shocks to their time-varying productivity. This reorganization is linked to complementarities in production between the time-varying firm productivity and worker types. As in Becker (1973), if the two are complements, the marginal effect of worker skills is higher at high type firms. This implies that high-skilled workers become more valuable to firms with positive productivity shocks, and these reorganize the workforce towards higher-skilled workers. With negative sorting, the exact opposite occurs. The marginal effect of skills decreases in firms with positive shocks to their transient productivity and therefore firms downgrade the skill composition of their workforce after positive shocks

[1] To the best of my knowledge, Lise, Meghir, and Robin (2016) is the only other study with assortative matching considering firm-level shocks.

[2] I follow an extensive literature in economics explaining differences in firm size by productivity differences, see e.g., Hopenhayn (1992), Melitz (2003), Luttmer (2007) and Lentz and Mortensen (2008)

and upgrade it after negative ones. Thus, the relationship between changes in average workforce quality and firm growth rates recovers the sign and degree of the complementarities between the time-varying firm productivity component and worker skills. On the other hand, the sorting pattern along the fixed firm productivity component is identified through the cross-sectional relationship between firm size and worker skills. I measure worker types by average annual earnings controlling for observable wage determinants. I show that this metric provides an accurate measure of worker types in my model.[3]

Worker productivity is estimated to be a complement to time-invariant firm productivity, which generates the positive cross-sectional sorting. On the other hand, worker skills are identified to be substitutes to time-varying firm productivity. The model is able to generate many salient facts of firm dynamics as well as all of the empirically documented relationships between firm growth and changes to the workforce skill composition. The model generates realistic relationships between (1) firm growth and firm size, (2) firm size and firm-level separation rates, (3) firm growth and changes in the within establishment dispersion of worker types, (4) firm growth and wage changes by low and high-skilled workers, (5) firm growth and the average quality difference between hires and separations, and in addition matches (6) contributions of hires and separations to overall changes in worker quality, (7) the employment elasticities over the worker skill distribution, (8) the autocorrelation of firm size over various lags, and (9) the job tenure profile closely. Furthermore I use the model to structurally decompose the large firm wage premium and show that that sorting of high-skilled workers into large firms explains most of the premium.

In the data as well as in the model, growing firms replace existing workers with less-skilled ones, whereas contracting firms use worker flows to upgrade worker skills. These adjustments generate hiring and separations at the same time and generate excess worker churn and the "hockey-stick" hire and separation rate dynamics documented in Davis et al. (2006). This behavior is puzzling in standard models, where firms want to quickly converge to their optimal size and therefore either shut down the hiring or the separation margin.[4]

This excess worker churn originating from quality adjustments has important implications for the welfare impact of dismissal taxes. Prior literature (see, e.g., Hopenhayn and Rogerson (1993); Alvarez and Veracierto (1999), among many others) has highlighted that dismissal taxes impede the optimal reallocation of workers from firms with negative

[3] Lopes de Melo (2018) shows in a similar model that worker fixed effects capture the corresponding true worker types closely.

[4] Two recent papers provide alternative theories for this pattern: In Borovicková (2016), firm productivity shocks are correlated with idiosyncratic match level productivity shocks, and in Bachmann et al. (2021), excess worker turnover is generated by persistent idiosyncratic separation rate shocks.

to firms with positive productivity shocks. However, in my model, firms not only adjust the quantity of their workforce in response to shocks, but also adjust the workforce quality due to worker-firm complementarities. Thus, in my setting, dismissal taxes impede not only the optimal adjustment in workforce size but also the adjustment in workforce quality, a channel missing in the dismissal cost literature so far. Introducing dismissal costs amounting to two months of labor productivity lowers employment by six percent and welfare by two percent. I show that my model predicts welfare losses from dismissal taxes that are twice as large as in a model where I turn off worker-firm complementarities and hence sorting. This highlights the importance of accounting for multi-dimensional sorting not only for understanding firm-level employment dynamics but also for evaluating the broader labor market impacts of dismissal regulations.

My paper contributes to a growing literature studying the sorting patterns in labor markets using structural search models. Eeckhout and Kircher (2011), Hagedorn et al. (2017), Bagger and Lentz (2018), Lopes de Melo (2018), Lise and Robin (2017), and Lise, Meghir, and Robin (2016), among others, study the identification problem in sorting models. My paper advances this work by including firm dynamics over time, which provides an alternative identification strategy using worker skill adjustments to uncover the complementarities in production and is key for the empirical puzzle I document. It also joins Lindenlaub (2017) and Lindenlaub and Postel-Vinay (2023) to study multi-dimensional sorting.

My paper also relates to the large empirical literature studying employment dynamics at the establishment level. Many papers study how firms adjust the number of hires and separations in response to shocks (e.g. Davis et al. (2006, 2012)), but there is surprisingly little evidence on how firms reorganize their workforce composition. Kaas and Kircher (2015), Schaal (2017), Bilal et al. (2022), Elsby and Gottfries (2021), among others develop search models with firm dynamics, but these models do not feature two-sided heterogeneity and sorting, therefore cannot study firm adjustments to the composition of their workforce over time. Caliendo, Monte, and Rossi-Hansberg (2015) find that French manufacturing firms grow by adding layers of management and expand preexisting layers with lower-skilled workers, whereas I find that German establishments grow by adding lower skilled workers. Crane (2014) studies the quality of hires over the firm growth distribution, but does not go beyond this. In addition to many other facts, I show that firms use both, hires and separations to change the composition of their workforce.

This paper also contributes to the growing literature on firms' labor supply elasticity, which plays a key role in understanding the extent of monopsony power in labor markets.[5] This body of work typically estimates how much a firm's labor input responds to wage

[5]See Sokolova and Sorensen (2021) for a full literature review.

increases, often under the assumption that the composition of the workforce remains constant in the face of firm-level shocks (e.g., Kroft et al., 2023; Kline et al., 2019). However, the evidence presented here challenges that assumption. A natural question that emerges is to what extent labor supply elasticity estimates are biased as a result. Using my estimated model, I find that accounting for changes in worker quality reduces the measured labor supply elasticity by up to 12 percent.

The structure of the paper is as follows: Section 2 presents the conceptual framework, while Section 3 documents the empirical regularities of workforce quality adjustments. Section 4 presents the full model that builds on the conceptual framework and the empirical findings. Section 5 discusses the identification of all parameters, and Section 6 provides the estimation results, Section 7 presents the model's applications, which comprise (i) a structural decomposition of the large-firm wage premium, (ii) estimates of labor-supply elasticities under changing worker compositions, and (iii) an analysis of the impact of dismissal taxes. The last section concludes.

2 Conceptual Framework

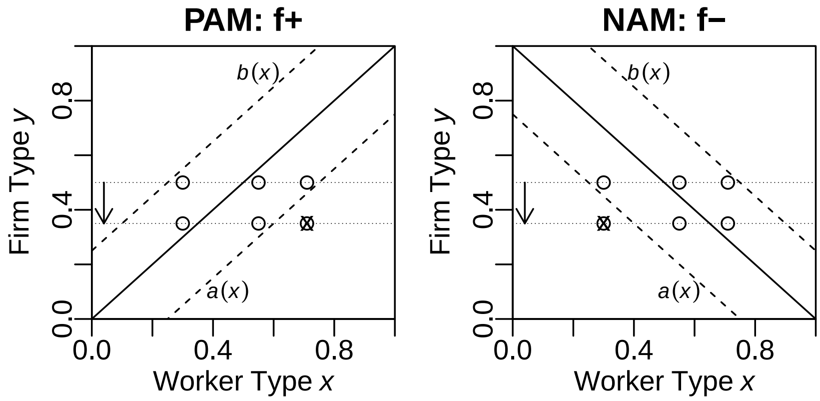
Why would firms adjust the quality of their workforce after productivity shocks? Before I lay out the full structural model in Section 4, I present Becker (1973) as a simple and widely used conceptual framework to understand the allocation of worker skills across firm types. Workers and firms are heterogeneous in productivity and match together to produce output. Worker productivity is denoted by $x \in [0, 1]$ and follows the CDF $\psi_x(x)$. Firms draw their productivity $y \in [0, 1]$ from distribution $\psi_y(y)$. When a firm of type y matches with a worker of type x, they produce output according to $f(x, y)$, with $f_x(x, y) > 0, f_y(x, y) > 0$, and the sign of the cross derivative $f_{xy}(x, y)$ is left unspecified. Firms, taking the wage schedule $w(x)$ as given, maximize profits $f(x, y) - w(x)$ by choosing the worker type x to be matched with, which yields the following first-order condition:

$$f_x(x, y) = w_x(x). \tag{1}$$

Intuitively, firms equate the marginal cost of an additional skill unit x with the marginal gain from hiring a marginally better worker. As wages increase with x due to $f_x(x, y) > 0$, the allocation of worker types across firms $\mu(x)$ will depend on complementarities in production, i.e. f_{xy}.[6] Under $f_{xy} > 0$, high-skilled workers have a higher marginal productivity at high-type firms and positive assortative matching (PAM) arises. In contrast, if worker and firm types are substitutes in production, i.e. $f_{xy} < 0$, negative assortative matching

[6] Appendix A derives the conditions formally.

Figure 1: Sorting Pattern



Notes: Matching sets for production functions implying PAM and NAM. With PAM, high type workers separate after negative shocks, whereas with NAM, it is the low type workers that move outside the matching bands and separate after negative productivity shocks.

(NAM) arises.

Shimer and Smith (2000) and Eeckhout and Kircher (2018) extend the frictionless Becker model to a setting with search friction. Intuitively, if hiring is a time- or resource intensive endeavor, firms will match with a range of worker types. Matching sets are now characterized by a lower bound $a(y)$ and an upper bound $b(y)$ on worker quality that a firm of type y is willing to match with in equilibrium. Under PAM, both of these bounds are increasing in y, as better firms match with better workers, whereas under NAM they are decreasing in y. Figure 1 plots stylized versions of such matching patterns.

How does workforce quality change after firm productivity shocks? With one-dimensional firm productivity, it is straightforward to see that if a firm redraws lower productivity, the adjustment pattern will mimic the cross-sectional pattern. Figure 1 clarifies this intuition. Under PAM, $f_{xy}(x, y) > 0$ implies that the marginal productivity declines more for high-skilled workers, and firms will instead match with lower skilled workers. Consequently, under PAM, there will be a *positive* relationship between productivity shocks and changes in worker quality, as firms *downgrade* worker quality after negative productivity shocks. In contrast, under NAM, there will be a *negative* relationship between productivity shocks and changes in worker quality, as firms *upgrade* worker quality after negative productivity shocks. The same holds true under search frictions. Firms now match with a range of workers, but the underlying logic remains the same. Under PAM, the bounds of the matching set $a(x)$ and $b(x)$ are upward sloping. Thus, after a negative productivity shock, it is the high-quality workers who drop out of the matching set and separate from the firm. These workers are then replaced by lower-type workers and average worker quality declines. In contrast, under NAM it is the high-type workers who move out of the matching sets and are replaced by better workers, raising average worker quality within the firm.

To summarize, the adjustments of workforce quality over time to firm-level shocks always mimic the cross-sectional sorting pattern. PAM implies a positive relationship between firm productivity shocks and changes in workforce quality. In addition, after positive shocks, the lower-type workers will be replaced by better workers, whereas after negative shocks, the higher type workers will be replaced. The next section uses German social security records to empirically document how firms adjust the quality of their workforce.

3 New Evidence on Worker Quality and Firm Growth

I use the *Linked-Employer-Employee data* (LIAB) longitudinal model provided by the Institute for Employment Research.[7] A key feature of this dataset is that it covers the work histories of all employees in a representative sample of 5000-6000 establishments. The dataset follows these establishments from 1999 to 2009 and contains complete social security records (1993–2010) for all employees who worked in them during that period. The panel structure of the data allows me to track changes in the worker skill composition in establishments over time. The exact working hours are not recorded, but the dataset indicates whether each worker is employed part-time or full-time. I restrict the sample to full-time employees, since hourly wages can only be constructed for these workers. Appendix B describes the data in more detail and outlines how I construct the variables used in the empirical analysis.

The main measure of worker quality is average lifetime earnings, adjusted for an education-specific age profile. The interpretation of worker quality follows an extensive applied literature, but is also grounded in theory, as more productive workers have higher lifetime earnings in standard search models (e.g., Lopes de Melo, 2018). Following Card et al. (2013) and Hagedorn et al. (2017), I estimate worker fixed effects from a Mincer wage regression. Yearly wages, deflated using the Consumer Price Index (CPI), are regressed on education fully interacted with a third-degree polynomial in age, using an 18-year panel. I then normalize all worker fixed effects to lie within the [0, 1] interval. In addition to this measure, I also use alternative proxies of worker quality, such as current and past wages, education level, and work experience.

An emerging body of literature examines sorting patterns in the cross-section. Most studies, including several focused on Germany, document positive sorting in the labor

[7] See Heining et al. (2013) for a detailed description.

Table 1: Cross-Sectional Regression Results

Worker Quality Measure	Fixed Effects	Wages	Wages last 2 yrs.	Wages UE spells	Edu.	Exp.
$\log(firm\_size_{jt})$	0.131	0.133	0.118	0.095	0.054	0.087
SE	0.003	0.002	0.002	0.003	0.002	0.003
N	60914	61460	60016	51971	61460	61460
Adj. R^2	0.183	0.265	0.256	0.087	0.075	0.081

Notes: Cross section regression of several worker quality measures (in logs) at the establishment level on log establishment size, as measured in workers employed. See text for details.

market, that is they find more successful firms match on average with better workers (Card et al., 2013; Hagedorn et al., 2017). In standard models of firms, firm size is directly linked to productivity (Hopenhayn and Rogerson, 1993). Thus, a simple way of studying the cross-sectional sorting pattern is to regress the log employment on worker quality measures with the following regression:

$$log(\overline{WFE}_{jt}) = \alpha + \beta \, \log(firm\_size_{jt}) + \epsilon_{jt} \tag{2}$$

Indeed, Table 1 shows that larger firms employ workers who, on average, have higher fixed effects, earn higher current wages, have higher average wages over the past two years, receive higher wages following unemployment spells, and possess more education and work experience.[8]

3.1 Firm Growth and Changes in Worker Quality

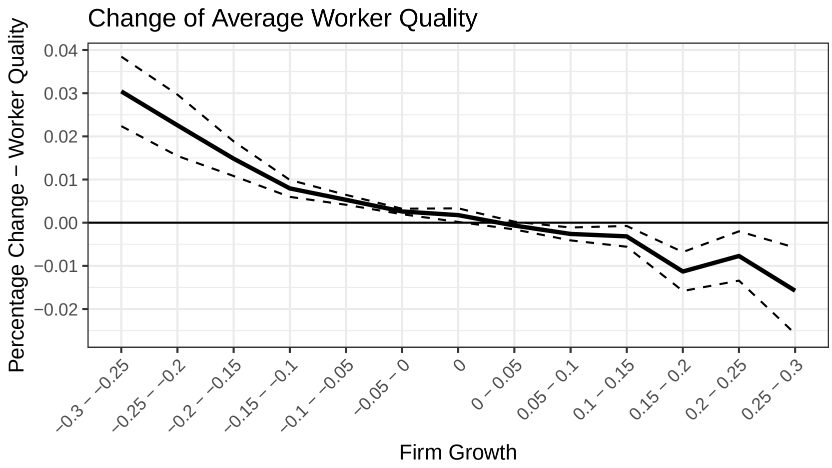
As described in the conceptual framework, and given the mounting evidence of positive sorting in the cross-section, one would expect that firms that become more successful also upgrade the quality of their workforce. A compact way to summarize how firms reorganize their workforce in response to shocks is to correlate the average worker quality changes with firm growth:

$$\Delta_{\%}\overline{WFE}_{jt} = \alpha + \sum_k \gamma^k growth^k_{jt} + \epsilon_{jt}. \tag{3}$$

Here, $\Delta_{\%}\overline{WFE}_{jt}$ denotes the percentage change in average worker type in establishment j between year t and $t-1$, measured at the end of the year. $growth^k_{jt}$ is a dummy that

[8]Borovičková and Shimer (2017) shows that wages after unemployment net out differential bargaining positions across workers. Table 12 shows that these relationships are robust to additional controls.

Figure 2: Workforce Skill Adjustments over Time



Notes: The figure shows the estimated relationship between firm growth rates and the percentage changes in average worker quality employed by firms using regression equation (3) in the German social security data, using growth bins. The sample consists of all establishments with size≥20. Standard errors are clustered at the establishment level. Broken lines indicate 95% confidence intervals. Establishment growth rates and percentage changes in average worker quality are yearly.

equals one if establishment j's employment growth in year t falls within the k-th growth bin.

Figure 2 shows the percentage change of average worker quality across bins of firm employment growth rates. There is a clear and statistically significant negative relationship between changes in average workforce quality and firm growth, implying that establishments downgrade worker quality as they expand and upgrade it as they contract in size.

Before discussing in detail how firms adjust worker quality, I briefly show that this negative relationship is robust to alternative worker quality measures, model specifications, and control variables. The detailed discussion of these robustness exercises is provided in Appendix C. Given the linear nature of the relationship, I use a continuous firm growth variable instead of firm growth rate bins on the right-hand side of the regression.

First, Table 13 in Appendix C shows that the negative relationship holds across various measures of worker quality, including current wages, the average of the last two years of wages, wages following unemployment spells, education level, and work experience. Furthermore, Table 14 in the appendix reports a battery of sensitivity and robustness checks. The estimated coefficient remains virtually unchanged when the regression is weighted by firm size, when controlling for firm size and age, firm fixed effects, or year $\times$ industry $\times$ location fixed effects which account for industry-wide shocks, and is robust

Table 2: Regression Results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$growth_{jt}$	-0.068	-0.049	-0.065	-0.059	-0.069	-0.000	-0.065	-0.044
SE	0.005	0.005	0.007	0.008	0.010	0.000	0.008	0.005
Sample/Model	Baseline	W/O Tenure	High Urate	Large Cities	Large Firms	Lagged	No Labor Shortage	Levels w/FEs
N	19912	19912	9518	4991	4837	17165	5654	19911
Adj. R^2	0.058	0.030	0.051	0.056	0.067	0.000	0.052	0.970

Notes: Establishment level regressions of firm growth rates on percentage change in average worker fixed effects. Regressions are weighted by establishment size. Standard errors are clustered at the firm level. The samples are restricted to establishments with more than 20 employees and growth rates between -0.75 and 0.75. See text and notes below for detailed explanation of the different specifications.
(1) Baseline regression
(2) Worker FEs computed including a control for tenure
(3) Baseline conditional on years with above average unemployment rate
(4) Baseline restricted to cities with population above 500,000
(5) Baseline regression restricted to establishment with size ≥ 190
(6) Worker quality adjustments regressed on previous years firm growth rate.
(7) Baseline restricted to establishment reporting neither 'Difficulties in finding the required specialized personnel on the labor market' nor 'Staff shortage' as expected hiring problems in the IAB establishment panel survey.
(8) $\hat{\gamma}$ from regression equation $log\bar{w}_{jt} = \psi_j + \gamma growth_{jt} + \epsilon_{jt}$

to different econometric specifications. In particular, when regressing the level of worker quality—rather than percentage changes— on firm growth while controlling for firm fixed effects, I continue to find that workforce quality declines as firms grow. This is shown in the final column of Table 2. In summary, the negative relationship between firm growth and changes in worker quality emerges as a salient and robust feature of firm dynamics.

Do firms prefer to lower the quality of their workforce as they expand, or are there alternative explanations for the negative relationship between worker quality changes and firm growth? Potentially, the relationship could be driven by firms outsourcing lower-skilled labor, for example, technology firms outsourcing to janitorial or security service firms. The growing firms providing these outsourcing services expand with lower-skilled workers, whereas shrinking outsourcing firms remain with the high-skilled workers. First, outsourcing likely accounts for only a small share of the overall firm growth distribution. Second, all industries show the negative relationship, thus a cross-industry reallocation is an unlikely explanation.[9] Third, a notable feature of the relationship between firm growth and changes in workforce composition is its linearity with no kink at zero growth. Thus, the motives behind these adjustments do not change abruptly between expanding

[9] Figure 10 in the appendix further shows that the negative relationship between growth rates and changes in worker quality holds for all Nace-1 industries except hotels and restaurants, which is imprecisely estimated.

and contracting firms.

A first-in-first-out layoff policy would explain why firms separate from lower-type workers[10] when they contract, although it is less clear why firms would choose to expand by hiring lower-type workers. Column 2 of Table 2 shows that this is not the driving force of the negative relationship, as the relationship remains negative after controlling for job tenure.

Another plausible explanation why firms expand with lower-type workers is that hiring firms face shortages of high-quality workers. A number of specifications provide evidence that this is not the driving force behind the estimated negative relationship. First, this concern should be less relevant in thick labor markets and during periods of high unemployment. But as columns 3 and 4 show, the relationship remains robust to restricting the sample to high unemployment years and firms in the largest cities. In addition, large firms should be more affected by labor shortages, however, the relationship remains unchanged, as shown in column 5. Moreover, if firms cannot fill their vacancies with their preferred worker types we would expect that they continue adjusting the quality of their workforce in the following periods. This again is not the case, as the relationship vanishes and drops to zero when considering worker quality changes with lagged firm growth, as reported in column 6.

Finally and perhaps most convincingly, a subsample of firms was directly surveyed about expected labor shortages when hiring. 75 percent of firms expected neither any shortage of skilled workers nor labor shortages in general when hiring.[11] Furthermore, estimated worker quality adjustments for firms not expecting labor shortages are nearly identical to the baseline results, as shown in column 7.

3.2 Cross-section versus Adjustments over Time

Given that firm growth is robustly negatively related to changes in worker quality, how does the positive relationship between worker quality and firm size in the cross-section emerge?

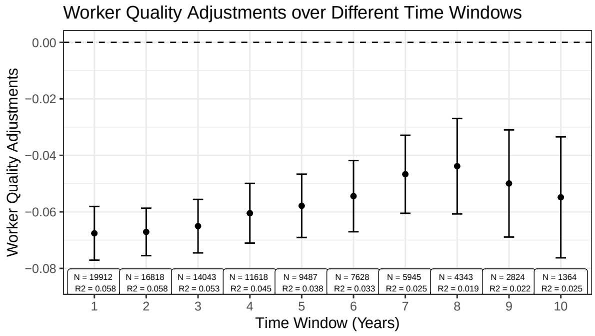
A first candidate explanation is that the year-to-year changes studied so far only reflect short-term adjustments which may not be representative for long-term adjustments. This would explain why in the short run worker quality changes are negatively correlated with firm growth, but in the long run and in the cross-section larger and more successful firms employ better workers. Figure 3a investigates this hypothesis by reporting the

[10] Assuming that low fixed effect workers have higher turnover rates and therefore lower tenure on average

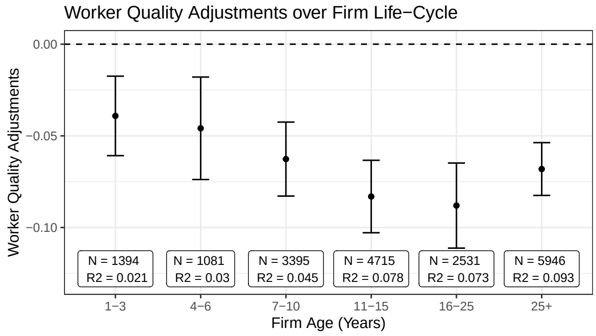
[11] This survey question comes from the IAB Establishment Panel. Firms were asked to indicate whether they expect "difficulties in finding the required specialized personnel on the labor market" and "staff shortage" in general in the next two years.

Figure 3: Firm Level Adjustments I

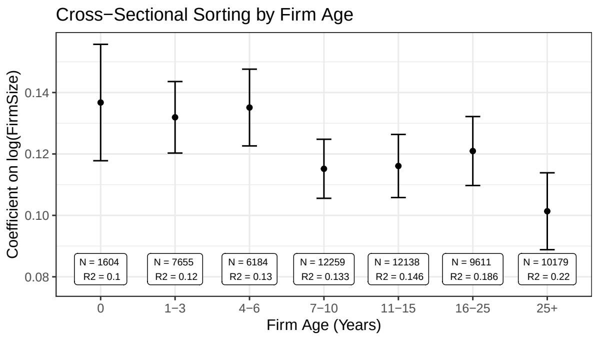
(a) Adjustments - Horizons



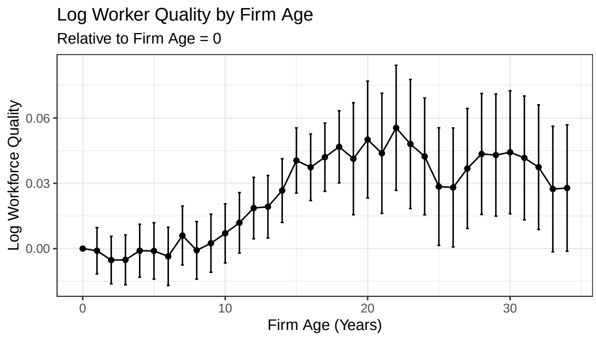
(b) Adjustments - Life Cycle



(c) Cross Sectional Sorting - Life Cycle



(d) Workforce Quality over the Life-Cycle



Notes: The top two panels show the estimated relationship between changes in worker quality and firm growth (equation (3)) for different time windows and firm ages. The bottom left panel shows the relationship between log of firm size and worker quality from equation (2) by firm age. The bottom right panel reports how average workforce quality changes over the life-cycle of firms. Except the cross-sectional regression, all samples consist of establishments with size≥20. Standard errors are clustered at the firm level. The error bars indicate 95% confidence intervals.

coefficients from regression equation (3) for horizons of up to 10 years. As the figure shows, the relationship remains negative over all horizons.

Second, I investigate whether the pattern of worker quality adjustments changes over the life cycle of firms. Perhaps the relationship is positive for younger firms, which are more volatile and change size more frequently, and once firms mature and become more stable, the relationship turns negative as documented above. As most of the firm size changes happen when firms are young, it could be that the positive correlation in the cross section emerges when firms are very young. Consequently, the potential negative relationship between firm growth and worker quality adjustments among older firms may have less impact on the cross-sectional pattern, given that these firms are already more stable and adjust employment less frequently.

Figure 3b shows the estimated relationship between firm growth and changes in worker quality from regression equation (3) over the life-cycle of the firm, i.e. conditional on firm age. Three facts emerge: First, the relationship is consistently and robustly negative over the entire life cycle of firms, at no point do expanding firms upgrade worker types. Second, the negative relationship between firm growth and changes in worker quality strengthens as firms age. The slope of firms older than 10 years is about twice as steep as that for the youngest firms in the sample. This suggests that adjustments of young firms may be influenced by additional forces, such as learning about their own type or finding the correct worker types. Figure 3d provides evidence for the latter, showing that the skill intensity increases over firms' life cycle. Last, as firms age, firm growth explains an increasing share of the variation in worker quality changes, as indicated by the increasing R^2 over the life cycle. Since any post-entry firm growth is negatively associated with changes in worker quality, the reason for the positive relationship in the cross-section between worker quality and firm size must originate from larger establishments being founded with higher-quality workers. This is confirmed in the data, as shown in Figure 3c. Over time, negative quality adjustments in response to firm dynamics weaken the cross-sectional sorting pattern. This is reflected in the declining correlation between firm size and workforce quality as firms age. The positive sorting in the cross-section can be sustained as long as ex-post shocks to firms are not too large compared to the ex-ante heterogeneity determining firm size in young establishments. This is consistent with the evidence in Sterk et al. (2021), who show that ex-ante heterogeneity accounts for a large share of the cross-sectional dispersion in employment across firms. Having established that the negative relationship is a salient and robust feature of firm dynamics, I now turn to the question of how firms adjust the quality of their workforce.

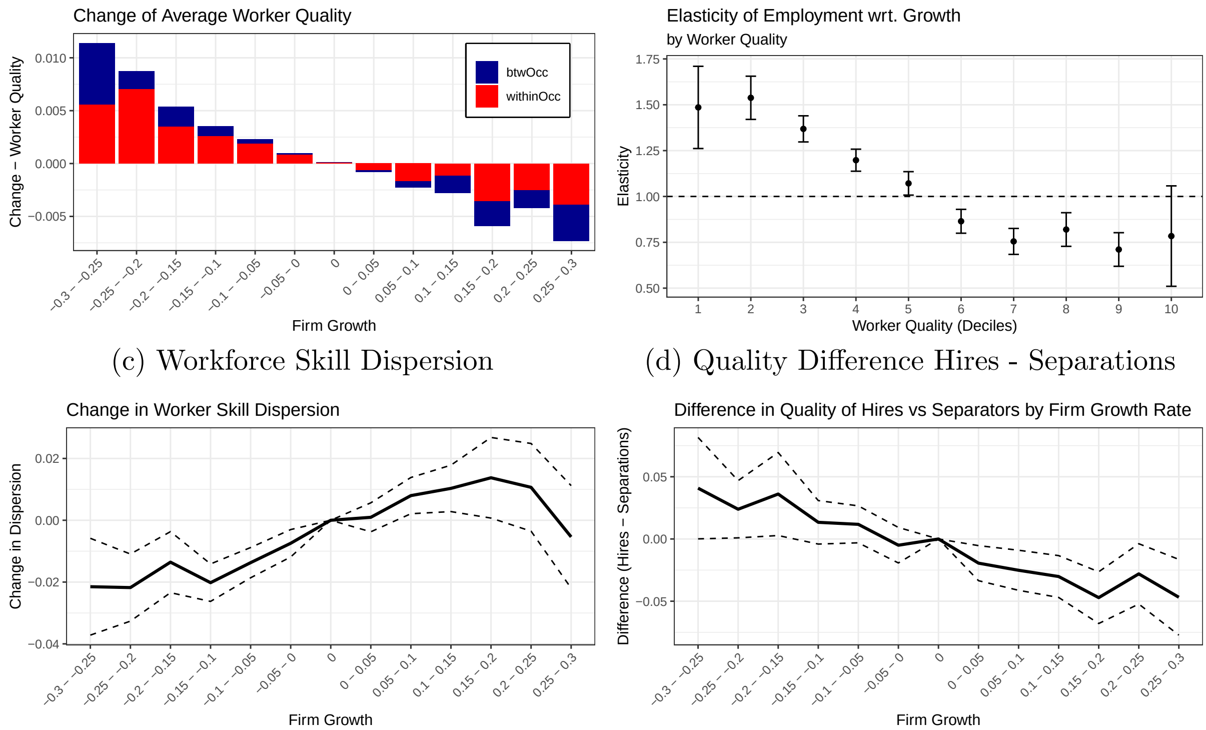
3.3 How do Firms Adjust Worker Quality?

First, I decompose the change in overall workforce quality into within- and between-occupation components:

$$\begin{aligned}\overline{WFE}_{jt} - \overline{WFE}_{jt-1} &= \frac{1}{N_{jt}} \sum_{i \in \mathcal{N}_{jt}} \left(w_i - \bar{w}_{o(i)}\right) - \frac{1}{N_{jt-1}} \sum_{i \in \mathcal{N}_{jt-1}} \left(w_i - \bar{w}_{o(i)}\right) \\ &\quad + \frac{1}{N_{jt}} \sum_{i \in \mathcal{N}_{jt}} \bar{w}_o(i) - \frac{1}{N_{jt-1}} \sum_{i \in \mathcal{N}_{jt-1}} \bar{w}_{o(i)}. \end{aligned} \tag{4}$$

The first line of equation (4) represents changes in average workforce quality driven by skill level changes within occupation o, while the second line represents changes in the occupational composition. Figure 4a plots this decomposition across the firm growth distribution. First, firms change workforce composition both within and across occupations. The average skill intensity of occupations in expanding firms declines, and within occupations the average skill level declines. Second, across the firm growth distribution, the

Figure 4: Firm Level Adjustments II



(a) Changes Within & Between Occupations (b) Adjustments by Worker Quality (c) Workforce Skill Dispersion (d) Quality Difference Hires - Separations

Notes: The top left panel shows the within and between decomposition of worker skill adjustments from equation (4). The top right panel shows the elasticity of employment of different worker skills with respect to growth, i.e. coefficient β from regression equation (5). The bottom left panel shows the relationship between changes in the standard deviation of worker types and firm growth. The bottom right figure shows the difference in the skill level of hires and separations by firm growth. All samples consist of all establishments with size$\geq$20. Standard errors are clustered at the firm level. The error bars indicate 95% confidence intervals.

majority of the change in workforce composition is driven by within-occupation adjustments, which account for roughly 80 percent of the overall change. Thus, the decline in workforce quality in expanding firms cannot be explained by expansions of lower skilled occupations, but rather by hiring lower skilled workers within occupations.

To move beyond changes in the average workforce skill level, I bin workers into deciles of the worker skill distribution and estimate the elasticity of employment for each decile with respect to firm growth using the following regression:

$$\Delta_{\%} Workers_{jt}^{k} = \alpha + \gamma_k \, growth_{jt} + \epsilon_{jt}, \tag{5}$$

where $\Delta_{\%} Workers_{jt}^{k}$ is the percentage change in the number of workers in skill decile k and $growth_{jt}$ is establishment-level employment growth. The coefficient γ_k thus measures by how much the number of workers in decile k grows within establishments when overall employment grows by one percent.

Figure 4b reports the estimated coefficients for each worker skill decile, along with 95 percent confidence intervals. The dashed line indicates a coefficient of 1, which corresponds to a one-to-one relationship between employment growth in a given decile and overall firm employment growth. For all skill deciles, the correlation with overall employment growth is positive. The more interesting aspect is that firms expand and contract more than one for one with low-skilled workers, while changes in high-skill employment are less responsive to overall firm growth. This is in contradiction to PAM, which would imply that firms expand more intensively with higher-skilled workers. This adjustment pattern also explains why the dispersion of worker skills increases with firm growth, as shown in Figure 4c.[12]

The overall change in worker quality is a result of how the quality of hires and separations differs, as shown in Figure 4d. It plots the difference between the quality of hires and separations by firm growth using equation (3) with a redefined left-hand-side variable. It shows that in shrinking firms, the quality of hires exceeds that of separations, whereas in expanding firms, the workers who separate are of higher quality. Table 3 provides an additional perspective by reporting whether firms hire or separate from better workers compared to their own historical average. This is estimated using the following regression:

$$\bar{w}^i_{jt} = \psi_j + \beta \mathbb{1}(growth_{jt} > 0) + \epsilon_{jt},\ i \in \{\text{hires}, \text{separations}\}, \tag{6}$$

where ψ_j is a firm fixed effect and $\bar{w}^i_{jt}$ is the average quality of hires or separations. The left panel shows that expanding firms hire lower-skilled workers than before, while also separating from higher-skilled workers. This pattern again contradicts PAM, which would imply that expanding firms hire better and separate from worse workers.[13]

How much do newly hired and separating workers contribute to the overall change in

[12]Here I use equation (3), but I redefine the left-hand side to the percentage change in the standard deviation of worker types within establishments.

[13]One could argue that the value on newly hired workers might be distorted because many of these workers might not stay long at their new employer. This is not the case, the average job duration of newly hired workers in my sample is 3.78 years.

Table 3

	Regression Worker Quality	Decomposition Share	
		Expanding	Contracting
Hires	-0.008 (0.001)	0.897	0.173
Separations	0.006 (0.002)	0.103	0.827

Notes: The left panel reports the regression coefficient from regressing firm growth dummy ($\mathbb{1}(growth_{it} > 0)$) on quality of hires and separations. The right two columns describe the decomposition results of equation (7). See text for details.

workforce quality? This is formally decomposed in equation (7):

$$\begin{aligned}\bar{w}_{t+1} - \bar{w}_t &= \frac{1}{N_{t+1}} \left(\sum_{i \in \mathcal{A}_t} w_i + \sum_{i \in \mathcal{H}_{t+1}} w_i - \sum_{i \in \mathcal{S}_{t+1}} w_i \right) - \bar{w}_t \\ &= \frac{H_{t+1}}{N_{t+1}} \bar{w}^H_{t+1} - \frac{S_{t+1}}{N_{t+1}} \bar{w}^S_{t+1} + \frac{N_t}{N_{t+1}} \bar{w}_t - \bar{w}_t \\ &= \frac{H_{t+1}}{N_{t+1}} \left(\bar{w}^H_{t+1} - \bar{w}_t \right) + \frac{S_{t+1}}{N_{t+1}} \left(\bar{w}_t - \bar{w}^S_{t+1} \right). \end{aligned} \tag{7}$$

The first line in the equation uses the fact that the average worker quality in $t+1$ consists of the set of workers from period t ($\mathcal{A}_t$) plus workers hired during period $t+1$ ($\mathcal{H}_{t+1}$) minus the set of workers that left the company ($\mathcal{S}_{t+1}$). The second line introduces the notation for the average quality of hires ($\bar{w}^H_t = \frac{1}{H_t} \sum_{i \in \mathcal{H}_t} w_i$) and separations ($\bar{w}^S_t = \frac{1}{S_t} \sum_{i \in \mathcal{S}_t} w_i$), where H_t and S_t denote the number of hires and separations during period t. Adding and subtracting $\bar{w}_t$ and applying the identity $N_{t+1} = N_t + H_{t+1} - S_{t+1}$ yields how much hires and separations contribute to the overall change in worker quality. Intuitively, the overall change in workforce quality is driven by how different hires and separations are compared to the workers previously employed, weighted by the number of hires and separations.

The right panel of Table 3 reports this decomposition. For expanding firms, 90 percent of the decline in average workforce quality is explained by the lower quality of newly hired workers, while the remaining 10 percent is due to the departure of higher-quality workers. In contrast, contracting firms separate from lower-skilled workers, which explains around 80 percent of the adjustment, whereas the inflow of higher-skilled workers contribute 20 percent to the increase in worker quality. Thus, both hiring and separations are in contradiction to PAM, as both contribute to the negative relationship between firm growth and changes in workforce quality.

Taken together, the evidence suggests that growing firms favor lower-quality workers, while contracting firms favor higher-type workers. Is this also reflected in the wage dy-

Table 4: Wage Changes by Firm Growth

Specification	(1)	(2)	(3)	(4)	(5)	(6)
growth	0.0085	0.0082	0.0084	0.0067	0.0090	0.0085
SE	0.0025	0.0023	0.0019	0.0022	0.0026	0.0025
growth × *skilled*	-0.0063	-0.0060	-0.0068	-0.0017	-0.0066	-0.0058
SE	0.0029	0.0027	0.0022	0.0006	0.0029	0.0031
Small firms excluded		✓				
No large wage changes			✓			
Firm FE				✓		
Firm controls					✓	
Worker FE						✓
N	3,850,992	3,955,049	3,700,564	3,955,049	3,850,992	3,850,992
Adj. R^2	0.0001	0.0001	0.0015	0.0025	0.002	-0.0336

Notes: Regression results of regression equation (8). Baseline specification conditions on workers staying at firms and on firm size $\geq$ 20. Standard errors are clustered at the firm level, except in the case of worker fixed effects, where the clustering is at the worker level. No large wage changes excludes wage changes ±20%. Firm controls include a 2 digit sector code and firm age.

namics of workers staying at their establishments? To answer this, I estimate whether low- and high-skilled workers staying at their employers experience differential wage growth:

$$\Delta_{\%}w_{it} = \alpha + \gamma_1 growth_{j(i,t),t} + \gamma_2 skilled_i \times growth_{j(i,t),t} + \beta_1 skilled_i + \beta_2 X_{it} + \epsilon_{it}, \quad (8)$$

where $\Delta_{\%}w_{it}$ is the percentage change of the wage of individual i in calendar year t, $skilled_i$ is an indicator for worker skills above the median, $growth_{j(i,t),t}$ denotes the growth rate of the establishment j employing individual i in year t, and X_{it} are potential worker and firm controls. γ_1 captures the wage elasticity with respect to firm growth for the reference group of low-skilled workers, whereas γ_2 measures the differential wage growth of high-skilled workers. Table 4 reports the coefficients γ_1 and γ_2 from this regression. As expected, wage growth is positively correlated with firm growth, as the positive coefficients on γ_1 show. But which workers benefit more from firm growth? The consistently negative estimates of γ_2 show that the wages of high-skill workers are less elastic with respect to firm growth than those of low-skill workers. In other words, low-skill workers gain more relative to high-skill workers in expanding firms, while the opposite holds in contracting firms. This again is consistent with the notion that expanding firms value low skilled workers more, whereas high-skilled workers become relatively more valuable for contracting firms. In the next section, I turn to the question of which model can rationalize these findings.

4 Model

The conceptual framework showed that a one-dimensional sorting model cannot explain the empirical facts documented in the previous section. The literature on firm dynamics emphasizes the need to model firms with both persistent and transitory productivity components, see e.g., Sterk et al. (2021).[14] Accordingly, I depart from most of the sorting literature and assume that firms are characterized by a persistent productivity component z, which remains constant over time, and a transitory component y. Firms face idiosyncratic shocks to their time-varying productivity with the transition rate given by $p(y'|y)$.[15] A priori, there is no reason to assume that these two firm productivity components interact the same way with worker skill x. For example, investments in modern capital equipment, which tend to be persistent, are typically thought to be complementary to worker skills (e.g. Krusell et al. (2000)). In contrast, a good management team, which temporarily increases productivity, or investments in new AI technology, which tend to depreciate quickly could be substitutes with worker skills. In order to capture these possibilities, I assume that when a worker of quality x and a firm with persistent productivity z and transitory productivity y match together, they produce according to the production function $f(x, y, z)$, where $f(x, y, z)$ is twice continuously differentiable with respect to all of its arguments, with $f_x(x, y, z) > 0$, $f_y(x, y, z) > 0$ and $f_z(x, y, z) > 0$. Thus, high types always have an absolute advantage over low types in production. At which firms high-skilled workers have a higher marginal productivity depends on the two cross derivatives $f_{xy}(x, y, z)$ and $f_{xz}(x, y, z)$. The sign and strength of the cross derivatives $f_{xy}(x, y, z)$ and $f_{xz}(x, y, z)$ are left unrestricted and will be estimated. I further assume that $f_{yz}(x, y, z) = 0$, i.e., there is no interaction between the different firm productivity components. Under this assumption, firm-level shocks to y affect workers' marginal product only through interactions with x. This assumption simplifies the identification of $f_{xy}(x, y, z)$ and $f_{xz}(x, y, z)$, the key objects of interest.

To provide intuition, consider the first order condition for worker types from the conceptual framework, now with persistent and transitory firm productivity:

$$f_x(x, y, z) = w'(x). \tag{9}$$

Suppose a firm experiences a positive idiosyncratic shock to its transitory productivity y. Since the wage schedule remains unchanged, whether the firm prefers to match with higher- or lower-skill workers after the shock depends on the sign of $f_{xy}(x, y, z)$. If

[14] See Syverson (2011) for a review about the determinants of firm productivity.

[15] see e.g. Eeckhout and Kircher (2011), Hagedorn et al. (2017), Bagger and Lentz (2018). Lindenlaub (2017) and Lise and Postel-Vinay (2020) provide notions of sorting based on multidimensional characteristics.

$f_{xy}(x, y, z) > 0$, the marginal return to worker skill increases with y, and the firm will upgrade the skill level of its workforce. Conversely, if $f_{xy}(x, y, z) < 0$, the marginal return to skill decreases, and the firm will prefer lower-type workers at the margin. Because z is assumed to be constant, this logic holds regardless of the firm's level of z and $f_{xz}(x, y, z)$. This feature gives the model the flexibility to reconcile the negative within-firm adjustments over time with the positive sorting observed in the cross section.[16] Adjustments over time are governed by the sign of $f_{xy}(x, y, z)$, while the sign of $f_{xz}(x, y, z)$ determines the cross-sectional sorting pattern. Although the sign of $f_{xy}(x, y, z)$ determines the direction of workforce adjustment after shocks, the magnitude of the adjustment depends on z. If complementarities between x and z are strong, and a given worker type is especially well-matched to z, then shocks to y will not affect the match value much, and changes to the workforce composition will be muted. In addition, the adjustment pattern also depends on the frequency of shocks, search frictions in the economy, but also on the distribution of firm and worker productivities, thus these will be estimated together.[17]

The empirical results are not in terms of the unobserved productivities y, z but rather firm size. To confront the model with the data, I develop a tractable multi-worker firm setup with two-sided heterogeneity in a frictional labor market. Time is discrete and the economy is populated by a unit mass of heterogeneous workers and a mass M of firms. Workers and vacant jobs match together in a frictional labor market to produce according to $f(x, y, z)$. Worker and firm productivities are distributed on the unit interval [0,1], with the stationary distributions of worker and firm types given by the probability distribution functions $\zeta_x(x)$ and $\zeta_f(y, z)$.

Firms consist of a continuum of identical jobs and produce with a linear production technology. Thus the total output of firm j with productivity (y, z) is given by the integral over the distribution $\psi_j(x)$ of worker types matched to the firm:

$$F_j(y, z) = \int f(x, y, z) d\psi_j(x). \tag{10}$$

The assumption of linear production rules out complementarities between workers within a firm and allows me to study the matching decision at the job level.[18] Firms can create new jobs v^N subject to a convex adjustment cost function $c(v^N)$, with $c'(v^N) > 0$ and

[16] Patel (2024) shows that following investments in automation and robotics, firms expand employment and substitute middle-skilled workers with both higher- and lower-skilled ones. My model is not well suited to study job polarization, as workforce adjustments in my framework are always unidirectional.

[17] Lindenlaub (2017) shows that in a multi-dimensional sorting setup, the sorting pattern could in theory also depend on the joint distribution of firm and worker attributes. I experimented with various worker and firm distributions, and the estimated sorting patterns remained qualitatively unchanged.

[18] Introducing such complementarities would render this model intractable as the surplus of each match would depend on other matches within the firm.

$c''(v^N) > 0$. These jobs are long-lived and are costlessly maintained, but expire at rate d per period. As newly created jobs start out vacant, firms will create new jobs until the marginal cost of establishing a new job is equal to the value of a vacant job $V(y, z)$, which will be described in detail later. Inverting this relationship yields the newly created jobs $v^N(y, z)$ for each firm of type (y, z):

$$c'(v^N) = V(y, z) \Leftrightarrow v^N(y, z) = c'^{-1}\left(V(y, z)\right). \tag{11}$$

Because of the linear job destruction rate and the convex job creation cost, sustaining larger firms becomes increasingly difficult. More productive firms value vacancies higher, create more jobs, and grow larger. In the absence of shocks, firms would converge to a certain steady-state size determined by (y, z).

The timing of the model is as follows: First, production takes place. Then a fraction d of jobs are exogenously destroyed, the idiosyncratic productivity shock is revealed, which can trigger endogenous separations, and new jobs are created. Workers who lost their job in a given period are not allowed to search again in the same period. After the separation stage, the search and matching takes place, which I describe next.

At the search and matching stage, workers and jobs meet and decide whether to form a match. Workers search both on and off the job: unemployed workers sample from the distribution of vacant jobs at rate λ_w, while employed workers do so at rate λ_e. Each vacant job samples from the distribution of workers at rate λ_f, who may be either unemployed or employed at another firm. Importantly, meetings occur at the job level, not the firm level. Because production is linear in jobs, jobs within a firm do not interact. This assumption allows me to model the acceptance decision entirely at the job level, independently of the firm's internal allocation of filled and vacant positions. The decision to match is therefore driven by fundamental scarcity: both workers and jobs face an opportunity cost when forming a match. Each party must decide whether to accept the current match or wait for a potentially better one in the future.[19] As a result, sorting arises endogenously in equilibrium, as in Shimer and Smith (2000), but the equilibrium job distribution is pinned down by convex job creation costs and linear job destruction.

Let u denote the mass of unemployed workers, e^s the number of employed workers at the matching stage, and v the total mass of vacant jobs. Conditional on a meeting, the probability that a vacancy contacts an unemployed worker is given by the number of

[19] One might worry that linear production combined with convex job creation would eliminate this scarcity. This is not the case. Due to the assumed timing, firms cannot create additional jobs in response to meetings.

searching unemployed workers divided by all searching workers:

$$p^u = \frac{\lambda_w u}{\lambda_w u + \lambda_e e^s}. \tag{12}$$

Since the total number of meetings must be equal on both sides of the market, the following condition holds:

$$\lambda_f v = \lambda_w u + \lambda_e e^s. \tag{13}$$

4.1 Wage Negotiation

When a job meets a suitable candidate, the two parties agree on a piece rate which is only renegotiated under certain conditions. The assumed wage-setting mechanism, combined with linear utility ensure bilateral efficiency. As a result, any match that generates a positive surplus will be formed and sustained. The current piece rate affects only the division of the surplus, not the surplus itself.

I denote the value of an unemployed worker of type x as $U(x)$. The value of an employed worker x matched together with a firm of type (y, z) and negotiated piece rate ϕ is $W(x, y, z, \phi)$. The value of a vacant job to a firm of type y is denoted as $V(y, z)$, whereas the value of a job occupied by a worker of type x with a piece rate ϕ is $J(x, y, z, \phi)$. The value functions are presented below in equations (21)-(24). The surplus of a match is consequently defined as

$$S(x, y, z) = W(x, y, z, w) - U(x) + J(x, y, z, w) - V(y, z). \tag{14}$$

Piece rates are negotiated at the beginning of each employment spell and might be renegotiated after outside offers or productivity shocks. At the start of the match, the worker's share of the surplus depends on whether they are hired from unemployment or poached from another firm. When a worker is hired from unemployment, the piece rate $\phi^U(x, y, z)$ is set according to Nash bargaining, with the worker's bargaining power α:

$$\phi^U(x, y, z) \quad : \quad W(x, y, z, \phi) - U(x) = \alpha S(x, y, z). \tag{15}$$

As in Postel-Vinay and Robin (2002), Dey and Flinn (2005), and Cahuc et al. (2006), when an employed worker meets another firm, the two firms engage in Bertrand competition. If the surplus of the poaching firm of type $(\tilde{y}, \tilde{z})$ is lower than the surplus currently appropriated by the worker, that is, if $S(x, \tilde{y}, \tilde{z}) < W(x, y, z, \phi) - U(x)$, then the meeting has no effect on the current piece rate, and the worker stays at their current employer. If the surplus offered is at least as high as the worker's current surplus, Bertrand competition drives the piece rate up to the point where the worker obtains the entire surplus from the lower surplus firm $S(x, y, z)$. The worker moves to (or stays with) the higher-surplus firm $(\tilde{y}, \tilde{z})$, and the new piece rate $\phi^E(x, y, z, \tilde{y}, \tilde{z})$ is set such that:

$$\phi^E(x, y, z, \tilde{y}, \tilde{z}) \quad : \quad W(x, \tilde{y}, \tilde{z}, \phi) - U(x) = S(x, y, z). \tag{16}$$

I denote the set of poaching firms that trigger a job-to-job transition as $\Upsilon^{EE}(x, y, z)$, and $\Upsilon^{BC}(x, y, z, \phi)$ the set that trigger a renegotiation without a job-to-job transition. Formally these are defined as

$$\Upsilon^{EE}(x, y, z) \;=\; \{\tilde{y}, \tilde{z} : S(x, \tilde{y}, \tilde{z}) \geq S(x, y, z)\} \tag{17}$$

$$\Upsilon^{BC}(x, y, z, \phi) \;=\; \{\tilde{y}, \tilde{z} : W(x, y, z, \phi) - U(x) < S(x, \tilde{y}, \tilde{z}) < S(x, y, z)\}. \tag{18}$$

After productivity shocks, I assume that piece rates are renegotiated if either the worker's value falls below their outside option $(W(x, y, z, \phi) - U(x) < 0)$ or the firm's value falls below that of a vacancy $(J(x, y, z, \phi) - V(y, z) < 0)$. The rationale is that renegotiation occurs only when one party has a credible threat to leave the match (MacLeod and Malcomson, 1993; Postel-Vinay and Turon, 2010). Intuitively, the side requesting renegotiation is in a weaker bargaining position than the side preferring the current piece rate. If a productivity shock pushes the value of a worker below their participation threshold, the firm extracts the full surplus and the piece rate $\phi^{NW}(x, y, z)$ is set such that

$$\phi^{NW}(x, y, z) \quad : \quad W(x, y, z, \phi) - U(x) = 0. \tag{19}$$

Conversely, if the current piece rate becomes unsustainable for the firm, the worker has the better bargaining position and receives the full surplus:

$$\phi^{NF}(x, y, z) \quad : \quad W(x, y, z, \phi) - U(x) = S(x, y, z). \tag{20}$$

This wage setting has two appealing features. First, wages feature partial, but limited pass-through of productivity shocks to wages, consistent with recent evidence (see for example Haefke et al. (2013), Balke and Lamadon (2022), Kroft et al. (2023)). Second, it prevents inefficient separations that would otherwise occur even when both parties have an incentive to renegotiate.

4.2 Matching Sets and Value Functions

The wage setting and the assumption of transferable utility ensures that acceptance decisions jointly maximize match surplus. Thus, agents are willing to match together if the match generates a positive surplus, and in case of job-to-job transitions, the prospective surplus is higher than the current one. This implies that the matching set for unemployed workers consists of all productivity combinations that yield a positive surplus, i.e $S(x, y, z) \geq 0$. A job-to-job transition occurs if a poaching firm $(\tilde{y}, \tilde{z})$ offers a higher surplus than the current employer (y, z), i.e. $S(x, \tilde{y}, \tilde{z}) \geq S(x, y, z)$. Equation (21) presents the value of a vacancy at the beginning of the period:

$$
\begin{aligned}
V(y,z) = \beta(1-d) \int_{y'} \Bigg\{ V(y',z) + \lambda_f \Bigg(p^u \int_x (1-\alpha) S(x,y',z)^+ \frac{\mu_x(x)}{u} dx + \\
+ \ (1-p^u) \int_{\tilde{z}} \int_{\tilde{y}} \int_x (S(x,y',z) - S(x,\tilde{y},\tilde{z}))^+ \frac{\psi^S(x,\tilde{y},\tilde{z})}{e^S} dx d\tilde{y} d\tilde{z} \Bigg) \Bigg\} p(y'|y) dy', \quad (21)
\end{aligned}
$$

where $x^+ = \max\{x, 0\}$. Jobs are destroyed with probability d, thus the effective discount rate is $\beta(1-d)$. The vacant job contacts an applicant with probability λ_f, who may be either employed or unemployed. The vacancy finds a suitable match if either the unemployed worker's type x is inside the matching bands ($S(x, y, z) \geq 0$) or if the surplus at the poaching firm is higher than at the current employer ($S(x, \tilde{y}, \tilde{z}) \geq S(x, y, z)$). In case of a successful match with an unemployed, the firm receives a fraction $(1-\alpha)$ of the surplus, whereas in the case of poaching, the firm obtains the surplus $S(x, y', z)$ minus the surplus which the worker generated at their old job $S(x, \tilde{y}, \tilde{z})$. The probability of meeting an unemployed worker of type x is equal to the probability of meeting an unemployed p_u times the probability of the unemployed being of type x. The latter is given by $\mu_x(x)$, the measure of unemployed of type x, divided by the total number of unemployed u. Similarly, the probability of an applicant being of type x employed for a firm $(\tilde{y}, \tilde{z})$ is given by the mass of employed types at the search stage $\psi^S(x, \tilde{y}, \tilde{z})$ divided by the total mass of employed workers at the search stage e^S. As will be described in more detail later, $\psi^S(x, \tilde{y}, \tilde{z})$ itself depends on the matching decision of workers and jobs and the job creation of all firms in the economy. Thus, the solution in this model amounts to a fixed point problem in solving the value functions together with the endogenous distributions of matched and unmatched agents.

Equation (22) represents the value of a filled job to a firm at the beginning of the production stage.

$$
\begin{aligned}
J(x,y,z,\phi) = (1-\phi)f(x,y,z) + \beta(1-d)\int_{y'} \Big\{ & V(y',z) + \mathbb{1}(S(x,y',z) \geq 0) \\
\times \int_{\tilde{z}}\int_{\tilde{y}} \Big\{ & \lambda_e \mathbb{1}\left((\tilde{y},\tilde{z}) \in \Upsilon^{BC}(x,y',z,\phi)\right)(S(x,y',z) - S(x,\tilde{y},\tilde{z})) \\
+ (1-\lambda_e & \left(\mathbb{1}(\tilde{y},\tilde{z} \in \Upsilon^{BC}(x,y',z,\phi) \cup (\tilde{y},\tilde{z}) \in \Upsilon^{EE}(x,y',z)\right) \\
\times \min\{\max\{ & J(x,y',z,\phi) - V(y',z), 0\}, S(x,y',z)\}\Big\} \times \frac{\mu_F(\tilde{y},\tilde{z})}{V} d\tilde{y}\Big\} p(y'|y)dy'.
\end{aligned} \tag{22}
$$

It consists of the flow output net of wages $(1-\phi)f(x,y,z)$ plus the discounted continuation value. Since jobs are destroyed with probability d, the effective discount factor is $\beta(1-d)$. The continuation value depends on the future value of the firm's transient productivity y', which conditional on today's productivity y, occurs with probability $p(y'|y)$. If the surplus becomes negative or the worker is poached, the firm is left with a vacancy, valued with $V(y',z)$. If the match surplus is positive after the productivity shock, several events affect the continuation value. First, a worker might meet a job of type $(\tilde{y},\tilde{z})$ from the distribution of vacant jobs $\mu_F(\tilde{y},\tilde{z})$ with intensity λ_e. The second line in equation (22) represents the case where the poaching firm's surplus is insufficient to poach the worker, but high enough to trigger a renegotiation, i.e. $(\tilde{y},\tilde{z}) \in \Upsilon^{BC}(x,y',z,\phi)$. In this case the current firm matches the surplus of the poaching firm, and its continuation value is $S(x,y',z) - S(x,\tilde{y},\tilde{z})$. If no poaching firm is contacted, or if none triggers a renegotiation or transition, the worker remains with the current employer. In this case, the final line captures renegotiation triggered by a productivity shock that violates the participation constraint of either party. If the renegotiation is demanded by the worker, the firm extracts the full surplus $S(x,y',z)$. If the firm requires the negotiation, the worker receives the full surplus, thus this case does not feature in the formula above. If no party has a credible threat to exit, the piece rate remains unchanged and the firm receives $J(x,y',z,\phi)$.

The worker's value functions are the mirror image of the firms' problems and are presented in equations (23) and (24). The value of unemployment (equation 23) consists of the flow value $b(x)$ during unemployment, which may depend on worker type x. Unemployed workers contact vacancies drawn from the distribution $\mu_F(y,z)$ at rate λ_w. If the surplus is positive, a match forms and the worker receives a share α of the surplus.

$$U(x) = b(x) + \beta \left(U(x) + \lambda_w \int_z \int_y \alpha S(x, y, z)^+ \frac{\mu_F(y, z)}{v} dydz \right), \tag{23}$$

$$\begin{aligned} W(x, y, z, \phi) = \phi f(x, z, y) + \beta \Bigg(U(x) + (1-d) \int_{y'} \Bigg\{ &\mathbb{1}(S(x, y', z) \geq 0) \\ \times \int_{\tilde{z}} \int_{\tilde{y}} \Bigg\{ &\lambda_e \left[\mathbb{1}\left((\tilde{y}, \tilde{z}) \in \Upsilon^{EE}(x, y', z) \right) S(x, y', z) \right. \\ &+ \left. \mathbb{1}\left((\tilde{y}, \tilde{z}) \in \Upsilon^{BC}(x, y', z, \phi) \right) S(x, \tilde{y}, \tilde{z}) \right] \\ &+ \left(1 - \lambda_e \mathbb{1}((\tilde{y}, \tilde{z}) \in \Upsilon^{EE}(x, y', z) \cup \Upsilon^{BC}(x, y', z)) \right) \\ &\times \min\{\max\{W(x, y', z, \phi) - U(x), 0\}, S(x, y', z)\} \Bigg\} \frac{\mu_F(\tilde{y}, \tilde{z})}{V} d\tilde{y} d\tilde{z} \Bigg\} p(y'|y) dy' \Bigg). \end{aligned} \tag{24}$$

I show in Appendix D that the surplus can be expressed as

$$\begin{aligned} S(x, y, z) = &f(x, y, z) - b(x) + \beta(1-d) \int_{y'} S(x, y', z)^+ p(y'|y) dy' \\ &- \beta \alpha \lambda_w \int_y \int_z S(x, y, z)^+ \frac{\mu_F(y, z)}{v} dz dy \\ &- \beta(1-d)\lambda_f \int_{y'} \Bigg(p^u \int_x (1-\alpha) S(x, y', z)^+ \frac{\mu_x(x)}{u} dx \\ &+ (1-p^u) \int_{\tilde{z}} \int_{\tilde{y}} \int_x (S(x, y', z) - S(x, \tilde{y}, \tilde{z}))^+ \frac{\psi^S(x, \tilde{y}, \tilde{z})}{e^s} dx d\tilde{y} d\tilde{z} \Bigg) p(y'|y) dy'. \end{aligned} \tag{25}$$

The first line represents the flow output of the match plus its continuation value, whereas the remaining terms originate from the outside options $V(y)$ and $U(x)$. The continuation value is independent of poaching events, as Bertrand competition implies that in job-to-job transitions, the worker appropriates the current surplus at the new job.

Three distributions emerge endogenously in my model. In a stationary equilibrium, the in- and outflows of the distributions of vacancies $\mu_F(y, z)$, unemployed workers $\mu_x(x)$ and employed workers across firm types $\psi(x, y, z)$ must balance each other. Equation (26) first presents the law of motion for the distribution of vacant jobs:

$$
\begin{aligned}
\mu_F(y,z) = \int_{y'} \Bigg(& (1-d)v^N(y',z)M\zeta(y',z) + (1-\lambda_f) + \lambda_f \left(p^u \int_x \mathbb{1}(S(x,y',z)<0)) \frac{\mu_x(x)}{u} dx \right. \\
& + \left. \left. (1-p^u) \int_{\tilde{z}} \int_{\tilde{y}} \int_x \mathbb{1}(S(x,y',z) < S(x,\tilde{y},\tilde{z})) \frac{\psi^S(x,\tilde{y},\tilde{z})}{e^S} dx d\tilde{y} d\tilde{z} \right) \right) p(y|y')\mu_y(y')dy' \\
& + \lambda_e \int_{\tilde{z}} \int_{\tilde{y}} \int_x \mathbb{1}\left(S(x,\tilde{y},\tilde{z}) \geq S(x,y,z)\right) \frac{\mu_F(\tilde{y},\tilde{z})}{v} \psi^S(x,y,z) dx d\tilde{y} d\tilde{z} \\
& + \int_{y'} \int_x (1-d)\mathbb{1}(S(x,y',z)<0))\psi(x,y',z)p(y|y')dxdy'. \qquad (26)
\end{aligned}
$$

The first two lines comprise the newly created jobs and the unfilled jobs carried over from the last period. The total mass of new jobs of type (y', z) created equals the number of jobs created by each firm $v^N(y', z)$ times the total mass of firms of the appropriate type $M\zeta(y', z)$. Since jobs are created at the beginning of the period, they are subject to job destruction and firm productivity shocks. The remaining terms in line one and two describe the events of unsuccessful search. The third and fourth line represent the inflow from previously filled jobs through endogenous separations.

Equation (27) describes the law of motion for the distribution of worker types across firm types at the production stage $\psi(x, y, z)$.

$$
\begin{aligned}
\psi(x,y,z) = & \lambda_w \mathbb{1}\left(S(x,y,z) \geq 0\right) \frac{\mu_F(y,z)}{v} \mu_x(x) \\
& + \lambda_e \int_{\tilde{z}} \int_{\tilde{y}} \mathbb{1}\left((y,z) \in \Upsilon^{EE}(x,\tilde{y},\tilde{z})\right) \psi^s(x,y,z) d\tilde{y} d\tilde{z} \frac{\mu_F(y,z)}{v} \\
& + \left(1 - \lambda_e \int_{\tilde{z}} \int_{\tilde{y}} \mathbb{1}\left((\tilde{y},\tilde{z}) \notin \Upsilon^{EE}(x,y,z)\right) \frac{\mu_F(\tilde{y},\tilde{z})}{v} d\tilde{y} d\tilde{z} \right) \psi^S(x,y,z), \qquad (27) \\
\psi^s(x,y,z) = & (1-d)\mathbb{1}(S(x,y,z) \geq 0) \int_{y'} \psi(x,y',z)p(y|y')d\tilde{y}, \qquad (28)
\end{aligned}
$$

where $\psi^s(x, y)$ is the distribution of matches at the search stage. The first and second lines describe the inflow through newly filled jobs from unemployment and from poaching, respectively. The third line captures retained jobs from the previous period. After production, firms receive productivity shocks and separate from the workers that now lie outside of the matching sets. In addition, a fraction d of jobs are destroyed. This process can be read off equation (28). All the remaining workers engage in on-the-job search.

The distribution of unemployed workers can be readily computed as the residual between the distribution of workers $\zeta_x(x)$ and distribution of employed workers $\psi(x, y, z)$:

$$
\mu_x(x) \quad = \quad \zeta_x(x) - \int_z \int_y \psi(x,y,z) dydz. \qquad (29)
$$

Table 5: Functional forms

Worker distribution	Log-normal(μ_x, σ_x)
Firm type z distribution	Beta(a_β,b_β)
Production function	$\nu_y \left(x^{1/\rho_y} + y^{1/\rho_y}\right)^{\rho_y} + \nu_z \left(x^{1/\rho_z} + z^{1/\rho_z}\right)^{\rho_z}$
Flow utility unemployed	$b(x) = f(x, 0, 0)$
Job creation cost function	$c_0 \left(\frac{v}{c_1}\right)^{c_1}$
Firm shocks	$y_{t+1} = \varphi y_t + \epsilon;\;\; \epsilon \sim N(0, \sigma_y^2)$

Notes: Log-normal distribution is truncated to [0,1].

5 Identification

The estimation follows an indirect inference approach and uses the German social security data described in section 3. First, I choose a set of auxiliary statistics from the data that identify the parameters of the model. Then, I estimate the parameters by minimizing the distance between these empirical moments and their model-simulated counterparts. This section describes the choice of functional forms and targeted moments and justifies their roles in the identification of the sorting pattern.

5.1 Functional Form Assumptions

The model is estimated at a monthly frequency. The functional form assumptions are summarized in Table 5. I use the sum of two CES production functions of the form $f(x, y, z) = \nu_y \left(x^{1/\rho_y} + y^{1/\rho_y}\right)^{\rho_y} + \nu_z \left(x^{1/\rho_z} + z^{1/\rho_z}\right)^{\rho_z}$, where ν_y and ν_z are normalizing constants, such that both parts are bounded between 0 and 1. ρ_y and ρ_z determine the cross partials $f_{xy}(x, y, z)$ and $f_{xz}(x, y, z)$, and thus the sorting patterns in this economy. If $\rho_y > 1$ (or $\rho_z > 1$), the production function is supermodular, implying that high-type workers are relatively more productive at firms with high transitory (or high persistent) productivity.

The worker distribution is assumed to be log-normal, with location parameter μ_x and scale parameter σ_x, truncated to the interval [0,1]. I assume that higher-type workers are more productive in home production. $b(x)$ is set to the lowest possible market productivity, i.e., $b(x) = f(x, 0, 0)$. The invariant firm type distribution is assumed to be beta, with shape parameters a_β and b_β. Firm productivity follows an AR(1) process with persistence parameter φ and the variance of the innovation σ_y^2. The distribution of jobs across firm productivity types is primarily governed by the job creation cost function. I assume the standard form $c(v) = c_0 v^{c_1}/c_1$, where c_1 determines the convexity and c_0 the scale of the job creation costs.[20]

Six parameters are preassigned. There is a unit mass of workers in the economy. The mass of firms M is set to match the firm-to-worker ratio in the German social security data. I set the discount rate to 0.995, which implies a yearly discount rate of about 6 percent. The bargaining power of unemployed workers is set to 0.2, which is similar to the values used in Bagger and Lentz (2018) and Lise et al. (2016). The shape parameter α_β is fixed at one. The remaining parameters are estimated by minimizing the distance between auxiliary statistics from the German social security data and those generated by the model.

I discretize the model with 20 worker types and 160 firm types.[21] First, I obtain the acceptance sets by solving for a fixed point in the surplus function $S(x, y, z)$ and the endogenous distributions $\psi(x, y, z)$, $\mu_F(y, z)$, and $\mu_x(x)$. I then simulate 534,502 workers and 5,587 firms over 18 years to construct a panel dataset with the same number of person-year and firm-year observations as in the German social security data. Appendix E describes the numerical implementation in detail. I compute a set of auxiliary statistics on the model-simulated data the same way as on the German social security data. Table 6 summarizes the target statistics, comparing them with those generated by the model. None of the parameters have a one-to-one relationship to the auxiliary statistics, but I provide a heuristic explanation of the underlying identification in the next subsections.

5.2 Identification of Parameters

The key identification challenge in my model is to recover the complementarities in production and thus the sorting pattern. Before I discuss these, I turn to the identification of the other parameters, which are more standard. I target the total hire rate, the unemployment, and job-to-job transition rate.[22] I compute the unemployment rate as the fraction of workers not employed each year on the reference day of December 31. I count every transition from one firm to another with an intermittent spell of non-employment shorter than 31 days as a job-to-job transition. These three moments inform the meeting rates for employed and unemployed workers, λ_e and λ_w, and the job destruction rate d.

To recover the worker productivity distribution, I leverage the insight that in models where higher-type workers have an absolute advantage in production, their utility level, and thus the discounted sum of all per-period earnings must be increasing with type.[23]

[20] Bagger and Lentz (2018), Coşar et al. (2016) and Merz and Yashiv (2007) among many others use this functional form

[21] 10 grid points along the z dimension, and 16 grid points along the y dimension

[22] In computing labor market transitions, I exclude temporary layoffs where the non-employment spell is shorter than 31 days and the worker joins the same firm again.

[23] Consider two types of agents with $x_1 < x_2$. Agent x_2 can achieve at least the utility level of x_1,

Therefore, higher-type workers will achieve higher average lifetime earnings, which motivates the use of worker fixed effects as a worker skill measure.[24] Per-period wages in the model depend not only on worker ability, but also on firm productivity and the worker's bargaining position at the time of hire. Over long time periods, these effects are washed out, but over shorter time periods, the worker fixed effect could be influenced by these factors. To address this, I use the estimated model to quantify how well the worker fixed effects recover worker types given the estimated labor mobility patterns and the 18 year horizon of the data. Since my model does not feature any life-cycle component, I follow Card et al. (2013) and Hagedorn et al. (2017) and compute wages after filtering out an education-specific age profile and year effects.[25] Further details are described in Appendix B. I recover the mean and standard deviation of the worker productivity distribution by targeting the mean and standard deviation of the empirical worker fixed effect distribution. This will identify the scale and shape parameters of the worker type distribution μ_x and σ_x.[26]

The remaining target moments primarily identify the parameters related to firms. In order to map unobservable changes in productivity to observable outcomes, I use the fact that firm employment expands following positive shocks, whereas it contracts after negative shocks. Since more productive firms value vacant jobs more[27], more productive firms create more jobs and grow larger.[28]

The parameters that affect the growth rate and establishment size distribution are c_0, c_1 of the job creation function, φ, σ_y that govern the persistence and dispersion of the productivity process, as well as the shape parameter of the invariant firm productivity

because x_2 could follow the acceptance and wage strategies of x_1. If all counter-parties will accept to match with her under these conditions, she will receive at least the value of the lower type. If firms are willing to hire x_1 agents, they will also be willing to hire x_2 agents under the same conditions since these agents produce more and yield strictly higher profits. And if workers are willing to match with x_1 firms, they will also be willing to match with x_2 because wages and separation probabilities are the same by construction. Thus, x_2 agents will always have weakly higher payoffs as x_1 agents.

[24]Strictly speaking, higher lifetime earnings do not necessarily imply higher utility if high-type workers spend more time unemployed. To address this, I calculate two alternative specifications: (1) using actual unemployment benefits as the flow value of non-employment; and (2) adding a 20% premium to these benefits to account for non-monetary components such as home production or leisure. The correlation between these alternatives and the baseline fixed-effect measure ranges from 0.9955 to 0.999. Since unemployment durations are short on average, the results are insensitive to the specification. I proceed with the baseline for simplicity.

[25]I compute the wage residual controlling for year effects and a cubic polynomial of age fully interacted with educational attainment.

[26]I normalize both the empirical worker quality measure and the one obtained from the model to [0,1].

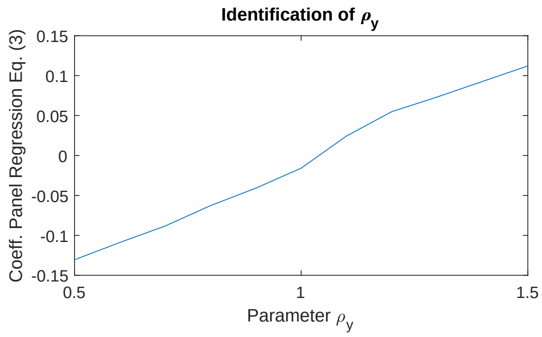
[27]The proof behind this follows the same logic as for workers. Lower type firms can always imitate the matching strategies of lower type firms, and thus obtain at least the same utility, as they have an absolute advantage in production

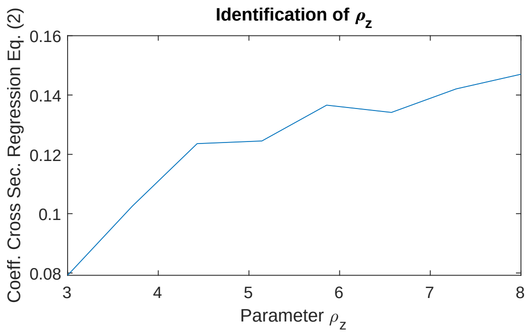
[28]It is theoretically possible that the job filling probability is lower for high-type firms as they might be "pickier". In practice, as high-type firms have higher opportunity costs of waiting, the matching bands are wider and thus high types firms will have higher job filling rates.

Figure 5: Identification of ρ_y and ρ_z

(a) Workforce Skill Adjustments

(b) Firm Size vs. Workforce Quality





Notes: The figure shows the estimated coefficient from the panel regression equation (3) for different values of ρ_y on model generated data (left panel), and the coefficient from the cross section regression equation (2) for different values of ρ_z (right panel). The rest of the parameters are held constant at the values reported in Table 7.

distribution b_β. To identify these parameters, I target the standard deviation of establishment growth rates, the autocorrelation of establishment size at lag one and ten years, five points of the firm size distribution, the standard deviation of the average worker fixed effect at the firm level, and the job filling rate. I compute the empirical job filling rate from the average time to fill a vacancy provided by the Institute for Employment Research, averaged over all available years.[29]

5.3 Identifying the Complementarity Parameters ρ_y and ρ_z

The only remaining parameters to be identified are the complementarity parameters ρ_y and ρ_z. The identification of production complementarities in search models has proven to be challenging (Eeckhout and Kircher, 2011). In my model, this difficulty is further compounded by two features: first, firms are characterized by multidimensional productivity; second, they are subject to productivity shocks. These productivity shocks provide a source of variation to study how firms adjust their workforce composition, which is governed by ρ_y— the complementarity between worker skill and transitory firm productivity. Firms respond to productivity shocks by adjusting both their scale of operations and the skill composition of their workforce. If worker and time-varying firm productivity are complements in the production function, i.e. $f_{xy}(x, y, z) > 0$ or $\rho_y > 1$, then the marginal productivity of high-skilled workers increases relative to low-skilled workers following a positive productivity shock, conditional on z. As a result, such firms expand and reorganize their workforce toward higher-skilled workers, creating a

[29]Data are available only from 2010 onward; I use the average from 2010–2015. Source: http://www.iab.de/stellenerhebung/download

Table 6: Target Moments

	Data	Model
Separation rate	0.019	0.019
Unemployment rate	0.137	0.138
Job-to-job transition rate	0.006	0.006
Job filling rate	0.388	0.399
Mean worker type distribution	0.496	0.537
Std. of worker type distribution	0.224	0.289
Std. of empl. weighted growth rates	0.169	0.095
Emp. weighted autocorr. (Lag 1) of firm size	0.998	0.999
Emp. weighted autocorr. (Lag 10) of firm size	0.948	0.947
Emp. weighted Std. of $\bar{W}_{j,t}$	0.160	0.162
Firm Size Distribution (5 Points)	see figure	
Panel Regression Coeff Equation (3)	-0.068	-0.068
Cross Sectional Reg Coeff Equation (2)	0.131	0.134

Notes: Fit of identifying moments. See text for details.

positive correlation between firm growth and changes in workforce quality. Conversely, if worker and time-varying firm productivity are substitutes, i.e., $f_{xy}(x, y, z) < 0$ or $\rho_y < 1$, firms expand while shifting towards lower-type workers, generating a negative correlation between growth and changes in workforce quality. This logic is presented in Figure 5a, which shows that the relationship between firm growth and changes in average workforce quality is negative when $\rho_y < 1$ and positive when $\rho_y > 1$.

The final parameter ρ_z, the complementarity between the time-invariant firm productivity z and worker skill, is identified using a similar logic. Conditional on all other parameters in the model, the strength of the relationship between firm size and the average worker quality from regression (2) informs ρ_z. As can be seen in Figure 5b, a higher value of ρ_z implies a stronger correlation between firm size and workforce quality.

6 Estimation Results

6.1 The Fit of the Moments

Table 6 presents the fit of all target moments and Table 7 displays the estimated parameter values. Overall, the model closely matches all moments, with the exception of the standard deviation of employment-weighted growth rates, which is underpredicted.

Figure 6 presents the model fit of the growth rate distribution and firm size distribution. As the left panel shows, the model implies a realistic growth-rate distribution with a symmetric bell shape with mean zero. The empirical growth rate distribution has a

Table 7: Parameter Estimates

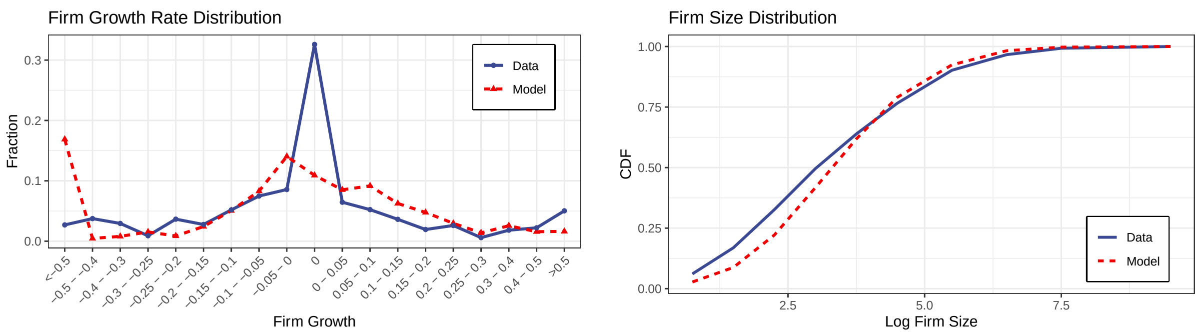
Preassigned Parameters			Calibrated Parameters		
Parameters	Symbol	Value	Parameters	Symbol	Value
Discount Factor	β	0.995	Complementarity y	ρ_y	0.864
Mass of firms	M	0.010	Complementarity z	ρ_z	5.360
Worker Bargaining Weight	α	0.200	Worker dist. location	μ_x	-0.316
Time-invariant firm prod	a_β	1.000	Worker dist. scale	σ_x	0.812
			Meeting rate workers	λ_w	0.178
			Job-to-job meeting rate	λ_e	0.021
			Job destruction rate	d	0.012
			Job creation cost, scale	c_0	14.800
			Job creation cost, convexity	c_1	1.119
			Firm shocks, persistence	φ	0.991
			Firm shocks, variance	σ_y	0.021
			Time invariant firm prod.	b_β	8.078

Notes: Estimated parameter values. See text for details.

Figure 6: Model Fit

(a) Distribution of Firm Growth

(b) Firm Size Distribution

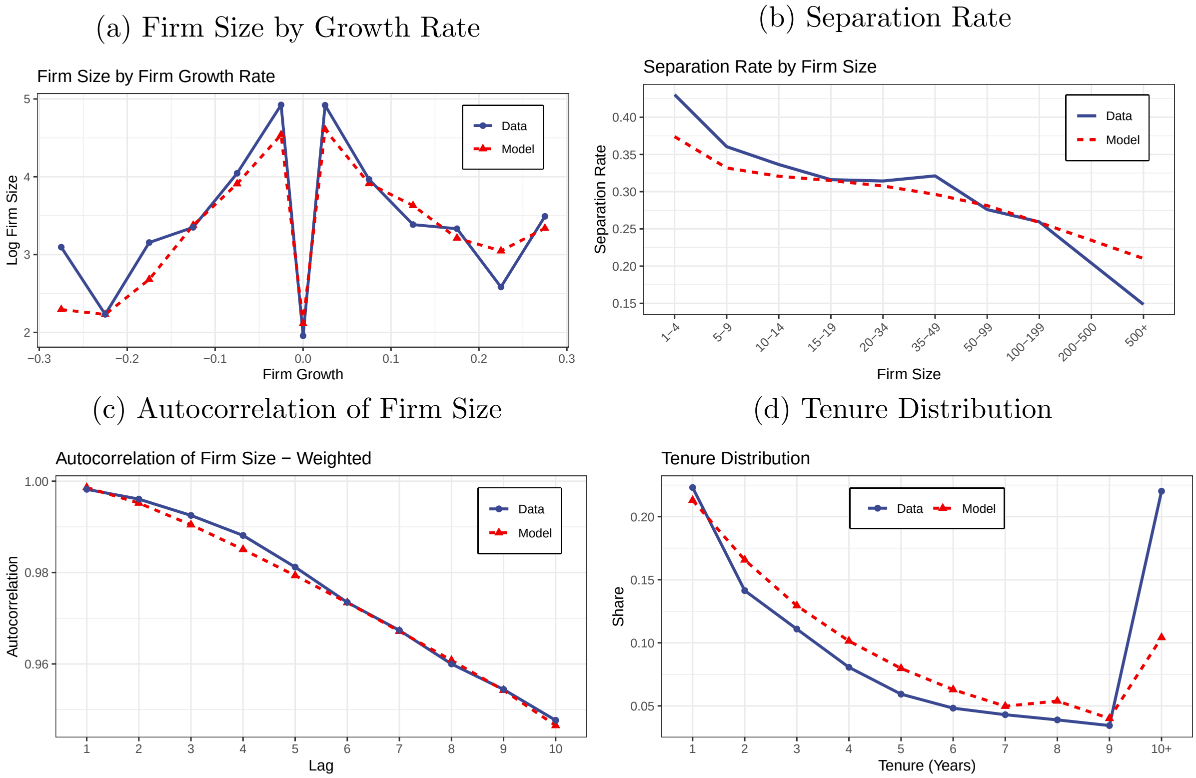


Notes: The left panel plots the firm growth rate distribution and the right panel represents the firm size distribution in the model versus the data.

mass point at zero, as many firms do not adjust their size in a given year. Because my model does not feature fixed costs of adjusting employment, it underpredicts the fraction of firms with zero growth. The shape of the firm size distribution is well matched, although large firms are slightly overrepresented in my model. Since the shape of the size distribution is constrained by the job creation cost function $c(v)$, some deviation from the empirical distribution is expected.

Figure 7a further shows the untargeted relationship between firm size and growth. The model matches the observed pattern in Germany closely. Smaller firms exhibit higher growth rates, while larger firms grow more slowly. In addition, the model replicates the notable dip in firm size among firms with zero employment growth. The well-matched relationship arises from the identified production function. Since constant and time-varying productivities enter additively, productivity shocks have a smaller relative effect

Figure 7: Untargeted Moments



(a) Firm Size by Growth Rate

(b) Separation Rate

(c) Autocorrelation of Firm Size

(d) Tenure Distribution

Notes: The top left figure plots the relationship between firm size and the firm growth rate. The top right panel presents the firm size and firm separation rate relationship. The bottom left figure shows the autocorrelation of firm size weighted by firm size over different yearly horizons. The bottom-right shows the tenure distribution.

on highly productive, larger firms. In turn, their growth path is more stable compared to smaller firms. This is also the reason why the model endogenously replicates another salient feature of firm dynamics, as shown in Figure 7b: Larger firms have lower separation rates. The model also closely tracks the autocorrelation of firm size across different time horizons.

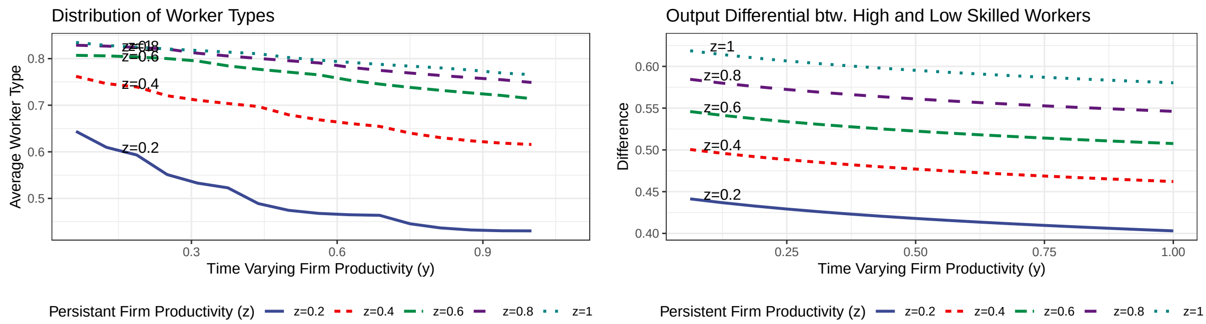
The estimated model provides a laboratory to quantify how precisely the worker fixed effects recover the true worker types. The correlation between the worker fixed effects and the true worker type x is 0.996. Thus, the 18-year time span of the social security data is enough to recover the true worker types almost perfectly.[30] In addition, the model matches the empirical job tenure distribution closely, as shown in Figure 7d.[31]

[30] Not all individuals in the dataset are observed for the full 18-year period. Table 15 in the appendix reports the distribution of time spent in the data and shows that most workers are observed either for the full 18 year period, or only slightly shorter. If I am reducing the number of years for workers to the median number 12, the correlation between fixed effects and true worker types is still very high at 0.9942.

[31] The average number of jobs per worker in my model is 4.5 versus 3.94 in the German social security data.

Figure 8: Sorting Pattern

(a) Worker distribution (b) $f(x^{\max}, y, z) - f(x^{\min}, y, z)$



Notes: The left figure plots the average worker type by time varying firm productivity y (x-axis) by different persistent firm productivity z. The right figure plots the output differential between high and low-skilled workers for the estimated production function.

Given the negative relationship between firm growth and changes in workforce quality over time, it is unsurprising that the estimated value of ρ_y is below 1. This implies that worker and *time-varying* firm types are weak substitutes in the production function. In contrast, ρ_z is estimated to be 5.4, indicating strong complementarity between worker skill and *time-invariant* firm productivity. Consistent with these estimates, the implied correlation between worker type x and transitory firm productivity y is relatively weak at -0.17, whereas the correlation between x and persistent firm productivity z is substantially stronger at 0.56. This pattern is in line with earlier studies that abstract from time-varying firm productivity and find positive assortative matching in Germany (e.g., Card et al. (2013) or Hagedorn et al. (2017)).

6.2 Complementarities and Workforce Adjustments

Before I discuss the sorting pattern and thus the adjustment patterns to workforce quality over time in more detail, I define the notion of sorting. I define positive (negative) sorting along x and y if the average quality of workers is increasing (decreasing) in transient firm productivity y, conditional on z, i.e. there is PAM (NAM) along x and y if $\frac{\partial \mathbb{E}[x|y,z]}{\partial y} > 0$ ($\frac{\partial \mathbb{E}[x|y,z]}{\partial y} < 0$). Conversely, there is PAM (NAM) along x and z if $\frac{\partial \mathbb{E}[x|y,z]}{\partial z} > 0$ ($\frac{\partial \mathbb{E}[x|y,z]}{\partial z} < 0$).[32]

Figure 8a clearly visualizes the resulting sorting pattern.[33] Conditional on y, higher values of the time-invariant firm productivity component z are associated with higher average worker types. Conversely, conditional on z, there is a negative relationship between

[32] This is a weaker definition of sorting compared to Shimer and Smith (2000). I use this definition, because it has a tighter link to the empirical moments documented in the empirical section.

[33] To avoid clutter, the figure displays only five levels of z, Appendix Figure 11 plots all.

the average worker type and the time-varying firm productivity component y.

What economic forces drive this sorting pattern? The right panel plots the output differential between high- and low-skilled workers, $f(x^{\max}, y, z) - f(x^{\min}, y, z)$, across different firm productivities. This differential approximates the marginal return to worker skill $f_x(x, y, z)$ and represents the firm's incentive to hire higher-type workers. The sorting pattern shown in the left panel closely mirrors this output differential. Focusing on the z-dimension, the figure reveals that firms with higher time-invariant productivity z benefit more from hiring high-type workers than firms with low z, conditional on y. This leads high-type workers to sort themselves into high z firms.

The opposite pattern emerges along the y dimension. Due to the estimated value of $\rho_y < 1$, the marginal return to worker skill declines with the transitory firm productivity component y, conditional on z. This leads to the negative sorting pattern along y. This mechanism also explains why the model reproduces the empirically observed negative relationship between firm growth and changes in workforce skill intensity: Following an increase in y, firms expand in size. However, the marginal effect of workers skills declines, which leads to a decrease in the skill level of its workforce. This insight explains how the model is able to reconcile the positive sorting in the cross section with the negative relationship between firm growth and changes to worker quality over time. Although only the linear relationship is targeted, the model reproduces the exact shape of the relationship over the firm growth distribution closely, as shown in Figure 9a.

The model not only matches the targeted moments but also closely replicates the broader empirical patterns documented in the data. Figure 9b demonstrates that, as in the data, employment of low-skilled workers is more elastic with respect to firm growth than that of high-skilled workers. This arises because low-quality workers become relatively more valuable following positive productivity shocks, leading firms to expand more intensively along the lower end of the skill distribution. This effect also generates the increasing dispersion of worker types with firm growth, which is also closely replicated by the model, as shown in Figure 9c.

The empirical section showed that both hires and separations contribute to changes in workforce quality. In expanding firms, the average quality of hires is lower than that of separations, while the opposite pattern holds in contracting firms. Figure 9d shows that the model closely replicates this pattern across the growth rate distribution. In addition, Figure 9e presents the decomposition of workforce quality changes into contributions from hires and separations, based on equation (7). Qualitatively, the model captures the key asymmetry: hires account for most of the change in workforce quality in growing firms, whereas separations play a larger role in shrinking firms. Quantitatively, however, the model somewhat underpredicts the magnitude of these contributions.

Figure 9: Workforce Adjustments - Untargeted Moments

(a) Change of Average Worker Quality

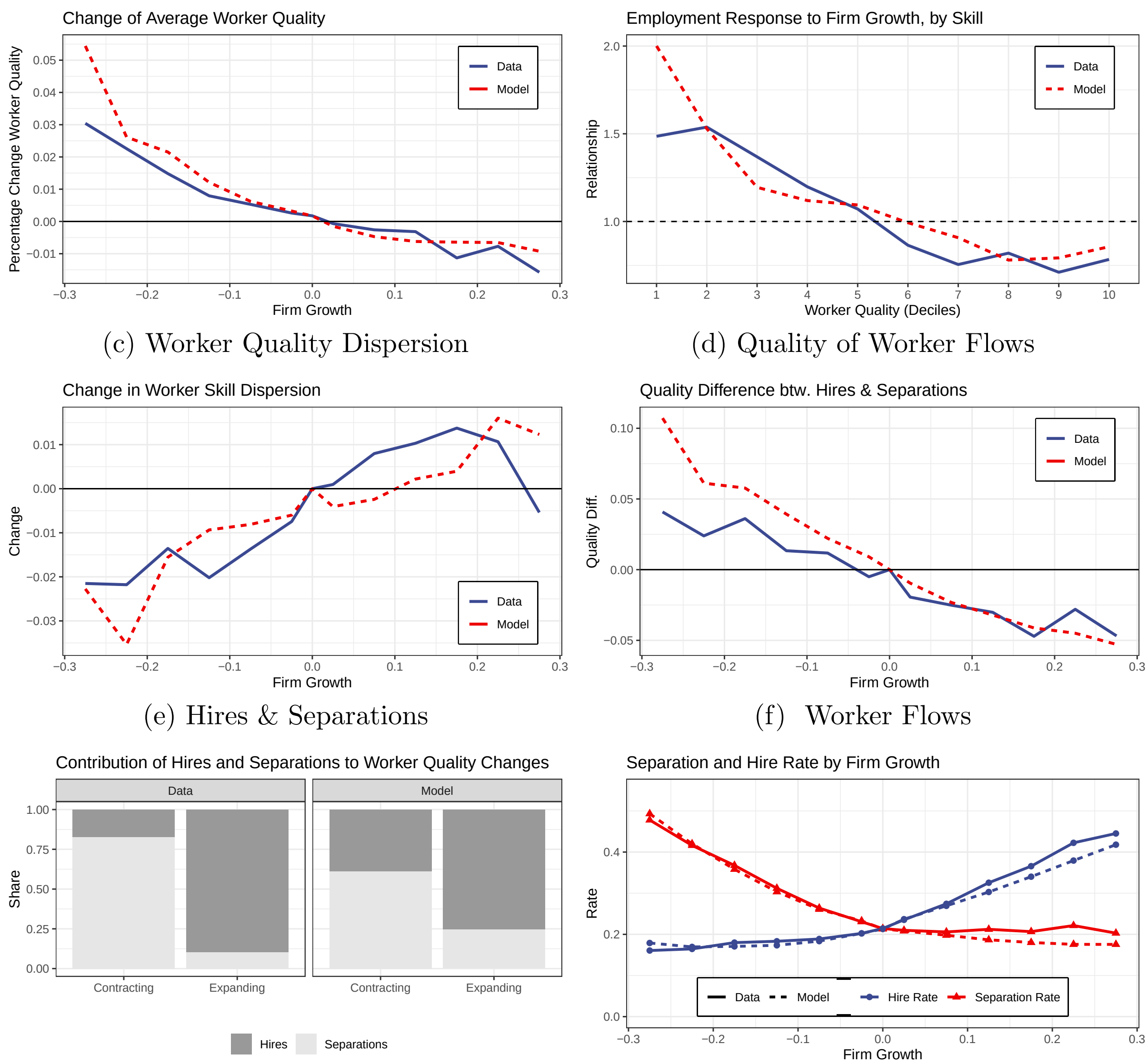
(b) Employment Elasticity

(c) Worker Quality Dispersion

(d) Quality of Worker Flows

(e) Hires & Separations

(f) Worker Flows



Notes: The left-top figure plots the relationship between year-to-year percentage changes in the average worker quality and the firm growth rate. The right-top panel presents the employment elasticity by different worker types from equation (5). The middle left panel shows how the firm level standard deviation of worker types changes with firm growth. The middle right panel shows the average quality difference between the hires and separations by firm growth. The bottom left figure shows the contribution of hires and separations to overall changes in the quality of the workforce. The bottom right panel shows the hire and separation rate by firm growth rate.

Table 8: Wage Changes by Firm Growth

	Model	Data
growth	0.0579	0.0085
growth × *skilled*	-0.0068	-0.0063

Notes: Regression results of equation (8), i.e.:
$\Delta_{\%} w_{it} = \alpha + \gamma_1 growth_{j(i,t),t} + \gamma_2 skilled_i \times growth_{j(i,t),t} + \beta_1 skilled_i + \beta_2 X_{it} + \epsilon_{it}$. Sample consists of workers employed and staying at firms of size ≥ 20.

A salient fact of firm dynamics is that both expanding and contracting firms have excess job turnover rates. Specifically, separation rates in growing firms remain positive rather than declining, and hire rates in shrinking firms do not drop to zero. This creates the characteristic "hockey-stick" shape of worker flows at the firm level, first documented by Davis et al. (2006). This is puzzling from the perspective of standard models of firm dynamics, in which rapidly growing or shrinking firms are expected to minimize worker churn in order to quickly converge to their optimal employment level. My model offers a natural explanation for this puzzle: firms not only adjust the number of employees in response to shocks, but also reorganize the composition of worker skills. This adjustment process involves separating from mismatched workers and hiring better-suited types, resulting in excess worker flows. As shown in Figure 9f, the model closely replicates the empirically observed "hockey-stick" pattern in both hires and separations.

The empirical section documented that wage growth in expanding firms is higher for low-type workers. Table 8 shows that the model successfully replicates this pattern. Firms expand after positive productivity shocks to y, and through the bargaining protocol this translates into higher wages, although my model implies a somewhat stronger relationship between wages and firm growth as in the data. Because worker skills are substitutes to transient firm productivity y, low-skilled workers become relatively more valuable to expanding firms. As a result, their wages grow faster than those of high-skilled workers—a pattern that the model replicates closely. In summary, the complex, multidimensional sorting structure identified in the model plays a central role in explaining the joint dynamics of employment and wages at the firm level.

7 Model Applications

7.1 Large Firm Wage Premium

A large empirical literature documents a positive relationship between firm size and wages.[34] In my model, wages at large firms can be higher for three structural reasons: First, larger firms poach more actively, which improves workers' bargaining positions at the time of hire and results in higher wages. Second, larger firms are more productive and thus offer higher wages to attract and retain workers. Third, due to positive assortative matching in the cross-section, larger firms employ higher-type—and therefore higher-paid—workers.

Table 9 presents a decomposition of the large-firm wage premium into these three channels. The first column reports the wage elasticity with respect to firm size, i.e., the regression coefficient β from the following regression:

$$\log \bar{w}_{jt} = \alpha + \beta \log size_{jt} + \epsilon_{jt}, \tag{30}$$

where $\bar{w}_{jt}$ denotes the average wage in firm j at time t, and $size_{jt}$ is the number of employees. To assess the contribution of different mechanisms, I conduct two counterfactual exercises: First, I equalize bargaining positions across all workers by imputing counterfactual wages $\phi^{NF}(x, y, z) \times f(x, y, z)$, and then recompute average firm-level wages. Column two shows that this has a minimal effect on the firm-size wage elasticity. Second, I equalize firm productivity by assigning to each worker a wage based on a reference firm type $(y^{\text{ref}}, z^{\text{ref}})$: $\phi^{NF}(x, y^{\text{ref}}, z^{\text{ref}}) \times f(x, y^{\text{ref}}, z^{\text{ref}})$.[35] Equalizing firm productivities reduces the wage firm size relationship and wages by 15 percent. The remaining 85 percent of the premium is attributable to the sorting of more productive workers into larger firms.[36] These findings are consistent with empirical studies that attribute the majority of the firm-size wage premium to worker sorting.[37]

7.2 Labor Supply Elasticity

The estimation of firm-level labor supply elasticity has received considerable attention recently. In a perfectly competitive labor market, firms face an infinitely elastic labor supply, whereas monopsonistic markets are characterized by lower elasticities (see Manning (2021) for a review). A common empirical strategy to estimate this elasticity is to exploit

[34] See Oi and Idson (1999) for a review.

[35] I have used $y^{\text{ref}} = 0.9375$ and $z^{\text{ref}} = 0.1$. This firm type matches with all worker types in the economy.

[36] The ordering of this exercise does not play a major role.

[37] e.g. Abowd et al. (1999); Winter-Ebmer and Zweimüller (1999); Gibson and Stillman (2009)

Table 9: Firm Size Wage Premium

	Overall correlation	With same bargaining position	With same firm productivity
Coefficient	0.174	0.174	0.150
Percentage	100	99.78	85.92

Notes: The Table shows regression coefficient of log average firm wage on log firm size. Column 1 uses the equilibrium wage, column 2 equal bargaining positions, and column 3 the wage under equal bargaining positions and equal firm types. See text for detail.

plausibly exogenous firm-level shocks, measure the resulting wage changes, and relate them to changes in labor input.[38] However, this assumption is at odds with the evidence presented in this paper, which documents systematic changes in workforce composition after productivity shocks. This raises an important question: how much would estimated labor supply elasticities change if adjustments in worker productivity were taken into account?

Conceptually, I am interested in quantifying how total labor input, measured in efficiency units, responds to changes in firm wage policies. Effective labor input $\tilde{N}_j = ALP_j \times N_j$ of a firm j is given by the number of workers N_j times their average labor productivity ALP_j. In my model, a positive firm-level productivity shock induces higher average wages and an expansion in firm size. However, the resulting workforce becomes less skill-intensive, implying that labor input in efficiency units increases by less than the headcount. To quantify the difference, I use the estimated model to simulate a time series for all firms types with a persistent positive and a negative shock, and a time series without shocks as the control group.[39] A key complication arises from the fact that average labor productivity changes for two reasons: First, time-varying firm productivity y affects $ALP_{j,t}$ in a non-separable way. Second, the workforce composition $\psi_{j,t}(x)$ changes. As the first factor is not an active choice of labor input, I isolate the later by computing the $ALP_{j,t}$ holding firm productivity fixed to its pre-shock level $y_{j,T_{Pre}}$:

$$\widetilde{ALP}_{j,t} = \frac{\int f(x, y_{j,T_{Pre}}, z_j)\psi_{j,t}(x)}{\int \psi_{j,t}(x)}. \tag{31}$$

I now contrast the labor supply elasticity estimation based on headcounts with one using labor input in efficiency units, $\widetilde{ALP}_{j,t} \times N_{j,t}$. I assume that firm productivity shocks in my model are observable to the researcher and follow the labor supply elasticity

[38] see Sokolova and Sorensen (2021) for a review on labor supply elasticity estimation.

[39] I model shocks as a one-grid-point change in y. The control group is necessary to net out baseline wage trends among stayers, which exist even in the absence of firm productivity shocks.

Table 10: Labor Supply Elasticity

	Head Count	Efficiency Units	Percentage Diff.
Positive Shock	2.16	2.05	0.05
Negative Shock	2.09	1.88	0.12

Notes: Model-based labor supply elasticity using head count or Efficiency units as labor input.

literature by estimating the response of wages and labor input over 2 years after the shock using a difference-in-differences (DiD) approach[40]:

$$y_{jt} = \alpha_1 Post_t + \alpha_2 Treat_j + \eta Post_t \times Treat_j + \epsilon_{jt}, \tag{32}$$

where y_{jt} is the outcome of interest—log average wage at the firm level, log headcount, or log efficiency-adjusted labor input. $Post_t$ is a dummy for periods after the shock, and $Treat_j$ is an indicator for whether firm j received a shock. I follow the literature and condition on the wages of stayers, so that the firm-level wage measure is not affected by compositional changes in the workforce.

The labor supply elasticity is given by the ratio of the DiD coefficient for labor input, η^{input}, to the DiD coefficient for wages, η^{wage}. This ratio measures how much firm level labor input changes with a change in firm level wages. Table 10 reports the results where I contrast using head count as labor input to using efficiency units. The estimated labor supply elasticity using head count is around 2.1 which is within the range of "best practice" estimates of the labor supply elasticities in the literature review of Sokolova and Sorensen (2021), and close to the value found in Kroft et al. (2023). Accounting for workforce-composition shifts lowers the estimated labor-supply elasticity by roughly 5 – 12 percent. Although firms expand headcount after a productivity shock, they disproportionately hire lower-skilled workers, so the increase in labor input measured in efficiency units is muted. Ignoring these quality adjustments therefore introduces a non-negligible downward bias in labor supply elasticity estimates.

7.3 Effects of Labor Market Institutions

This paper has shown that firms not only adjust the quantity of their workforce in response to idiosyncratic shocks, but also adjust the workforce quality due to worker-firm complementarities. Many labor market institutions affect hiring and separation margins,

[40] Or the equivalent event study specification; see, for example, Kroft et al. (2023) or Kline et al. (2019).

Table 11: Effects of Dismissal Costs

		Full Model	No Sorting
	Level	Percentage Change	
Welfare	0.299	-0.019	-0.010
Employment	0.862	-0.066	-0.040
EU rate	0.013	-0.041	-0.010
Job-finding Rate	0.094	-0.285	-0.192
Excess Churn	0.348	-0.030	-0.006
Size Elasticity	1.554	-0.018	-0.013

Notes: Effect of dismissal costs $\tau = 2$. Table shows the percentage change of key labor market variables in the baseline model, and in a model with no sorting, i.e. $\rho_y = 1$. See text for details.

potentially distorting a firm's ability to reoptimize its workforce composition after shocks. Among these, dismissal regulations—such as layoff taxes, mandatory notice periods, and legal protections against termination—directly increase the cost of separations. While such institutions are often intended to protect workers, they may have broader welfare implications, as they impede not only the optimal adjustment in workforce size, but also the adjustment in workforce quality.

To study how dismissal regulations interact with the firm's adjustments to shocks, I use the estimated model as a laboratory to compute counterfactual economies with dismissal taxes. The dismissal taxes are not redistributed and should be interpreted not only as literal taxes, but more generally as wedges generated by the costs imposed by dismissal regulation. First, I compute the effects of dismissal taxes equal to twice the level of labor productivity. Each time a worker and a firm separate, except in the case of a job-to-job transition, a tax $\tau \times f(x, y, z)$ must be paid. The resulting new surplus equation is described in Appendix G.

Second, I contrast this exercise with a model without sorting along the y dimension, i.e. I set $\rho_y = 1$. Without complementarities between workers and time-varying productivity, firms only adjust the quantity of employment after shocks, and not the quality of their workforce.

Table 11 shows the impact of increasing τ from zero to two in both models. Dismissal taxes fulfill their intended effects and reduce the separation rate into unemployment (EU). In the baseline model, the EU rate declines by over 4 percent. However, firms anticipate the additional costs associated with separations and respond by creating fewer jobs, which leads to a nearly 30 percent drop in the job-finding rate. Because the decline in job-finding outweighs the reduction in separations, overall employment falls by more than 6 percent.

Even though the employment rate declines, the overall effect on welfare is theoretically ambiguous, since vacancy creation also consumes resources. Welfare in the model is defined as total output plus home production, minus the resources spent on job creation:

$$\begin{aligned}\text{Welfare} \quad = \quad & \int_x \int_y \int_z f(x,y,z)\psi(x,y,z)dzdydx + \int_x b(x)d\mu_x(x) \\ & - \int_y \int_z c_0 \left(\frac{v^N(y,z)}{c_1}\right)^{c_1} M\zeta(y,z)dzdy \end{aligned} \tag{33}$$

Dismissal taxes reduce welfare by approximately two percent in the baseline model. This decline is driven by three main mechanisms: First, fewer workers are employed and thus fewer contribute to output. Second, dismissal taxes lead to a misallocation of jobs across firms. Optimally, firms that receive positive productivity shocks to y should expand, while those with negative shocks should contract. Dismissal taxes hinder this adjustment by raising the cost of separations, leading to inefficiently rigid firm sizes. As a result, the elasticity of firm size with respect to y declines by three percent. As documented in this paper, worker-firm complementarities imply that firms not only adjust the number of employees in response to shocks but also the composition of worker skills. This reorganization generates excess worker churn—firms hire and separate from more workers than necessary to change firm size alone. Dismissal taxes also impede this adjustment margin, as evidenced by a three percent decline in excess worker churn. In sum, while dismissal taxes save on vacancy creation costs, the resulting output losses from distorted employment levels, job misallocation, and reduced reoptimization of workforce composition dominate. The net effect is a two percent decline in welfare.

In contrast, in the model without sorting, dismissal taxes equivalent to two times labor productivity result in welfare losses of only one percent. These smaller welfare losses arise from two main factors. First, in the absence of sorting, firms are less selective about the worker types they match with. This reduces the impact of dismissal taxes on separations, since fewer workers fall outside the matching sets following productivity shocks. As a result, the employment response is dampened, with employment falling by just 4 percent. Second, without sorting along the y-dimension, firms adjust only the number of employees, not their composition. Although the relationship between firm size and productivity still weakens, excess worker churn remains largely unchanged. Since workforce quality adjustments are absent, dismissal taxes cannot interfere with this margin, resulting in lower output losses and smaller welfare effects.

In summary, my baseline model with sorting predicts a greater responsiveness of both employment and welfare to dismissal taxes. This highlights that accounting for multi-dimensional sorting is crucial not only for understanding firm-level employment dynamics, but also for evaluating the broader labor market impacts of dismissal regulations.

8 Conclusion

I document a puzzling pattern in firm-level workforce quality adjustments over time. Standard labor market sorting models predict that workforce quality adjustments should mirror the cross-sectional sorting pattern. However, using German social security records, I show that the opposite occurs. While larger firms match with higher-quality workers, expanding firms downgrade workforce quality, whereas contracting firms upgrade it.

I develop a search and matching model with heterogeneous workers and firms, job-to-job transitions, firm dynamics driven by idiosyncratic firm-level shocks, and multi-dimensional sorting. The model generates realistic relationships across several empirical dimensions: (1) firm growth and firm size, (2) firm size and firm-level separation rates, (3) firm growth and changes in the within establishment dispersion of worker types, (4) firm growth and wage changes by low and high-skilled workers, (5) firm growth and the average quality difference between hires and separations, (6) the hire and separation rates by firm growth rate and in addition matches (7) the contributions of hires and separations to overall changes in worker quality, (8) the employment elasticities over the worker skill distribution, (9) the autocorrelation of firm size over various lags, and (10) the job tenure profile closely.

I use the model to structurally decompose the large-firm wage premium and show that high-skilled workers' sorting into large firms is the dominant driver. Moreover, ignoring post-shock changes in workforce composition introduces a non-negligible bias in labor-supply estimates. The model also reveals that firm-level adjustments in workforce quality carry significant policy implications. Because dismissal taxes impede both the optimal resizing of the workforce after shocks and its quality adjustment, they generate higher welfare costs in my framework than in models without sorting.

This paper also opens several avenues. I focus on firm-level shocks and assumes worker types to be constant. But a related symmetry point could be made about worker types. How does the sorting pattern change after productivity shocks to workers, i.e., after health shocks or with the accumulation of human capital by learning-by doing, on-the-job training, additional education, or active labor market programs? How does the sorting pattern change as workers' productivity changes over their life cycle? These are interesting avenues for future research.

Overall, the evidence presented in this paper shows that labor market sorting is key to understand the firm level dynamics of employment and the impact of dismissal regulation.

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A Sorting in Becker (1973)

This yields the FOC:

$$f_x(x,y) - w_x(x) = 0 \tag{34}$$

Let us denote the equilibrium allocation of workers to firms by $\mu(x)$. The equilibrium allocation follows from the SOC:

$$f_{xx}(x,y) - w_{xx}(x) < 0 \tag{35}$$

We can get $w_{xx}(x)$ from the derivative of the FOC at the optimal allocation $y = \mu(x)$

$$f_{xx}(x,\mu(x)) + f_{xy}(x,\mu(x))\mu'(x) - w_{xx}(x) = 0 \tag{36}$$

Substitution this into the SOC gives us

$$f_{xy}(x,\mu(x))\mu'(x) > 0 \tag{37}$$

Now we can read off the conditions for positive assortative matching (PAM) versus negative assortative matching (NAM) from equation (37). For the case where higher type firms match with higher type workers, which implies that $\mu(x)$ is increasing in x, or $\mu'(x) > 0$, we can see that it must be that $f_{xy} > 0$. Thus

$$\text{PAM: } \mu'(x) > 0 \text{ if } f_{xy} > 0 \qquad \text{NAM: } \mu'(x) < 0 \text{ if } f_{xy} < 0 \tag{38}$$

B Appendix - Data Description:

The German social security data used in the empirical analysis is provided by the Research Data Centre of the German Federal Employment Agency. It is based on notifications of employers and several social insurance agencies for all workers and establishments covered by social security. This includes virtually every employees except of government employees. The particular dataset is the longitudinal model of the Linked-Employer-Employee Data (LIAB LM 9310). Heining et al. (2013) provide a detail data documentation.

This data set contains the complete work history of every worker that was employed at one of the selected establishments. The sample of establishments is based on the sample from IAB Establishment Survey. It is stratified according to industry, firm size, and federal state. In total, the dataset contains 2,702 to 11,117 establishments per year, and 1,090,728 to 1,536,665 individuals per year. It includes information on the foundation year of the establishment and a 3 digit industry identifier. For each worker employed at one of the establishments in the sample, the whole work history during 1993 and 2010 is recorded. This contains a 3 digit occupation identifier, part time and full time status, the beginning and end of all employment and unemployment spells precise to the day and the total daily wages and unemployment benefits received. All labor income is recorded that is subject to social security contribution. Only earnings that lie above the marginal part-time income threshold[41] and below the upper earnings limit for statutory pension insurance are not reported. In addition the dataset contains a number of socio demographic variables such as age, gender, nationality and education.

The exact working hours are not reported, only whether the employee is working part or full time. Since wages are recorded as daily wages, the hourly wage rate cannot be identified for part time employees.[42] Because of this, I focus on full time employees only in my analysis.

I use the following definitions for labor market transitions. I consider every worker transition from one employer to another firm as a job-to-job transition if the spell of non-employment between the two jobs was less than 30 days. In the computation of transition rates, I disregard any transition into unemployment and subsequent rehire if the person is rejoining the same firm within 30 days.[43]

I compute worker quality the following way. First, I deflate wages by the CPI index. Then, I compute annual earnings from full time jobs. I estimate a Mincer regression of the following form:

$$e_{it} = \alpha_i + \beta X_{it} + \epsilon_{it}. \tag{39}$$

Here e_{it} denotes the total anual earnings derived from employment and also potentially unemployment beneifts of individual i in year t. α_i represents the worker fixed effect and X_{it} a set of time-varying worker controls. I follow Card et al. (2013) and include a set of year dummies and quadratic and cubic terms in age fully interacted with educational attainment. The coding of the education variable follows exactly Card et al. (2013). The social security data does not have information on the labor force status of workers. Thus, I assume that everyone with zero earnings from employment for a full calendar year (i.e. from 1st of January until 31st of December) is not part of the labor force. Years not spent in the labor force are excluded from the regression since my model does not feature

[41]So called marginal part time jobs are not subject to social security contributions if the earnings do not exceed around 400 Euros a month

[42]The strict labor laws in Germany restrict the working week usually to around 40 hours. I therefore assume that the daily wages are a good measure for the true wage rate.

[43]This is in line with recent evidence shown in Fujita and Moscarini (2017) and Nekoei and Weber (2015)

a labor force participation margin. I trim the resulting fixed effects below the 0.5 and above the 99.5 percentile and normalize them to lie between 0 and 1.

C Additional Empirical Analysis

Table 12: Cross-Sectional Regression Results

Worker Quality Measure	Fixed Effect	Wages	Wages last 2 yrs.	Education	Experience	Wages after UE
$Log(FirmSize)$	0.092	0.090	0.083	0.035	0.030	0.060
SE	0.003	0.002	0.002	0.002	0.003	0.003
N	51581	51941	51351	51941	51941	44485
Adj. R^2	0.357	0.424	0.436	0.157	0.257	0.294

Notes: Cross section regression of several worker quality measures (in logs) at the establishment level on log establishment size, as measured in workers employed. Each regression controls for 2-digit industry, firm age, and region size.

Table 13: Panel Regression Results

LHS Variable	Fixed Effect	Wages	Wages last 2 years	Education	Experience	Wages post UE Spells
$Firmgrowth$	-0.068	-0.089	-0.075	-0.037	-0.180	-0.072
SE	0.005	0.006	0.004	0.006	0.009	0.007
N	19912	19912	19911	19912	19912	10890
Adj. R^2	0.058	0.052	0.075	0.011	0.113	0.039

Notes: Establishment level regressions of firm growth rates on percentage change in average worker fixed effects. Standard errors are clustered at the firm level. The samples are restricted to establishments with more than 20 employees and growth rates between -0.75 and 0.75. See text and notes below for detailed explanation of the different specifications.

Table 14: Regression Results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Firmgrowth	-0.068	-0.068	-0.069	-0.073	-0.047	-0.032	-0.043
SE	0.005	0.009	0.006	0.007	0.010	0.004	0.005
Sample/Model	Baseline[1]	Weighted[2]	Additional Controls[3]	Firm FE[4]	Trends[5]	Core Occup[6]	Worker FE pre 1999[7]
N	19912	4257830	19912	19912	2066	14423	18375
Adj. R^2	0.058	0.059	0.082	0.087	0.010	0.016	0.025

Notes: Establishment level regressions of firm growth rates on percentage change in average worker fixed effects. Standard errors are clustered at the firm level. The samples are restricted to establishments with more than 20 employees and growth rates between -0.75 and 0.75. See text and notes below for detailed explanation of the different specifications.

[1] Baseline: Yearly changes

[2] Weighted by Firm Size, otherwise baseline specification. In the table N is higher because of the weights, the true value is 19912.

[3] With Controls: Additionally controlling for firm age, firm size, 2 digit industry x year x location (state) fixed effects

[4] Baseline specification, in addition controlling for firm fixed effect.

[5] Regression using 5 year trends in firm growth and worker FE changes.

[6] Regression conditioning on core 2-dig Occupation

[7] Regression using Worker FEs pre 1999

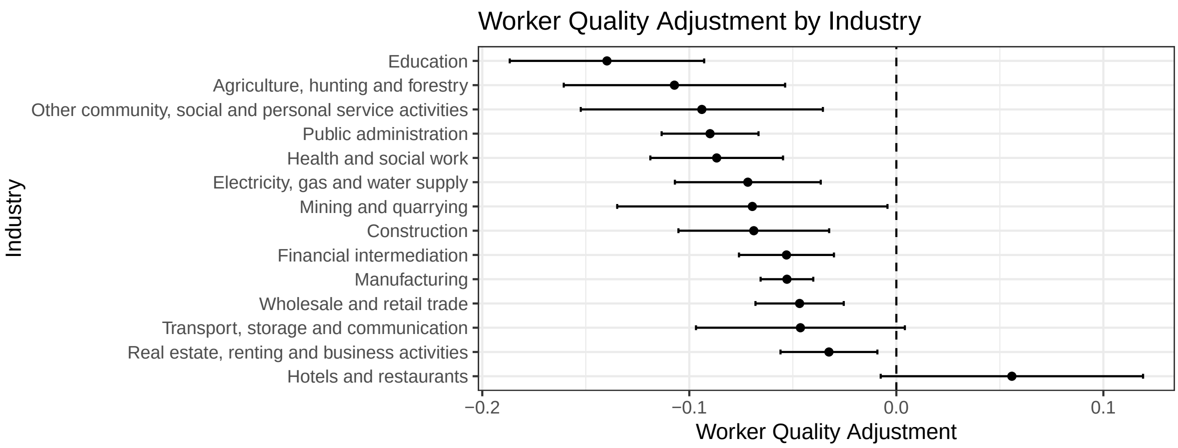

Figure 10: The figure shows the worker quality change (coefficient from regression equation 3) by NACE-1 industry classification. The sample consists of all establishments with size$\geq$20 and growth rates $\in (-.75, 0.75)$. Establishment growth rates and percentage changes in worker quality are yearly.

D Value Functions, Derivation of Surplus

This appendix section presents the value functions and the derivation of the surplus function. To compute the value of a vacancy we have to first integrate over all possible future time-varying firm productivity types. Second, we have to take into account whether the firm meets a suitable match with either an unemployed or employed worker. $A^U(x, y)$ is the indicator function that takes on the value of 1 if the match between an unemployed worker of type x and a firm of type y, z is consummated and zero otherwise. Similarly, if a worker x employed at a type y, z firm is contacted by a poaching firm of type $\tilde{y}, \tilde{z}$, $A^E(x, y, z, \tilde{y}, \tilde{z})$ is one if the job offer is accepted and zero otherwise. The wage setting mechanism and the assumption of transferable utility assures that acceptance decisions jointly maximize the total surplus. Thus, agents are willing to match together if the match generates a positive surplus, and in case of job-to-job transitions, the prospective surplus is higher than the current one. Formally,

$$A^U(x,y,z) = \begin{cases} 1 & \text{if } S(x,y,z) \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{40}$$

$$A^E(x,y,\tilde{y}) = \begin{cases} 1 & \text{if } S(x,\tilde{y},\tilde{z}) \geq S(x,y,z) \\ 0 & \text{otherwise.} \end{cases} \tag{41}$$

The value function of a vacant firm then reads

$$\begin{aligned} V(y,z) =& \beta(1-d)\int_{y'}\Bigg\{(1-\lambda_f)V(y',z) \\ &+ \lambda_f\Bigg(p^u\int_x A^U(x,y',z)J(x,y',z,\phi^U(x,y',z) + (1-A^U(x,y',z))V(y',z)\frac{\mu_x(x)}{u}dx+ \\ &+ (1-p^u)\int_{\tilde{y}}\int_{\tilde{z}}\int_x A^E(x,y',z,\tilde{y},\tilde{z})J(x,y',z,\phi^E(x,y',z,\tilde{y},\tilde{z})) \\ &+(1-A^E(x,y',z,\tilde{y},\tilde{z}))V(y',z)\frac{\psi^S(x,\tilde{y},\tilde{z})}{e^S}dxd\tilde{y}d\tilde{z}\Bigg)\Bigg\}p(y'|y)dy', \end{aligned}$$

An unemployed worker either remains unemployed, or finds a suitable match next period:

$$\begin{aligned} U(x) =& b(x) + \beta\Bigg((1-\lambda_w)U(x) \\ &+\lambda_w\int_z\int_y\left(A^U(x,y,z)W(x,y,z,\phi^U(x,y,z)) + (1-A^U(x,y,z))U(x)\right)\frac{\mu_F(y,z)}{V}dydz\Bigg). \end{aligned} \tag{42}$$

For ongoing matches, several different outcomes might occur. First, the firm productivity shock might be such that the match is not viable anymore ($A^U(x, y', z) = 0$). Second, if the worker meets another firm of type $\tilde{y}, \tilde{z}$, the poaching offer might either trigger a job-to-job transition ($\tilde{y}, \tilde{z} \in \Upsilon^{EE}(x, y', z)$) or a renegotiation ($\tilde{y}, \tilde{z} \in \Upsilon^{BC}(x, y', z)$). If the worker stays with its current employer, the firm productivity shock might lead to a violation of the participation constraint of one of the parties. This triggers a renegotiation as described in the main text. The set of y that violate the participation constraint for a worker of type x working for a firm of type z for a piece rate ϕ is $\Upsilon^{NW}(x,z,\phi)$, whereas the corresponding set for the violation of the firms participation constraint is $\Upsilon^{NF}(x,z,\phi)$. Their formal definitions are

$$\begin{aligned}\Upsilon^{NW}(x,z,\phi) &= \{y : S(x,y,z) \geq 0 \geq W(x,y,z,\phi) - U(x)\}\\ \Upsilon^{NF}(x,z,\phi) &= \{y : S(x,y,z) \geq 0 \geq J(x,y,z,\phi) - V(y,z)\}.\end{aligned}$$

Equation (43) presents the value of a filled job to a firm.

$$\begin{aligned}J(x,y,z,\phi) = f(x,y,z)(1-\phi) + \beta \int_{y'} \Big\{ &(1-(1-d)A^U(x,y',z))V(y',z)\\ &+ (1-d)(A^U(x,y',z)) \int_{\tilde{z}} \int_{\tilde{y}} \left\{ \lambda_e \left[\mathbb{1}\left((\tilde{y},\tilde{z}) \in \Upsilon^{EE}(x,y',z)\right) V(y',z) \right.\right.\\ &+ \mathbb{1}\left((\tilde{y},\tilde{z}) \in \Upsilon^{BC}(x,y',z)\right) J\left(x,y',z,\phi^E(x,\tilde{y},\tilde{z},y',z)\right)\\ &+ (1-\lambda_e \mathbb{1}\left((\tilde{y},\tilde{z}) \in \Upsilon^{BC}(x,y',z) \cup \Upsilon^{EE}(x,y',z)\right)\\ &\times \left[\mathbb{1}\left(y' \in \Upsilon^{NW}(x,z,\phi)\right) J\left(x,y',z,\phi^{NW}(x,y',z)\right)\right.\\ &+ \mathbb{1}\left(y' \in \Upsilon^{NF}(x,z,\phi)\right) J\left(x,y',z,\phi^{NF}(x,y',z)\right)\\ &\left.\left.\left. + \mathbb{1}\left(y' \notin \Upsilon^{NF}(x,z,\phi) \cup \Upsilon^{NF}(x,z,\phi)\right) J(x,y',z,\phi)\right]\right\} \frac{\mu_F(\tilde{y},\tilde{z})}{v} d\tilde{y} \right\} p(y'|y)dy'.\end{aligned} \tag{43}$$

Equation (44) presents the workers' valuation of a match, which is the mirror image of the firm's value of a filled job.

$$\begin{aligned}W(x,y,z,\phi) = \phi f(x,y,z) + \beta \int_{y'} \Big\{ &(1-(1-d)A^U(x,y',z))U(x)\\ &+ (1-d)(A^U(x,y',z)) \int_{\tilde{z}} \int_{\tilde{y}} \left\{ \lambda_e \left[\mathbb{1}\left((\tilde{y},\tilde{z}) \in \Upsilon^{EE}(x,y',z)\right) W\left(x,\tilde{y},\tilde{z},\phi^E(x,y',z,\tilde{y},\tilde{z})\right) \right.\right.\\ &+ \mathbb{1}\left((\tilde{y},\tilde{z}) \in \Upsilon^{BC}(x,y',z)\right) W\left(x,y',z,\phi^E(x,\tilde{y},\tilde{z},y',z)\right)\\ &+ (1-\lambda_e \mathbb{1}\left((\tilde{y},\tilde{z}) \in \Upsilon^{BC}(x,y',z) \cup \Upsilon^{EE}(x,y',z)\right)\\ &\times \left[\mathbb{1}\left(y' \in \Upsilon^{NW}(x,z,\phi)\right) W\left(x,y',z,\phi^{NW}(x,y',z)\right)\right.\\ &+ \mathbb{1}\left(y' \in \Upsilon^{NF}(x,z,\phi)\right) W\left(x,y',z,\phi^{NF}(x,y',z)\right)\\ &\left.\left.\left. + \mathbb{1}\left(y' \notin \Upsilon^{NF}(x,z,\phi) \cup \Upsilon^{NF}(x,z,\phi)\right) W(x,y',z,\phi)\right]\right\} \frac{\mu_F(\tilde{y},\tilde{z})}{v} d\tilde{y} \right\} p(y'|y)dy'.\end{aligned} \tag{44}$$

The value function in the main text can be simply derived by using the specific bargaining rules defined in the wage setting mechanism. For deriving the surplus we first use the definition of the surplus $S(x, y) = J(x, y, \phi) - V(y) + W(x, y, \phi) - U(x)$. Then after some simplifications one can arrive at the surplus function:

$$\begin{aligned}
S(x,y,z) =& f(x,y,z) - b(x) + \beta(1-d) \int_{y'} S(x,y',z)^{+} p(y'|y) dy' \\
& - \beta\alpha\lambda_w \int_y \int_z S(x,y,z)^{+} \frac{\mu_F(y,z)}{v} dz dy \\
& - \beta(1-d)\lambda_f \int_{y'} \Bigg(p^u \int_x (1-\alpha) S(x,y',z)^{+} \frac{\mu_x(x)}{u} dx \\
& + (1-p^u) \int_{\tilde{z}} \int_{\tilde{y}} \int_x (S(x,y',z) - S(x,\tilde{y},\tilde{z}))^{+} \frac{\psi^S(x,\tilde{y},\tilde{z})}{e^s} dx d\tilde{y} d\tilde{z} \Bigg) p(y'|y) dy'.
\end{aligned} \tag{45}$$

D.1 Appendix - Definition of Equilibrium

In steady state, all the decisions in the model can be derived from the surplus $S(x, y, z)$, the distribution of workers across job types $\psi(x, y, z)$, the distribution of unemployed $\mu_x(x)$, and the distribution of vacant job types $\mu_f(y, z)$. Consequently, all other variables of interest such as unemployment rate or wages can be derived from those objects.

The surplus function is sufficient to determine all decision regarding worker mobility, and is defined by the unique solution to the contraction mapping in equation (45). The unique steady state equilibrium in this model then consists of the surplus function $S(x, y, z)$, the distribution of workers across job types $\psi(x, y, z)$, the distribution of unemployed $\mu_x(x)$, and the distribution of vacant job types $\mu_f(y, z)$ such that $S(x, y, z)$, $\psi(x, y, z)$, $\mu_x(x)$, and $\mu_f(y, z)$ joint solve equations (25), (26), (28), and (29).

The unemployment rate u is defined by $\int_x \mu_x(x)dx$, the employment rate e as $1 - u$, the vacancy rate as $\int \mu_f(y, z)dydz$. The job-to-job rate is defined as

$$JJr = \frac{\int_z \int_z \lambda_e \int_{\tilde{z}} \int_{\tilde{y}} \mathbb{1}\left(y, z \in \Upsilon^{EE}(x, \tilde{y}, \tilde{z})\right) \psi^s(x, y, z) d\tilde{y} d\tilde{z} \frac{\mu_F(y,z)}{v} d\psi^S(x, y, z)}{\int_z \int_z \psi(x, y, z) dy dz}, \tag{46}$$

The separation rate is given by:

$$Sepr = \frac{\int_x \int_z \int_y \psi(x,y,z) - \psi^s(x,y,z) dy dz dx + \lambda_e \int_{\tilde{z}} \int_{\tilde{y}} \mathbb{1}\left(y, z \in \Upsilon^{EE}(x,\tilde{y},\tilde{z})\right) \psi^s(x,y,z) d\tilde{y} d\tilde{z} \frac{\mu_F(y,z)}{v}}{\int \int \int \psi(x,y,z) dy dz dx} \tag{47}$$

Given the surplus function and the distributions, one can simulate firm size, workforce composition for firms and wages and labor market histories at the individual level for a given sequence of shocks.

E Appendix - Numerical Implementation

I apply the following numerical procedure to solve the model. First, I discretize the state space by using a equidistant grid of 20 worker types and 10 grid points over the support of z and 16 over the support of y. The solution algorithm is the following iterative process:

1. Guess $S^0(x, y, z)$, $\psi^0(x, y, z)$, $\mu_x^0(x)$ and $\mu_F^0(y, z)$
2. Update $S^{i+1}(x, y, z)$ using equation (25)
3. Using the new value of $S(x, y, z)$, update acceptance policies $A^U(x, y, z)$ and $A^E(x, y, z, \tilde{y}, \tilde{z})$. The indicator functions are updated slowly.
4. Update the distributions $\psi(x, y, z)$, $\mu_x(x)$ and $\mu_F(y, z)$ using the updated acceptance policies. The distributions are updated by using the law of motion equations (27), (29) and (26).
5. Compute the sup norm of the absolute values of differences between the iteration outcomes and set $i = i + 1$
6. Repeat steps 2-5 the until the surplus, acceptance strategies and the distributions converged. I use 10^{-6} as the convergence criteria for the surplus and acceptance strategies and 10^{-7} for the distributions.

Due to the discretization, infinitesimal changes in $S(x, y, z)$ lead to discontinuous changes in the distributions of agents. This could cause the algorithm to not converge at the desired convergence criteria. In order to smooth I assume that agents very close to the decision thresholds randomize between acceptance and rejection. I use the following randomization strategies:

$$
\begin{aligned}
A^U(x, y, z) &= \begin{cases} 1 & \text{if } S(x, y, z) \geq 10^{-2} \\ \frac{1-\left(10^{-2}-S(x,y,z)\right)}{10^{-2}} & \text{if } 0 \leq S(x, y, z) < 10^{-2} \\ 0 & \text{if } S(x, y, z) < 0 \end{cases} \\
A^E(x, y, \tilde{y}, z, \tilde{z}) &= \frac{1}{1 + \exp(-100(S(x, \tilde{y}, \tilde{z}) - S(x, y, z)))}
\end{aligned}
$$

These randomizations only affect a tiny fraction of the state space. With the estimated parameters from section 6, only around 5 percent of all possible $A^E(x, y, z, \tilde{y}, \tilde{z})$ and no

$A^U(x, y, z)$ are deviating from 0 or 1 by more than 10^{-6}. Similar smoothing strategies have been applied by Lopes de Melo (2018) and Hagedorn et al. (2017).

After obtaining the equilibrium solutions to value functions, acceptance rules and steady state distributions I simulate the evolution of 6126 firms and 511239 individuals. This exactly corresponds to the sample sizes in the German social security data. I use the stationary distribution as initial conditions and simulate labor market outcomes for 50 years. The first 32 years are burned in, thus the target moments are computed with the data of the remaining 18 years, which corresponds to the time frame of the German social security dataset. The calibration procedure minimizes the average percentage deviation from the target moments. I use the particle swarm optimization method. Swarm size is set to 96. In order to find a good initial swarm, I compute 15000 points using a Sobol sequence and use the best 96 as starting points.

F Additional Model Results

Figure 11: Sorting Pattern

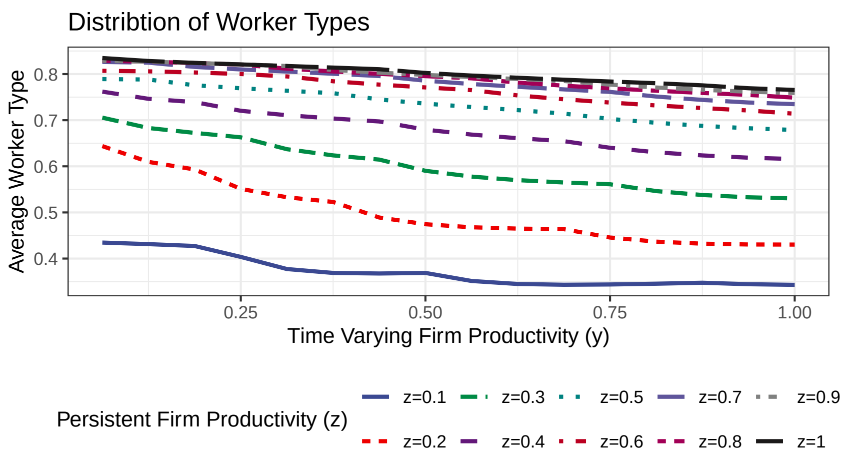

Table 15: Distribution of Years in Sample - Data

Percentile	10	25	50	75	90
Years in Sample	3	7	12	18	18

G Firing Taxes

This section describes the dismissal taxes in detail. The surplus function with firing taxes $\tau(x, y, z)$ is now given by:

$$
\begin{aligned}
S(x,y,z) = & f(x,y,z) - b(x) - \beta d \tau(x,y,z) \\
& - \beta(1-d) \int_{y'} \tau(x,y',z) \mathbb{1}\left(S(x,y',z) \leq -\tau(x,y',z)\right) p(y'|y) dy' \\
& + \beta(1-d) \int_{y'} \mathbb{1}\left(S(x,y',z) > -\tau(x,y',z)\right) S(x,y',z) p(y'|y) dy' \\
& - \beta \alpha \lambda_w \int_y \int_z S(x,y,z)^+ \frac{\mu_F(y,z)}{v} dz dy \\
& - \beta(1-d)\lambda_f \int_{y'} \left(p^u \int_x (1-\alpha) S(x,y',z)^+ \frac{\mu_x(x)}{u} dx \right. \\
& \left. + (1-p^u) \int_{\tilde{z}} \int_{\tilde{y}} \int_x \left(S(x,y',z) - S(x,\tilde{y},\tilde{z})\right)^+ \frac{\psi^S(x,\tilde{y},\tilde{z})}{e^s} dx d\tilde{y} d\tilde{z} \right) p(y'|y) dy'.
\end{aligned} \tag{48}
$$