Wage and Layoff Risk Across Tenure

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1 Introduction

2 Model

2.1 Environment

Time is discrete and indexed by t. The economy is populated by a continuum of firms with measure 1, indexed $j \in [0, 1]$, and workers with measure I, indexed $i \in [0, I]$. Both types of agents are ex ante homogeneous and infinitely lived, with time-separable preferences and a discount factor β. Firms are owned by outside investors, able to diversify any potential risk from firm-level productivity shocks. Thus, in practice, firms maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \pi_{jt}$$

Workers are risk-averse with no access to financial markets. They consume home production b when unemployed and wage w when employed. Their utility is

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}), u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

Production

Firms may pay κ_e to start producing, and will have to pay κ_f every period to stay open. Open firms employ a measure[1] n of workers to produce. Each worker-firm match may be of high

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[1]Law of large numbers thus applies and is extensively used throughout the model

or low quality, fully persistent during existence of the match. Only the firm is aware of the quality of any individual match, but the average match quality in the firm z(equivalently, the proportion of high-quality matches) is common knowledge. Firm production exhibits decreasing returns to scale in size n and potentially quality z[2]. Lastly, production is subject to shocks $y \in \mathcal{Y}$ at the firm level. The overall production function is

$$yF(n,z), \quad F_n', F_z' > 0, F_n'' < 0$$

Labor Market

Every period, firms just entering the market immediately hire $\tilde{n} = 1$ workers. Incumbent firms hire $\tilde{n} \geq 0$ workers. Workers, both employed and unemployed, search for a job. Workers and hiring firms meet in a frictional labor market with directed search, as in Moen (1997). There is a continuum of submarkets indexed by the promised value v owed to the workers. Firms choose in which submarket to post vacancies at the cost c and workers choose where to search. Within each submarket, matches are formed according a constant returns to scale matching function. Due to the CRS nature of the function, tightness of a submarket θ_v is a sufficient statistic for matching probabilities. Denote these probabilities $p(\theta_v), q(\theta_v) \leq 1$ for a worker and a vacancy, respectively.

Firms are not restricted in fielding a discrete number of vacancies, and thus can deterministically hire $\tilde{n}$ workers from submarket v at the cost $\tilde{n}\frac{c}{q(\theta_v)}$. The average quality of these workers is always z_0. Upon hiring these workers, the firm commits to delivering expected discounted utility v. The core trade-off in firm hiring is between the cost of hiring $\frac{c}{q(\theta_v)}$ and the cost of employing the worker, which increases with v. Firms are capable of downsizing via layoffs s and by incentivizing incumbent workers to find jobs elsewhere.

I consider two cases of this economy: in the steady-state or, under an additional assumption (see Appendix A.5), in a Block-Recursive equilibrium (following Menzio and Shi (2011) and Schaal (2017)). Either way, agents do not need to keep track of the aggregate distribution of the economy.

Timing

Each period is divided into 4 stages, as illustrated in Figure 1. First, production takes place. Firm collects the output and pays wage w to each worker it employs. Next firm lays off a fraction $s \geq 0$ of its workforce. Fired workers become unemployed, but may not search until the next period. Remaining workers, both employed and unemployed, then search for a job.

[2] In the quantitative exploration, I will restrict attention to decreasing returns in quality-adjusted quantity $g(z)n$

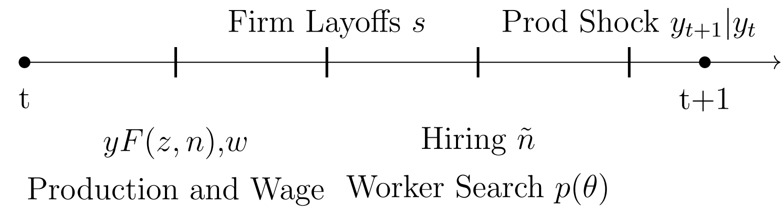

Figure 1: Within-period time line

This coincides with all the firm hiring $\tilde{n}$, from both entering and incumbent firms. All the hiring and search choices happen before the next prod-ty y_{t+1} realizes and thus have to rely on the expectation operator $E_{y_{t+1}|y_t}$.

Information Structure and Contracts

Upon hiring a worker, the firm commits to deliver expected utility v via a contract. A contract defines the wage and actions for a matched worker and firm for all future firm productivity histories $y^\tau \equiv (y_1, ..., y_\tau)\mathcal{Y}^\tau$. The future history of firm productivity is common knowledge to both agents and is thus fully contractible. However, the match-specific production z_{ij} is private information of the firm and worker's search decision $\hat{v}$ is private information of the worker. The contract $\mathcal{C}$ is then represented by

$$\mathcal{C} = \{w_\tau, s_\tau, \hat{v}_\tau\}_{\tau=t}^{\infty} \tag{1}$$

The first components captures firm's wage policy w for each future productivity history. The second component captures *expected* layoff probability s, given that the worker does not know the quality of their match. Note that these probabilities are not ex-ante: in histories where worker's information about their match quality updates, it will be reflected in all the corresponding layoff probabilities. An example of that could be a history where firm has faced multiple negative productivity shocks and was forced to lay off a vast majority of its workforce. The remaining workers will bayesian update that their match quality is now more likely to be high, and any future layoff probabilities will reflect that. The last component is worker's search decision. Although this action is unobserved by the firm, I focus on contracts where the contractual recommendations are incentive-compatible. The firm thus chooses workers' search decisions, subject to the incentive compatibility constraint that the decisions match the workers' optimal response.

The contract space is completely flexible in how wages and layoffs respond to productivity histories. In a setting with a continuum of contracts at the same time, this allows the firm to choose how to treat its heterogeneous (in quality and contracts) workforce: when a negative shock hits, who to fire and for whom to cut wages. This property is central to the paper and unique to the setting: unlike model with CRS production functions, these decisions depend on the state of the entire firm. Unlike other models of firm dynamics, with Nash Bargaining (McCrary (2022)) or Sequential Bargaining (Bilal et al. (2022)), the workers in the same firm may end up with different wages, layoffs, and their responses to productivity shocks. By taking the model to the data, I are able to quantify how different firms, depending on their size and productivity, discriminate between their junior and senior workers.

2.2 Value functions

The above contract, and thus the problems of all the agents, can be described recursively. I start with the individual workers' problem and move on to firms managing contracts with a continuum of workers. I show that the state-space of the firm problem is discrete and, under a relatively weak approximation, bounded.

Worker's Problem

Unemployed workers consume home production b. Each period, they search on the submarket that offers the best tradeoff between promised future utility and job finding probability. Dropping all time subscripts and focusing on a stationary equilibrium, the value of being unemployed U can be written as:

$$U = \max_{v} u(b) + \beta[(1 - p(\theta_v))U + p(\theta_v)v] \tag{2}$$

Consider an employed worker with an owed value v. Suppose a firm pays wage w this period, will fire with probability s and offers a lifetime expected utility v' from tomorrow into the future. Then a worker faces the following search problem:

$$v = \max_{\hat{v}} u(w) + \beta[sU + (1 - s)[(1 - p(\theta_{\hat{v}}))v' + p(\theta_{\hat{v}})\hat{v}]] \tag{3}$$

The optimal worker policy $\hat{v}$ depends only on the future offered utility v'. By raising v', firm incentivizes its worker to search in higher $\hat{v}$, thus lowering the probability that the worker will leave. Note that this can be equivalently rewritten as

$$v = u(w) + \beta[sU + (1 - s)R(v')]$$

where $R(v') \equiv \max_{\hat{v}}[(1 - p(\theta_{\hat{v}}))v' + p(\theta_{\hat{v}})\hat{v}]$ the optimal future value the worker gets upon being promised v' and not being laid off.

Firm's Problem

For now, consider the version of the model with no match heterogeneity ($z_0 = 0$ or 1). A firm employs a measure n of workers. Denote the distribution of their promised values v that the firm owes to its workers as $P(v)$. Then for each of these values v, the firm has to choose the wage to pay w_v, the layoff rate s_v, and tomorrow, future productivity state-contingent promised value $v'_{v,y'}$. Firm may also hire $\tilde{n}$ workers at the value $\tilde{v}$. The firm's problem can then be formulated recursively as follows:

$$J(y,n,P(v)) = \max_{\tilde{n},\tilde{v},\{w_v,s_v,v'_{v,y'}\}} yF(n,z_0) - \int_v w_i dP(v_i) - \tilde{n}\frac{c}{q(\tilde{v})} - \kappa_f + \beta E_{y'|y} J(y',n',P'(v))$$

$$\begin{aligned} s.t.\ & u(w_v) + \beta[s_v U + (1-s_v)R(v'_v) = v]\ \forall v \\ & v'_v = E_{y'|y} v'_{v,y'}\ \forall v \\ & n' = n\int_v (1-s_v)(1-p(v'_v))dP(v) + \tilde{n} \\ & n'P'(v) = n\int_v E_{y'|y}\mathbb{1}_{v'_{v,y'}\le v}(1-s_v)(1-p(v'_v))dP(v) + \mathbb{1}_{\tilde{v}\le v}\tilde{n} \end{aligned}$$

Firm has to maximize its net present value of profits subject to fulfilling every worker's promised value. Note that because the search decision happens before the next productivity state realizes, workers only care about the expected average promised value v'_v when making their decisions rather than about any of the $v'_{v,y'}$ in particular. The latter two conditions specify the law of motion for firm size and the distribution of promised values.

Discretizing the Problem In the current formulation, this problem is intractable as it involves a probability distribution, an uncountably infinitely-dimensional object, in the state-space. I first show that this state space can be discretized, thus bringing it "down" to countably infinite states. Then I explain that,under a weak approximation, the state space is actually bounded.

First, note that, when hiring, a firm chooses just one value at which to hire, $\tilde{v}$. This is an outcome of the directed structure of the labor market. Since a firm optimally decides at which submarket v to post vacancies in [3], it will only hire from that submarket and thus at that value. This means that all the workers hired at the same time by the same firm are going to be at the same value, both at the time of hiring and in all the future periods. Therefore, it is equivalent to work with the cdf $P(v)$ or with the related probability mass function $\mathbb{P}(V = v)$: $P(v) = \sum_{v' \le v} \mathbb{P}(V = v')$. Furthermore, for a firm of age $K < \infty$, there is at most K different values v such that $\mathbb{P}(V = v) > 0$. These values correspond to the

[3] Assume no mixed strategies: in case a firm happens to be indifferent across multiple submarkets, it will only post vacancies in one of them.

values owed to workers hired at different time periods, thus of different tenure at the firm $k = t - t_{hired} \leq K$. One can then redefine the state space using tenure:

Lemma 1. *A decision problem $J(y, n, P(v))$ of a firm of age K can be equivalently represented as*

$$
\begin{aligned}
J(y, \{n_k, v_k\}_{k \leq K}) = \max_{\tilde{n}, \tilde{v}, \{v'_{y',k}, w_k, s_k\}_{k \leq K}} & yF(\sum_k n_k, z_0) - \sum_k w_k n_k \\
& - \tilde{n}\frac{c}{q(\tilde{v})} - \kappa_f + \beta E_{y'|y} J(y', \{n'_k, v'_k\}_{k \leq K+1}) \\
s.t.\ & u(w_k) + \beta[s_k U + (1 - s_k) R(v'_{k+1}) = v_k]\ \forall k \leq K \\
& v'_{k+1} = E_{y'|y} v'_{k+1,y'}\ \forall k \leq K \\
& n'_{k+1} = n_k(1 - s_k)(1 - p(v'_{k+1})) + \tilde{n}\ \forall k \leq K \\
& n'_0 = \tilde{n}, v'_0 = \tilde{v}
\end{aligned}
$$

Contracts Converge This tenure-based formulation of the problem allows me to work with a discrete, although expanding, state-space. Finally, to make this fully tractable, I note that wages in contracts tend to converge, suggesting that $v_k \approx v_{k+1} \forall k \geq \bar{k}$. I derive this result in the following propositions. The idea follows a theoretical result from Balke and Lamadon (2022)'s Proposition 3 that wages in dynamic contracts tend to always follow a "target wage". I adapt their result to my model with a continuum of contracts. Empirically, I show in Appendix B.1 that wage growth in France stagnates after about 10 years of tenure. This allows me to restrict attention to finite and constant K for all the firms in my quantitative exploration. Note that this is only an approximation since, as $K \to \infty$, the model approaches the problem described in Lemma 1.

To show that the wages converge, I start by characterizing the wage growth.

Proposition 1. *For any current state $(y, \{n_k, v_k, z_k\})$, wages change according to the following relationship:*

$$
\frac{1}{u'(w'_{k+1})} - \frac{1}{u'(w_k)} = \eta(v'_{k+1}) E_{y'|y} \frac{\partial J(y', \{n'_k, v'_k, z'_k\})}{\partial n'_{k+1}} \tag{4}
$$

where $\eta(v'_{k+1}) = \frac{\partial log(1-p(v'))}{\partial v'}$ is the semi-elasticity of the job-finding probability with respect to the promised value v'_{k+1}.

Proof. See Appendix A.1. □

The relationship is at the core of the firm's insurance vs incentive provision trade-off: while the marginal value of the worker $\frac{\partial J(y', \{n'_k, v'_k, z'_k\})}{\partial n'_{k+1}}$ is positive, the firm is intent on keeping its workers, and thus chooses to backload wages, thus incentivizing the workers to stay at the cost of higher total wage payments. On the flip side, if the marginal worker value is negative, the firm will choose to lower wages, incentivizing workers to leave. This brings us to the result on contract convergence:

Proposition 2. *Fix firm's state* $(y, \{n_k, v_k\}) \equiv (y, n, P(v))$ *and define* v^* *such that* $E_{y'|y}\frac{\partial J(y',n',P(v'))}{\partial P(v^*)} = 0$. *Then*

$$|w'_{k+1} - w^*_{v^*}| < |w_k - w^*_{v^*}| \; \forall k$$

Moreover, defining $\bar{k} \equiv arg\max_k v_k$,

$$w'_{\bar{k}+1} - w'_{k+1} < w_{\bar{k}} - w_k \; \forall k \neq \bar{K}$$

Proof. See Appendix A.1 □

This proposition suggests that, across all tenure steps, wages change in the direction of the target wage $w^*_{v^*}$. Moreover, they do so in a way that contracts the incumbent wage space, lowering the difference between the highest and all other wages. Then, for workers that have been at the firm for long enough, this process of constant movement towards the target wage will result in convergence of wages.

Introducing Heterogeneity I stick to the theoretical formulation in Lemma 1 and introduce heterogeneity in match quality. With $0 < z_0 < 1$, matches employed by the firm may be of both high and low quality. I do not allow the firm to choose quality-contingent wages, thus the firm can only influence match quality via layoffs [4].

Although any particular worker does not know their own match quality, they know the proportion of high quality matches in their cohort. All new hires start with a proportion z_0 of high matches, and, although it may evolve, all the workers of the same tenure k will have the same probability of having a high quality match $z_k \geq z_0$. This probability is common knowledge to the firm and all its workers and depends on layoffs in the corresponding cohort to $z'_{k+1} = min(\frac{z_k}{1-s_k}, 1) \; \forall k \leq K$.

[4] I show in Appendix A.3 that, under a sufficiently low elasticity of on-the-job search, this is in fact an outcome of the firm's optimal information allocation problem.

Definition 1. *The complete firm decision problem involves*

$$
\begin{aligned}
J(y, \{n_k, v_k, z_k\}_{k\leq K}) = \max_{\tilde{n}, \tilde{v}, \{v'_{y',k}, w_k, s_k\}_{k\leq K}} & yF(\sum_k n_k, \frac{\sum n_k z_k}{\sum n_k}) - \sum_k w_k n_k \\
& - \tilde{n}\frac{c}{q(\tilde{v})} - \kappa_f + \beta E_{y'|y} J(y', \{n'_k, v'_k, z'_k\}_{k\leq K+1}) \\
s.t.\ & u(w_k) + \beta[s_k U + (1-s_k)R(v'_{k+1}) = v_k]\ \forall k \leq K \\
& v'_{k+1} = E_{y'|y} v'_{k+1,y'}\ \forall k \leq K \\
& n'_{k+1} = n_k(1-s_k)(1-p(v'_{k+1})) + \tilde{n}\ \forall k \leq K \\
& z'_{k+1} = min(\frac{z_k}{1-s_k}, 1)\ \forall k \leq K \\
& n'_0 = \tilde{n}, v'_0 = \tilde{v}, z'_0 = z_0
\end{aligned}
$$

Free-entry and exit

Firms are free to enter the market and start producing upon paying an entry cost κ_e. Upon entry, firms draw a productivity shock and start with a single worker. Thus, the free-entry condition pins down the expected profits of firms upon entry:

$$
\kappa_e \geq \max_{v_0} -\frac{c}{q_{v_0}} + \beta E_y J(y, \{1, 0, ...\}, \{v_0, ...\}, \{z_0, ...\}) \tag{5}
$$

When taking the model to the data, this results in a negative connection between the cost of entry κ_e and the probability to fill a vacancy $q(\theta)$: the cheaper it is to enter, the tighter will the labor market be, thus making it easier for workers to find jobs and harder for firms to fill vacancies.

Similarly to new firms, incumbent firms have to pay an operating cost κ_f every period to stay open. With κ_f already included into the firm value function, firms stay open if

$$
J(y, \{n_k\}, \{v_k\}, \{z_k\}) \geq 0 \tag{6}
$$

2.3 Equilibrium

A complete equilibrium is a set of value functions, policies, matching rates, and distribution of workers and firms for each labor market v such that

- Firms solve the problem from Definition 1
- Workers solve search problems from Equations 2 and 3
- Free-entry and free exit conditions 5,6 are satisfied

- Job-finding and vacancy-filling probabilities are consistent with the matching function
- Tightness function θ_v is consistent with the firm posting and worker search strategies
- Labor market clears

Under an additional assumption in Appendix A.5, I show that the equilibrium may be block recursive, meaning independent of the distribution of workers and firms. I use that assumption in the quantitative exploration, but not in the theoretical discussion, where instead I focus my attention on the steady-state of the economy described above.

2.4 Model Mechanism

I now show how the model delivers the heterogeneity in wage and layoff passthrough across tenure. To do this, I first characterize the FOC for layoffs:

Proposition 3. *Fix a firm state $(y, \{n_k, v_k, z_k\})$, layoffs then follow:*

3 Conclusion

References

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A Model Appendix

A.1 Proofs

Proof of Proposition 4

Proof of Proposition 2

A.2 Tenure-specific Severance Payments

I allow the firm to offer tenure-specific severance payments sev_k to its workers. The severance is constant over time and paid perpetually upon firing and before finding a new job. I show that the severance structure involves higher payments for longer tenured workers (if those workers are on a higher promised value).

Proposition 4. *Fix a firm state. Its severance payments for each tenure k are given by*

$$\frac{u'(b+sev_k)}{u'(w_k)} = 1 - \frac{\beta sev_k \frac{\partial p(\theta_{sev_k})}{\partial sev_k}}{1-\beta(1-p(\theta_{sev_k}))}$$

$$\theta_{sev_k} = \theta(arg\max_v[(1-p(v))U(sev_k)+p(v)v])$$

Proof. I start by describing the unemployment value of a worker with severance payment sev_k:

$$U(sev_k) = u(b + sev_k) + \beta \max_v [(1 - p(\theta_v))U(sev_k) + p(\theta_v)v]$$

Denote the probability of finding a job with severance payment sev_k as $p(\theta_{sev_k})$. The extra value to the unemployed from the severance payment is then given by

$$\frac{\partial U(sev_k)}{\partial sev_k} = u'(b + sev_k) + \beta(1 - p(\theta_{sev_k}))U'(sev_k) = \frac{u'(b + sev_k)}{1 - \beta(1 - p(\theta_{sev_k}))}$$

Then the total benefit to the firm from raising the severance payment is the slackening of the promised-keeping constraint thanks to this rise in the unemployment value:

$$\lambda_k n_k \beta s_k \frac{\partial U(sev_k)}{\partial sev_k} = \frac{n_k}{u'(w_k)} \beta s_k \frac{u'(b + sev_k)}{1 - \beta(1 - p(\theta_{sev_k}))}$$

On the cost side, the firm internalizes the net present value of the severance payments when firing $n_k s_k$ workers:

$$\frac{\partial}{\partial sev_k} \left[n_k s_k \beta \frac{sev_k}{1 - \beta(1 - p(\theta_{sev_k}))} \right] = n_k s_k \beta \frac{[1 - \beta(1 - p(\theta_{sev_k}))] - \beta sev_k \frac{\partial p(\theta_{sev_k})}{\partial sev_k}}{[1 - \beta(1 - p(\theta_{sev_k}))]^2}$$

The optimal severance payment then follows from the first-order condition:

$$\frac{n_k}{u'(w_k)} \beta s_k \frac{u'(b + sev_k)}{1 - \beta(1 - p(\theta_{sev_k}))} = n_k s_k \beta \frac{[1 - \beta(1 - p(\theta_{sev_k}))] - \beta sev_k \frac{\partial p(\theta_{sev_k})}{\partial sev_k}}{[1 - \beta(1 - p(\theta_{sev_k}))]^2}$$

Rearranging gives the result.

$$\frac{u'(b + sev_k)}{u'(w_k)} = 1 - \frac{\beta sev_k \frac{\partial p(\theta_{sev_k})}{\partial sev_k}}{1 - \beta(1 - p(\theta_{sev_k}))}$$

□

Note that, besides $u'(w_k)$, all the components of the severance payment are independent of both the firm state and the worker tenure. It is immediate to notice then that higher paid workers will have higher severance payments: as $\frac{1}{u'(w_k)}$, the value to the firm of the severance payment goes up, while costs stay the same. Therefore, the firm will optimally choose to offer higher severance payments to higher paid workers.

This equation is also easy to implement numerically: using the formulation in Appendix A.4, where $\rho_k \equiv u'(w_k)$ is a state variable, I can immediately compute the payments for all the firm states, before solving the rest of the firm problem.

A.3 Microfounding the Wage Noncontractability

Consider a simplified, 2-period version of the model. The firm starts with measure $n = 2$ of workers, half of them of high quality and the other half of low. I allow the firm to offer quality-contingent wages in whichever way it likes.

I show that, under a sufficiently small elasticity of job search probability with respect to promised value v', $\eta(v') \equiv \frac{\partial(1-p(v'))/\partial v'}{(1-p(v'))}$, the firm will optimally choose to keep wages constant across matches of different quality.

A.4 Recursive Lagrangian Approach

The original design of the problem would require solving promised values $v'_{y',k}$ for both each tenure step and each future productivity state. Following Balke and Lamadon (2022), I solve the following Pareto problem:

$$\begin{aligned}\mathcal{P}(y, \{n_k, \rho_k, z_k\}) = &\inf_{\omega_k} \sup_{\tilde{n}, \tilde{v}, \{w_k, s_k, v'_k\}} yF(n, z) - \sum_k n_k w_k - \kappa_f - \tilde{n}\frac{c}{q(\theta_{\tilde{v}})} \\ &+ \sum_k \rho_k n_k (u(w_k) + \beta[s_k U + (1-s_k)R(v'_{k+1})] \\ &- \beta \sum_k \omega_k n'_{k+1} v'_{k+1} + \beta E_{y'|y} \mathcal{P}(y', \{n'_k, \omega_k, z'_k\})\end{aligned}$$

where

$$\mathcal{P}(y, \{n_k, \rho_k, z_k\}) \equiv \sup_{\{v_k\}} J(y, \{n_k, v_k, z_k\}) + \sum_k \rho_k n_k v_k$$

The following proof (for $K \to \infty$ but the proof extends trivially to finite K) establishes its equivalence with the initial problem. It follows the steps of Balke and Lamadon (2022), extending it to the case of a multi-worker firm.

Proof. We have the following recursive formulation for J:

$$\begin{aligned}J(y, \{n_k, v_k, z_k\}_{k \le K}) = &\max_{\tilde{n}, \tilde{v}, \{v'_k, v'_{y',k}, w_k, s_k\}_{k \le K}} yF(\sum_k n_k, \frac{\sum n_k z_k}{\sum n_k}) - \sum_k w_k n_k - \tilde{n}\frac{c}{q(\tilde{v})} - \kappa_f \\ &+ \beta E_{y'|y} J(y', \{n'_k, v'_k, z'_k\}_{k \le K+1}) \\ &(\lambda_k)\, u(w_k) + \beta[s_k U + (1-s_k)R(v'_{k+1}) = v_k] \, \forall k \le K \\ &(\omega_k) v'_{k+1} = E_{y'|y} v'_{k+1,y'} \, \forall k \le K \\ &n'_{k+1} = n_k(1-s_k)(1-p(v'_{k+1})) + \tilde{n} \, \forall k \le K \\ &z'_{k+1} = min(\frac{z_k}{1-s_k}, 1) \, \forall k \le K \\ &n'_0 = \tilde{n}, v'_0 = \tilde{v}, z'_0 = z_0\end{aligned}$$

Consider the Pareto problem

$$\mathcal{P}(y,\{n_k,\rho_k,z_k\}) = \sup_{\{v_k\}} J(y,\{n_k,v_k,z_k\}) + \sum_k \rho_k n_k v_k$$

I first substitute the definition of J together with its constraints into $\mathcal{P}$:

$$\begin{aligned}
\mathcal{P}(y,\{n_k,\rho_k,z_k\}) = &\sup_{\tilde{n},\tilde{v},\{v_k,v'_k,v'_{y',k},w_k,s_k\}_{k\leq K}} yF(\sum_k n_k, \frac{\sum n_k z_k}{\sum n_k}) - \sum_k w_k n_k - \tilde{n}\frac{c}{q(\tilde{v})} - \kappa_f \\
&+ \beta E_{y'|y} J(y',\{n'_k,v'_k,z'_k\}_{k\leq K+1}) + \sum_k \rho_k n_k v_k \\
&(\lambda_k)\ u(w_k) + \beta[s_k U + (1-s_k)R(v'_{k+1}) = v_k]\ \forall k\leq K \\
&(\omega_k)\ v'_{k+1} = E_{y'|y} v'_{k+1,y'}\ \forall k \leq K \\
&n'_{k+1} = n_k(1-s_k)(1-p(v'_{k+1})) + \tilde{n}\ \forall k\leq K \\
&z'_{k+1} = min(\frac{z_k}{1-s_k},1)\ \forall k\leq K \\
&n'_0 = \tilde{n}, v'_0 = \tilde{v}, z'_0 = z_0
\end{aligned}$$

I now substitute in the promise-keeping constraint:

$$\begin{aligned}
\mathcal{P}(y,\{n_k,\rho_k,z_k\}) = &\sup_{\tilde{n},\tilde{v},\{v'_k,v'_{y',k},w_k,s_k\}_{k\leq K}} yF(\sum_k n_k, \frac{\sum n_k z_k}{\sum n_k}) - \sum_k w_k n_k - \tilde{n}\frac{c}{q(\tilde{v})} - \kappa_f \\
&+ \beta E_{y'|y} J(y',\{n'_k,v'_k,z'_k\}_{k\leq K+1}) + \sum_k \rho_k n_k (u(w_k) + \beta[s_k U + (1-s_k)R(v'_{k+1})]) \\
&(\omega_k)\ v'_{k+1} = E_{y'|y} v'_{k+1,y'}\ \forall k \leq K \\
&n'_{k+1} = n_k(1-s_k)(1-p(v'_{k+1})) + \tilde{n}\ \forall k\leq K \\
&z'_{k+1} = min(\frac{z_k}{1-s_k},1)\ \forall k\leq K \\
&n'_0 = \tilde{n}, v'_0 = \tilde{v}, z'_0 = z_0
\end{aligned}$$

I introduce the ω_k-constraints with weights $\beta n'_{k+1}$ into the problem:

$$\begin{aligned}
\mathcal{P}(y,\{n_k,\rho_k,z_k\}) = &\inf_{\{\omega_k\}} \sup_{\tilde{n},\tilde{v},\{v'_k,v'_{y',k},w_k,s_k\}_{k\leq K}} yF(\sum_k n_k, \frac{\sum n_k z_k}{\sum n_k}) - \sum_k w_k n_k - \tilde{n}\frac{c}{q(\tilde{v})} - \kappa_f \\
&+ \beta E_{y'|y} J(y',\{n'_k,v'_k,z'_k\}_{k\leq K+1}) + \sum_k \rho_k n_k (u(w_k) + \beta[s_k U + (1-s_k)R(v'_{k+1})]) \\
&+ \sum_k \beta\omega_k n'_{k+1}(E_{y'|y} v'_{y',k+1} - v'_{k+1}) \\
&n'_{k+1} = n_k(1-s_k)(1-p(v'_{k+1})) + \tilde{n}\ \forall k\leq K \\
&z'_{k+1} = min(\frac{z_k}{1-s_k},1)\ \forall k\leq K \\
&n'_0 = \tilde{n}, v'_0 = \tilde{v}, z'_0 = z_0
\end{aligned}$$

I then rearrange the value function by moving $E_{y'|y}\sum_k \beta\omega_k n'_{k+1}v'_{y',k+1}$ (additional constraints are dropped to simplify notation):

$$\begin{aligned}\mathcal{P}(y,\{n_k,\rho_k,z_k\}) = \inf_{\{\omega_k\}} \sup_{\tilde{n},\tilde{v},\{v_k,v'_{y',k},w_k,s_k\}_{k\leq K}} & yF(\sum_k n_k, \frac{\sum n_k z_k}{\sum n_k}) - \sum_k w_k n_k - \tilde{n}\frac{c}{q(\tilde{v})} - \kappa_f \\ & + \beta E_{y'|y}[J(y',\{n'_k,v'_k,z'_k\}_{k\leq K+1}) + \sum_k \omega_k n'_{k+1}v'_{y',k+1}] \\ & \sum_k \rho_k n_k(u(w_k) + \beta[s_k U + (1-s_k)R(v'_{k+1})]) - \sum_k \beta\omega_k n'_{k+1}v'_{k+1}\end{aligned}$$

Lastly, I split the sup:

$$\begin{aligned}\mathcal{P}(y,\{n_k,\rho_k,z_k\}) = \inf_{\{\omega_k\}} \sup_{\tilde{n},\tilde{v},\{v'_k,w_k,s_k\}_{k\leq K}} & yF(\sum_k n_k, \frac{\sum n_k z_k}{\sum n_k}) - \sum_k w_k n_k - \tilde{n}\frac{c}{q(\tilde{v})} - \kappa_f \\ & + \beta E_{y'|y}[\sup_{v'_{y',k+1}} J(y',\{n'_k,v'_k,z'_k\}_{k\leq K+1}) + \sum_k \omega_k n'_{k+1}v'_{y',k+1}] \\ & \sum_k \rho_k n_k(u(w_k) + \beta[s_k U + (1-s_k)R(v'_{k+1})]) - \sum_k \beta\omega_k n'_{k+1}v'_{k+1}\end{aligned}$$

From this, one can note that, by definition of $\mathcal{P}$

$$\sup_{v'_{y',k+1}} J(y',\{n'_k,v'_k,z'_k\}_{k\leq K+1}) + \sum_k \omega_k n'_{k+1}v'_{y',k+1} = \mathcal{P}(y',\{n'_k,\omega_k,z'_k\}$$

We thus arrive to the formulationof the problem as described at the beginning, not involving finding future state-specific promised values $v'_{y',k}$. □

A.5 Block Recursivity

I introduce an assumption that would allow for a block recursive eqiulibrium under the same conditions as in Schaal (2017). Block recursivity requires an indifference condition, either on the side of the firms or on the side of the workers. Under two-sided ex-post heterogeneity, that is not immediately achievable.

Schaal (2017) shows that, in a setting similar to mine, but with transferable utility between workers and firms, which he achieves due to the risk-neutral worker utility function, firms all have the same preferences across all the submarkets that they may post vacancies in. Define the minimal hiring cost as

$$k = \min_v [v + \frac{c}{q_v}]$$

Due to transferable utility, the cost of employing the worker from submarket v becomes simply the value v. Thus, the optimal entry of vacancies in Schaal (2017) can be summarized by

$$\theta_v[v + \frac{c}{q_v} - k] = 0$$

Meaning that either a submarket v minimizes the hiring cost or it is closed. This condition is completely independent of the distribution of firms and workers, exactly because the one component where the firm type might come through, the cost of employing a worker from submarket v, is completely independent from the firm's state due to transferable utility.
Utility is not transferable in my model, and thus different firms may face different costs of employing a worker at some value v (for example, fixing y and z, small firms prefer high values v due to their intention to upsize). To get around that, I split the value v that the worker would get upon getting hired into two components, the sign-on wage w_v and the remaining value v_0 such that

$$u(w_v) + \beta v_0 = v$$

This additional wage payment is incurred immediately upon hiring, allowing the remaining value that the firm owes to its worker, v_0, to be completely independent of the submarket v. Essentially, from the firm's perspective, submarkets now differ not in the value that firms would owe to the workers, but in this sign-on wage. The cost minimization problem then becomes

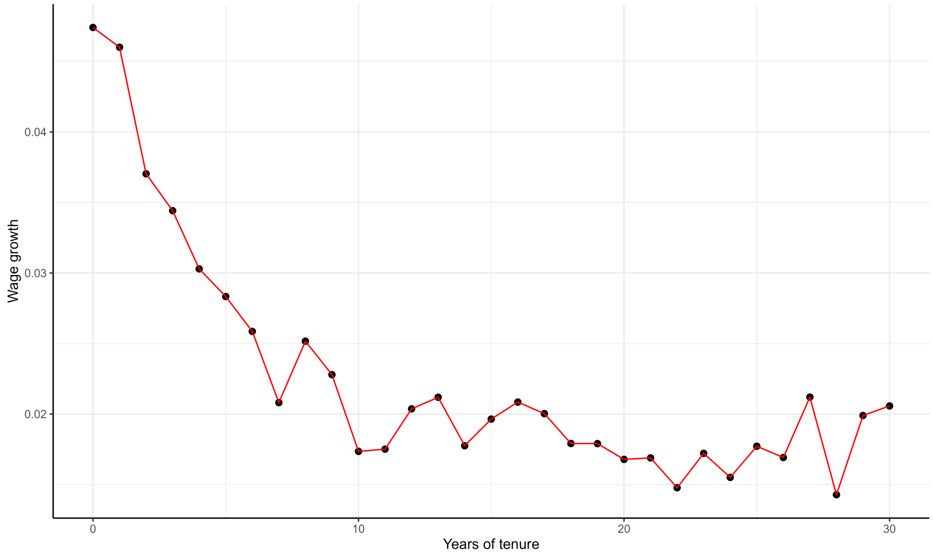
$$k = \min_v [w_v + \frac{c}{q_v}]$$

This problem is now again completely independent of the firm's state, and thus the distribution of firms and workers no longer affects the tightness function q_v.Schaal (2017) shows that, in a setting similar to mine, but with transferable utility between workers and firms, which he achieves due to the risk-neutral worker utility function, firms all have the same preferences across all the submarkets that they may post vacancies in. Then setting θ_v such that

B Data Appendix

B.1 Wage Growth

I use the same sample to plot the log (real) wage growth across first 30 years of tenure.



The wage growth appears to flatten after about 10 years of tenure, suggesting that it is not quantitatively costly to use $K = 10$ as an approximation of the firm problem from Definition 1.