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UNCERTAINTY AND UNEMPLOYMENT

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This paper studies the impact of time-varying idiosyncratic risk at the establishment level on unemployment fluctuations over 1972–2009. I build a tractable directed search model with firm dynamics and time-varying idiosyncratic volatility. The model allows for endogenous separations, entry and exit, and job-to-job transitions. I show that the model can replicate salient features of the microeconomic behavior of firms and that the introduction of volatility improves the fit of the model for standard business cycle moments. In a series of counterfactual experiments, I show that time-varying risk is important to account for the magnitude of fluctuations in aggregate unemployment for past U.S. recessions. Though the model can account for about 40% of the total increase in unemployment for the 2007–2009 recession, uncertainty alone is not sufficient to explain the magnitude and persistence of unemployment during that episode.

KEYWORDS: Uncertainty, volatility, search-and-matching, business cycles.

1. INTRODUCTION

THE RECESSION THAT FOLLOWED the 2007–2008 collapse of the financial markets resulted in one of the deepest downturns in post-war U.S. labor markets. While GDP contracted by up to 6.8% in the fourth quarter of 2008, the unemployment rate grew from 5% in January 2008 to 10% in October 2009 according to the Bureau of Labor Statistics (BLS).

A large and growing body of literature has advanced the hypothesis that the heightened level of uncertainty over the period 2007–2012, as suggested by various measures at the macro[1] and micro[2] levels, may be partly responsible for the unusual magnitude and persistence of the slump. This paper examines to what extent fluctuations in uncertainty over the business cycle can shed light on the U.S. labor market experience over various past recessions, including the Great Recession of 2007–2009.

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[1]Some typical measures of aggregate uncertainty are the volatility of aggregate TFP as measured by a GARCH model, aggregate stock market volatility, survey-based measures of disagreement in forecasts, or ex ante forecast errors over aggregate variables such as output or inflation. Other more recent contributions include the measures proposed by Jurado, Ludvigson, and Ng (2015) using common factor analysis or the news-based index of economic and policy uncertainty of Baker, Bloom, and Davis (2016).

[2]Measures of micro-level risk suggested in the literature include the volatility of establishment-level TFP using Census data (Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), Kehrig (2015)), the cross-sectional dispersion of various firm-level proxies such as sales growth rates (Bloom (2009)), prices (Vavra (2014)), employment growth (Bachmann and Bayer (2013)), business forecasts (Bachmann, Elstner, and Sims (2013)), and stock prices (Campbell, Lettau, Malkiel, and Xu (2001)), including the implied stock market volatility as captured in the VIX/VXO series (Bloom (2009)). Figure 16 in the Supplemental Material (Schaal (2017)) compares the VIX/VXO series and sales growth dispersion from Compustat to the volatility of plant-level TFP from Bloom et al. (2012).

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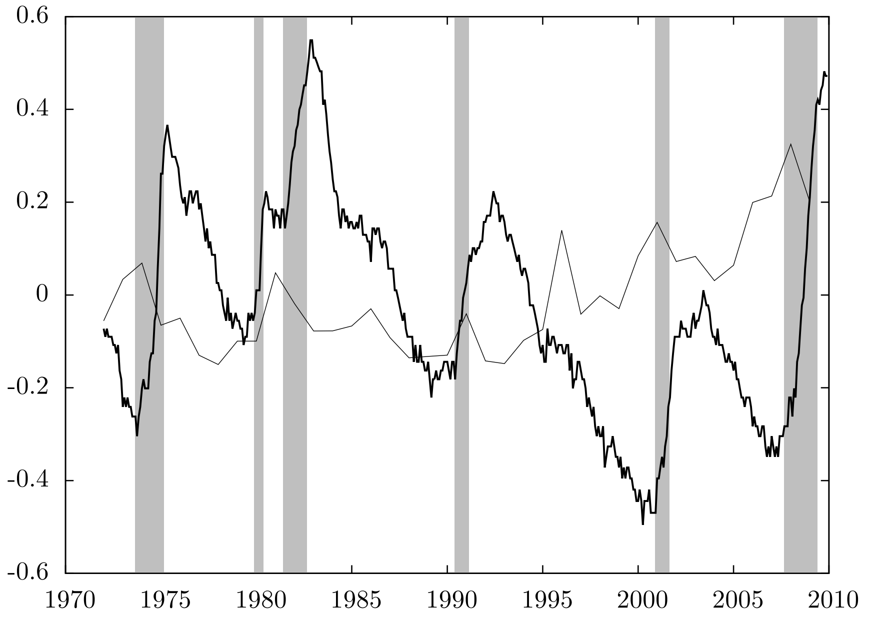

FIGURE 1.—Unemployment and establishment-level volatility in TFP. *Notes*: Data are shown in log deviations from their long-run averages. The thick curve shows seasonally adjusted civilian unemployment from the BLS; the thin curve displays the interquartile range of establishment-level TFP shocks constructed by Bloom et al. (2012) from the Census of Manufactures and the Annual Survey of Manufactures. Shaded areas correspond to NBER recessions. See Appendix B for details.

Uncertainty is a broad concept that encompasses notions as diverse as risk and ambiguity. This paper focuses on a particular source of uncertainty, namely, time-varying establishment-level volatility in TFP.[3] While being largely overlooked in labor market studies, fluctuations in micro-level volatility are large in the data. This suggests that volatility may be an important determinant of employment decisions and labor market reallocation. Figure 1 compares the evolution of the U.S. unemployment rate to a measure of establishment-level volatility constructed from Census data by Bloom et al. (2012) over 1972–2009. Using annual input-output data from the Census of Manufactures and the Annual Survey of Manufactures, this series presents the cross-sectional interquartile range (IQR) of innovations to establishment-level TFP, estimated from an AR(1) process.[4] Two important facts can be drawn from that figure. First, as it has been widely noted in previous literature using other measures of micro-level volatility,[5] idiosyncratic risk is countercyclical and rises during recessions. This particular measure peaks early as the economy enters a downturn and then declines relatively quickly as the recession unfolds. Second, fluctuations in idiosyncratic risk are large, reaching peaks as high as 30% above its long-run average at the end of 2007.

[3]In particular, I do not consider macro-level risk. A previous version of this paper experimented with the effects of time-varying volatility in aggregate TFP. However, because volatility in aggregate TFP is small in the data, its quantitative impact was negligible, while significantly increasing the computational cost. This finding is consistent with Leduc and Liu (2016) and, more recently, Backus, Ferriere, and Zin (2015).

[4]The estimation controls for time and plant-level fixed effects and four-digit price deflators. See Appendix B for additional details and discussion.

[5]See Figure 16 in the Supplemental Material for a comparison with other measures.

How does uncertainty affect the level of economic activity? How does it contribute to aggregate unemployment fluctuations? Several channels have been put forward in the literature. The first one, on which a large part of the literature has focused, is the *real option* channel (Bernanke (1983), McDonald and Siegel (1986), Dixit and Pindyck (1994)). Firms usually face a substantial amount of uncertainty when making major investments decisions, such as buying new equipment, purchasing land and buildings, or expanding their workforce. These decisions frequently entail large sunk costs that are, at least partially, irreversible. The interaction of irreversibility and uncertainty creates an option value of waiting. In times of heightened uncertainty, firms have an incentive to postpone investment until conditions improve in order to avoid costly mistakes. A second important channel is the *risk premium* channel (Arellano, Bai, and Kehoe (2016), Gilchrist, Sim, and Zakrajšek (2014)). When uncertainty is high, risk premia rise: the cost of external financing increases and the ability of firms to undertake large investments or expand is reduced. A third channel is the *risk aversion* channel. Because of risk aversion, investors and managers may turn away from risky, high return projects, potentially resulting in low growth and slow recovery. Precautionary motives on the household side may further negatively affect an economy subject to nominal rigidities as aggregate demand may fall due to an increase in savings (Basu and Bundick (2017), Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015)).

Employment decisions display several features likely to produce large real option effects: they typically involve important sunk costs (advertisement, search, screening, and training costs); employment contracts are usually long-term relationships that cannot be easily ended, because of both frictions and institutional reasons (labor contracts, dismissal costs, etc.). Because of these characteristic features of labor markets, I focus my analysis on real option effects. In times of high uncertainty, hiring is risky because it is costly, and because productivity may revert quickly. As a result, the *option value of waiting* increases and firms should delay hiring. Hence, high uncertainty may induce a drop in the number of vacancies and in the job finding rate, ultimately resulting in a rise in unemployment.

This, however, only captures part of the story. First, not only hirings but also separations should be subject to an option value: with higher uncertainty, firms should become more reluctant to separate from their workforce, as it would be costly to search for new workers in the case of a rise in future productivity. The combined effect of lower hiring and separation flows on unemployment is thus ambiguous. Moreover, volatility shocks are known in the literature to produce additional effects that could affect the response of unemployment. For instance, volatility, by raising the actual dispersion across establishments, tends to increase reallocation on the labor market: more workers are laid off, but some firms hit by large positive shocks also substantially expand. To evaluate the impact of uncertainty on labor market fluctuations, I propose an equilibrium model that allows to disentangle and quantify this variety of forces, as well as understand the importance of general equilibrium effects and other characteristic features associated to time-varying risk.

The concept of establishment is often absent from search-and-matching models. In order to address the relevant aspects of the response of the labor market to establishment-level risk, I develop an equilibrium search-and-matching model with firm dynamics and heterogeneity in productivity and size. The concept of establishment is introduced through the assumption of decreasing returns to scale. The model allows for aggregate productivity shocks and time-varying volatility in idiosyncratic productivity. Despite being a large heterogeneous agent economy, the model retains its tractability, and dynamics can be easily computed. The model is estimated by simulated method of moments using a set of standard business cycle moments as well as targets drawn from labor market flow data. First, as a validation exercise, I show that the model is consistent with a range of cross-sectional and establishment-level facts. Second, I demonstrate that the introduction of time-varying idiosyncratic risk improves the general fit of search-and-matching models for a range of business cycle moments. Then, I analyze and decompose the response of the economy to aggregate productivity and idiosyncratic volatility shocks. A general lesson from my analysis is that search frictions do not lead to strong real option effects. This result stems from the fact that gross U.S. labor market flows are large, implying that the estimated search costs are too low to create strong irreversibilities. My findings suggest, however, that volatility is still a major determinant of labor market flows through its impact on reallocation. For instance, I find that an increase in volatility leads to a large rise in unemployment, due to an increase in layoffs at firms hit by unusually negative shocks. This increase is accompanied by a modest rise in hiring, dampened by general equilibrium and real option effects, which turns out to be insufficient to compensate for the rise in layoffs. I finally run various counterfactual experiments to evaluate the ability of the model to replicate the U.S. experience across all the recessions included in the 1972–2009 period. The model is quite successful in replicating output dynamics in general. In terms of unemployment, the model can account for about 80% of the 1973–1975, 1980–1982, and 2001 episodes, and virtually 100% of the 1990–1991 recession. Idiosyncratic volatility allows to explain up to 40% of the total increase in unemployment in the 2007–2009 recession, but a large fraction of the magnitude and persistence remains unexplained.

Beyond the analysis of the role of idiosyncratic risk on the labor market, this paper contributes to the search-and-matching literature by developing a model of firm dynamics and search frictions that is fully tractable under a rich structure of aggregate shocks. Dynamic models featuring heterogeneous firms usually raise a number of technical issues. In particular, one must keep track of the infinite-dimensional distribution of firms to solve the model. To address this issue, I use the directed search structure of Menzio and Shi (2010, 2011) in order to exploit the convenient property of *block recursivity*, which allows for an easy and complete characterization of the economy's out-of-steady-state dynamics. While they established this property in an environment with single-worker firms only, I show that block recursivity continues to hold with multiworker firms and decreasing returns to scale, under some additional conditions. In particular, a specific trick allows me to obtain this property despite the presence of two-sided heterogeneity. The model features realistic firm dynamics and a rich description of labor markets flows. In the model, heterogeneous firms can endogenously expand/contract and enter/exit over the business cycle. Workers search for new job opportunities both off and on the job. On-the-job search is especially important for quantitative applications to business cycles as it allows distinction between quits and layoffs, which have notably different cyclical properties. In Section 3, I show that the model is able to reproduce a range of facts at the establishment and cross-sectional levels. For instance, it matches a number of features of the micro-level employment policies of establishments in terms of hires, layoffs, and quits. It also does reasonably well in replicating the cross-sectional distribution of establishment growth rates as reported in Davis, Faberman, and Haltiwanger (2006, 2012). Finally, I explore the wage predictions of the model in Appendix F of the Supplemental Material (Schaal (2017)) and conclude that the model can generate a substantial wage dispersion in line with empirical estimates. The model can also produce a sizeable size-wage differential.

Related Literature

This paper is related to several strands in the literature. It first relates to the growing literature on uncertainty-driven business cycles.[6] This paper studies the role of time-varying risk in interaction with search frictions in explaining labor market dynamics, unlike most of the literature with the exception of Leduc and Liu (2016) and Lin (2014). The first paper added search frictions to the New-Keynesian DSGE framework of Basu and Bundick (2017) and concluded that labor market imperfections provide strong amplification to uncertainty shocks. Lin (2014) built on the more traditional RBC search-and-matching tradition of Merz (1995) and Andolfatto (1996) and found that uncertainty shocks help explain jobless recovery episodes. While their DSGE frameworks allow for an easier comparison to standard RBC and New-Keynesian models as well as to examine, for instance, the role of uncertainty on inflation, the representative agent approach of these two papers restricts their analysis to macroeconomic uncertainty, whose size and impact are relatively small (see Leduc and Liu (2016)). This paper is more closely related to the firm dynamics and heterogeneous agent literature in the spirit of Hopenhayn (1992). This approach allows me, in particular, to examine the impact of the large empirical fluctuations in establishment-level risk and use micro-data to discipline the quantitative exercise. My model also relates to the rest of the literature that uses non-convex adjustment costs in labor to create a real option value (Bloom (2009), Bachmann and Bayer (2013), Bloom et al. (2012)). Search frictions manifest themselves, in my model, as an endogenous linear hiring cost at the firm level, a feature reminiscent of the kinked adjustment cost model of Bertola and Caballero (1990) and Hopenhayn and Rogerson (1993). This hiring cost creates a kink in the objective function, which leads to an inaction region able to generate the irreversibility essential to real option effects. The search approach of this paper opens up the possibility of using rich labor market flow data to discipline the size of this cost and the frequency of adjustment.

This paper also relates to the recent strand in the literature that has sought to introduce search-and-matching frictions to models of firm dynamics. For instance, Acemoglu and Hawkins (2014) and Elsby and Michaels (2013) extended the Mortensen and Pissarides (1994) model to firms with decreasing returns using the Stole and Zwiebel (1996) bargaining procedure.[7] Acemoglu and Hawkins (2014) emphasized the time-consuming aspect of matching to generate persistence in unemployment. Elsby and Michaels (2013) showed that the gap between average and marginal products of labor resulting from the decreasing returns allows a reasonable calibration of the model to generate large fluctuations in unemployment and vacancies. However, computing the out-of-steady-state dynamics in these models is difficult and requires the use of approximation methods. My paper explores another more tractable approach that exploits directed search and block recursivity. This tractability enables me to enrich the model further by adding job-to-job transitions and endogenous firm entry/exit, which play an important role in business cycles. Kaas and Kircher (2015) developed a model that applies a similar idea but differs

[6]See, for example, the time-varying volatility and real option value models of Bloom (2009), Bachmann and Bayer (2013), Bloom et al. (2012); the fiscal volatility paper of Fernández-Villaverde et al. (2015); the uncertainty and financial friction models of Christiano, Motto, and Rostagno (2009), Gilchrist, Sim, and Zakrajšek (2014), and Arellano, Bai, and Kehoe (2016); the New-Keynesian DSGE papers of Basu and Bundick (2017) and Leduc and Liu (2016); the uncertainty and ambiguity aversion paper of Bianchi, Ilut, and Schneider (2014); or the measurement papers of Bachmann, Elstner, and Sims (2013), Baker, Bloom, and Davis (2016), Jurado, Ludvigson, and Ng (2015).

[7]See Brugemann, Gautier, and Menzio (2015) for a corrected version of the Stole and Zwiebel (1996) bargaining procedure.

in the techniques used. Addressing the question of efficiency of search models with large firms, they built a model in which firms offer long-term contracts and used a device similar to block recursivity for tractability. Block recursivity usually requires some indifference condition on either side of the labor market. By assuming that workers are homogeneous and cannot search on the job, Kaas and Kircher (2015) obtained this indifference condition on the worker side. As a result, firms are not indifferent between contracts, and their model can replicate the empirical fact that growing firms have higher job-filling rates. They cannot, however, address issues related to job-to-job transitions, which have very specific cyclical properties and account for the largest part of hires and separations in the data. In my model, there is heterogeneity on both sides of the market because workers with different contracts are allowed to search on the job and firms differ in size and productivity. Block recursivity still obtains, because firms, despite being heterogeneous, value workers in the same way, giving rise to an indifference condition on the firm side. As a consequence, workers in my model are not indifferent between contracts, and the model can replicate some new features of the data, in particular the optimal firm policy in terms of quits and layoffs (Figure 4), as evidenced in Davis, Faberman, and Haltiwanger (2012), and can study the dynamics of job-to-job transitions over the business cycle.

The paper is structured as follows. Section 2 introduces the model and presents results on the existence and efficiency of the equilibrium. In Section 3, I calibrate the model and evaluate the performance of the model using some establishment-level and cross-sectional facts. Section 4 analyzes and discusses the impact of aggregate productivity and idiosyncratic volatility shocks, before evaluating the ability of the model to account for the U.S. labor market experience over the 1972–2009 period. Section 5 concludes.

2. MODEL

In order to study the role of time-varying firm-level volatility in explaining fluctuations in unemployment over the business cycle, I build a dynamic search model with (i) heterogeneous firms that operate a decreasing returns to scale technology, (ii) idiosyncratic risk with time-varying volatility, (iii) aggregate fluctuations in productivity, and (iv) endogenous separations and on-the-job search to allow for the most complete description of the labor market. The assumption of decreasing returns is crucial, in particular, to provide a well-defined notion of firm size, which enables me to study the dynamics of employment in the cross-section of firms in response to aggregate productivity and volatility shocks. The model builds on the directed search framework of Menzio and Shi (2010) in order to exploit the property of block recursivity, defined below.

2.1. *Population and Technology*

Time is discrete. The economy is populated by a continuum of measure 1 of equally productive, infinitely-lived workers and an endogenous measure of firms with free entry. Firms and workers are risk neutral and share the same discount rate β. Firms all produce an identical homogeneous good. The aggregate state of nature is described by a variable s that takes a finite number of values in $\mathcal{S}$ and follows a Markov process with transition matrix $\pi_s(s'|s)$. Aggregate productivity depends on the state of nature and is given by $y(s)$. Firms differ in their idiosyncratic productivity z, independent across firms, that lies in the finite set $\mathcal{Z}$ and follows a Markov process $\pi_z(z'|z, s)$, which I allow to depend on the aggregate state of nature. A firm with a measure n of workers operates the production technology,

$$Y(s, z, n) = e^{y(s)+z} F(n),$$

where F is a strictly increasing, concave production function with $F(0) = 0$. Upon entry, firms must pay a sunk entry cost k_e. Finally, I follow Hopenhayn (1992) in assuming that firms must pay a fixed operating cost $k_f > 0$ every period in order to use the production technology. This operating cost is crucial to generate endogenous exits in the model. It can be interpreted in two ways: either as the fixed cost of using some resources, or similarly as the value of some outside option for firms.

The aggregate state space in this economy comprises the current aggregate state of nature s and should, in principle, include the distribution of employment across firms. Fortunately, the aggregate state space reduces to the variable s under the property of block recursivity. I assume below that this property holds and derive conditions in Section 2.9 under which such an equilibrium exists.

2.2. *Labor Market*

Search is directed on the worker and firm sides. Firms announce contracts to attract workers. Since utility is transferable between firms and workers, a sufficient statistics for each contract is the utility x that it delivers to the worker upon matching. Firms offering identical contracts compete on the same market segment, and we therefore describe the labor market as a continuum of submarkets indexed by the utility $x \in [\underline{x}, \overline{x}]$ that firms promise to workers. Firms must pay a cost c for each vacancy that they post. Workers can direct their search and choose in which submarket to look for a job. Match creation on each market segment is governed by a standard matching function with constant returns to scale. Denote $\theta(s, x)$ the vacancy-unemployment ratio or tightness of submarket x in state s. On a submarket with tightness θ, workers find jobs with probability $p(\theta)$, while firms find candidates with probability $q(\theta) = p(\theta)/\theta$. As is common in the literature, we assume that p is increasing, while q is decreasing, and that $p(0) = 0$, $q(0) = 1$. Workers and firms must solve a trade-off between the level of utility of a given contract and the corresponding probability of being matched. Search takes time and I assume that firms and workers can only visit one submarket at a time.

Employed workers are allowed to search on the job, but are less efficient in doing so than unemployed workers. Denoting λ as the relative search efficiency of the employed compared to the unemployed, the job-finding probability of employed workers is $\lambda p(\theta)$. The equilibrium tightness can be written as $\theta(s, x) = \nu/\mu$, where ν stands for the number of vacancies posted on submarket x and μ the corresponding efficiency-weighted number of searching workers.[8]

The amount of vacancies ν that a firm posts is not restricted to be discrete and should be interpreted as a mass. As a result, a law of large numbers applies and firms do not face uncertainty about the number of workers that they recruit. In particular, a firm that posts ν vacancies meets exactly a measure $\nu q(\theta)$ of workers.

2.3. *Contracting and Timing*

Contracts specify various elements relevant to the firm and its workers. To simplify the exposition, let us assume for now that contracts are complete, state-contingent, and that there is full commitment from both the worker and firm sides. A contract specifies

[8]In particular, $\mu = \mu_u + \lambda\mu_e$, where μ_u is the number of unemployed workers and μ_e the corresponding number of employed workers searching on that market.

$\{w_{t+j}, \tau_{t+j}, x_{t+j}, d_{t+j}\}_{j=0}^{\infty}$, where w is a wage, τ a layoff probability, x the submarket where the worker searches while employed,[9] and d an exit dummy. Each element at time $t+j$ is contingent on the entire history of shocks (s^{t+j}, z^{t+j}). I maintain the assumptions of completeness and full commitment throughout this section, but show, however, in Section E.2 of the Supplemental Material that completeness and commitment from the worker side can be relaxed along some dimensions.

The contracts offered by firms are large objects, but can be written in their recursive form. As a convention, the contracts are rewritten every period after matching occurs and when production takes place (stage B on Figure 2). At this stage, the firm starts with some utility W, promised in the past to its incumbent workers or new recruits. A recursive contract $\omega = \{w, \tau, x, d, W'\}$ for the current period specifies the current wage w and next period's quantities $\{\tau(s', z'), x(s', z'), d(s', z'), W'(s', z')\}$, contingent on next period's state, where W' is some future promised utility. Because of commitment, contract ω is required to deliver at least the promised utility W to the worker.

The timing is illustrated in Figure 2. At the beginning of period t, the aggregate state of nature s is realized. Firms then decide whether to enter or not. Immediately after, incumbent and entering firms learn their idiosyncratic productivity z and decide whether to exit ($d = 1$) or stay. In the following stage, separations occur at probability τ. Search and matching follows with new and incumbent firms on one side and unemployed/employed workers on the other side. Production takes place in the final stage of the period.

2.4. *Worker's Problem*

As a convention, the following value functions are expressed at stage B of the period, when production takes place. We write the value of unemployment as follows:

$$\begin{aligned}\mathbf{U}(s) = \max_{x_u(s')} b + \beta\mathbb{E}\big[&p\big(\theta\big(s', x_u\big(s'\big)\big)\big)x_u\big(s'\big) \\ &+ \big(1 - p\big(\theta\big(s', x_u\big(s'\big)\big)\big)\big)\mathbf{U}\big(s'\big)\big].\end{aligned} \tag{1}$$

When unemployed, workers enjoy a utility b from home production or leisure. In the following period, they choose a market segment, $x_u(s')$, for their job search. In doing so, they must solve a trade-off between the offered utility, x_u, and the likelihood to get a job, $p(\theta(s', x_u))$, which also depends on the aggregate state of the economy. When successful, they enjoy the promised utility x_u, but remain unemployed otherwise.

In the case of a worker employed in a firm with productivity z under the contingent contract $\omega = \{w, \tau(s', z'), x(s', z'), d(s', z'), W'(s', z')\}$, the value can be written

$$\begin{aligned}\mathbf{W}(s, z, \omega) = w + \beta\mathbb{E}\big[&\big(d + (1-d)\tau\big)\mathbf{U}\big(s'\big) \\ &+ (1-d)(1-\tau)\big(\lambda p\big(\theta\big(s', x'\big)\big)x + \big(1 - \lambda p\big(\theta\big(s', x\big)\big)\big)W'\big(s', z'\big)\big)\big].\end{aligned} \tag{2}$$

The worker first receives the wage w. The following period may then lead to three different outcomes, which correspond to three terms in brackets: (i) in the case of exit, d, or layoff, τ, the worker goes back to unemployment with value $\mathbf{U}(s')$; (ii) he finds a job in a different firm under a contract with value x at probability $\lambda p(\theta(s', x))$; or (iii) he stays in

[9]The fact that the contract specifies x, the submarket on which the worker should be looking for a job, is a consequence of the assumption of completeness. Section E.2 in the Supplemental Material shows that this feature can be relaxed as the firm can write an incentive compatible contract that makes the worker search on the optimal market segment.

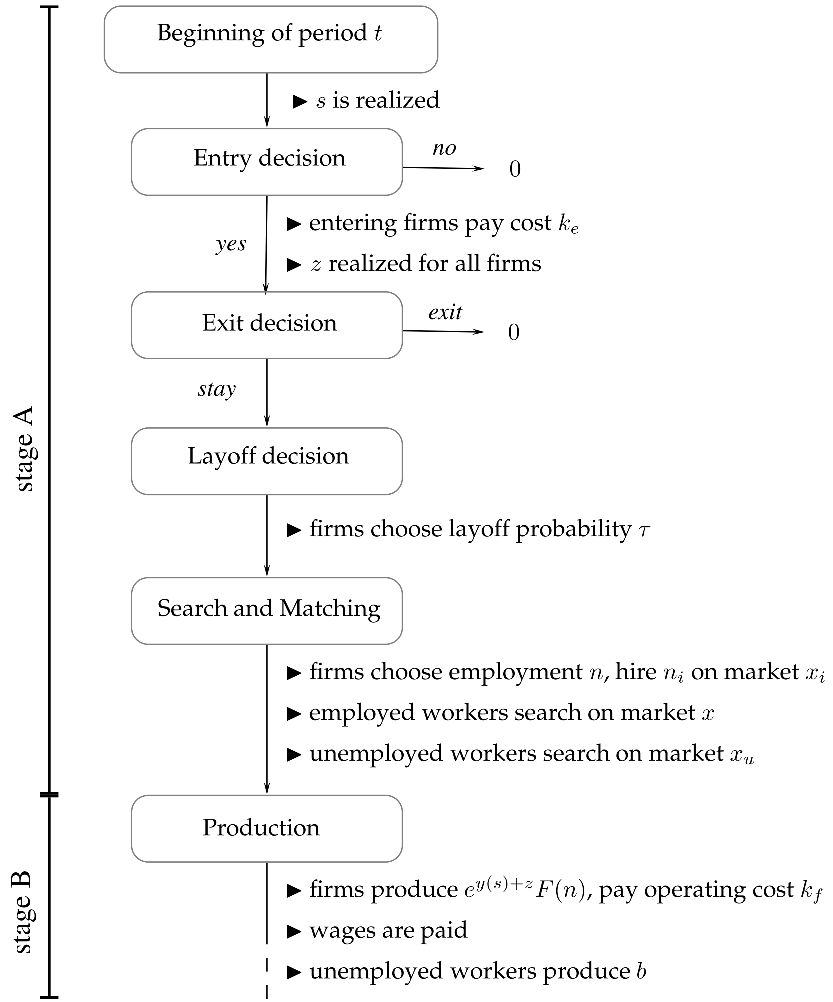

FIGURE 2.—Timing.

the firm and receives the promised utility $W'(s', z')$ in the following period. Notice that laid-off workers have to spend one period unemployed before looking for a job.

2.5. *Firm's Problem*

Consider the problem of a firm at the production stage with a measure n of employed workers. Workers within the same firm may differ in their levels of promised utility. Each worker is identified by an index $j \in [0, n]$ and a corresponding level of promised utility $W(j)$.

The problem of a firm consists of choosing a list of contracts for its incumbent workers,

$$\omega(j) = \{w(j), \tau(s', z'; j), x(s', z'; j), W'(s', z'; j), d(s', z')\}, \quad \forall j \in [0, n].$$

In addition, the firm must decide on a submarket for its hiring in the next period $x_i(s', z')$ and choose a number of workers to hire $n_i(s', z')$. We may describe the problem faced by firms as follows:

$$\begin{aligned}
&\mathbf{J}\big(s, z, n, \{W(j)\}_{j\in[0,n]}\big) \\
&\quad = \max_{\substack{n_i(s',z'),x_i(s',z'),\\ \{\omega(j)\}_{j\in[0,n]}}} \mathrm{e}^{y(s)+z}F(n) - k_f - \int_0^n w(j)\,\mathrm{d}j \\
&\qquad + \beta\mathbb{E}\bigg\{-n_i\frac{c}{q\big(\theta(s',x_i)\big)} + \mathbf{J}\big(s',z',n',\{\hat{W}(s',z';j')\}_{j'\in[0,n']}\big)\bigg\}^+
\end{aligned} \tag{3}$$

subject to the laws of motion, for all (s', z'):

$$n'(s',z') = \int_0^n \big(1-\tau(s',z';j)\big)\big(1-\lambda p\big(\theta\big(s',x(s',z';j)\big)\big)\big)\,\mathrm{d}j + n_i(s',z'), \tag{4}$$

$$\hat{W}(s',z';j') = \begin{cases} W'(s',z';j) & \text{for } j'\in[0,n'-n_i'] \text{ and } j'=\Phi(s',z';j), \\ x_i(s',z') & \text{for } j'\in[n'-n_i',n'], \end{cases} \tag{5}$$

where $\Phi(s', z'; j) = \int_0^j(1 - \tau(s', z'; k))(1 - \lambda p(\theta(s', x(s', z'; k))))\,\mathrm{d}k$.

In the current period, the firm earns revenue from production, $\mathrm{e}^{y(s)+z}F(n)$, minus the fixed operating cost k_f and wage bill $\int_0^n w(j)\,\mathrm{d}j$. In the following period, the firm chooses whether to exit or not. The $\{\cdot\}^+$ notation, standing for $\max(\cdot, 0)$, captures this decision, which we summarize in the dummy $d(s', z') \in \{0, 1\}$ ($d = 1$ for exit). Following this decision, the firm then chooses a number of workers to hire $n_i(s', z')$ and a submarket $x_i(s', z')$ for their recruitment. Because each vacancy has a probability $q(\theta(s', x_i))$ to be filled, the total vacancy cost incurred for these new recruits is $n_i c/q(\theta(s', x_i))$.

Constraint (4) is the law of motion of total employment. Employment $n'(s', z')$ in the next period is the sum of the new hires $n_i(s', z')$ with the remaining workers after the departure of those laid off with probability $\tau(s', z'; j)$ and of those moving to other jobs with probability $\lambda p(\theta(s', x(s', z'; j)))$. Equation (5) keeps track of the promised utilities across workers. Because the measure of workers within the firm evolves over time, I use the mapping Φ to reindex the job stayers and make sure that a worker with an original index $j \in [0, n]$ is assigned a new index $\Phi(s', z'; j) \in [0, n' - n_i]$ in the next period. New hires with promised utility, $x_i(s', z')$, are assigned an index in $[n' - n_i, n']$.

In addition to these constraints, because of commitment, the firm is subject to the following *promise-keeping* constraint:

$$\forall j \in [0, n], \quad W(j) \le \mathbf{W}\big(s, z, \omega(j)\big). \tag{6}$$

Constraint (6) checks that the contract $\omega(j)$, assigned to worker j, delivers at least the promised lifetime utility $W(j)$. Note finally that there is no non-negativity constraint on the firm's value, implying that firms have deep pockets and no limited liability.

2.6. *Joint Surplus Maximization*

The structure of the economy allows us to greatly simplify the firm's problem. The completeness of contracts, the commitment assumption, and the transferability of utility guarantee that optimal policies always maximize the joint surplus of a firm and its workers. The model can thus be solved in two stages: a first stage in which we maximize the surplus and a second step in which we can design the contracts that implement the allocation.

Define the joint surplus maximization problem for a firm and its current workers by the following Bellman equation:

$$\begin{aligned}\mathbf{V}(s,z,n) = \max_{\substack{d(s',z'),n_i(s',z'),x_i(s',z'),\\ \{\tau(s',z';j),x(s',z';j)\}_{j\in[0,n]}}} & \mathrm{e}^{y(s)+z}F(n) - k_f + \beta\mathbb{E}\Big\{nd\mathbf{U}(s') \\ & + (1-d)\Big[\mathbf{U}(s')\int_0^n \tau\,\mathrm{d}j + \int_0^n (1-\tau)\lambda p\big(\theta(s',x)\big)x\,\mathrm{d}j \\ & - \Big(\frac{c}{q\big(\theta(s',x_i)\big)} + x_i\Big)n_i + \mathbf{V}(s',z',n')\Big]\Big\}\end{aligned} \tag{7}$$

subject to, $\forall(s',z')$

$$n'(s',z') = \int_0^n \big(1-\tau(s',z';j)\big)\big(1-\lambda p\big(\theta\big(s',x(s',z';j)\big)\big)\big)\,\mathrm{d}j + n_i(s',z').$$

The surplus maximization problem characterizes the optimal allocation of physical resources within a firm: the optimal amount of layoffs, job-to-job transitions, new hires, and the decision whether to exit or not. Since utility is transferable, transfers between the firm and its workers leave the surplus unchanged. Elements of the contracts that describe the way profits are split, such as wages and continuation utilities, thus disappear in the surplus maximization problem. In particular, the distribution of promised utilities, $\{W(j)\}_{j\in[0,n]}$, is not part of the state space and only the size of employment at the production stage n matters.

The first element in the surplus maximization problem is production followed by the payment of the operating cost k_f. In the next period, the firm chooses whether to exit or not, a decision captured by the exit dummy $d(s',z')$. If a firm chooses to exit, all the workers return to unemployment while the firm's value is set to zero, yielding a total utility of $n\mathbf{U}(s')$. If it chooses not to exit, the firm may then proceed with its layoffs. The total mass of layoffs is $\int_0^n \tau(s',z';j)\,\mathrm{d}j$, which provides a total expected utility of $\mathbf{U}(s')\int_0^n \tau\,\mathrm{d}j$ to the worker-firm group. After searching, some workers move to other jobs with value $x(s',z';j)$ and contribute the amount $\int_0^n (1-\tau(s',z';j))\lambda p(\theta(s',x(s',z';j)))x(s',z';j)\,\mathrm{d}j$ to the total surplus. Simultaneously, the firm proceeds with its hiring. For each new hire on the labor market segment $x_i(s',z')$, the firm incurs a total vacancy cost of $c/q(\theta(s',x_i(s',z')))$ and must offer the lifetime utility-wage $x_i(s',z')$ to its new recruits, which appears as a cost to the current worker-firm group.

The following proposition formally establishes the equivalence between the firm's problem and the joint surplus maximization.

PROPOSITION 1: *The firm's problem and joint surplus maximization are equivalent in the following sense*:

(i) *The surplus and firm's profit verify*

$$\mathbf{V}(s,z,n) = \mathbf{J}\big(s,z,n,\{W(j)\}_{j\in[0,n]}\big) + \int_0^n W(j)\,\mathrm{d}j.$$

(ii) *For a profit maximizing policy* $\{\{\omega(j)\}_{j\in[0,n]}, d(s',z'), n_i(s',z'), x_i(s',z')\}$, *firm policy* $\{\{\tau(s',z';j), x(s',z';j)\}_{j\in[0,n]}, d(s',z'), n_i(s',z'), x_i(s',z')\}$ *maximizes the joint surplus*.

(iii) *Conversely, if* $\{\{\tau(s', z'; j), x(s', z'; j)\}_{j\in[0,n]}, d, n_i, x_i\}$ *maximizes the joint surplus, there exists a set of contracts* $\{\omega(j)\}_{j\in[0,n]}$ *with wages and continuation utilities* $\{w(j), W'(s', z'; j)\}_{j\in[0,n]}$ *such that the proposed policy* $\{\{\omega(j)\}_{j\in[0,n]}, d(s', z'), n_i(s', z'), x_i(s', z')\}$ *maximizes profits*.

Proposition 1 tells us that it is possible to find the optimal allocation of physical resources $\{\{\tau(s', z'; j), x(s', z'; j)\}_{j\in[0,n]}$, $d(s', z')$, $n_i(s', z')$, $x_i(s', z')\}$ first and solve for the contracts that implement that allocation later, in a second stage. This proposition establishes, in particular, that such contracts always exist and are, in fact, easy to construct once the allocation is known. This result is of particular interest in practice since equation (7) is a Bellman equation that can easily be solved with standard numerical methods.

The fact that one can maximize the joint surplus regardless of the distribution of promised utilities is an important result, which stems from the transferability of utility, the completeness of contracts, and the assumption of commitment. Transferability ensures that a firm and its workers evaluate the benefits from their actions using a common utility scale and agree on a definition of a joint surplus. Completeness guarantees that there always exists sophisticated enough schemes of transfers, through wages and continuation utilities, that can implement the surplus maximizing allocation by suitably redistributing the benefits between the firm and its workers for any initial distribution of promises. Finally, commitment ensures that firms cannot extract a larger part of the surplus by reneging on their promises, for instance by laying off workers with high utility-wages. Since promises have to be fulfilled in all circumstances, including upon separation, tweaking the allocation away from the surplus maximizing allocation cannot deliver higher profits: it is optimal for firms to maximize the physical size of the "cake," that is, the surplus, pay the workers their dues, and enjoy the remaining profits, which are then maximized. The distribution of promises is thus irrelevant to the determination of the physical allocation of resources, but only matters for the way the surplus is split between the firm and its workers, in particular for wages.

Notice, finally, from equation (7), that since all the contracting aspects have disappeared, the surplus maximization problem is purely forward looking and the firm's current state (s, z) has no impact on the optimal policy $\{\{\tau, x\}_{j\in[0,n]}, d, n_i, x_i\}$ chosen by a firm in state (s, z, n). As a result, while the equilibrium policy should in principle depend on the entire state space $(s', z'; s, z, n)$, it solely depends on the firm's state at the beginning of next period $(s', z'; n)$. This result is assumed throughout the rest of the paper.

2.7. *Free Entry*

Every period after the aggregate shock s is realized, potential entrants decide whether or not to enter. Upon entry, firms must pay an entry cost k_e, after which they draw their idiosyncratic productivity z from some distribution g_z. Depending on the outcome, firms may decide to exit or stay, in which case they can start hiring and producing as any normal firm.

We define the problem faced by an entering firm of type z as follows:

$$\mathbf{J}_e(s, z) = \max_{n_e(s,z), x_e(s,z)} \left[\mathbf{V}(s, z, n_e) - n_e x_e - n_e \frac{c}{q\big(\theta(s, x_e)\big)} \right]^+. \tag{8}$$

Having drawn the idiosyncratic productivity z, the potential entrant first decides whether or not to exit, a decision captured by the notation $\{\cdot\}^+$ and summarized in the dummy

$d_e(s, z)$. If it stays, the firm chooses a measure of workers to hire, $n_e(s, z)$, and a market for recruitment, $x_e(s, z)$, in order to maximize its profits net of the total vacancy cost $n_e c/q(\theta(s, x_e))$. Using Proposition 1, these profits can be written as the joint surplus $\mathbf{V}(s, z, n_e)$ minus the total utility $n_e x_e$ that the firm must deliver to its new recruits.[10]

An important feature of this economy is that the submarket x_e, in which workers are hired, solely appears through the term $\frac{c}{q(\theta(s,x_e))} + x_e$, which we can describe as a *hiring cost per worker*, common to both entering and incumbent firms. The first term, $c/q(\theta(s, x_e))$, captures the total vacancy cost of hiring exactly one worker. The second term, x_e, is the utility-wage that firms offer to their new recruits. As a result, the decision of entering firms can be decomposed as a two-stage problem: a first stage, during which firms choose where to search for their workers; a second stage, in which firms decide on the number of workers to recruit. In the first stage, firms choose the submarkets that minimize hiring costs per worker. Define the minimal hiring cost κ as[11]

$$\kappa(s) = \min_{\underline{x} \le x \le \overline{x}} \left[x + \frac{c}{q\big(\theta(s, x)\big)} \right]. \tag{9}$$

Optimal entry further imposes the requirement that only the submarkets that minimize this hiring cost are open in equilibrium, which we summarize in the following complementary slackness condition:

$$\forall x, \quad \theta(s, x)\left[x + \frac{c}{q\big(\theta(s, x)\big)} - \kappa(s) \right] = 0. \tag{10}$$

Equation (10) expresses that submarkets either minimize the hiring cost, $\kappa(s) = x + c/q(\theta(s, x))$, or remain unvisited, $\theta(s, x) = 0$. In equilibrium, active submarkets will have the same hiring cost $\kappa(s)$ and firms will be indifferent between them. Notice that equation (10) provides us with an expression for the equilibrium market tightness on every active market,

$$\theta(s, x) = q^{-1}\left(\frac{c}{\kappa(s) - x} \right). \tag{11}$$

The job-filling probability q being a decreasing function, this expression tells us, in particular, that equilibrium tightness must decrease with the level of utility promised to workers as these offers succeed in attracting more workers, while firms refrain from posting such expensive contracts. The probability of finding a job for workers thus declines with the attractiveness of the offer.

We may now describe the full free-entry condition in this economy. Firms enter the economy as long as expected profits exceed the entry cost k_e, driving these profits down to k_e. Therefore, expected surplus from entering must be equal to k_e in equilibrium:

$$k_e = \sum_{z \in \mathcal{Z}} \mathbf{J}_e(s, z) g_z(z), \quad \forall s. \tag{12}$$

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[10]The ex post profits after entry for a firm of type z coincide with the surplus net of the promised utility x_e, $\mathbf{J}(s, z, n_e, \{x_e\}_{j \in [0, n_e]}) - n_e \frac{c}{q(\theta(s,x_e))} = \mathbf{V}(s, z, n_e) - n_e x_e - n_e \frac{c}{q(\theta(s,x_e))}$.

[11]Note in particular that the cost minimization problem is the same across firms. This property is key to obtain the indifference condition on the firm side required for block recursivity, as we discuss in Section 2.10.

Note that the free-entry condition is crucial to guarantee the existence of a block-recursive equilibrium. Section 2.10 discusses this property in more detail and explains why it obtains in this setup.

2.8. *Unemployment and Firm Distribution Dynamics*

Using the optimal decision of firms, we may now describe the evolution of employment over time. Let u be the unemployment rate and $g(z, n)$ the distribution of employment across firms in stage B of the current period, when production takes place.

Given a current aggregate state (s, g), the evolution of the unemployment rate is governed by the following equation:

$$\begin{aligned} u' = {} & \big(1 - p\big(\theta\big(s', x_u(s')\big)\big)\big)u \\ & + \sum_{z,z',n} n\big[d\big(s', z'; n\big) + \big(1 - d\big(s', z'; n\big)\big)\tau\big(s', z'; n\big)\big]\pi_z\big(z' \mid z, s\big)g(z, n). \end{aligned} \tag{13}$$

Equation (13) states that unemployment at the start of the next period corresponds to the fraction $1 - p(\theta(s', x_u(s')))$ of unemployed workers that do not find a job next period in addition to the workers that lose their jobs because of exits, d, or layoffs, τ.

The dynamics of the distribution of employment across firms can be described by

$$\begin{aligned} g'\big(z', n'\big) = {} & \sum_{z,n} \mathbb{1}\big\{n'\big(s', z'; n\big) = n'\big\}\big(1 - d\big(s', z'; n\big)\big)\pi_z\big(z' \mid z, s\big)g(z, n) \\ & + m_e\big(s', g\big)\mathbb{1}\big\{n_e\big(s', z'\big) = n'\big\}\big(1 - d_e\big(s', z'\big)\big)g_z\big(z'\big), \end{aligned} \tag{14}$$

where $\mathbb{1}\{\cdot\}$ denotes an indicator function. Equation (14) defines the number of firms with individual state (z', n') in the next period as the sum of the surviving incumbents and entering firms that end up in this state. The term m_e is the endogenous measure of new entrants, which depends on the aggregate state of nature s' and distribution g. It is defined as the number of entering firms required to reach the equilibrium market tightness on every market segment. Fortunately, because firms are indifferent between the various submarkets, these equilibrium conditions can be summarized by a unique aggregate condition which states that the total number of jobs found by workers has to equal the number of jobs created by firms. More formally, m_e is implicitly defined by the equation

$$JF_{\text{total workers}}\big(s', g\big) = JC_{\text{total incumbents}}\big(s', g\big) + m_e\big(s', g\big)JC_{\text{entrant}}\big(s'\big), \tag{15}$$

where JF holds for the number of jobs found by workers across all submarkets and JC for the number of jobs created by firms. In particular, JC_{entrant} is the number of jobs created by a measure 1 of entrants. An explicit formula for m_e is derived in Appendix A.

2.9. *Existence and Efficiency*

We may now define a block-recursive equilibrium in this economy. For this purpose, I proceed in a constructive way and introduce the notion of a quasi-equilibrium as a candidate equilibrium. I define a quasi-equilibrium as a block-recursive solution to both the workers' problems (1)–(2) and firms' problem (3), which further satisfies the free-entry condition (12). Unfortunately, without further restrictions on the parameters, the labor market equilibrium condition as described by equation (15) may imply negative entry in some cases. Under such circumstances, the assumption of free-entry is not valid and block recursivity does not obtain. For a quasi-equilibrium to be a well-defined block-recursive equilibrium, one must verify that entry is non-negative in every possible state of the world.

DEFINITION 1: Define the following concepts:

(i) A quasi-equilibrium of this economy is (a) a set of value functions $\mathbf{U}(s)$, $\mathbf{W}(s, z, \omega)$, $\mathbf{J}(s, z, n, \{W(j)\}_{j\in[0,n]})$, $\mathbf{V}(s, z, n)$, and $\mathbf{J}_e(s, z)$, (b) a decision rule for unemployed workers $\{x_u(s')\}$, for entering firms $\{d_e(s, z), n_e(s, z)\}$, and for incumbent firms $\{d(s', z'; n)$, $n_i(s', z'; n)$, $x_i(s', z'; n)$, $\{\omega(j; s, z, n)\}_{j\in[0,n]}\}$, (c) a hiring cost $\kappa(s)$ and corresponding labor market tightness $\theta(s, x)$ such that equations (1)–(12) are satisfied.

(ii) A block-recursive equilibrium is a quasi-equilibrium such that entry is non-negative in any state of the world.

Proposition 2 below establishes existence and efficiency results.

PROPOSITION 2:

(i) *Under weak regularity conditions stated in Appendix* G.2, *a quasi-equilibrium always exists*.

(ii) *A block-recursive equilibrium*, *when it exists*, *is efficient*.

First, Proposition 2 shows that a quasi-equilibrium always exists under some weak regularity conditions. In particular, it uses Schauder's fixed point theorem to show the existence of a joint solution to the surplus maximization, free-entry, and unemployed workers' problems. The existence of a solution to the firm's profit maximization problem and corresponding contracts ensues from Proposition 1. Unfortunately, the existence of a full block-recursive equilibrium, namely, a quasi-equilibrium with positive entry, cannot be easily proved. The key issue is that the measure of entrants m_e, implicitly defined in (15), depends on the infinite-dimensional distribution g and cannot be put into a recursive form with a low-dimensional state space. Although one can derive sufficient conditions on parameters to guarantee that entry is always non-negative, it is easier to check this condition numerically in practice. Proposition 2 thus provides us with a constructive way to solve for block-recursive equilibria: (1) solve for a quasi-equilibrium in a first stage, and (2) check that the obtained policy functions imply positive entry in every state of the world. Note, in addition, that the non-negative entry condition is a weak restriction for empirically relevant cases as data from the United States and other developed economies always display positive entry, even in the midst of large recessions.

Turning to (ii), this proposition establishes the efficiency of block-recursive equilibria. It guarantees, in particular, that a quasi-equilibrium with positive entry, once found, maximizes welfare and must be, as such, unique in a payoff-equivalent sense. This extends standard results in competitive search models with single-worker firms. This model thus offers an efficient benchmark in which unemployment is efficient and there is no mispricing, nor inefficient separations.

Appendix E in the Supplemental Material characterizes additional properties of the optimal contracts and provides an alternative version of the model relaxing commitment and completeness of the contracts.

2.10. *Block-Recursivity*

In this section, I explain the intuition behind the property of block recursivity as it appears in the literature and discuss why it obtains in this setup. Readers interested in the quantitative exercise may directly skip to Section 3.

In search-and-matching models with sufficient heterogeneity, the distribution of workers across firms is, in general, required for agents to forecast wages and the labor market tightness. This feature is problematic when the distribution is an infinite-dimensional object, which standard numerical techniques cannot handle. To address that problem, Menzio and Shi (2011) introduced the concept of block recursivity using two modeling tricks. The first trick is the use of directed search instead of random search. In random search, wages are negotiated and depend on workers' and firms' outside options, which usually depend on the distribution of workers across firms in equilibrium. In a directed search setup, firms and workers do not need to forecast wages (or contracts) because wages are choice variables, which, as such, do not depend on who they meet: firms choose the wage that they offer; workers choose where to apply. However, in such an environment, workers and firms still need to forecast the vacancy-unemployment ratio or market tightness on each market. The second trick comes into play at this stage and relies on a clever use of the free-entry condition. This condition equalizes the cost of opening a vacancy to the value of a job. This value depends on the probability at which a job is created—a function of the market tightness—and on the surplus of the match, but does not directly depend on the distribution of employment in the economy. Since the cost of opening a vacancy is a constant in these models, the free-entry condition pins down the value of the market tightness as a function of the value of a new job. Likewise, the value of a new job is not directly affected by the employment distribution, but only indirectly through the expectation of future market tightness. If the free-entry condition pins down future market tightness independently from the distribution of firms, it is then possible to construct a full equilibrium in which *neither* the value functions *nor* the market tightness depend on the employment distribution across firms: the equilibrium is *block-recursive*.

Unfortunately, this reasoning does not easily apply to a setup with rich heterogeneity. The free-entry condition only pins down the equilibrium market tightness on a single market: the one chosen by entering firms. To characterize the tightness on the other submarkets, homogeneity is often assumed on either side of the labor market. With homogeneous workers or firms, an indifference condition arises that can be used to ensure that the free-entry condition pins down the tightness on *every* active submarket in the economy. In the environment proposed in this paper, there is heterogeneity on both sides of the market. Firms differ in productivity and sizes. Workers differ in their employment statuses—employed or unemployed—and in their current utility levels depending on whether they work in high-paying jobs or not. A contribution of this paper is to show that block recursivity may still obtain in the presence of two-sided heterogeneity and proves the existence of such equilibria. This result relies on two assumptions: the transferability of utility—which guarantees that all contracts are viewed in an identical way by agents—and the fact that firms hire a continuum of workers. Under these two conditions, the decision over the market for hiring can be summarized by the minimization of the cost $\kappa(s)$, which is the same to every firm. Therefore, despite heterogeneity on the firm side, firms are effectively indifferent across submarkets because they face the same hiring cost $\kappa(s)$. Even though firms differ in productivity and sizes, they all seek to minimize this cost and thus post their offers on the same markets. As a consequence, indifference on the firm side in combination with the free-entry condition allows to characterize the equilibrium tightness of every active submarket and generalizes the block-recursive property to the whole economy.

3. BUSINESS CYCLE AND ESTABLISHMENT-LEVEL PROPERTIES

In this section, I calibrate the model and evaluate its predictions at various levels of aggregation. Starting at the aggregate level, I present some standard business cycle statistics from model simulations and compare them to the same model with aggregate productivity shocks only. I show that the presence of time-varying idiosyncratic volatility generally leads to more realistic fluctuations in unemployment and other variables. Turning to the establishment-level implications of the model, I discuss some of its properties in terms of growth rates, and show that it can replicate salient features of the employment behavior of firms.

3.1. *Calibration*

3.1.1. *Functional Forms and Stochastic Processes*

I parameterize the model as follows. The production function is the concave function $F(n) = An^{\alpha}$, where α governs the amount of diminishing returns in the economy. Since time is discrete, I must choose a job-finding probability function bounded between 0 and 1, which rules out Cobb–Douglas matching functions. Following Menzio and Shi (2010), I pick the CES contact rate functions

$$p(\theta) \equiv \theta\big(1 + \theta^{\gamma}\big)^{-1/\gamma}, \qquad q(\theta) \equiv p(\theta)/\theta = \big(1 + \theta^{\gamma}\big)^{-1/\gamma}.$$

In addition to providing a good fit to the data on job-finding rates and labor market tightness, these functions satisfy all the regularity conditions required for the existence of an equilibrium stated in Supplemental Material Section G.2.1.

The aggregate and idiosyncratic productivity shocks follow the AR(1) processes[12]

$$y_t = \rho_y y_{t-1} + \sigma_y \sqrt{1 - \rho_y^2}\,\varepsilon_{y,t}, \quad \varepsilon_{y,t} \sim \mathcal{N}(0, 1),$$

$$z_t = \rho_z z_{t-1} + v_{t-1} \sqrt{1 - \rho_z^2}\,\varepsilon_{z,t}, \quad \varepsilon_{z,t} \sim \mathcal{N}(0, 1),$$

where v_t denotes the time-varying volatility of idiosyncratic productivity. I assume that its log follows the AR(1) process with mean $\log \overline{v}$:

$$\log v_t = (1 - \rho_v) \log \overline{v} + \rho_v \log v_{t-1} + \sigma_v \sqrt{1 - \rho_v^2}\,\varepsilon_{v,t}, \quad \varepsilon_{v,t} \sim \mathcal{N}(0, 1),$$

which ensures that idiosyncratic volatility remains positive. In the data, idiosyncratic volatility is countercyclical. I therefore allow the innovations $\varepsilon_{y,t}$ and $\varepsilon_{v,t}$ to be correlated and denote $\sigma_{yv} = \operatorname{cor}(\varepsilon_{y,t}, \varepsilon_{v,t})$. Innovations to z_t are independent across agents. The aggregate state of nature is $s_t = (y_t, v_t)$.

[12]Under this timing assumption common in uncertainty literature (Bloom (2009), Bloom et al. (2012)), volatility shocks have a delayed impact on the distribution of idiosyncratic productivity shocks. By allowing real option effects to take place before volatility is actually realized, this timing favors wait-and-see effects. Relaxing this delay assumption would only reinforce my later findings that real option effects due to search frictions are small and dwarfed by realized volatility effects.

3.1.2. *Calibration Strategy*

The model is estimated using a method of simulated moments. For the largest part, I follow the search-and-matching literature in choosing the moments to target. The chosen calibration strategy mostly targets aggregate labor market flows as in Shimer (2005). It is conservative in the sense that such a strategy usually leads to the unemployment volatility puzzle.

The time period is set to one month. I set the discount rate β to 0.996 so that the annual interest rate is about 5%. I set the decreasing returns to scale parameter $\alpha = 0.85$ in the middle of the range of empirical estimates in the literature (Basu and Fernald (1995), Basu (1996), Basu and Kimball (1997)).[13] Without firm-level panel data, I do not have observations on the idiosyncratic productivity process of firms. I thus follow the investment literature and set $\rho_z = (0.95)^{\frac{1}{3}}$ in order to match an approximate quarterly autocorrelation of 0.95 as in Khan and Thomas (2008) and Bloom et al. (2012).

The parameters left to estimate are the following: the productivity parameters (ρ_y, σ_y, $\overline{v}$, ρ_v, σ_v, σ_{yv}), the home production b, the vacancy posting cost c, the matching function parameter γ, the entry cost k_e, the fixed operating cost k_f, and the relative search efficiency of employed workers compared to unemployed ones λ.

To discipline the choice of the aggregate productivity parameters (ρ_y, σ_y), I target the autocorrelation and standard deviation of log-detrended output, using seasonally adjusted quarterly real GDP from the Bureau of Economic Analysis.[14] Regarding the idiosyncratic productivity parameters ($\overline{v}$, ρ_v, σ_v, σ_{yv}), I select moments from the establishment-level volatility series constructed by Bloom et al. (2012). I target, in particular, the average interquartile range (IQR) of innovations to idiosyncratic TFP, its autocorrelation, standard deviation, and correlation with aggregate output. To inform the estimation of the labor market parameters (c, b, λ), I include in my moments the following historical averages of the monthly transition rates: an Unemployment-Employment (UE) rate of 45%, an Employment-Unemployment (EU) rate of 2.6% according to Shimer (2005), and an Employment-Employment (EE) rate of 2.9% following estimates by Nagypál (2007). The matching function parameter γ is set to match an elasticity of the UE rate with respect to the vacancy-unemployment ratio of 0.72 as estimated by Shimer (2005). To discipline the entry cost k_e, I target an average fraction of jobs created by opening establishments of 21%, according to the Business Employment Dynamics (BED) over the period 1992Q3–2009Q4. Finally, because the operating cost k_f governs the rate of exit in the economy and the degree of dynamic selection, I target an average establishment size of 15.6, as in the 2002 Economic Census.

The parameters are jointly estimated using a search algorithm in the parameter space that minimizes the distance between the empirical and simulated moments, with weights chosen to yield relative errors of the same amplitude for each moment. Section D in the Supplemental Material describes the numerical implementation. Table I summarizes the parameter values that result from the calibration. Table II shows the fit of the model with the targeted moments. The fit is, overall, quite satisfactory. Note that the autocorrelation of output produced by the model is slightly below its empirical counterpart, because the less persistent volatility shocks introduce extra variation in output.

[13] I choose to match the total decreasing returns at the firm level because I am interested in explaining firm dynamics, despite the absence of capital in the model. A previous version of this paper targeted a wage share of 0.66, with little difference on the final results.

[14] During the estimation procedure, all time series are computed in log deviations from an HP-trend with parameter 1600 for quarterly data and 100 for annual data.

TABLE I

ESTIMATED PARAMETERS

Parameter	Value	Description
Calibrated:		
A	1	Technology parameter
β	0.996	Monthly discount factor
α	0.85	Decreasing returns to scale coefficient
ρ_z	$0.95^{\frac{1}{3}}$	Autocorrelation of idiosyncratic productivity z
Estimated:		
ρ_y	0.990	Autocorrelation of aggregate productivity y
σ_y	0.042	Standard deviation of aggregate productivity y
$\overline{v}$	0.533	Standard deviation of idiosyncratic productivity z
ρ_v	0.979	Autocorrelation of volatility process v
σ_v	0.132	Standard deviation of volatility process v
ρ_{yv}	−0.400	Correlation between $\varepsilon_{y,t}$ and $\varepsilon_{v,t}$
b	1.403	Home production
c	1.789	Vacancy posting cost
λ	0.366	Relative search efficiency of employees
γ	1.599	Matching function parameter
k_e	14.21	Entry cost
k_f	1.956	Operating cost

Targeting an annual interquartile range of 0.393, the long-run standard deviation $\overline{v}$ of idiosyncratic volatility is estimated to be 0.533, about ten times as large as that of output, a result in line with Bloom et al. (2012). The standard deviation of the volatility process is large, $\sigma_v = 13.2\%$, but still relatively low compared to the 30% increase in the IQR observed from 2004 to 2008. The labor market parameters can be interpreted as follows. The estimated home production b represents about 63% of the average output per person

TABLE II

TARGETED MOMENTS[a]

Moment	Empirical Value	Simulated
$\rho[Y_t]$	0.839	0.781
$\sigma[Y_t]$	0.016	0.016
$IQR(e_{z,t})$	0.393	0.396
$\rho[IQR(e_{z,t})]$	0.760	0.758
$\sigma[IQR(e_{z,t})]$	0.049	0.051
$\text{corr}[IQR(e_{z,t}), Y_t]$	−0.092	−0.127
UE rate	0.450	0.435
EU rate	0.026	0.026
EE rate	0.029	0.028
$\varepsilon_{\text{UE}/\theta}$	0.720	0.743
Average establishment size	15.6	15.2
Entry/Total job creation	0.21	0.27

[a]UE, EU, and EE are monthly transition rates. The notation ρ stands for autocorrelation and σ for standard deviation. Y_t denotes output. The autocorrelation and standard deviation of log-detrended output are quarterly. $IQR(e_{z,t})$ denotes the interquartile range of annual innovations to idiosyncratic productivity. $\varepsilon_{\text{UE}/\theta}$ is the elasticity of the UE rate with respect to the aggregate vacancy-unemployment ratio.

in the economy, consistent with the 71% found by Hall and Milgrom (2008). The vacancy cost c is about 32% of the average quarterly compensation of workers, which is about twice as much as the 14% estimated in Silva and Toledo (2009). There is, unfortunately, no widely accepted empirical estimate for the entry and operating costs. Comparing their values to the average output produced by a single firm in a month, my estimated costs represent about 38% for the entry cost k_e and about 5% for the operating cost k_f.

3.2. *Business Cycle Statistics*

To evaluate the performance of the model at the aggregate level, I simulate it for a large number of periods and compute various business cycle moments. In particular, I calculate the standard deviation and contemporaneous correlation with output of several variables. These variables include unemployment, total vacancies, and various labor market flows such as total hirings, quits, and layoffs. In order to understand the contribution of idiosyncratic volatility shocks, I further compute the same moments in a version of my model with aggregate productivity shocks only.

The results are presented in Table III. A first striking result is that the model proposed in this paper explains about 50% of the volatility in unemployment with aggregate productivity shocks only (column 3). This finding suggests that the introduction of heterogeneous multiworker firms and the presence of a slow-moving distribution of employment across establishments adds amplification to search-and-matching models, which are known to produce little volatility in aggregate unemployment when calibrated to match moments as those chosen in my estimation.[15,16]

Most importantly, column 5 shows that the addition of stochastic idiosyncratic volatility makes substantial progress in explaining the volatility of labor market variables. With

TABLE III

BUSINESS CYCLE STATISTICS[a]

	Data		Model (y Only)		Model ($y + v$)	
	(1) Std Dev.	(2) cor(Y, x)	(3) Std Dev.	(4) cor(Y, x)	(5) Std Dev.	(6) cor(Y, x)
Y	0.016	1	0.017	1	0.016	1
Y/L	0.012	0.590	0.014	0.993	0.013	0.977
U	0.121	−0.859	0.067	−0.954	0.090	−0.725
V	0.138	0.702	0.034	0.680	0.053	0.264
Hirings	0.058	0.677	0.033	0.544	0.049	0.199
Quits	0.102	0.720	0.070	0.881	0.072	0.649
Layoffs	0.059	−0.462	0.048	−0.969	0.087	−0.606

[a]Time series are aggregated to a quarterly frequency and presented in log-deviation from an HP trend with parameter 1600. Y is output, Y/L output per person, U unemployment, V vacancies. Quits are identified as job-to-job transitions in the model. See Appendix B for data sources.

[15]To emphasize this point, Table IV evaluates the same business cycle moments to a standard Diamond–Mortensen–Pissarides model, calibrated along the strategy described in Shimer (2005). The calibration is identical to the one in the original article except that I target the autocorrelation (0.839) and standard deviation of output (0.016) instead of output per person to harmonize it with my estimation.

[16]Curvature in the production function is not responsible for this result either. When the model is recalibrated with α set to 0.75 instead, the model without aggregate productivity shocks explains only 44% of the fluctuations in unemployment. This result had already been pointed out in the case of bargaining models by Hawkins (2011).

TABLE IV

COMPARISON WITH STANDARD DIAMOND–MORTENSEN–PISSARIDES MODEL[a]

	Data		Shimer (2005)		Model (y Only)	
	Std Dev.	cor(Y, x)	Std Dev.	cor(Y, x)	Std Dev.	cor(Y, x)
Y	0.016	1	0.016	1	0.017	1
Y/L	0.012	0.590	0.016	1	0.014	0.993
U	0.121	−0.859	0.006	−0.984	0.067	−0.954
V	0.138	0.702	0.019	0.993	0.034	0.680
Hirings	0.058	0.677	0.003	0.441	0.033	0.544
Quits	0.102	0.720	—	—	0.070	0.881
Layoffs	0.059	−0.462	0.001	0.936	0.048	−0.969

[a]Time series are presented in logs. Quarterly time series detrended using an HP filter with parameter 1600. Y is output, Y/L output per person, U unemployment, V vacancies. Quits are identified as job-to-job transitions in the model. See Appendix B for data sources. I compare simulated moments from a standard DMP model calibrated as in Shimer (2005) to my model with productivity shocks only.

these additional shocks, the model accounts for 75% of the total volatility in unemployment as well as a greater fraction of the volatility in other variables, improving the general fit of the model with the exception of an excessive volatility in layoffs. Turning to comovements, aggregate productivity shocks being the only source of business cycle fluctuations, column 4 displays in general excessively high contemporaneous correlations with output. The introduction of time-varying volatility in column 6 breaks this result and helps the model produce slightly lower correlations more in line with the data, as evidenced by quits and layoffs. The correlation of vacancies and hirings with output are, however, too weak compared to the data because they tend to rise with volatility shocks, as we will see in the next section. Qualitatively, the cyclicality of each variable is in general correctly predicted by both versions of the model, with various degrees of quantitative success.

Overall, the introduction of heterogeneous multiworker firms allows the model to have more realistic predictions than a typical search-and-matching model. In addition, volatility shocks generate larger fluctuations in unemployment and other labor market variables, offering a plausible mechanism to account for the volatility unexplained by standard models.

3.3. *Establishment-Level Properties*

Because I do not target any establishment-level or cross-sectional moment other than the interquartile range of idiosyncratic productivity, I now examine several implications of the model in the cross-section of establishments as a validation exercise. For that purpose, it is convenient to introduce the following measure of establishment growth rates as used by Davis, Haltiwanger, and Schuh (1996). Denoting $n_{i,t}$ as the total employment of establishment i at date t, define growth rate $g_{i,t}$ as

$$g_{i,t} = \frac{n_{i,t} - n_{i,t-1}}{\frac{1}{2}(n_{i,t} + n_{i,t-1})}.$$

This measure takes the ratio of net employment growth to the average size of the establishment between periods $t-1$ and t. This measure is convenient in that it can account for the entry and exit and treats them in a symmetric fashion. A growth rate of 2 means entry, while -2 stands for exit.

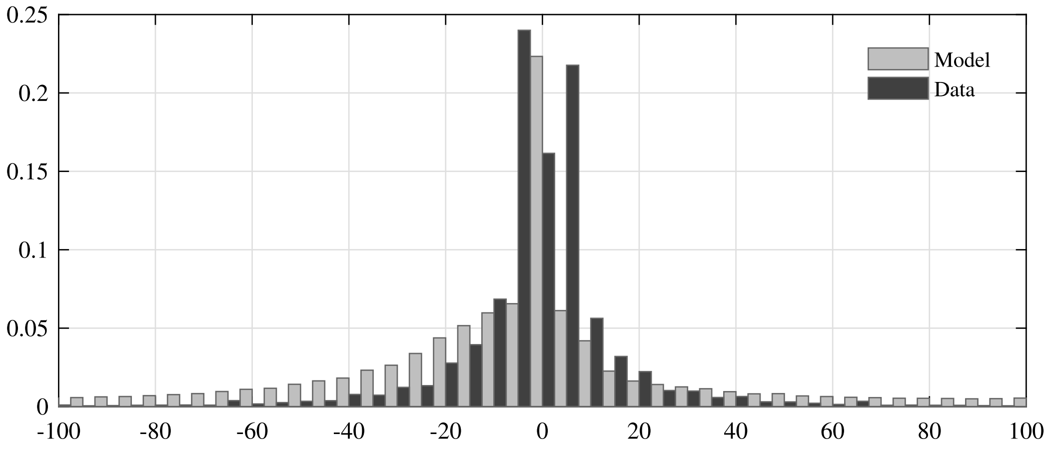


FIGURE 3.—Distribution of quarterly establishment growth rates. *Notes*: Quarterly data from 2008 tabulated from the BED data set by Davis, Faberman, and Haltiwanger (2012). Simulated distribution aggregated over a three-month interval.

3.3.1. *Growth Rate Distribution*

Davis, Faberman, and Haltiwanger (2012)[17] reported the quarterly employment growth rate distribution of establishments using data from the BED data set in 2008. I simulate the model for a large number of periods, aggregate the data over three-month periods, and compare the empirical and simulated growth rate distributions.

Figure 3 displays the two distributions. Given that the only cross-sectional moment in the estimation was the IQR of idiosyncratic productivity shocks, the model-generated distribution displays a reasonable fit to its empirical counterparts. Yet, the fit is imperfect and the reason is worth highlighting for future extensions. On the positive side, both present a large peak at 0 (16% in the data, 22% in the model), which indicates that a substantial number of establishments do not adjust their employment at all in a quarter. The model can replicate this feature because the search frictions manifest themselves as a kink in the firm's problem, thereby producing a region of inaction for firms. On the negative side, the distribution generated by the model is more left-skewed than its empirical counterpart. This result stems, in the model, from the endogenous exit and dynamic selection of firms. Since mostly unproductive firms exit, large productive firms tend to be overrepresented in the sample of surviving firms. At the same time, these productive firms have a stronger tendency to contract over time because of mean reversion in their fundamentals. The combination of these two facts explains why the simulated distribution is asymmetric. A possible way to improve this dimension would be to introduce permanent productivity differences across firms, so that transitory productivity shocks would have a lower impact on exits and firm sizes.

3.3.2. *Employment Policy*

Empirical evidence shows that firms with different growth rates have different hiring, layoff, and quit rates. The composition of hirings against separations and the balance between layoffs and quits present some important regularities at the establishment

[17]I would like to thank Steven Davis, Jason Faberman, and John Haltiwanger for allowing me to use their tabulations from the BED data set.

level. Davis, Faberman, and Haltiwanger (2012) showed that capturing these regularities may be important to improve the time-series predictions of search models. Being one of the few models in the literature with multiworkers firms and a meaningful distinction between quits and layoffs, I examine my model's predictions along this dimension.

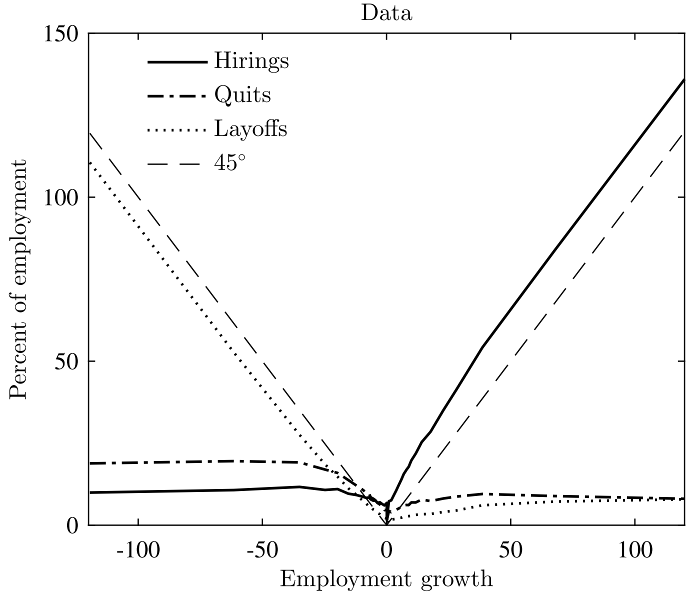
Figure 4 displays the empirical and simulated employment-weighted levels of hirings, quits, and layoffs as a function of establishment growth. To produce this graph, I simulate the model for a large number of periods and compute the corresponding series by aggregating over three-month periods. Quite surprisingly, without targeting any of these observations in the cross-section, the model can replicate a number of qualitative and quantitative features of hiring, quit,, and layoff rates at the establishment level. In particular, it is able to match the change in the composition of quits versus layoffs for contracting firms. Establishments that contract by a small amount tend to favor quits over layoffs, as they internalize the fact that workers can be directly employed without experiencing unemployment. However, the job-to-job transition technology becomes congested at some point, and firms that contract by a significant amount use layoffs more intensively.[18] The key qualitative feature that the model misses is *churning*: expanding establishments in the data separate from a non-negligible fraction of their workforces; contracting establishments, on the other hand, hire a positive amount of workers. The model is able to generate churning to some extent through time aggregation, as evidenced in the nonzero amount of quits for expanding establishments, but too much churning is suboptimal in the model since workers are homogeneous. Accounting for the observed level of churning in the data would likely require adding worker heterogeneity in productivity to the model.

4. UNDERSTANDING THE FORCES AT WORK

With time-varying idiosyncratic volatility and multiworker firms heterogeneous in productivities and sizes, this paper introduces two important dimensions to standard search-and-matching models. Before running the final counterfactual experiments, I pause in this section to describe the workings of the model in detail and explore how each of these dimensions affect the labor market.

I first describe the equilibrium and, in particular, how search frictions affect the employment decision of firms as a function of their productivities and sizes. The optimal policy takes the form of various action thresholds—or *triggers*—in the spirit of the kinked adjustment cost literature. I then examine the impact of aggregate productivity and idiosyncratic volatility shocks. The response of the economy to these shocks hides a rich variety of effects that I decompose between first moment, general equilibrium, option value, and realized volatility effects.

[18]Note that the levels at which the quit rate settles for contracting establishments differ in the model and the data. This result is an artifact of the estimation. Because we are ultimately interested in the aggregate predictions of the model, the estimation targets the aggregate Employment-to-Employment (EE) rate in the data. However, since expanding establishments in the model do not use quits, the estimation compensates with larger rates for contracting ones.



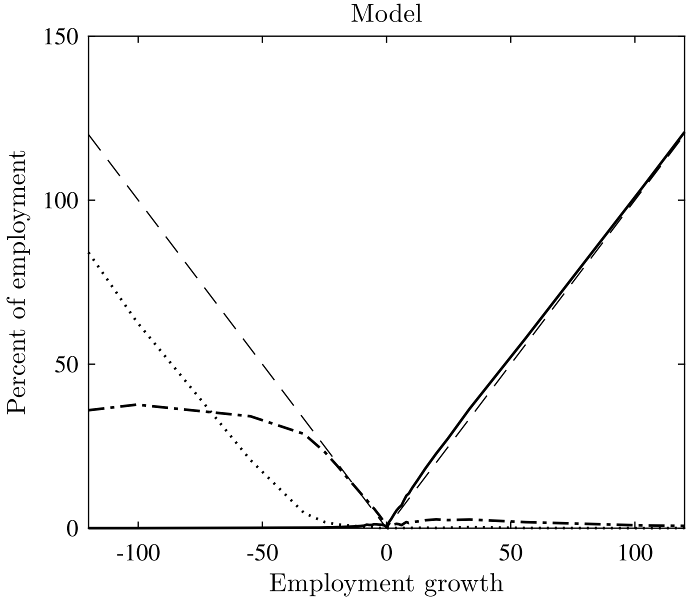


FIGURE 4.—Empirical and simulated employment policies as a function of growth. *Notes*: Tabulations from the BED data set by Davis, Faberman, and Haltiwanger (2012). Simulations aggregated over a three-month period. The averages are employment weighted. The dashed lines are the $-45°$ and $+45°$ lines to show the minimal level of separations and hirings needed to achieve the corresponding growth rate.

4.1. *Equilibrium Description*

4.1.1. *Labor Market Equilibrium*

The labor market is organized in a continuum of submarkets, indexed by the contracts that firms offer. We take a closer look in this section at how firms and workers allocate themselves across these submarkets in equilibrium.

Figure 5 depicts the different labor market segments on the axis $[\underline{x}, \overline{x}]$ with the equilibrium market tightness $\theta(s, x)$. An important feature is that the market tightness decreases with the value of the contract. To maximize profits, firms prefer offering low utility contracts and post more vacancies in markets with low x. However, as these markets become more crowded, the job-filling probability declines and the cost of searching rises. As a result, some firms find it profitable to raise their offers, trading off lower profits from higher utility-wages for a greater probability of filling the vacancy, until they become effectively indifferent across markets. The equilibrium tightness, captured in equation (11), is consequently a decreasing function of x with the implication that the job-filling probability for firms rises with the value of the contract, while the job-finding probability for workers declines.

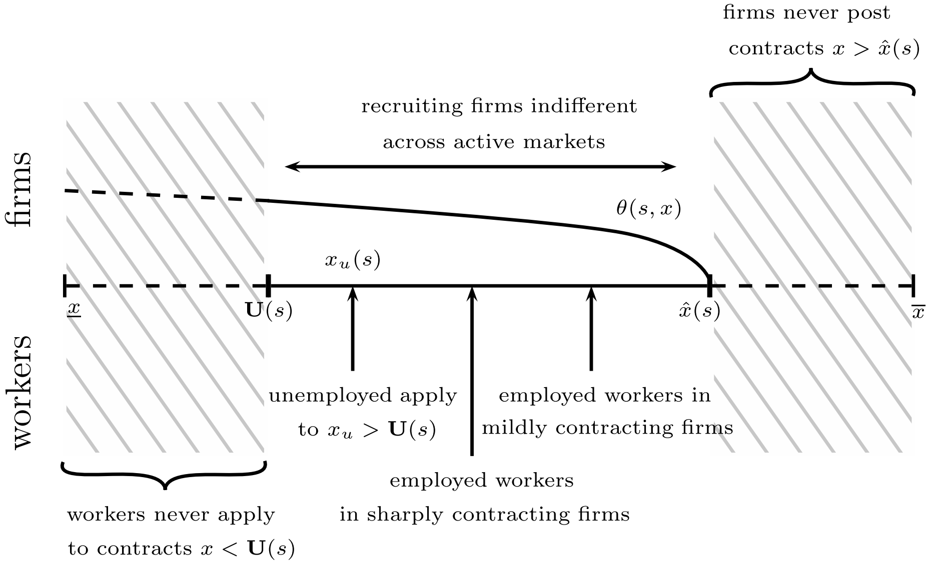


FIGURE 5.—Description of labor market equilibrium. *Notes*: (1) The equilibrium market tightness $\theta(s, x)$ decreases with the value of the contract x. Because offering lower-paying contracts yields higher profits to firms, more vacancies are posted for low-x markets until the job-filling probability drops sufficiently so that recruiting firms are effectively indifferent between active markets. (2) Workers are not indifferent between markets, because they have different outside options. Having the lowest outside option, that is, unemployment, unemployed workers are less willing to tolerate low job-finding probability and apply to markets with a low wage-utility x but high job-finding probability. Because of efficiency, the relevant concept of outside option for employed workers is the shadow value of maintaining employment. It is thus possible to rank where employed workers apply for jobs: workers in sharply contracting firms have a lower outside option and apply to lower paid jobs than workers in mildly contracting firms. (3) Markets such that $x < \mathbf{U}(s)$ are inactive in equilibrium because unemployed workers never apply to jobs with a value below that of unemployment. Similarly, firms never post vacancies in markets with $x > \hat{x}(s)$, a point at which tightness is 0 and the job-filling probability is 1, because offering higher-paying contracts cannot increase the job-filling probability further.

While recruiting firms are indifferent across the various submarkets, workers are not. Having different outside options, unemployed and employed workers search on different market segments as illustrated on the graph: unemployed workers tend to apply to low-paid jobs, while employed workers apply to higher-paid jobs, as they are more willing to tolerate low job-finding probabilities.

4.1.2. *Employment Policy at the Establishment Level*

Establishments can use various margins—hires, quits, layoffs, or exit—to adjust employment over the cycle. I examine in this section how the decision of firms to use these margins varies as a function of their individual characteristics (z, n) at the beginning of a period.

Figure 6 displays the optimal policy of firms, as it appears in my baseline calibration. As one would naturally expect, hirings take place at small productive firms, whose marginal value of adding jobs is high, while separations—quits and layoffs—occur at large unproductive firms. Interestingly, because search frictions show up in the surplus (7) as a linear hiring cost, $\kappa(s) = c/q(s, x_i) + x_i$, a wedge appears in the adjustment cost faced by firms at $n' = n$. More specifically, laying a worker off earns a value of $\mathbf{U}(s)$ to the worker-firm

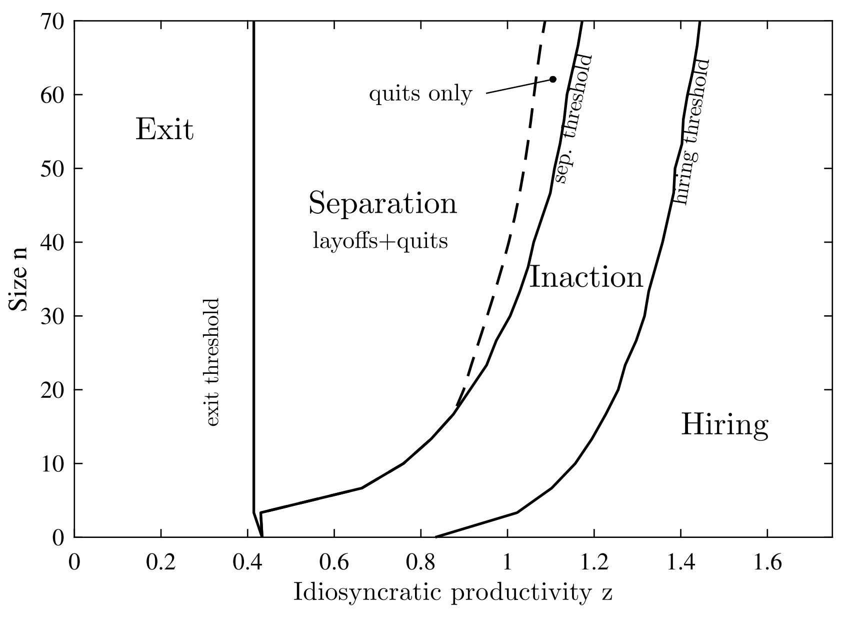


FIGURE 6.—Firm's action thresholds in the space of (z, n). *Notes*: The optimal policies depicted on this figure correspond to the baseline calibration, holding the aggregate productivity y and volatility v to their mean values. Several points are worth noticing. (1) The areas corresponding to the different margins of adjustment are distinct and do not overlap, with the exception that firms separating from some of their workers tend to use, in general, a mix of quits and layoffs. However, hires and separations never occur at the same time because it is more costly for firms to hire new workers than retain the current workforce. (2) There exists a narrow band between the dashed line on the figure and the separation threshold, where firms exclusively separate from their workforce using quits. This feature is due to the fact that workers are strictly better off switching jobs directly, instead of going through a painful spell of unemployment. Firms successfully internalize this fact and send their workers looking for jobs outside before laying them off. However, the job-to-job transition technology is limited and quickly crowds out, so that firms willing to separate from a larger fraction of their workforce also use layoffs.

group, while hiring incurs the cost $\kappa(s)$, strictly greater than the value of unemployment in equilibrium.[19] Arising from this kink in adjustment costs, a band of inaction emerges between two thresholds, a hiring and a separation threshold, which play the role of triggers in the firm's employment strategy. Whenever a firm falls in the hiring region, in the lower right area, its optimal strategy consists in hiring workers up until it reaches the hiring threshold—a point at which the marginal value of adding jobs equals the hiring cost. Symmetrically, whenever a firm finds itself in the separation region, its optimal decision is to separate from workers, using a mix of quits and layoffs, until it reaches the separation threshold, at which the marginal value of employment equals the marginal value of quitting.[20] The presence of an inaction region implies the existence of a non-negligible mass of firms not adjusting employment within a period, a fact well supported in the data as evidenced by Davis, Haltiwanger, and Schuh (1996).

Exits take place at small unproductive firms. Indeed, the operating cost k_f being fixed, the decision to exit mostly affects small firms with low productivity, as their current production and expected future surpluses fall short of the total operating costs. This feature is consistent with empirical observations, as evidenced in Evans (1987).

4.2. *Productivity Shocks*

Aggregate productivity shocks are the common source of business cycle fluctuations in the search-and-matching literature. In this section, I analyze the impact of negative aggregate productivity shocks at the macroeconomic level and use the model to study at a deeper level how these shocks affect firms in the cross-section. The response of the economy reflects the combination of various effects from partial to general equilibrium, that affect entrants and incumbents in remarkably different ways. In what follows, *partial equilibrium* refers to the direct response of an individual firm to the shock in isolation from the endogenous response of aggregate variables such as the labor market tightness and the value of unemployment, which are held constant during the experiment. *General equilibrium* is the total response when the tightness and the value of unemployment are allowed to adjust to their equilibrium levels.

4.2.1. *Employment Policy*

Figure 7 illustrates how a negative one standard deviation productivity shock affects the employment strategy of firms in partial equilibrium (upper panel) and in general equilibrium (lower panel). The black continuous lines depict the hiring, separation, and exit thresholds before the shock, when aggregate productivity and volatility are set to their means; the dashed lines describe how these thresholds are affected when the shock hits.

How a negative productivity shock affects the employment strategy of firms in partial equilibrium is straightforward. When productivity declines, the marginal value of a job decreases. As a result, expanding firms cut on hiring and grow less—the hiring threshold

[19] Because the market $x_u(s) > \mathbf{U}(s)$ where unemployed workers search is active in equilibrium, we know that $\kappa(s) = c/q(\theta(s, x_u(s))) + x_u(s) > \mathbf{U}(s)$. Similarly, a wedge appears between the value of quitting and the cost of hiring, as the best contract offered in equilibrium is $\hat{x}(s) = \kappa(s) - c < \kappa(s)$.

[20] Notice here that the optimal policy takes the form of a "barrier control" policy in which the hiring and separation thresholds both play the role of *triggers* and *return points*, in accordance to the kinked adjustment cost literature (see Bertola and Caballero (1990)). Despite important similarities, this strategy is different from the Ss-type policies that arise in the fixed adjustment cost literature, where the trigger and return point differ (see Khan and Thomas (2008)).

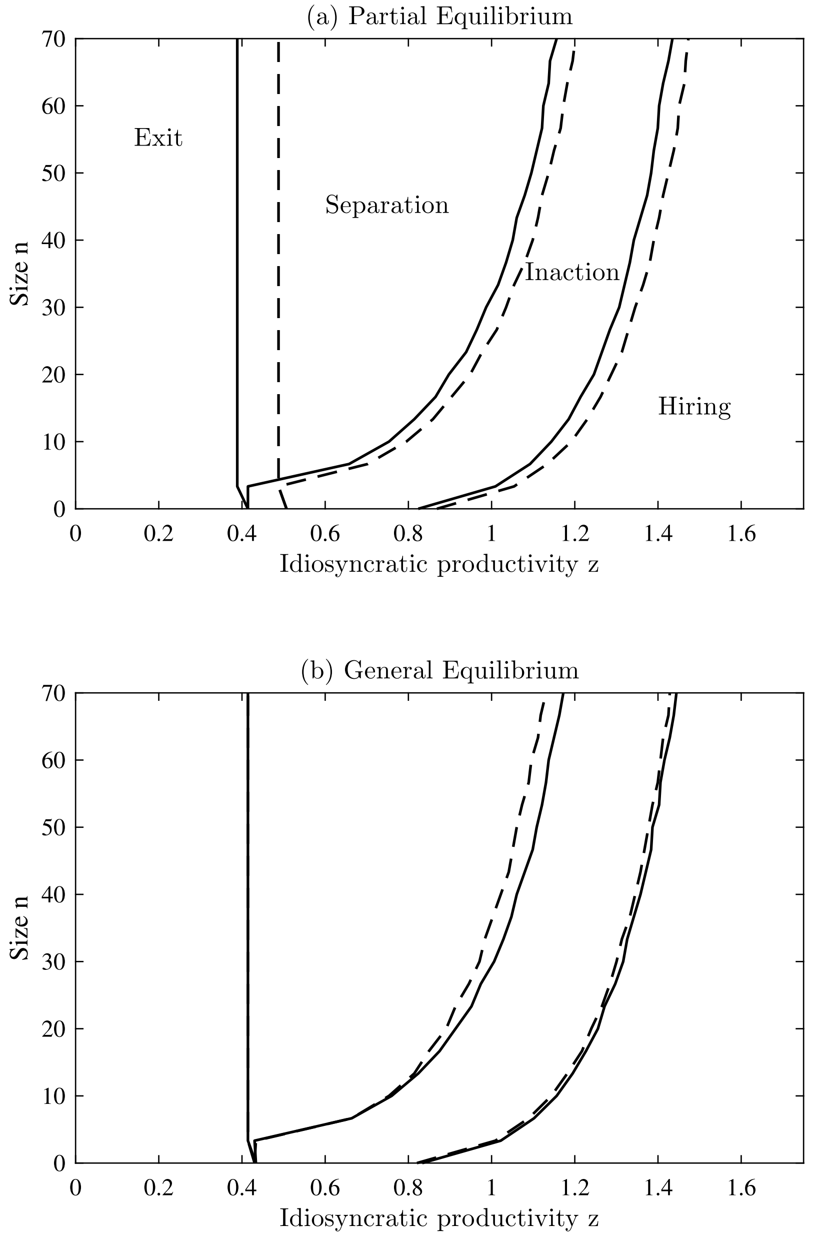


FIGURE 7.—Firm's optimal policy after a negative one-standard-deviation shock to *y*. *Notes*: The black continuous line corresponds to the firm's optimal policy before the shock, and the dashed line is after the shock. The general equilibrium panel corresponds to the full model. The partial equilibrium is computed holding the hiring cost and the value of unemployment constant after the shock.

shifts down—while separations rise as the value of jobs in less productive firms falls below the value of switching to another job and the value of unemployment. The separation threshold shifts down and the separation region widens. An increase in exits is simultaneously observed as the decline in production makes firms at the margin of profitability unable to cover the costs of operation, causing a rightward shift of the exit threshold.

This picture is, however, incomplete without considering general equilibrium effects. As the value of firms falls with productivity, entry declines, and the tightness falls on active segments of the labor market. Consequently, the job-finding rate of workers dips, causing a fall in the value of unemployment as job prospects deteriorate. At the same time, as the degree of competition on the labor market diminishes, the cost of hiring drops and firms find it easier to hire workers. Resulting from these two general equilibrium effects, relatively low productivity firms have weaker incentives to separate, while productive ones are encouraged to hire more. Which of these first moment or general equilibrium effects dominate is, in principle, ambiguous. The total effect of productivity shocks in the baseline calibration is displayed in the lower panel of Figure 7. The general equilibrium effects dominate on these margins as the hiring and separation thresholds shift leftward, while the opposite forces affecting the exit threshold exactly cancel out.[21]

4.2.2. *Impulse Responses*

Figure 8 displays the impulse responses of several variables after the economy is hit by a negative 1% transitory shock to aggregate productivity. As one would expect, output and output-per-worker drop on impact and recover slowly, closely tracking the recovery in productivity. Total vacancies and hirings decrease, largely driven by a fall in entry that dominates the mild increase in hiring by incumbents. Consistent with Figure 7, total separations decrease, hiding two opposite behaviors from quits and layoffs. As entry falls and unemployed workers start flooding the labor markets, the probability of finding a job decreases for workers, making job-to-job (quits) transition less appealing. As a result, contracting firms reduce their use of quits, but intensify layoffs. The joint increase in layoffs with a reduction in hiring results in an overall rise in unemployment of about 4%, confirming our previous finding that the addition of firm heterogeneity to search-and-matching models may provide some amplification to aggregate productivity shocks. Turning to exits, even though the exit threshold is unaffected by aggregate productivity, total exits rise because exiting firms are on average larger. Figure 9 breaks down the response of the economy in three categories by simulating (i) the partial equilibrium response of a population of firms with the number of entrants held constant, (ii) the response of the same population of firms with constant entry but allowing for general equilibrium objects to adjust, (iii) the total response with flexible entry.

4.3. *Volatility Shocks*

We are now ready to address the main question that motivated this study: what is the impact of uncertainty or volatility shocks on the economy as a whole, the labor market,

[21]The key to understanding why general equilibrium effects are so strong in this economy lies in the fact that the general equilibrium objects are determined by the infinitely elastic entry margin through the free-entry condition. The requirement that the value of firms remains constantly equal to the entry cost necessitates a strong reactivity of general equilibrium objects. For instance, the fall in the value of entering firms must be largely compensated by a decline in the hiring cost and tightness, sufficiently so to offset the more muted response from incumbent firms. As a result, incumbent firms benefit on net from the fall in hiring costs and grow in response to a fall in productivity. Entrants, on the other hand, being determined as the residual that adjusts to satisfy equilibrium on each labor market segment, take a large hit, and entry falls significantly.

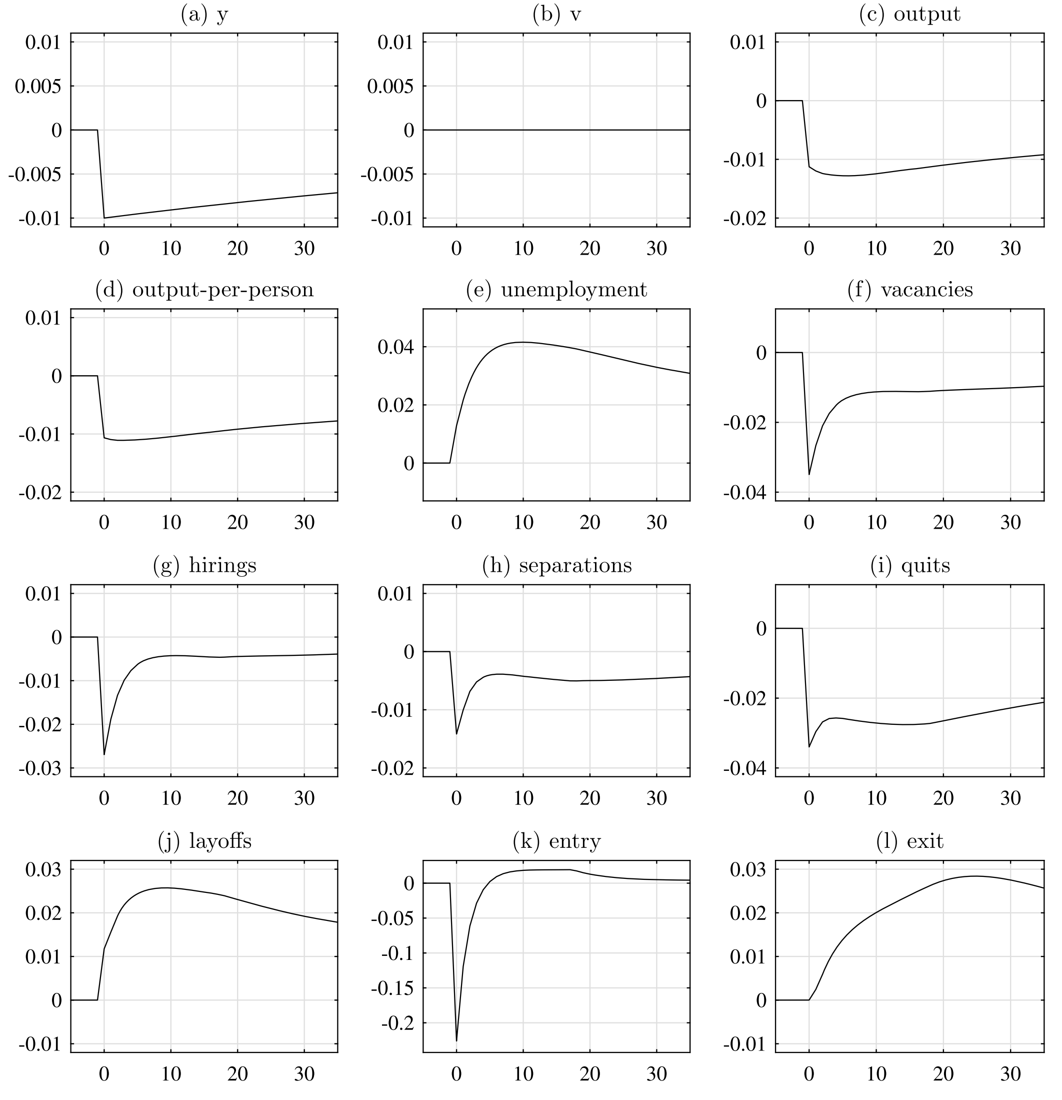


FIGURE 8.—Response to a -1% transitory shock to aggregate productivity y. *Notes*: Series presented in log deviation from their steady-state values when aggregate productivity and volatility are set to their means. The time period is a month and the shock hits at time $t = 0$. Separation is the sum of quits and layoffs. Entry and exit are expressed in total employment.

and the cross-section of firms? Volatility shocks produce a variety of effects that are, in general, difficult to disentangle, including real option effects, Oi–Hartman–Abel effects, realized volatility, and general equilibrium effects.

4.3.1. *Employment Policy*

Let us first examine how an increase in idiosyncratic volatility affects the optimal employment policy of firms as a function of their individual characteristics. Using the same convention as in the previous section, Figure 10 presents the impact of a positive one standard deviation shock to volatility v in both partial (upper panel) and general equilib-

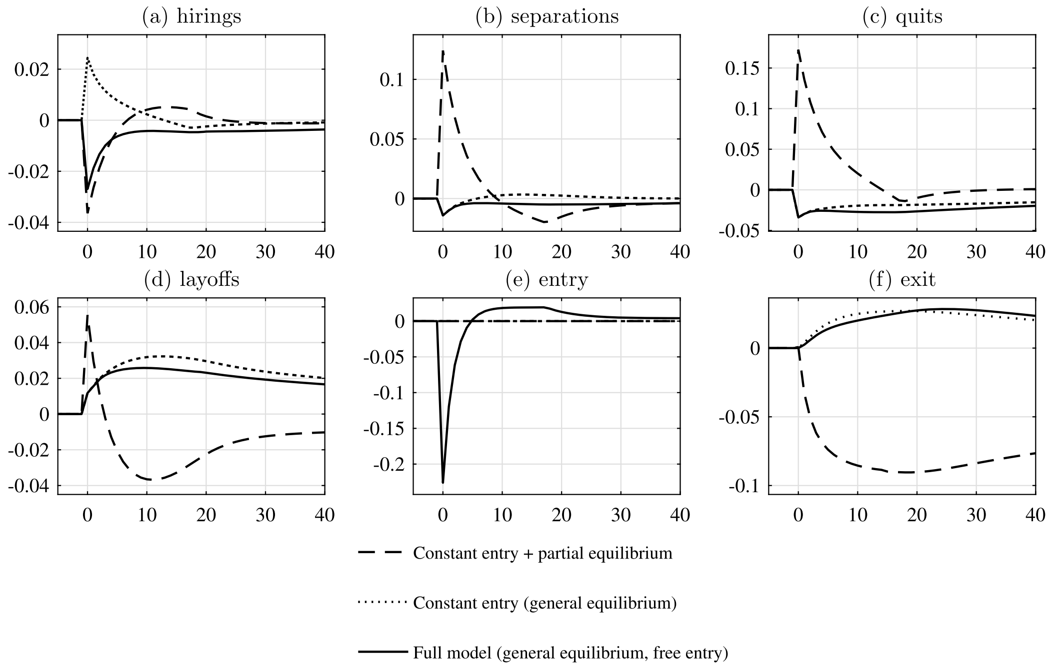


FIGURE 9.—Breakdown of response to -1% transitory shock to aggregate productivity y. *Notes*: The black continuous line corresponds to the full economy; the dotted line to an economy with constant entry set to its steady-state value; the dashed line to an economy with constant entry and partial equilibrium. The series are presented in log deviation from the steady state when aggregate productivity and volatility are set to their means. The time period is a month and the shock hits at time $t = 0$. The shock is identical to that in Figure 8. These series can be interpreted as follows. In partial equilibrium with constant entry (dashed line), we know from Figure 7 that firms hire less and separate more, leading to greater quits and layoffs. While the exit threshold shifts out, total exits slowly decline, reflecting the fact that exiting firms are on average smaller after the shock. Turning to the dotted line, which adds the general equilibrium effects, hirings shoot up due to the lower cost of hiring for incumbents. Quits and layoffs decline, caused by worsened job prospects for job movers and a lower value of unemployment. Exits rise mildly because exiting firms are on average larger with lower hirings costs. See the main text for the full model.

rium (lower panel), where partial equilibrium describes the response of an individual firm when labor market tightness and the value of unemployment are held fixed and general equilibrium is the total response.

The partial equilibrium figure allows us to isolate the real option effects. Consistent with previous literature, an increase in volatility raises the option value of waiting, and firms have stronger incentives to delay decisions that involve irreversibilities. Because search incurs sunk costs, the decision to hire a worker is partially irreversible and firms have a tendency to defer recruitment to future periods.[22] Likewise, firms may prefer to delay laying off workers, in order to avoid repaying the search costs if conditions were to improve. Consequently, the hiring and separation regions shrink, leading to a widening of

[22]It should be noted here that such real option effects on the hiring margin are absent from standard one-worker/one-firm search models with free entry as the infinitely elastic entry margin eliminates any option value embedded in vacancies by pushing their value to zero. In this model, however, the free-entry condition equalizes the value of firms to the entry costs, but the value of jobs can vary, leading to an optimal timing decision for vacancy posting.

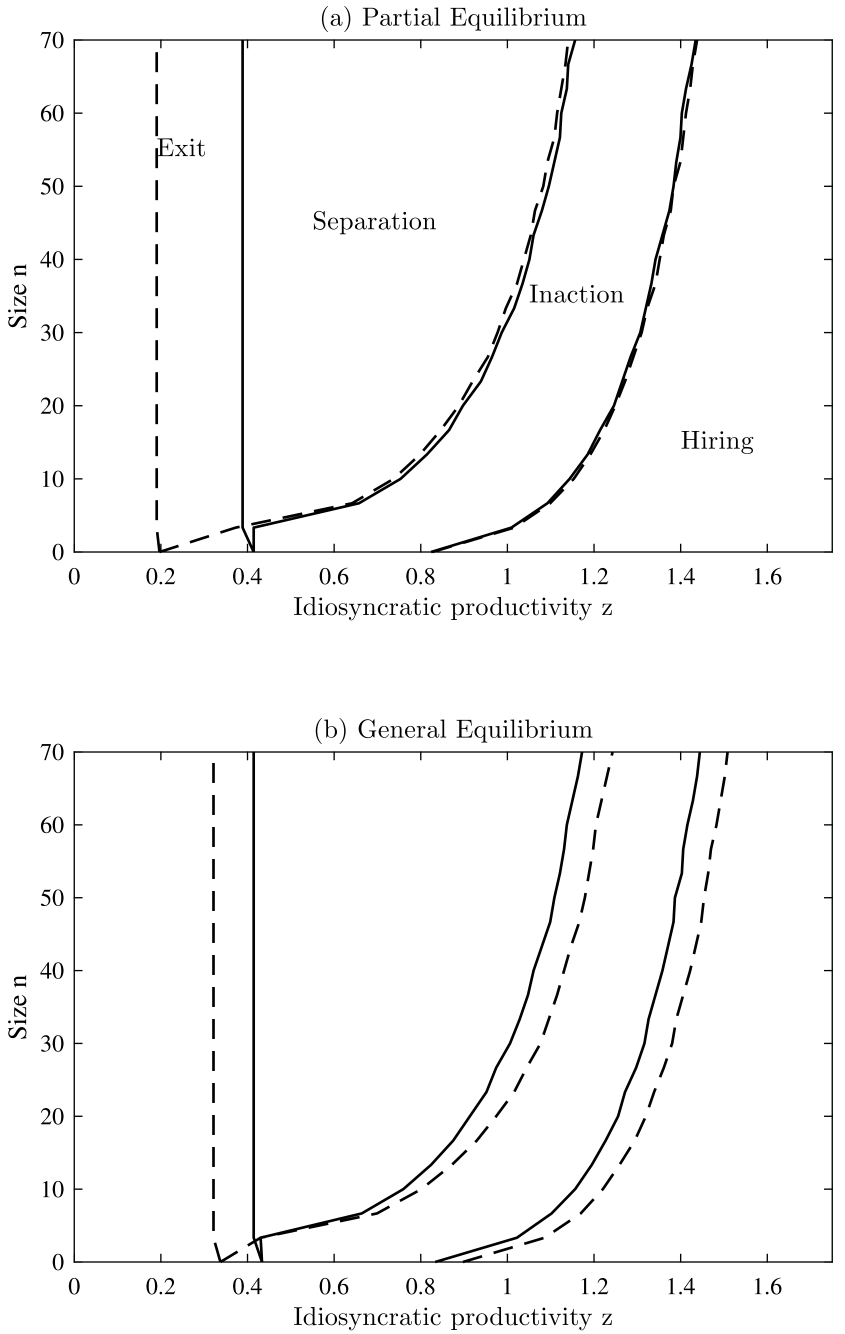


FIGURE 10.—Firm's optimal policy after a positive one-standard-deviation shock to v. *Notes*: The black continuous line corresponds to the firm's optimal policy before the shock, and the dashed line is after the shock. The general equilibrium panel corresponds to the full economy. The partial equilibrium is computed holding the hiring cost and the value of unemployment constant after the shock.

the inaction band. For the same reason, exits, being fully irreversible, subside substantially and the exit threshold falls back left.

The upper panel of Figure 10 reveals an important finding: search frictions alone do not seem large enough to generate strong option value effects. Despite being qualitatively consistent with the uncertainty literature, the *wait-and-see* effects, visible in the widening of the inaction band, are surprisingly small. This finding stems from the fact that labor market mobility in the United States is high. For instance, Davis, Faberman, and Haltiwanger (2013) estimated the job-filling rate probability to be 5.2% per day, about 80% per month, while the average job-finding probability per worker is about 45% per month. Because these numbers are high, the degree of irreversibility of a hire or a layoff cannot be too large. Hence, any model calibrated to match average labor market flows in the United States would have difficulty generating strong option value effects, unless additional costs or heterogeneity among workers were considered.[23] One should not, however, jump too quickly to the conclusion that volatility is unimportant to explain the dynamics of the labor market. As we will see in the next section, time-varying volatility will prove to be important to explain several episodes in the data, mostly through its impact on reallocation.

The lower panel of Figure 10 incorporates the general equilibrium effects on hiring costs (tightness) and the value of unemployment. The movements in the various thresholds, in this case, are mostly due to an effect commonly called the *Oi–Hartman–Abel effect*.[24] Because of the Oi–Hartman–Abel effect and an embedded real option value, idiosyncratic volatility shocks increase the value of firms, causing a large flow of firms to enter the economy. Consequently, the labor market tightness rises, the cost of hiring shoots up, making incumbent firms hire less and pushing the hiring threshold further down. Simultaneously, a higher tightness leading to a greater job-finding probability, the value of quitting, and the value of unemployment rise, leading firms to separate more and the separation region to expand, effectively overriding the option value effect. Finally, the exit region widens in comparison to the partial equilibrium case, as the greater hiring costs reduce the expectation of future surpluses.

4.3.2. *Impulse Responses*

Figure 11 displays the aggregate impulse responses of several variables to a transitory +5% idiosyncratic volatility shock. The response of the economy reflects the combination of various components: (i) partial equilibrium (isolating real option effects), (ii) general equilibrium, (iii) entry, and (iv) realized volatility. The term *realized volatility* designates the fact that dispersion across firms actually increases once the volatility shock is realized. As a result, even though the inaction band may widen, firms may hit the action thresholds more often and become more active in response to an increase in uncertainty. The model predicts that this effect is strong, and I thus attempt to quantify the relative importance of each component.

As with our previous decomposition in the case of a productivity shock, Figure 12 offers a decomposition of the response of the economy to a transitory volatility shock according

[23]To explore the robustness of this claim, I run the same exercise in partial equilibrium by increasing the vacancy posting cost c and lowering the efficiency of the matching function m, such that $p(\theta) = \theta q(\theta) = m \cdot \theta(1+\theta^\gamma)^{-\frac{1}{\gamma}}$. The option value only becomes sizeable for extreme value of c and m that would be difficult to reconcile with the data.

[24]This effect, described in Oi (1961), Hartman (1972), Abel (1983), is well known in the uncertainty literature. Because firms increase employment when idiosyncratic productivity rises, while they reduce employment when productivity is low, the value of a firm is in general a convex function of productivity. As a result, a mean-preserving spread of idiosyncratic productivity tends to increase the value of firms.

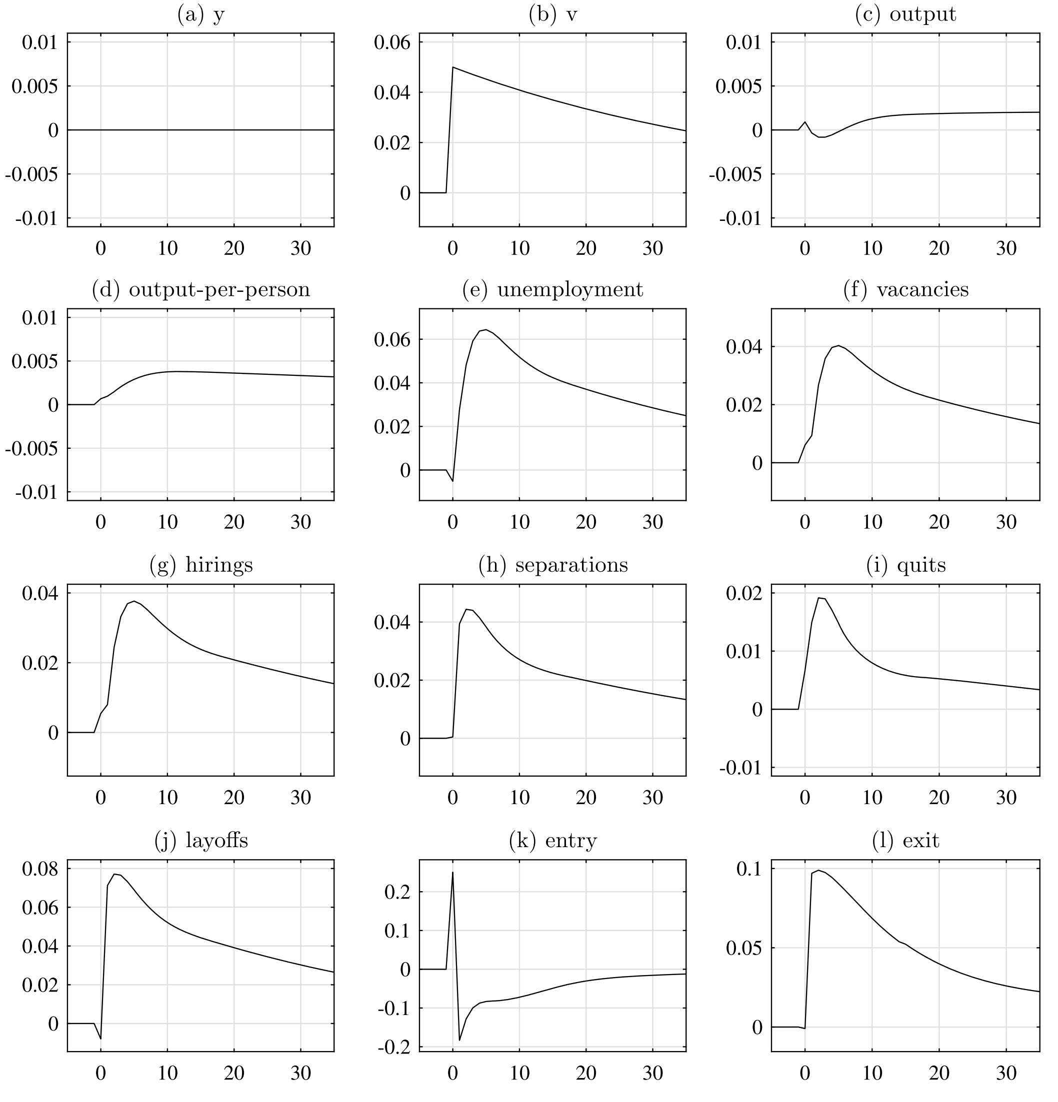


FIGURE 11.—Response to $+5\%$ transitory shock to idiosyncratic volatility v. *Notes*: Series presented in log deviation from their steady-state values when innovations to aggregate shocks are set to 0 for a long time. The time period is a month and the shock hits at time $t = 0$. Separation is the sum of quits and layoffs. Entry and exit are expressed in total employment.

to partial versus general equilibrium, flexible versus constant entry and, additionally, realized volatility. For the latter, I simulate an economy in which only the beliefs of firms are hit by the volatility shock, while the actual realization of the shock remains constant to its steady-state level, in order to isolate the effect of realized volatility. The black continuous line presents the response of the full economy. Starting from the simplest, the dash-dotted line presents the direct partial equilibrium response of firms, holding the number of entrants and realized volatility constant. The dashed line is identical but presents the firms' total response when general equilibrium objects adjust to their equilibrium levels (tightness and the value of unemployment). From the dashed line to the dotted line, I relax the

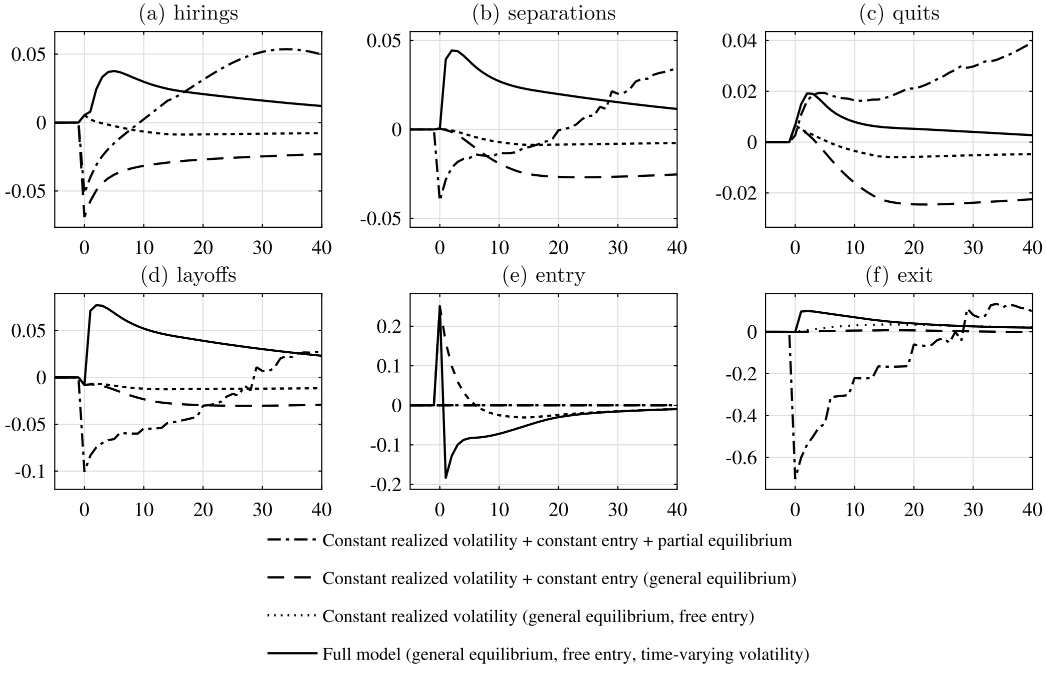


FIGURE 12.—Breakdown of response to +5% transitory shock to volatility v. *Notes*: The black continuous line corresponds to the full economy; the dotted line to an economy with constant realized volatility fed but the same entry process as the full economy; the dashed line to an economy with constant realized volatility and entry set to its steady-state value; the dash-dotted line to an economy with constant realized volatility, entry set to its steady-state value, and partial equilibrium. Series presented in log deviation from the steady state when innovations to aggregate shocks are set to 0 for a long time. The time period is a month and the shock hits at time $t = 0$. The shock is identical to that in Figure 11. These series can be interpreted as follows. The dash-dotted curve, which presents the partial equilibrium response with constant entry and constant realized volatility, isolates the real option effects with a fall in hirings, layoffs, and exits. Quits rise slightly as firms substitute away from layoffs to the milder form of separation that job-to-job transitions represent. The positive trend observed in these series is an artifact of the exercise because the panel of firms is unbalanced: entry is kept constant, while exits decrease, leading to a slow increase in employment over time. Adding the general equilibrium effects, the dashed curve displays a fall in hirings due to an increase in hiring costs. Despite the outward shift of the separation threshold, separations (quits and layoffs) decrease because firms operate on a smaller scale after the shock. Adding free entry, the dotted line shows a similar behavior with the addition of a surge in entry and a mild increase in hirings due to the Oi–Hartman–Abel effects. See the main text for the full model.

entry margin and allow the number of entrants to freely adjust. The difference between the dotted and black continuous lines identifies the contribution of the realized volatility effects.

The dash-dotted line shows the importance of the real option effects. Consistent with our findings from Figure 10, hirings, separations, and exits drop on impact because of an increase in the option value of waiting. Firms sensibly turn away from layoffs and substitute with quits to the point that quits rise, while layoffs plunge. Note that hirings and separations (quits and layoffs) end up rising to a point above their initial levels, an effect largely driven by the fact that the sample of firm grows as exits decline. Taking into account the endogenous response of the general equilibrium objects, the dashed line reflects our findings from Figure 10. Hirings fall deeper. Despite incentives for firms to separate more from workers, separations and layoffs decrease because firms operate at a smaller scale on average. Due to the higher job-finding probability, quits increase slightly on impact. The dotted line, which allows the number of entrants to adjust, shows a similar pattern to the dashed line except that hirings pick up immediately because of the surge in entry caused by the Oi–Hartman–Abel effect, discouraging future entrants and lowering hirings in the subsequent periods. The difference between the full model (black continuous line) and the dotted lines captures the effect of realized volatility. As the figure illustrates, the realized volatility effects are extremely large and dominate all the previously mentioned effects. Because they are hit by more dispersed shocks, firms hit their action thresholds more often and pure volatility shocks result in more turnover across firms: hirings, quits, layoffs, and exits all rise.

With our decomposition of labor market flows, we may now return to the aggregate impulse responses of Figure 11 to analyze the overall contribution of volatility to output and unemployment. Because of the Oi–Hartman–Abel effect, total output and output-per-person, aggregated over the cross-section of firms, rise as volatility increases. Vacancies increase, mirroring the evolution of total hires. Unemployment rises quite substantially because unemployment inflows (layoffs) dominate the outflows (hires from unemployment). Indeed, even though hirings increase in response to a volatility shock, a large part of that increase is accounted for by job-to-job transitions, as workers reallocate from low to high productivity firms. Therefore, unemployment surges, unambiguously reflecting the fact that a greater number of firms receive bad shocks.

5. COUNTERFACTUAL EXERCISES

After our detailed analysis on the impact of productivity and idiosyncratic volatility on the labor market, we are now prepared to conduct the main quantitative exercise. I ask, in this section, whether the model can account for the U.S. labor market experience over the period 1972–2009 and how much variation can be attributed to fluctuations in productivity and idiosyncratic volatility.

5.1. *Description*

I jointly estimate two series of shocks for aggregate productivity $\{y_t\}_{1972:1}^{2009:12}$ and idiosyncratic volatility $\{v_t\}_{1972:1}^{2009:12}$ by matching two natural empirical counterparts: (i) the quarterly output-per-person series from the BLS, and (ii) the annual cross-sectional IQR of innovations to idiosyncratic TFP from the Census. Since these two series are endogenous in the model, I use a procedure of search in the space of productivity and volatility shocks, which minimizes the distance between the empirical and simulated series. In both cases, the simulated series are computed following the same steps as in the data.[25]

Instead of using a standard HP filter to detrend the data in this exercise, I use the band-pass filter developed by Christiano and Fitzgerald (2003) and restrict my attention to fluctuations in the range of 6 to 32 quarters, as is commonly done in the business cycle literature. I adopt this method in order to remove high-frequency noise components from the empirical series.[26] To illustrate the difference between the two detrending approaches,

[25]Since the IQR measure controls for selection and productivity shocks have an impact on the selection of firms, the IQR responds to productivity. This effect is, however, small and the two series of shocks are well identified.

[26]Since an i.i.d. process has a flat frequency spectrum, note also that the band-pass filter reduces the incidence of i.i.d. measurement errors.

Figure 17 presents the output-per-person series in panel (a) and the IQR series in panel (b), detrended using both methods. As the figure shows, the two series are very close, but the HP-detrended series display more high-frequency variations, which turn out to be difficult to match with the model without extremely volatile, negatively autocorrelated shocks that cause a spurious amount of reallocation in the labor market.[27]

I perform two counterfactual experiments. In the first experiment, I use the full model calibrated as in Section 3 with both aggregate productivity and idiosyncratic volatility shocks. In the second experiment, in order to isolate the contribution of volatility, I run the same exercise in the version of the model with productivity shocks only and fit the output-per-person series alone, while volatility is kept constant to its mean. Figure 17 in the Supplemental Material shows the fit with the empirical series on panels (a) and (b). As the figure illustrates, the fit of the simulated series with their empirical counterparts is almost perfect. Panels (c) and (d) report the imputed aggregate productivity and idiosyncratic volatility shock series.

5.2. *Results*

I now analyze the ability of the model to account for the various NBER-dated recessions over the period 1972–2009.[28] Figures 13 and 14 report output and unemployment in the data and in the model across the five episodes. Because the labor market flow data from the Job Openings and Labor Turnover Survey (JOLTS) is unavailable before 2001, the recession of 2007–2009 is the unique episode entirely covered by the data set. Figure 15 displays the fit of the model for the various labor market flows provided by the JOLTS during this episode. The series are presented in log deviation from the peak (or trough for countercyclical variables) preceding the recession. Peaks (troughs) are identified as the local maxima (minima) that precede recessions.[29] Peak-to-trough measures in both variables are detailed on Table V for the various episodes.

The model is quite successful at explaining fluctuations in output with either version of the model. Early recessions in particular, including the recession of 1990–1991, display very little difference between the versions with and without volatility shocks. Productivity thus appears to be the main force driving variation in output. During the more recent recessions, the presence of volatility shocks help explain an additional 0.5% to 1% decline in peak-to-trough measures. The recession of 2001 displays the largest discrepancy with the data, but the overall fit is nonetheless satisfactory.

The unemployment series suggest a more important role for volatility. The model with productivity shocks explains in general between 40% and 60% of the total increase in unemployment in the early recessions of 1972 to 1991. The contribution of productivity to unemployment variation then falls to about 20% in the last two recessions. The introduction of volatility shocks, as we know, contributes to unemployment through a combination

[27] A caveat of this filtering approach is that the IQR series to display non-negligible low-frequency fluctuations (see Figure 1), which could in principle matter for the average level of unemployment, are totally eliminated from the detrended series. My analysis is thus limited to the business cycle frequencies of 6 to 32 quarters.

[28] These recessionary episodes are 1973Q4–1975Q1, 1980Q1–Q3, 1981Q3–1982Q4, 1990Q3–1991Q1, 2001Q1–Q4, and 2007Q4–2009Q2. I group together the recessions of 1980Q1–Q3 and 1981Q3–1982Q4 as a single recessionary episode because the two events are too close in time to allow the identification of separate turning points for peak-to-trough analysis.

[29] I also impose that a peak (trough) must be preceded by at least three quarters of consecutive growth (decline) to avoid selecting blips in the data.

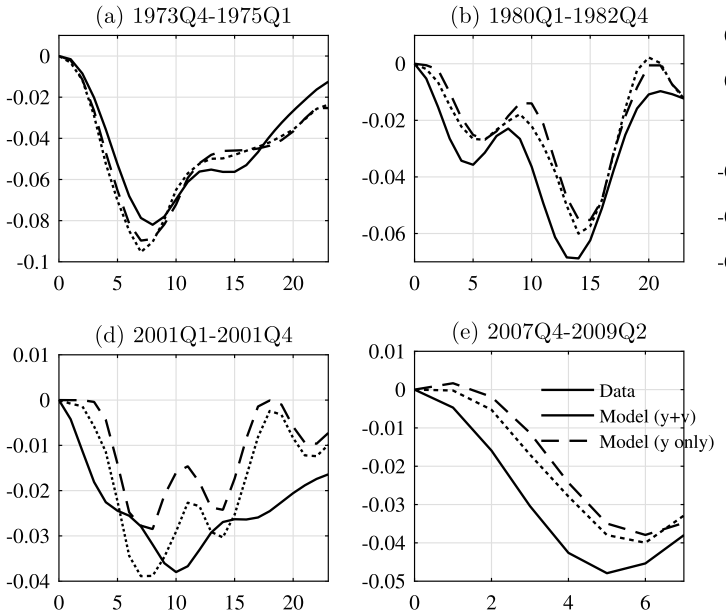


FIGURE 13.—Counterfactual time series for output. *Notes*: The black continuous line presents the data, the dotted line is the model with both aggregate productivity and volatility shocks, the dashed line is the model with productivity shocks only and constant volatility. Responses shown in log deviation from the peak preceding the recession. The aggregate productivity shock and volatility shock series are estimated to match the empirical output per person series and the IQR series from the Census.

of various effects, including real option effects, but mostly by intensifying the reallocation of labor across firms. The simulated series confirm the importance of volatility shocks to explain variation in unemployment. The full model explains between 60% and 80% of the total increase in unemployment during the recessions of 1973–1975 and 1980–1982. It captures reasonably well the rise and subsequent fall in unemployment during 1990–1991, though the reversal in employment takes place earlier in the model than in the data. Volatility appears to have played a major role during the recession of 2001 as it explains about 50% of the total increase in unemployment, while only 30% is attributable to productivity. The model, however, cannot justify the slow decline in unemployment that took place after 2003.

Surprisingly, despite a large peak in volatility in 2007, the presence of volatility shocks only increases the explanatory power of the model from 20% to 40% of the total rise in unemployment. Part of the reason stems from the fact that volatility rose slowly from 2005 to 2007 in the Census data, as shown in Figure 1. As a result, the reallocation of labor in the model occurs progressively during that period and only few workers have to experience unemployment before finding new jobs. The labor market flow series of Figure 15, however, provide encouraging support that volatility is essential to understand the U.S. labor market experience. As the figure illustrates, the full model does a very satisfactory job at explaining the evolution of hirings, layoffs, and quits during the 2007–2009 period, and clearly outperforms the model with productivity shocks only. In particular, the model with volatility shocks accounts for more than 80% of the total peak-to-trough variations in these variables. It misses, however, the evolution of total vacancies which do not fall as much as in the data. Since the model captures most of the fall in hirings, the discrepancy

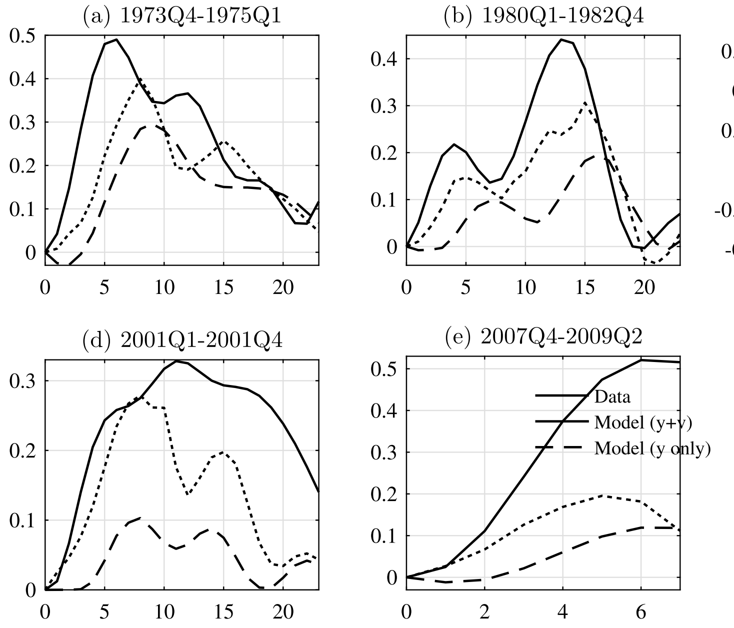


FIGURE 14.—Counterfactual time series for unemployment. *Notes*: The black continuous line presents the data, the dotted line is the model with both aggregate productivity and volatility shocks, the dashed line is the model with productivity shocks only and constant volatility. Responses shown in log deviation from the peak preceding the recession. The aggregate productivity shock and volatility shock series are estimated to match the empirical output per person series and the IQR series from the Census.

between the model and the data must result from a larger decline in the model's vacancy yield and an insufficient decrease in the labor market tightness. While my findings suggest that time-varying volatility has played a non-negligible role in the 2007–2009 recession, it also shows that volatility alone does not seem sufficient to account for the total variation in unemployment.

Various reasons may explain the relatively minor role that the model attributes to volatility in the recession of 2007–2009. The main reason, suggested by the model, is that search costs, estimated from aggregate labor market flows and micro-level firm employment patterns, are too small to generate large wait-and-see effects. While this finding appears fairly robust from the perspective of the model, one may also question the validity of the model itself, which misses some important dimensions. A first example is the absence of non-search related costs such as hiring and training costs, that could magnify real option effects. The absence of capital, possibly associated to more severe irreversibilities, is another likely candidate as capital is susceptible to produce larger real option effects through its complementarity with labor. The absence of risk aversion may also be a concern, even though idiosyncratic firm-level risk is unlikely to matter for households without additional capital market imperfections. Finally, recent evidence from Caldara, Fuentes-Albero, Gilchrist, and Zakrajšek (2016) suggests that the interaction of uncertainty with financial frictions played a particularly important role during the 2007–2009 recession, suggesting that incorporating financial frictions alongside time-varying uncertainty is another avenue for future research. For all these reasons, my findings should be taken as a first pass on the question, not the definitive answer concerning the role of volatility shocks for labor markets.

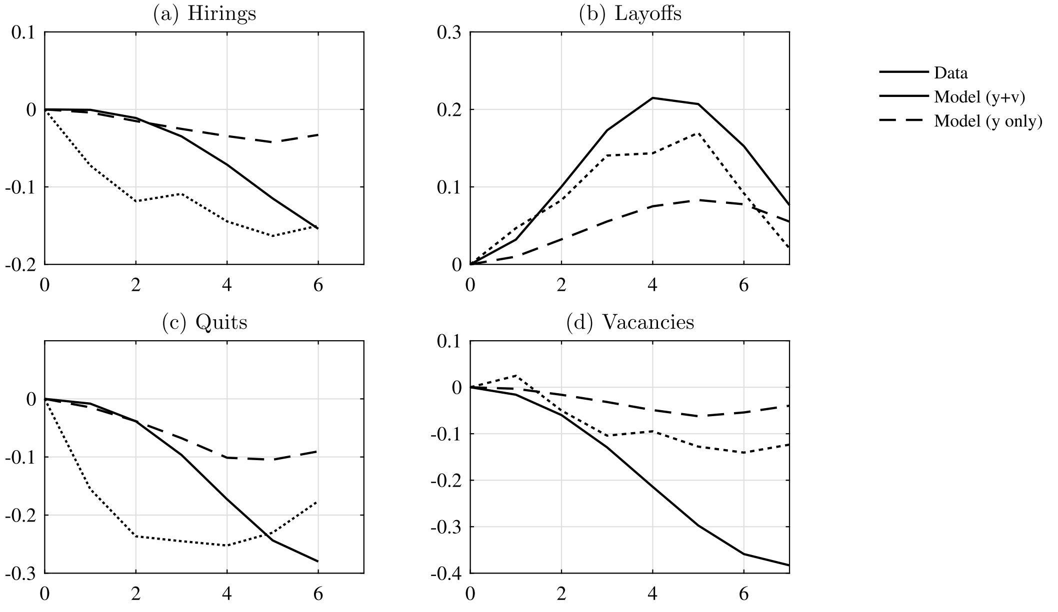


FIGURE 15.—Counterfactual time series for labor market flows in the 2007–2009 recession. *Notes*: The black continuous line presents the labor market flow data from the JOLTS data set, the dotted line is the model with both aggregate productivity and volatility shocks, the dashed line is the model with productivity shocks only and constant volatility. The aggregate productivity shock and volatility shock series are estimated to match the empirical output per person series and the IQR series from the Census. The procyclical variables (hirings, quits, and vacancies) are shown in log deviation from the peak preceding the recession, the countercyclical variables (layoffs) from the preceding trough.

TABLE V

PEAK-TROUGH VARIATIONS ACROSS VARIOUS RECESSIONS[a]

	1973–1975	1980–1982	1990–1991	2001	2007–2009
Output					
Data	−0.082	−0.069	−0.019	−0.038	−0.048
Model ($y+v$)	−0.095	−0.060	−0.021	−0.039	−0.040
Model (y only)	−0.089	−0.055	−0.019	−0.028	−0.038
Unemployment					
Data	0.490	0.441	0.124	0.328	0.521
Model ($y+v$)	0.399	0.300	0.168	0.268	0.195
Model (y only)	0.296	0.195	0.068	0.103	0.119
Labor wedge					
Data	−0.059	−0.046	−0.015	−0.056	−0.069
Model ($y+v$)	−0.016	−0.043	−0.025	−0.034	−0.028
Model (y only)	−0.016	−0.010	−0.006	−0.007	0.000

[a]The peak-trough measures are computed in log deviation. The time series are detrended using a band pass filter for fluctuations from 6 to 32 quarters. Peaks (troughs) are identified as the first local maximum (minimum) preceding the recessionary period which follows at least three quarters of growth (decline). Simulated data are aggregated at the quarterly level.

To summarize our conclusions, we have seen that the model with both productivity and volatility shocks can reasonably account for the joint evolution of output and unemployment across various past episodes. Time-varying volatility, mostly through its impact on the reallocation of labor, appears to be an important driver of labor market flows and contributes to offer a more complete view of the labor market. We found, however, only partial support for the role of productivity and volatility in the 2007–2009 recession, as the combination of both shocks explains at most 40% of the total rise in unemployment.

5.3. *Labor Wedge*

Several authors have reported large movements in the labor wedge over past recessions, including in particular the recession of 2007–2009. I conclude this section with an exploration of the model's predictions for the labor wedge, namely, the ratio of the marginal rate of substitution of consumption for leisure and the marginal product of labor. Following Chari, Kehoe, and McGrattan (2007), I define the labor wedge as the implicit labor tax,

$$1-\tau_{l,t}=-\frac{u_H}{u_C}(C_t,H_t)/F_H(K_t,H_t)=\frac{\xi}{1-\alpha}\frac{C_t}{Y_t}H_t^{1+\nu},$$

assuming $u(C,H)=\log C-\xi\frac{H^{1+\nu}}{1+\nu}$ and $F(K,H)=K^{\alpha}H^{1-\alpha}$, where C holds for total consumption, H hours, and K capital.[30] I first compute the response of the labor wedge to aggregate productivity and volatility shocks and report the results on Figure 18 in the Supplemental Material. Interestingly, the labor wedge increases in the case of a negative productivity shock as output falls more than consumption, which benefits from a decline in entry costs. On the other hand, the labor wedge declines, equivalent to an implicit increase in a tax on labor income, in the case of a positive volatility shock. This decline is due to the fact that volatility shocks imply an increase in unemployment, mostly through intensified reallocation of labor, as well as an increase in output through the Oi–Hartman–Abel effect, which both push the labor wedge down.

Going back to the counterfactual exercises, I compute the labor wedge both in the model and in the data and report the peak-to-trough measures in Table V. As the table shows, recessions are usually followed by a worsening of the labor wedge, implying that the implicit tax on labor rises in the aftermath of a recession. As was pointed out before, the last recession appears as the worst episode with a fall of about 7% in the labor wedge under the chosen specification. On the other hand, the recession of 1990–1991 appears as the mildest episode. Table V shows that the full model with volatility shocks is, in general, more successful at explaining movements in the labor wedge than the version with productivity shocks only, as one could have expected from the previous paragraph. The model does reasonably well for the recessions of 1980–1982 to 2001, during which uncertainty seems to have played a larger role, but only explains a fraction of the decline in the wedge for the last recession, mirroring the fact that uncertainty explains a limited part of unemployment during this episode. Surprisingly, the model fails at replicating the deterioration of the labor wedge during the recession of 1973–1975. This result is, however, driven by the fact that the model overpredicts the fall in output during this recession, limiting the decline in the wedge, and because the imputed volatility shock during this episode is rather small as Figure 17 shows.

[30]See Appendix B for more details on the parameterization and data sources. Consumption is defined by the resource constraint in the model as total output net of costs (vacancy, entry, and operating costs). Since there is no intensive margin of labor, I use total employment instead of hours in the model.

6. CONCLUSION

In this paper, I have developed a dynamic search-and-matching model of the labor market with firm dynamics and heterogeneity in productivity and sizes. The model is based on directed search and allows for endogeneous separations, on-the-job search, and endogenous entry and exit of firms. Despite the amount of heterogeneity, the model is highly tractable and can accommodate a variety of aggregate shocks, thanks to the property of block recursivity, which I exploit to analyze the out-of-steady-state dynamics of the model.

After showing that the model can replicate salient features of firm behavior at the establishment level, I use this framework to analyze the role of time-varying idiosyncratic risk on aggregate unemployment fluctations and on the labor market. I show that the response of the economy to productivity and volatility shocks is complex and hides a variety of effects. The response of the economy to volatility shocks, in particular, is the combination of various effects ranging from real option, Oi–Hartman–Abel to general equilibrium effects. My findings suggest, however, that the real option effects are mild and dominated by realized volatility effects. In other words, volatility shocks intensify the reallocation process, inducing larger gross labor market flows and higher unemployment.

In a series of counterfactual experiments, I examine the ability of the model to account for the U.S. labor market experience during past historical episodes. Feeding the model with a series of shocks that match the productivity and volatility data, I show that the model offers a quite satisfactory account of various past recessions. Time-varying volatility appears as an important driver of labor market fluctuations, in particular for the recessions of 1990–1991, 2001, and 2007–2009. The success is, however, only partial for the last recession, as the joint combination of productivity and volatility explains at most 40% of the observed increase in unemployment.

The model is quite flexible and could be used in a variety of setups with aggregate shocks or transitional dynamics. For instance, because it allows for decreasing returns, a possible extension would be to introduce monopolistic competition and study the model's dynamic implications for international trade. Applications to markets other than the labor market may also provide interesting insights. For instance, Boualam (2014) proposed an application to the banking industry and studied the dynamics of the credit market. Blanco and Navarro (2016) and Sepahsalari (2016) extended the model to introduce financial frictions. Other extensions, such as the introduction of concave utility or skill heterogeneity among workers, also seem promising.

Regarding the role of uncertainty, the model has the implication that real option effects are weak. This result stems from the fact that employment decisions can be easily reversed when search frictions are the only costs associated to the reallocation of labor. This conclusion may, however, change with the introduction of additional sunk costs, such as job-specific human capital investments. Uncertainty may also affect employment through other channels. For instance, adding stronger discount factor effects could attenuate the Oi–Hartman–Abel effects and lower the response in entry and hiring. Other sources of uncertainty not considered in this paper could also be important, for instance, policy uncertainty as studied in Baker, Bloom, and Davis (2016) and Fernández-Villaverde et al. (2015). Financial frictions, in interaction with uncertainty shocks, could also improve the response of the model during the recession of 2007–2009.

APPENDIX A: COMPUTING THE MEASURE OF ENTRANTS

This section explains how to compute the measure of entering firms in every period. The number of entering firms is implicitly determined by the equilibrium conditions on each labor market segment. More specifically, recall that the equilibrium market tightness on a given submarket x is such that

$$\mu(s', g, x)\theta(s', x) = \nu(s', g, x), \quad \forall x,$$

where $\nu(s', g, x)$ is the measure of vacancies posted on that submarket and $\mu(s', g, x)$ the efficiency-weighted measure of searching workers. Multiplying both sides by $q(\theta(s', x))$ and using the identity $p(\theta) = \theta q(\theta)$, this condition is equivalent to

$$JF(s', g, x) \equiv \mu(s', g, x) p(\theta(s', x)) = \nu(s', g, x) q(\theta(s', x)) \equiv JC(s', g, x), \quad \forall x, \tag{16}$$

where $JF(s', g, x)$ is the total number of jobs found by workers on submarket x and $JC(s', g, x)$ is the total number of jobs created by firms on the same submarket. Since firms are indifferent between the various submarkets, the continuum of equilibrium conditions (16) can be summarized by a unique aggregate condition which guarantees that the total number of jobs found by workers across the various submarkets is equal to the total number of jobs created:

$$JF_{\text{total workers}}(s', g) \equiv \int_{\underline{x}}^{\overline{x}} JF(s', g, x)\,\mathrm{d}x = \int_{\underline{x}}^{\overline{x}} JC(s', g, x)\,\mathrm{d}x \equiv JC_{\text{total firms}}(s', g).$$

To compute the number of entrants m'_e, calculate the total number of jobs found by workers in the economy for a given period,

$$\begin{aligned} JF_{\text{total workers}}(s', g) = {} & p(\theta(s', x'_u(s')))u \\ & + \sum_{z,z',n} \pi_z(z' \mid z, s) g(z, n)(1 - d'(s', z'; n)) \\ & \times \int n(1 - \tau'(s', z'; j, n)) \lambda p(\theta(s', x'(s', z'; j, n)))\,\mathrm{d}j, \end{aligned}$$

which includes the number of successful hires from unemployment and the number of successful job-to-job transitions. Then, compute the total number of jobs created by incumbent firms,

$$JC_{\text{total incumbents}}(s', g) = \sum_{z,z',n} \pi_z(z' \mid z, s) g(z, n)(1 - d'(s', z'; n)) n_i(s', z'; n),$$

and the number of jobs created by a measure 1 of entrants,

$$JC_{\text{entrant}}(s') = \sum_{z'} g_z(z')(1 - d_e(s', z')) n_e(s', z').$$

The measure of entrants may finally be computed using our aggregate condition:

$$\begin{aligned} JF_{\text{total workers}}(s', g) &= JC_{\text{total firms}}(s', g) \\ &= JC_{\text{total incumbents}}(s', g) + m_e(s', g) JC_{\text{entrant}}(s'). \end{aligned}$$

APPENDIX B: DATA DESCRIPTION

This section details the construction and sources of the empirical time series used throughout the paper.

B.1. *Measures of Micro-Level Risk*

B.1.1. *Establishment-Level Volatility of TFP*

The establishment-level volatility of TFP is taken from Bloom et al. (2012) constructed using data from the Census of Manufactures and the Annual Survey of Manufactures by the Census Bureau. This data set contains output and inputs data for more than 50,000 establishments. Frequency is annual and the data set covers the period 1972 to 2009. Establishments with less than 25 years of data are excluded. Establishment-level TFP $\hat{z}_{j,t}$ is calculated using a standard approach, controlling for demand side effects with four-digit industry price deflators. TFP shocks are then estimated using

$$\log(\hat{z}_{j,t}) = \rho \log(\hat{z}_{j,t-1}) + \mu_j + \lambda_t + e_{j,t},$$

where μ_j is an establishment fixed effect and λ_t a year fixed effect. The base measure for micro-level risk is then defined as the cross-sectional interquartile range of the residual $e_{j,t}$. See Bloom et al. (2012) for additional details on the construction of this measure.

A potential concern is whether the variation in the cross-sectional dispersion captured by this measure should be interpreted as time-varying volatility in TFP. This measure controls for (i) demand side effects using price deflators, (ii) unobservable heterogeneity using establishment-level fixed effects, and (iii) selection by choosing only establishments with 25+ years of data. One remaining concern lies in the possibility that unobservable heterogeneity could lead to differences in cyclical sensitivity across firms. In that case, an increase in cross-sectional dispersion could simply reflect the heterogeneous response of firms to a first-moment shock. This effect is, however, difficult to control for and, despite this caveat, the proposed measure is arguably the best that can be constructed with available data and I therefore use it throughout the paper as my benchmark idiosyncratic volatility measure.

B.1.2. *Alternative Measures of Micro-Level Risk*

The Compustat sales growth dispersion measure is constructed using quarterly sales (SALEQ) in dollars for active U.S. firms over the period 1972Q1–2009Q4. I keep firms that have 100+ observations. Annual sales growth is computed according to $g_{i,t} = \frac{s_{i,t}-s_{i,t-4}}{1/2(s_{i,t}+s_{i,t-4})}$. The growth measures are detrended with time-industry dummies (two-digit NAICS). The micro-level risk measure derived from this series is the cross-sectional interquartile range of detrended $\hat{g}_{i,t}$.

The VIX measure is the monthly average of the implied volatility (new method) of stock market returns constructed by the CBOE over 1990–2009.

B.2. *Other Series*

- Output is taken from the NIPA tables constructed by the Bureau of Economic Analysis. I use quarterly GDP in 2005 dollars from 1972Q1 to 2009Q4.
- Productivity Y/L is seasonally adjusted real average output per person in the nonfarm sector over the period 1972Q1–2009Q4 from the Bureau of Labor Statistics.
- Unemployment is the seasonally adjusted monthly unemployment rate constructed by the BLS from the Current Population Survey over the period January 1972–December 2009 (for people aged 16 and over). Similarly, I use the total civilian labor force for people aged at least 16 from the BLS over the same period. The series are averaged over quarters.

- Vacancy is the quarterly average of the monthly vacancy measure from the Job Openings and Labor Turnover Survey.
- Historical UE and EU monthly transition rates are taken from Shimer (2012) over the period 1972Q1–2007Q1. For later periods, I use the monthly series on labor force status flows from the Current Population Survey constructed by the BLS over February 1990 to March 2010.
- EE is constructed by taking the ratio of quits from JOLTS over employment $(1-U)$ from January 2001 to December 2009.
- Labor market flows for hiring, quits, and layoffs are quarterly sums of the JOLTS measures from January 2001 to December 2009. The series are normalized by total labor force.
- The empirical labor wedge was constructed using quarterly, seasonally adjusted, chained 2009 dollars "Real Personal Consumption Expenditure" from Fred (PCECC96), total hours worked tabulated from the CPS by Cociuba, Prescott, and Ueberfeldt (2012) normalized by total population aged 16–64 from the BLS, and the output measure from the NIPA described above. The wedge was computed following Chari, Kehoe, and McGrattan (2007) with the expression

$$1-\tau_{l,t} = -\frac{u_H/u_C}{F_H(K_t,H_t)} = \frac{C_t H_t^{1+\nu}}{Y_t}$$

derived under the assumptions of $u(C,H) = \log C - \xi\frac{H^{1+\nu}}{1+\nu}$ and $F(K,H) = K^\alpha H^{1-\alpha}$ with the normalization $\xi = 1$ and assuming $\nu = 0.25$, which implies a Frisch elasticity of 4, a value within the range of standard macro estimates.

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