Wage and Layoff Risk Across Tenure

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1 Introduction

2 Model

2.1 Environment

Time is discrete and indexed by t. The economy is populated by a continuum of firms with measure 1, indexed $j \in [0, 1]$, and workers with measure I, indexed $i \in [0, I]$. Both types of agents are ex ante homogeneous and infinitely lived, with time-separable preferences and a discount factor β. Firms are owned by outside investors, able to diversify any potential risk from firm-level productivity shocks. Thus, in practice, firms maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \pi_{jt}$$

Workers are risk-averse with no access to financial markets. They consume home production b when unemployed and wage w when employed. Their utility is

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}), u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

Production

Firms may pay κ_e to start producing, and will have to pay κ_f every period to stay open. Open firms employ a measure[1] n of workers to produce. Each worker-firm match may be

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[1]Law of large numbers thus applies and is extensively used throughout the model (Sun and Zhang (2009))

of high or low quality, fully persistent during the existence of the match. Only the firm is aware of the quality of any individual match, but the proportion of high-quality matches in the firm, denoted by z , is common knowledge. Firm production exhibits decreasing returns to scale in size n and potentially quality z.[2] Lastly, production is subject to shocks $y \in \mathcal{Y}$ at the firm level. The overall production is a function of a number of high quality matches n_H, low quality matches n_L, and the firm-level productivity shock y. Equivalently, I will write it as a function of total measure of workers n and the proportion high quality matches z.

$$yF(n_H, n_L) \equiv yF(n, z), \quad n = n_H + n_L, z = \frac{n_H}{n_H + n_L}$$

Labor Market

Every period, firms just entering the market immediately hire $\tilde{n} = 1$ workers. Incumbent firms hire $\tilde{n} \geq 0$ workers. Workers, both employed and unemployed, search for a job. Workers and hiring firms meet in a frictional labor market with directed search, as in Moen (1997). There is a continuum of submarkets indexed by the promised value v owed to the workers. Firms choose in which submarket to post vacancies at the cost c and workers choose where to search. Within each submarket, matches are formed according a constant returns to scale matching function. Due to the CRS nature of the function, tightness of a submarket θ_v is a sufficient statistic for matching probabilities. Denote these probabilities $p(\theta_v), q(\theta_v) \leq 1$ for a worker and a vacancy, respectively.

Firms are not restricted in fielding a discrete number of vacancies, and thus can deterministically hire $\tilde{n}$ workers from submarket v at the cost $\tilde{n}\frac{c}{q(\theta_v)}$. The probability of being a high quality match upon being hired is $0 \leq z_0 \leq 1$, constant across agents and time. Upon hiring these workers, the firm commits to delivering expected discounted utility v. The core trade-off in firm hiring is between the cost of hiring $\frac{c}{q(\theta_v)}$ and the cost of employing the worker, which increases with v. Firms are capable of downsizing via laying off proportion $0 \leq s \leq 1$ of its workforce and by incentivizing incumbent workers to find jobs elsewhere.

I consider two cases of this economy: in the steady-state or, under an additional assumption (see Appendix A.5), in a Block-Recursive equilibrium (following Menzio and Shi (2011) and Schaal (2017)). Either way, agents do not need to keep track of the aggregate cross-sectional distribution of the economy.

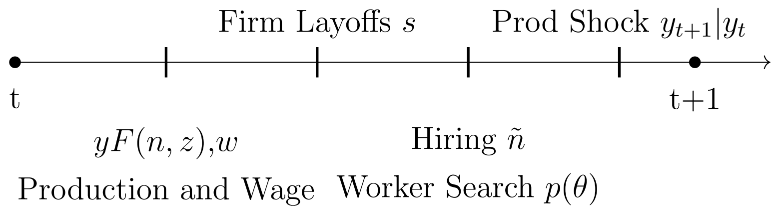


Figure 1: Within-period time line

Timing

Each period is divided into 4 stages, as illustrated in Figure 1. First, production takes place. Firm collects the output and pays wage w to each worker it employs. Next, each firm lays off a fraction $s \geq 0$ of its workforce. Fired workers become unemployed, but may not search until the next period. The remaining workers, both employed and unemployed, then search for a job. This coincides with all the firm hiring $\tilde{n}$, from both entering and incumbent firms. All the hiring and search choices happen before the next productivity y_{t+1} realizes and thus the agents have to rely on the expectation operator $E_{y_{t+1}|y_t}$.

Information Structure and Contracts

Upon hiring a worker, the firm commits to deliver expected utility v via a contract. A contract defines the wage and actions for a matched worker and firm for all future firm productivity histories $y^\tau \equiv (y_1, ..., y_\tau) \in \mathcal{Y}^\tau$. The future history of firm productivity is common knowledge to both agents and is thus fully contractible. However, the match-specific productivity z_{ij} is private information of the firm and worker's search decision $\hat{v}$ is private information of the worker. The contract $\mathcal{C}$ is then represented by

$$\mathcal{C} = \{w_\tau, s_\tau, \hat{v}_\tau\}_{\tau=t}^{\infty} \tag{1}$$

The first component captures firm's wage policy w for each future productivity history. The second component captures *expected* layoff probability s, given that the worker does not know the quality of their match. Note that these probabilities are not ex-ante: in histories where worker's information about their match quality updates, it will be reflected in all the corresponding layoff probabilities. An example of that could be a history where firm has faced multiple negative productivity shocks and was forced to lay off a vast majority of its workforce. The remaining workers will bayesian update that their match quality is now more likely to be high, and any future layoff probabilities will reflect that. The last

[2] In the quantitative exploration, I will restrict attention to decreasing returns in quality-adjusted quantity $g(z)n$

component is worker's search decision. Although this action is unobserved by the firm, I focus on contracts where the contractual recommendations are incentive-compatible. The firm thus chooses workers' search decisions, subject to the incentive compatibility constraint that the decisions match the workers' optimal response.

The contract space is completely flexible in how wages and layoffs respond to productivity histories. In a setting with a continuum of contracts at the same time, this allows the firm to choose how to treat its heterogeneous (in quality and contracts) workforce: when a negative shock hits, who to fire and for whom to cut wages. This property is central to the paper and unique to the setting: unlike model with CRS production functions, these decisions depend on the state of the entire firm. Unlike other models of firm dynamics, with Nash Bargaining (McCrary (2022)) or Sequential Bargaining (Bilal et al. (2022)), the workers in the same firm may end up with different wages, layoffs, and their responses to productivity shocks.

2.2 Value functions

The above contract, and thus the problems of all the agents, can be described recursively. I start with the individual workers' problem and move on to firms managing contracts with a continuum of workers. I show that the firm problem can be reformulated with a discrete state-space.

Worker's Problem

Unemployed workers consume home production b. Each period, they search on the submarket that offers the best tradeoff between promised future utility and job finding probability. Dropping all time subscripts and focusing on a stationary equilibrium, the value of being unemployed U can be written as:

$$U = \max_{v} u(b) + \beta[(1 - p(\theta_v))U + p(\theta_v)v] \tag{2}$$

Consider an employed worker with an owed value v. Suppose a firm pays wage w this period, will fire with probability s and offers a lifetime expected utility v' from tomorrow into the future. Then a worker faces the following search problem:

$$v = \max_{\hat{v}} u(w) + \beta[sU + (1 - s)[(1 - p(\theta_{\hat{v}}))v' + p(\theta_{\hat{v}})\hat{v}]] \tag{3}$$

The optimal worker policy $\hat{v}$ depends only on the future offered utility v'. By raising v', firm incentivizes its worker to search in higher $\hat{v}$, thus lowering the probability that the worker will leave. Note that this can be equivalently rewritten as

$$v = u(w) + \beta[sU + (1 - s)R(v')]$$

where $R(v') \equiv \max_{\hat{v}}[(1-p(\theta_{\hat{v}}))v' + p(\theta_{\hat{v}})\hat{v}]$ the optimal future value the worker gets upon being promised v' and not being laid off.

Firm's Problem

A firm employs a measure n of workers. Denote the distribution of their promised values v that the firm owes to its workers as $P(v)$. Then for each of these values v, the firm has to choose the wage to pay w_v, the layoff rate s_v, and tomorrow, future productivity state-contingent promised value $v'_{v,y'}$. Firm may also hire $\tilde{n}$ workers at the value $\tilde{v}$. Each worker-firm match can be of high or low quality. I do not allow the firm to choose quality-contingent wages, thus the firm can only influence match quality via layoffs [3]. The firm's problem can then be formulated recursively as follows:

$$J(y,n,P(v),z) = \max_{\tilde{n},\tilde{v},\{w_v,s_v,v'_{v,y'}\}} yF(n,z) - \int_v w_i dP(v_i) - \tilde{n}\frac{c}{q(\tilde{v})} - \kappa_f + \beta E_{y'|y} J(y',n',P'(v))$$

$$s.t.\ u(w_v) + \beta[s_v U + (1-s_v)R(v'_v) = v]\ \forall v$$

$$v'_v = E_{y'|y} v'_{v,y'}\ \forall v$$

$$n' = n\int_v (1-s_v)(1-p(v'_v))dP(v) + \tilde{n}$$

$$n'P'(v) = n\int_v E_{y'|y}\mathbb{1}_{v'_{v,y'}\leq v}(1-s_v)(1-p(v'_v))dP(v) + \mathbb{1}_{\tilde{v}\leq v}\tilde{n}$$

Firm has to maximize its net present value of profits subject to fulfilling every worker's promised value. Note that because the search decision happens before the next productivity state realizes, workers only care about the expected average promised value v'_v when making their decisions rather than about any of the $v'_{v,y'}$ in particular. The latter two conditions specify the law of motion for firm size and the distribution of promised values.

Discretizing the Problem In the current formulation, this problem is intractable as it involves a probability distribution, an uncountably infinitely-dimensional object, in the state-space. I show that this state space can be discretized, thus bringing it "down" to countably infinite states.

First, note that, when hiring, a firm chooses just one value at which to hire, $\tilde{v}$. This is an outcome of the directed structure of the labor market. Since a firm optimally decides in which submarket v to post vacancies [4], it will only hire from that submarket and thus at that value. This means that all the workers hired at the same time by the same firm

[3] I show in Appendix A.3 that, under a sufficiently low elasticity of on-the-job search, this is in fact an pooling equilibrium in a signaling game, where the firm is allowed to set quality-contingent wages.

[4] I assume no mixed strategies by a single firm: in case a firm happens to be indifferent across multiple submarkets, it will only post vacancies in one of them.

are going to be owed the same expected utility, both at the time of hiring and, due to not allowing the firm to choose quality-contingent wages, in all the future periods. Therefore, it is equivalent to work with the cdf $P(v)$ or with the related probability mass function $\mathbb{P}(V = v)$: $P(v) = \sum_{v' \leq v} \mathbb{P}(V = v')$. Furthermore, for a firm of age $K < \infty$, there are at most K different values v such that $\mathbb{P}(V = v) > 0$. These values correspond to the values owed to workers hired at different time periods, thus of different tenure at the firm $k = t - t_{hired} \leq K$. One can then redefine the state space using tenure:

Lemma 1. *A decision problem $J(y, n, P(v))$ of a firm of age K can be equivalently represented as*

$$
\begin{aligned}
J(y, \{n_k, v_k, z_k\}_{k \leq K}) = \max_{\tilde{n}, \tilde{v}, \{v'_{y',k}, w_k, s_k\}_{k \leq K}} & yF(\sum_k n_k, \frac{\sum n_k z_k}{\sum n_k}) - \sum_k w_k n_k \\
& - \tilde{n} \frac{c}{q(\tilde{v})} - \kappa_f + \beta E_{y'|y} J(y', \{n'_k, v'_k, z'_k\}_{k \leq K+1}) \\
s.t.\ & u(w_k) + \beta[s_k U + (1 - s_k) R(v'_{k+1}) = v_k] \ \forall k \leq K \\
& v'_{k+1} = E_{y'|y} v'_{k+1,y'} \ \forall k \leq K \\
& n'_{k+1} = n_k (1 - s_k)(1 - p(v'_{k+1})) + \tilde{n} \ \forall k \leq K \\
& z'_{k+1} = min(\frac{z_k}{1 - s_k}, 1) \ \forall k \leq K \\
& n'_0 = \tilde{n}, v'_0 = \tilde{v}, z'_0 = z_0
\end{aligned}
$$

This redefinition, besides discretizing the state-space, is also useful for understanding how layoffs and wages interact within a cohort. Although any particular worker does not know their own match quality, they know the proportion of high quality matches in their cohort. All new hires start with a proportion z_0 of high quality matches, and, although it may evolve, all the workers of the same tenure k will have the same probability of having a high quality match $z_k \geq z_0$. This probability is common knowledge to the firm and all its workers and depends on layoffs in the corresponding cohort. As I will show later, the average quality within a cohort has a first-order effect on its wage growth, allowing me to explore the connection between wage passthrough and layoffs within a firm. Unclear. Clarify how using the cohort definition of the problem helps.

Free-entry and exit

Firms are free to enter the market and start producing upon paying an entry cost κ_e. Upon entry, firms draw a productivity shock and start with a single worker. Thus, the free-entry condition pins down the expected profits of firms upon entry:

$$\kappa_e \geq \max_{v_0} -\frac{c}{q_{v_0}} + \beta E_y J(y, \{1, 0, ...\}, \{v_0, ...\}, \{z_0, ...\}) \tag{4}$$

When taking the model to the data, this results in a negative connection between the cost of entry κ_e and the probability to fill a vacancy $q(\theta)$: the cheaper it is to enter, the tighter will the labor market be, thus making it easier for workers to find jobs and harder for firms to fill vacancies.

Similarly to new firms, incumbent firms have to pay an operating cost κ_f every period to stay open. With κ_f already included into the firm value function, firms stay open if

$$J(y, \{n_k, v_k, z_k\}) \geq 0 \tag{5}$$

2.3 Equilibrium

A complete equilibrium is a sequence of policies, matching rates, and distribution of workers and firms for each labor market v such that each period

- Firms solve the problem from Lemma 1
- Workers solve search problems from Equations 2 and 3
- Free-entry and free exit conditions 4,5 are satisfied
- Job-finding and vacancy-filling probabilities are consistent with the matching function
- Tightness function θ_v is consistent with the firm posting and worker search strategies
- Labor market clears

Under an additional assumption in Appendix A.5, I show that the equilibrium may be block recursive, meaning independent of the distribution of workers and firms. I use that assumption in the quantitative exploration, but not in the theoretical discussion, where instead I focus my attention on the steady-state of the economy described above.

2.4 Mechanism

Contracts Converge

The tenure-based formulation of the problem allows me to work with a discrete, although expanding, state-space. Finally, to make the problem fully tractable, I note that wages in contracts tend to converge, suggesting that $w_k \approx w_{k+1} \forall k \geq \bar{k}$. I derive this result in the following propositions. The core idea follows Balke and Lamadon (2022)'s Proposition 3, which states that wages in dynamic contracts tend to always follow a "target wage". I extend their result to my model with a continuum of contracts.

To show that the wages converge, I start by characterizing the wage growth.

Proposition 1. *For any current state $(y, \{n_k, v_k, z_k\})$, wages change according to the following relationship:*

$$\frac{1}{u'(w'_{k+1})} - \frac{1}{u'(w_k)} = \eta(v'_{k+1}) E_{y'|y} \frac{\partial J(y', \{n'_k, v'_{y',k}, z'_k\})}{\partial n'_{k+1}} \tag{6}$$

where $\eta(v'_{k+1}) = \frac{\partial log(1-p(v'))}{\partial v'}$ is the semi-elasticity of the job-finding probability with respect to the promised value v'_{k+1}.

Proof. See Appendix A.1. □

The relationship is at the core of the firm's insurance vs incentive provision trade-off: while the marginal value of the worker $\frac{\partial J(y', \{n'_k, v'_k, z'_k\})}{\partial n'_{k+1}}$ is positive, the firm is intent on keeping its workers, and thus chooses to backload wages, thus incentivizing the workers to stay at the cost of higher total wage payments. On the flip side, if the marginal worker value is negative, the firm will choose to lower wages, incentivizing workers to leave. This can be formalized by definining a target wage:

Definition 1. *The target wage for cohort k, $w^*_k(y, \{n_k, v_k, z_k\}/v_k)$, is defined as the wage associated with the promised value $v^*_k(y, \{n_k, v_k, z_k\}/v_k)$, for which $M(y, \{n_k\}, v_0, ..., v^*_k, ..., \{z_k\}) = 0$, where*

$$M(y, \{n_k\}, v_0, ..., v_k, ..., \{z_k\}) = E_{y'|y} \frac{\partial J(y', \{n'_k, v'_k, z'_k\})}{\partial n'_{k+1}},$$

where $\{n'_k, v'_k, z'_k\}$ – future states from solving Problem 1 given current state of $(y, \{n_k\}, v_0, ..., v_k, ..., \{z_k\})$. The target wage is equal to:

$$w^*_k(y, \{n_k, v_k, z_k\}/v_k) = u'^{-1}\Big(- \frac{n'_{k+1}}{\partial J(y', \{n'_k, v'_k, z'_k\})/\partial v'_{y',k+1}} \Big)$$

The target wage captures the point where the marginal profit of a cohort k reaches zero, thus making the firm indifferent between keeping or losing these workers to poaching (it may still want to lose the low quality matches within the cohort and keep the high quality ones). I use the target wage to show convergence of wages across cohorts:

Proposition 2. *For each firm state $(y, \{n_k, v_k, z_k\})$ and cohort k there exists a target wage w^*_k. To first-order:*

1. *Wage in each cohort chases the corresponding target wage:*

$$w_k \le w^*_k \implies w_k \le w'_{k+1} \le w^*_k$$

$$w_k \ge w^*_k \implies w^*_k \le w'_{k+1} \le w_k$$

2. *Wage growth is faster the further wage is from the target wage:* Recheck

$$|w_k - w_k^*| \geq |w_{k'} - w_{k'}^*| \implies |w'_{k+1} - w_k| \geq |w'_{k'+1} - w'_{k'}|$$

3. *Target wage increases in* z_k*, but does not change in* $z_{k'}, k' \neq k$.

4. *All target wages change the same in response to all the other state variables:*

$$\frac{\partial w_k^*}{\partial s} = \frac{\partial w_{k'}^*}{\partial s} \, \forall k, k', s \neq z_k, z_{k'}$$

Proof. See Appendix A.1

□

At any state and cohort, the wage adjusts towards w_k^*. This allows me to show that, fixing a quality of the cohort, the wages will indeed converge.

Corollary 1. *Cohorts with the same average quality have the same target wage:*

$$z_k = z'_{k'} \implies w_k^* = w_{k'}^* \, \forall k, k' \leq K$$

Cohorts of the same quality have the same target wage. Results 1 and 2 from Proposition 2 then tell us that, no matter their promised values $v_k, v_{k'}$, the two cohorts will converge towards the same wage, with the cohort farther behind catching up over time. To confirm that wages converge in all cohorts, inclding of different qualities, I show that the qualities across cohorts also converge. To do that, I show how quality in lower value cohorts may rise and catch up via layoffs.

Proposition 3. *Consider a firm state* $(y, \{n_k, v_k\})$ *and a cohort* $k, z_k < 1$. *Layoffs are characterized by the following first-order condition:*

$$-E_{y'|y} \frac{\partial J'}{\partial n'_{k+1}} (1 - p(v'_{k+1})) + E_{y'|y} \frac{\partial J'}{\partial z'_{k+1}} \frac{\partial z'_{k+1}}{\partial s_k} \frac{1}{n_k} - \frac{R(v'_{k+1}) - U}{u'(w_k)} \leq 0 \tag{7}$$

and $s_k \geq 0$ *with complementary slackness.*
Moreover, cohorts with lower promised value are more subject to layoffs:

$$v_k \leq v_{k'} \implies s_k(1 - z_k)n_k \geq s_{k'}(1 - z_{k'})n_{k'} \, \forall k' \leq K, z_{k'} < 1$$

And, to first-order, layoffs are equally dependent on quality of any cohort:

$$\frac{\partial s_k}{\partial z_k} = \frac{\partial s_k}{\partial z_{k'}} \, \forall k' \leq K$$

Proof. See Appendix A.1 □

Lower value cohorts are more subject to layoffs, no matter their quality (if below 1). However, from point 3 of Proposition 2, we know that cohort quality has a long-term effect on wages via the target wage. Therefore, lower quality cohorts will catch up to higher quality ones via layoffs, due to having lower target wage prior to layoffs. Lower value cohorts will catch up to higher value cohorts naturally, and with the "help" of layoffs if they're also lower quality.

Empirically, I show in Appendix C.1 that wage growth in France stagnates after about 10 years of tenure. This allows me to restrict attention to finite and constant K for all the firms in my quantitative exploration. Note that this is only an approximation since, as $K \to \infty$, the model approaches the problem described in Lemma 1.

Wage and Layoffs Across Tenure

I conclude the section by showing how the model delivers the heterogeneity in wage and layoff passthrough across tenure.

I first come back to the layoffs as described in Propositon 3. Just like wages, the trade-off for layoffs primarily revolves around the value of the marginal worker $E_{y'|y}\frac{\partial J'}{\partial n'_{k+1}}$ and the cost of having to compensate the worker $\frac{R(v'_{k+1})-U}{u'(w_k)}$. The truly unique component to layoffs is the quality effect $E_{y'|y}\frac{\partial J'}{\partial z'_{k+1}}\frac{\partial z'_{k+1}}{\partial s_k}\frac{1}{n_k}$ since, while the firm is not allowed to set quality-contingent wages, it is allowed to fire whoever it wants.

Workers with higher value are more likely to be laid off. Higher tenure workers will generally have higher promised value than junior workers due to wage backloading, as described in result 1 of Proposition 2. Therefore, junior workers are generally more subject to layoffs than seniors.

I now connect the layoffs and wage cuts. Survivors in cohorts subjected to layoffs will receive lower wage cuts than workers in other cohorts:

Proposition 4. *Let $K_s \equiv \{k \leq K | s_k > 0\}$. In states where K_s is non-empty:*

$$w'_{k+1} - w_k > w'_{k'+1} - w_{k'} \; \forall k \in K_s, k' \notin K_s$$

Proof. See Appendix A.1 □

The intuition behind this result is that layoffs give a second-order push-up effect on wages: since the remaining workers are now of higher quality, there is less incentive to cut those workers' wages. And, although higher tenure steps may be of even higher quality, because their quality effect has already been internalized in wages (following Proposition 2), their wage cuts will in fact be larger than the cuts of just recently fired workers.

3 Conclusion

References

1. Balke, Neele and Thibaut Lamadon (2022). "Productivity shocks, long-term contracts, and earnings dynamics". In: *American Economic Review* 112.7, pp. 2139–2177.

2. Bilal, Adrien et al. (2022). "Firm and worker dynamics in a frictional labor market". In: *Econometrica* 90.4, pp. 1425–1462.

3. Cho, In-Koo and David M. Kreps (May 1987). "Signaling Games and Stable Equilibria*". In: *The Quarterly Journal of Economics* 102.2, pp. 179–221. ISSN: 0033-5533. DOI: 10.2307/1885060. eprint: https://academic.oup.com/qje/article-pdf/102/2/179/5441738/102-2-179.pdf. URL: https://doi.org/10.2307/1885060.

4. McCrary, Sean (2022). "A Job Ladder Model of Firm, Worker, and Earnings Dynamics". In: *Worker, and Earnings Dynamics (November 4, 2022)*.

5. Menzio, Guido and Shouyong Shi (2011). "Efficient search on the job and the business cycle". In: *Journal of Political Economy* 119.3, pp. 468–510.

6. Moen, Espen R (1997). "Competitive search equilibrium". In: *Journal of political Economy* 105.2, pp. 385–411.

7. Schaal, Edouard (2017). "Uncertainty and unemployment". In: *Econometrica* 85.6, pp. 1675–1721.

8. Sun, Yeneng and Yongchao Zhang (2009). "Individual risk and Lebesgue extension without aggregate uncertainty". In: *Journal of Economic Theory* 144.1, pp. 432–443. ISSN: 0022-0531. DOI: https://doi.org/10.1016/j.jet.2008.05.001. URL: https://www.sciencedirect.com/science/article/pii/S0022053108000768.

A Model Appendix

A.1 Proofs

Proof of Proposition 1

The proposition is derived from the first-order conditions of the firm's problem.

Proof. Consider the problem from Lemma 1. The FOC for v'_{k_1} yields (denote ρ_k the shadow cost of PK_k and ω_k the shadow cost of the expectation condition):

$$\rho_k\beta(1-s_k)(1-p(v'_{k+1}))-\omega_k+\beta n_k(1-s_k)\frac{\partial(1-v(v'_{k+1}))}{\partial v'_{k+1}}E_{y'|y}\frac{\partial J(y',\{n'_k,v'_{y',k}\}_{k\leq K+1})}{\partial n'_{k+1}}=0$$

To develop this further, I use the FOC for w_k:

$$-n_k + \rho_k u'(w_k) = 0 \iff \rho_k = \frac{n_k}{u'(w_k)}$$

One can similarly develop ω_k by applying the FOC for $v'_{y',k+1}$:

$$\beta Pr_{y'|y} \frac{\partial J(y', \{n'_k, v'_{y',k}\}_{k \leq K+1})}{\partial v'_{y',k+1}} + Pr_{y'|y}\omega_k = 0 \iff \omega_k = -\beta \frac{\partial J(y', \{n'_k, v'_{y',k}\}_{k \leq K+1})}{\partial v'_{y',k+1}} \forall y'$$

Applying the envelope theorem one can then note that

$$-\frac{\partial J(y', \{n'_k, v'_{y',k}\}_{k \leq K+1})}{\partial v'_{y',k+1}} = \frac{n'_{k+1}}{u'(w'_{k+1})}$$

Putting all this together, we get

$$\frac{n_k}{u'(w_k)}\beta(1-s_k)(1-p(v'_{k+1})) - \beta \frac{n'_{k+1}}{u'(w'_{k+1})} + \beta n_k(1-s_k)\frac{\partial(1-v(v'_{k+1}))}{\partial v'_{k+1}} E_{y'|y} \frac{\partial J(y', \{n'_k, v'_{y',k}\}_{k \leq K+1})}{\partial n'_{k+1}} = 0$$

All that is left is to note that $n'_{k+1} = n_k(1-s_k)(1-p(v'_{k+1}))$, divide by the common terms $(\beta n'_{k+1})$ and rearrange.

$$\frac{1}{u'(w'_{k+1})} - \frac{1}{u'(w_k)} = \eta(v'_{k+1}) E_{y'|y} \frac{\partial J(y', \{n'_k, v'_{y',k}\})}{\partial n'_{k+1}}$$

□

Proof of Proposition 2

Proof. □

Proof of Proposition 3

I start by describing the FOC with respect to layoffs s_k:

Proof. Consider the case where $s_k > 0$. Then

$$-\beta n_k(1-p(v'_{k+1}))E_{y'|y}\frac{\partial J(y', \{n'_k, v'_{y',k}, z'_k\})}{\partial n'_{k+1}} + \beta E_{y'|y}\frac{\partial J(y', \{n'_k, v'_{y',k}, z'_k\})}{\partial z'_{k+1}}\frac{\partial z'_{k+1}}{\partial s_k} - \rho_k\beta(U - R(v'_{k+1})) = 0$$

Note that the FOC with respect to w_k yields $\rho_k = \frac{n_k}{u'(w_k)}$ and divide by βn_k to get the FOC in the Propositon.

For the further results, I rewrite the firm state $(y, \{n_k, v_k, z_k\})$ into $(y, \{\underline{n}_k, \bar{n}_k, v_k\})$, where $\bar{n}_k = z_k n_k$ and $\underline{n}_k = (1-z_k)n_k$.

I focus on the case where $\underline{n}_k > 0$ and $s_k(\underline{n}_k + \bar{n}_k) \leq \underline{n}_k$, that is, firm has not yet fired all the bad matches with the cohort (in the previous FOC, this is equivalent to $\frac{\partial z'_{k+1}}{\partial s_k} > 0$). The marginal value of firing a worker is then

$$-\frac{R(v'_{k+1}) - U}{u'(w_k)} - (1 - p(v'_{k+1}))E_{y'|y}\frac{\partial J(y', \{\underline{n'}_k, \bar{n'}_k, v'_k\})}{\partial \underline{n}'_{k+1}}$$

To show that cohorts with lower promised values are more subject to layoffs, I take a derivative of this FOC with respect to the promised value v_k.The only component in the FOC directly dependent on v_k is $\frac{1}{u'(w_k)}$, where $w_k = u^{-1}(v_k - \beta[s_k U + (1-s_k)R(v'_{k+1})])$. Due to the $CRRA$ utility function, we find that $\frac{1}{u'(w_k)}$ is increasing in v_k, and, therefore, the marginal profit of firing workers is decreasing in v_k.

Lastly, I show that layoffs are equally dependent on the quality of any cohort (as long as $z_k < 1$). First, I note that quality directly appears only in the marginal value of low quality workers

$$\frac{\partial J(y', \{\underline{n}'_k, \bar{n}'_k, v'_k\})}{\partial \underline{n}'_{k+1}} = y'[F'_1(\sum n'_k, \frac{\sum n'_k z'_k}{\sum n'_k}) - F'_2(\sum n'_k, \frac{\sum n'_k z'_k}{\sum n'_k})\frac{\sum n'_k}{(\sum n'_k z'_k)^2} - w'_{k+1} + \beta E_{y''|y'}\frac{\partial J(y'', ...)}{\partial \underline{n}''_{k+2}}\frac{\partial \underline{n}''_{k+}}{\partial \underline{n}'_{k+}}$$

Next, I note that all the quality states $\{z_k\}$ contribute in the same manner to $\frac{\partial J(y', \{\underline{n}'_k, \bar{n}'_k, v'_k\})}{\partial \underline{n}'_{k+1}}$, no matter the cohort k. Therefore, to first-order, the marginal value of firing a worker is equally dependent on all the quality states.

I don't think I need to assume $s_k(\underline{n}_k + \bar{n}_k) \leq \underline{n}_k$. I can write the proof for both cases, and the results should be there for either one. □

Proof of Proposition 4

Start with the wage growth equation from Proposition 1, extended to the case of heterogeneous matches.

$$\frac{1}{u'(w'_{k+1})} - \frac{1}{u'(w_k)} = \eta(v'_{k+1})E_{y'|y}\frac{\partial J(y', \{n'_k, v'_{y',k}, z'_k\})}{\partial n'_{k+1}}$$

One can now apply the Envelope Theorem to extend the RHS of the equation:

$$\begin{aligned} E_{y'|y}\frac{\partial J(y', \{n'_k, v'_{y',k}, z'_k\})}{\partial n'_{k+1}} =& E_{y'|y}\Big[y'\frac{\partial F(\sum n'_k, \frac{\sum n'_k z'_k}{\sum n'_k})}{\partial n'_{k+1}} - w'_{k+1} + \\ & \beta(1 - p(v''_{k+2}))(1 - s'_{k+1})E_{y''|y'}\frac{\partial J(y'', \{n''_k, v''_{y'',k}, z''_k\})}{\partial n''_{k+2}})\Big] \end{aligned}$$

One can note that the production derivative $\frac{\partial F(\sum n'_k, \frac{\sum n'_k z'_k}{\sum n'_k})}{\partial n'_{k+1}}$ is increasing in z'_{k+1}. And, moreover, that $\frac{\partial^2 F(\sum n'_k, \frac{\sum n'_k z'_k}{\sum n'_k})}{\partial n'_{k+1}\partial z'_{k+1}} > \frac{\partial^2 F(\sum n'_k, \frac{\sum n'_k z'_k}{\sum n'_k})}{\partial n'_{k'}\partial z'_{k+1}}, k' \neq k+1$. This implies that, the workers in steps $k \in K_s$ will experience the largest rise in marginal productivity. This doesn't mean that they are at the highest marginal productivity though! Part of my intuition is predicated on the fact that the other workers, ones not fired, are already quite close to their target wages! Otherwise those guys would still have the best wage growth. Essentially, my intuition is that the fired cohorts receive a spike in marginal productivity, so, if they're on the similar-ish wage path to the other cohorts, they should get the best wage growth. But this needs to be formalized some more.

A.2 Tenure-specific Severance Payments

I allow the firm to offer tenure-specific severance payments sev_k to its workers. The severance is constant over time and paid perpetually upon firing and before finding a new job. I show that the severance structure involves higher payments for longer tenured workers (if those workers are on a higher promised value).

Proposition 5. *Fix a firm state. Its severance payments for each tenure k are given by*

$$\frac{u'(b+sev_k)}{u'(w_k)} = 1 - \frac{\beta sev_k \frac{\partial p(\theta_{sev_k})}{\partial sev_k}}{1-\beta(1-p(\theta_{sev_k}))}$$
$$\theta_{sev_k} = \theta(arg\max_v[(1-p(v))U(sev_k)+p(v)v])$$

Proof. I start by describing the unemployment value of a worker with severance payment sev_k:

$$U(sev_k) = u(b+sev_k) + \beta\max_v[(1-p(\theta_v))U(sev_k)+p(\theta_v)v]$$

Denote the probability of finding a job with severance payment sev_k as $p(\theta_{sev_k})$. The extra value to the unemployed from the severance payment is then given by

$$\frac{\partial U(sev_k)}{\partial sev_k} = u'(b+sev_k) + \beta(1-p(\theta_{sev_k}))U'(sev_k) = \frac{u'(b+sev_k)}{1-\beta(1-p(\theta_{sev_k}))}$$

Then the total benefit to the firm from raising the severance payment is the slackening of the promised-keeping constraint thanks to this rise in the unemployment value:

$$\lambda_k n_k \beta s_k \frac{\partial U(sev_k)}{\partial sev_k} = \frac{n_k}{u'(w_k)}\beta s_k \frac{u'(b+sev_k)}{1-\beta(1-p(\theta_{sev_k}))}$$

On the cost side, the firm internalizes the net present value of the severance payments when firing $n_k s_k$ workers:

$$\frac{\partial}{\partial sev_k}\left[n_k s_k \beta \frac{sev_k}{1-\beta(1-p(\theta_{sev_k}))}\right] = n_k s_k \beta \frac{[1-\beta(1-p(\theta_{sev_k}))] - \beta sev_k \frac{\partial p(\theta_{sev_k})}{\partial sev_k}}{[1-\beta(1-p(\theta_{sev_k}))]^2}$$

The optimal severance payment then follows from the first-order condition:

$$\frac{n_k}{u'(w_k)}\beta s_k \frac{u'(b+sev_k)}{1-\beta(1-p(\theta_{sev_k}))} = n_k s_k \beta \frac{[1-\beta(1-p(\theta_{sev_k}))] - \beta sev_k \frac{\partial p(\theta_{sev_k})}{\partial sev_k}}{[1-\beta(1-p(\theta_{sev_k}))]^2}$$

Rearranging gives the result.

$$\frac{u'(b+sev_k)}{u'(w_k)} = 1 - \frac{\beta sev_k \frac{\partial p(\theta_{sev_k})}{\partial sev_k}}{1-\beta(1-p(\theta_{sev_k}))}$$

□

Note that, besides $u'(w_k)$, all the components of the severance payment are independent of both the firm state and the worker tenure. It is immediate to notice then that higher paid workers will have higher severance payments: as $\frac{1}{u'(w_k)}$, the value to the firm of the severance payment goes up, while costs stay the same. Therefore, the firm will optimally choose to offer higher severance payments to higher paid workers.

This equation is also easy to implement numerically: using the formulation in Appendix A.4, where $\rho_k \equiv u'(w_k)$ is a state variable, I can immediately compute the payments for all the firm states, before solving the rest of the firm problem.

A.3 Microfounding the Wage Noncontractability

A version of my model where the firm is allowed to choose quality-specific wage is in fact a signalling game: firm's action of choosing the wage signals the workers their quality. In this section I show that a pooling Perfect Bayesian Equilibrium of such a game exists, and thus it is plausible that no information is conveyed.

As a full infinite-horizon, multiple-receivers, complex sender model is too complicated of a game to solve, I restrict attention to a simplified model.

Consider a 3-period version of the model. The firm starts with measure $n = 2$ of workers, half of them of high quality and the other half of low. I allow the firm to offer quality-contingent wages in whichever way it likes.

I show that, under a sufficiently small elasticity of job search probability with respect to promised value v', $\eta(v') \equiv \frac{\partial(1-p(v'))/\partial v'}{(1-p(v'))}$, the pooling Perfect Bayesian Equilibrium exists. Moreover, under a stronger condition on the elasticity, this PBE survives the Intuitive Criterion (Cho and Kreps (1987)).

A.4 Recursive Lagrangian Approach

The original design of the problem would require solving promised values $v'_{y',k}$ for both each tenure step and each future productivity state. Following Balke and Lamadon (2022), I solve the following Pareto problem:

$$\mathcal{P}(y, \{n_k, \rho_k, z_k\}) = \inf_{\omega_k} \sup_{\tilde{n},\tilde{v},\{w_k,s_k,v'_k\}} yF(n,z) - \sum_k n_k w_k - \kappa_f - \tilde{n}\frac{c}{q(\theta_{\tilde{v}})}$$
$$+ \sum_k \rho_k (u(w_k) + \beta[s_k U + (1-s_k) R(v'_{k+1})]$$
$$- \beta \sum_k \omega_k v'_{k+1} + \beta E_{y'|y} \mathcal{P}(y', \{n'_k, \omega_k, z'_k\})$$

where

$$\mathcal{P}(y, \{n_k, \rho_k, z_k\}) \equiv \sup_{\{v_k\}} J(y, \{n_k, v_k, z_k\}) + \sum_k \rho_k v_k$$

The following proof (for $K \to \infty$ but the proof extends trivially to finite K) establishes its equivalence with the initial problem. It follows the steps of Balke and Lamadon (2022), extending it to the case of a multi-worker firm.

Proof. We have the following recursive formulation for J:

$$J(y, \{n_k, v_k, z_k\}_{k\leq K}) = \max_{\tilde{n},\tilde{v},\{v'_k, v'_{y',k}, w_k, s_k\}_{k\leq K}} yF(\sum_k n_k, \frac{\sum n_k z_k}{\sum n_k}) - \sum_k w_k n_k - \tilde{n}\frac{c}{q(\tilde{v})} - \kappa_f$$
$$+ \beta E_{y'|y} J(y', \{n'_k, v'_k, z'_k\}_{k\leq K+1})$$
$$(\lambda_k)\ u(w_k) + \beta[s_k U + (1-s_k) R(v'_{k+1}) = v_k]\ \forall k \leq K$$
$$(\omega_k) v'_{k+1} = E_{y'|y} v'_{k+1,y'}\ \forall k \leq K$$
$$n'_{k+1} = n_k(1-s_k)(1-p(v'_{k+1})) + \tilde{n}\ \forall k \leq K$$
$$z'_{k+1} = min(\frac{z_k}{1-s_k}, 1)\ \forall k \leq K$$
$$n'_0 = \tilde{n}, v'_0 = \tilde{v}, z'_0 = z_0$$

Consider the Pareto problem

$$\mathcal{P}(y, \{n_k, \rho_k, z_k\}) = \sup_{\{v_k\}} J(y, \{n_k, v_k, z_k\}) + \sum_k \rho_k v_k$$

I first substitute the definition of J together with its constraints into $\mathcal{P}$:

$$\mathcal{P}(y,\{n_k,\rho_k,z_k\}) = \sup_{\tilde{n},\tilde{v},\{v_k,v'_k,v'_{y',k},w_k,s_k\}_{k\le K}} yF(\sum_k n_k, \frac{\sum n_k z_k}{\sum n_k}) - \sum_k w_k n_k - \tilde{n}\frac{c}{q(\tilde{v})} - \kappa_f$$
$$+ \beta E_{y'|y} J(y',\{n'_k,v'_k,z'_k\}_{k\le K+1}) + \sum_k \rho_k v_k$$
$$(\lambda_k)\ u(w_k) + \beta[s_k U + (1-s_k)R(v'_{k+1}) = v_k]\ \forall k \le K$$
$$(\omega_k)\ v'_{k+1} = E_{y'|y} v'_{k+1,y'}\ \forall k \le K$$
$$n'_{k+1} = n_k(1-s_k)(1-p(v'_{k+1})) + \tilde{n}\ \forall k \le K$$
$$z'_{k+1} = min(\frac{z_k}{1-s_k},1)\ \forall k \le K$$
$$n'_0 = \tilde{n}, v'_0 = \tilde{v}, z'_0 = z_0$$

I now substitute in the promise-keeping constraint:

$$\mathcal{P}(y,\{n_k,\rho_k,z_k\}) = \sup_{\tilde{n},\tilde{v},\{v'_k,v'_{y',k},w_k,s_k\}_{k\le K}} yF(\sum_k n_k, \frac{\sum n_k z_k}{\sum n_k}) - \sum_k w_k n_k - \tilde{n}\frac{c}{q(\tilde{v})} - \kappa_f$$
$$+ \beta E_{y'|y} J(y',\{n'_k,v'_k,z'_k\}_{k\le K+1}) + \sum_k \rho_k(u(w_k) + \beta[s_k U + (1-s_k)R(v'_{k+1})])$$
$$(\omega_k)\ v'_{k+1} = E_{y'|y} v'_{k+1,y'}\ \forall k \le K$$
$$n'_{k+1} = n_k(1-s_k)(1-p(v'_{k+1})) + \tilde{n}\ \forall k \le K$$
$$z'_{k+1} = min(\frac{z_k}{1-s_k},1)\ \forall k \le K$$
$$n'_0 = \tilde{n}, v'_0 = \tilde{v}, z'_0 = z_0$$

I introduce the ω_k-constraints with weights β into the problem:

$$\mathcal{P}(y,\{n_k,\rho_k,z_k\}) = \inf_{\{\omega_k\}} \sup_{\tilde{n},\tilde{v},\{v'_k,v'_{y',k},w_k,s_k\}_{k\le K}} yF(\sum_k n_k, \frac{\sum n_k z_k}{\sum n_k}) - \sum_k w_k n_k - \tilde{n}\frac{c}{q(\tilde{v})} - \kappa_f$$
$$+ \beta E_{y'|y} J(y',\{n'_k,v'_k,z'_k\}_{k\le K+1}) + \sum_k \rho_k(u(w_k) + \beta[s_k U + (1-s_k)R(v'_{k+1})])$$
$$+ \sum_k \beta\omega_k(E_{y'|y} v'_{y',k+1} - v'_{k+1})$$
$$n'_{k+1} = n_k(1-s_k)(1-p(v'_{k+1})) + \tilde{n}\ \forall k \le K$$
$$z'_{k+1} = min(\frac{z_k}{1-s_k},1)\ \forall k \le K$$
$$n'_0 = \tilde{n}, v'_0 = \tilde{v}, z'_0 = z_0$$

I then rearrange the value function by moving $E_{y'|y}\sum_k \beta\omega_k n'_{k+1} v'_{y',k+1}$ (additional constraints are dropped to simplify notation):

$$\mathcal{P}(y,\{n_k,\rho_k,z_k\}) = \inf_{\{\omega_k\}} \sup_{\tilde{n},\tilde{v},\{v_k,v'_{y',k},w_k,s_k\}_{k\le K}} yF(\sum_k n_k, \frac{\sum n_k z_k}{\sum n_k}) - \sum_k w_k n_k - \tilde{n}\frac{c}{q(\tilde{v})} - \kappa_f$$
$$+ \beta E_{y'|y}[J(y',\{n'_k,v'_k,z'_k\}_{k\le K+1}) + \sum_k \omega_k v'_{y',k+1}]$$
$$\sum_k \rho_k(u(w_k) + \beta[s_k U + (1-s_k)R(v'_{k+1})]) - \sum_k \beta\omega_k v'_{k+1}$$

Lastly, I split the sup:

$$\mathcal{P}(y,\{n_k,\rho_k,z_k\}) = \inf_{\{\omega_k\}} \sup_{\tilde{n},\tilde{v},\{v'_k,w_k,s_k\}_{k\le K}} yF(\sum_k n_k, \frac{\sum n_k z_k}{\sum n_k}) - \sum_k w_k n_k - \tilde{n}\frac{c}{q(\tilde{v})} - \kappa_f$$
$$+ \beta E_{y'|y}[\sup_{v'_{y',k+1}} J(y',\{n'_k,v'_k,z'_k\}_{k\le K+1}) + \sum_k \omega_k v'_{y',k+1}]$$
$$\sum_k \rho_k(u(w_k) + \beta[s_k U + (1-s_k)R(v'_{k+1})]) - \sum_k \beta\omega_k v'_{k+1}$$

From this, one can note that, by definition of $\mathcal{P}$

$$\sup_{v'_{y',k+1}} J(y',\{n'_k,v'_k,z'_k\}_{k\le K+1}) + \sum_k \omega_k v'_{y',k+1} = \mathcal{P}(y',\{n'_k,\omega_k,z'_k\})$$

We thus arrive to the formulationof the problem as described at the beginning, not involving finding future state-specific promised values $v'_{y',k}$. □

A.5 Block Recursivity

I introduce an assumption that would allow for a block recursive eqiulibrium under the same conditions as in Schaal (2017). Block recursivity requires an indifference condition, either on the side of the firms or on the side of the workers. Under two-sided ex-post heterogeneity, that is not immediately achievable.

Schaal (2017) shows that, in a setting similar to mine, but with transferable utility between workers and firms, which he achieves due to the risk-neutral worker utility function, firms all have the same preferences across all the submarkets that they may post vacancies in. Define the minimal hiring cost as

$$k = \min_v [v + \frac{c}{q_v}]$$

Due to transferable utility, the cost of employing the worker from submarket v becomes simply the value v. Thus, the optimal entry of vacancies in Schaal (2017) can be summarized by

$$\theta_v[v + \frac{c}{q_v} - k] = 0$$

Meaning that either a submarket v minimizes the hiring cost or it is closed. This condition is completely independent of the distribution of firms and workers, exactly because the one component where the firm type might come through, the cost of employing a worker from submarket v, is completely independent from the firm's state due to transferable utility.
Utility is not transferable in my model, and thus different firms may face different costs of employing a worker at some value v (for example, fixing y and z, small firms prefer high values v due to their intention to upsize). To get around that, I split the value v that the worker would get upon getting hired into two components, the sign-on wage w_v and the remaining value v_0 such that

$$u(w_v) + \beta v_0 = v$$

This additional wage payment is incurred immediately upon hiring, allowing the remaining value that the firm owes to its worker, v_0, to be completely independent of the submarket v. Essentially, from the firm's perspective, submarkets now differ not in the value that firms would owe to the workers, but in this sign-on wage. The cost minimization problem then becomes

$$k = \min_{v}[w_v + \frac{c}{q_v}]$$

This problem is now again completely independent of the firm's state, and thus the distribution of firms and workers no longer affects the tightness function q_v.Schaal (2017) shows that, in a setting similar to mine, but with transferable utility between workers and firms, which he achieves due to the risk-neutral worker utility function, firms all have the same preferences across all the submarkets that they may post vacancies in. Then setting θ_v such that

B Quantitative Appendix

B.1 Variables

Endogenous state variables

$$\{n_k\}_{k\leq K}, \{\rho_k, z_k\}_{1\leq k\leq K}$$

Code: `states: size, rho, q`

Exogenous state variables

$$y$$

Code: `z`

Control variables

$$\tilde{n}, \{\rho'_k\}_{1\leq k\leq K}, \{s_k\}_{k\leq K}$$

Code: `hiring, rho_star, sep_star`

Value function

$$\mathcal{P}(y, \{n_k, \rho_k, z_k\})$$

Code: `ERho_star, EJ_star`

B.2 Equations

Equations (1)–(21): Value function in olive (given by value and its gradient guesses from future states), states in green (given), controls in orange (given by policy guess from current states), next period exogenous states in magenta (to be summed over), next period states in blue, parameters in black.

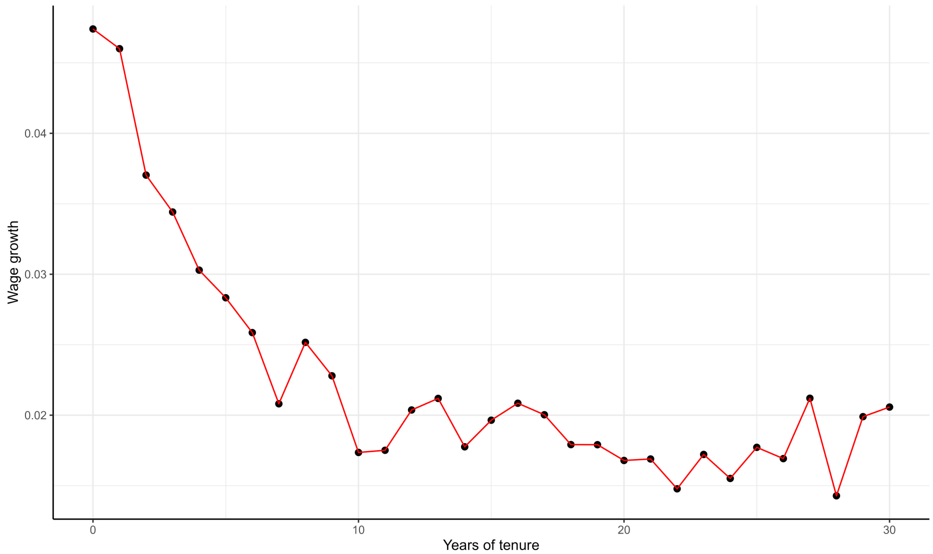
$$0 = \rho'_{k+1} - \rho_k - \eta(E\frac{\partial P(y', \{n'_k, \rho'_k, z'_k\})}{\partial \rho'_{k+1}})E\frac{\partial \mathcal{P}(y', \{n'_k, \rho'_k, z'_k\})}{\partial n'_{k+1}} \tag{8}$$

$$\tag{9}$$

C Data Appendix

C.1 Wage Growth

I use the same sample to plot the log (real) wage growth across first 30 years of tenure.



The wage growth appears to flatten after about 10 years of tenure, suggesting that it is not quantitatively costly to use $K = 10$ as an approximation of the firm problem from Definition 1.