Heterogeneous Wage Cyclicality and Unemployment Fluctuations*

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Abstract

Firms hire workers from different pools. Some firms hire more unemployed workers than others, making their demand for labor more important for unemployment. I differentiate jobs based on their hiring pool and estimate their wage cyclicality. The key finding is that wages in jobs hiring from unemployment are half as cyclical as wages in other jobs, for both incumbent workers and new hires. To measure the effects of this on unemployment volatility, I develop a labor search model with separation of search and heterogeneous wage rigidity and show that accounting for this heterogeneity increases the volatility of unemployment by $14\% - 34\%$.

1 Introduction

Standard labor search models, calibrated to business cycle frequency, significantly underdeliver in the volatility of unemployment when compared to the data (Shimer 2005). Wage rigidity, the inverse of the sensitivity of wages to aggregate productivity, has been widely proposed as a solution: if wages are rigid, during the recession, wages are stuck too high, thus

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firms are disincentivized from searching for workers, increasing unemployment. But what type of rigidity matters for the volatility of unemployment? Estimating aggregate wage cyclicality may not be the right approach if all wages do not matter to the same degree.

This paper suggests the job hiring pool as the key source of heterogeneity. Firms do not all hire the same workers, nor do all the firms hire the same number of unemployed workers. Ranking jobs based on the ratio of unemployed workers they hire, I find that jobs above the 90th percentile hire more than twice as many unemployed workers as jobs below the 10th percentile. From the perspective of importance for the volatility of unemployment, the rigidity of the latter jobs matters significantly less as those jobs simply do not hire as many unemployed workers as the jobs above the 90th percentile. More generally, data shows that jobs are heterogeneous in their pools, and thus the importance of rigidity of their wages is also heterogeneous. Jobs hiring primarily unemployed workers have a fundamentally different connection to the unemployment pool than jobs poaching workers. If wages are rigid in a job hiring unemployed workers, then the standard intuition follows: during recessions wages will be high, thus the job will hire fewer workers, increasing the unemployment level. On the other hand, jobs that poach workers can only impact unemployment indirectly, by affecting hiring incentives of the jobs actually looking for unemployed workers.

This dimension of heterogeneity is new to the literature. There are random search models that study the effect of wage rigidity on unemployment volatility, but they are unable to account for heterogeneous hiring pools, as all firms successfully hire unemployed workers. In directed search, Balke and Lamadon (2022) and Souchier (2022) endogenize wage rigidity via the dynamic contracting framework, but do not take into account implications for unemployment volatility. Rudanko (2023) incorporates a within-firm equity constraint on wages, resulting in amplified responses of firms to shocks. However, these papers do not incorporate potential heterogeneity in wage rigidity. Empirical papers brush over the distinction completely by estimating wage rigidity aggregated across all jobs. In contrast, I explicitly make this distinction in both theoretical and empirical settings by exploring the differences in levels of rigidity across these jobs and their effects on unemployment volatility. The key empirical finding is that there is significant heterogeneity in wage cyclicality across jobs of different hiring pools: jobs that hire the most unemployed workers have half as cyclical wages as the jobs hiring mostly job-to-job transitioners. To evaluate the importance of this distinction, I then build a quantitative labor search model with a separation of search and heterogeneous wage rigidity. Accounting for this heterogeneity in a quantitative model increases the volatility of unemployment by $14\% - 34\%$ compared to the baseline case of homogeneous rigidity.

I use the French matched employer-employee panel data DADS to separately estimate the wage rigidity of jobs hiring from unemployment and employment. The matched nature of the data is crucial for measuring hiring pools. The employee side of the data allows tracking the history of each worker, allowing one to identify, each time a worker finds a new job, whether she was previously employed (employment-to-employment, or EE) or unemployed (unemployment-to-employment, or UE). Matching that with the employer side, it is now possible to calculate how many UE and EE workers each job hires, providing information on each job's hiring pool. I classify jobs based on their proportion of UE workers and then estimate wage rigidity separately for each of the brackets. The rigidity differences across jobs are found for both incumbent workers and new hires.

Focus on job-level heterogeneity, rather than match- or worker-level is important for two reasons. First, it is job-side wage rigidity that primarily matters for the volatility of unemployment, rather than the worker side, as the firms are the ones making hiring decisions. Second, rather than interpreting the results as differences in workers' abilities to leverage their outside options, the job-level nature of the result implies that heterogeneity occurs at the level of firms' hiring practices. To argue for this interpretation, in Table 6 I redo the main regression with worker dimension (whether a worker is a job-to-job transitioner) instead of the job dimension and find that it fails to show the same kind of heterogeneity. More precisely, the increment for the worker being a job-to-job transitioner has an almost zero coefficient for incumbents. This does not come as a surprise though as, past the initial stage of hiring, workers' previous outside options usually have little effect on their wages (Addario, Kline, Saggio, and Sølvsten 2022).

A quantitative labor search model is developed to measure the relative importance of rigidity across jobs of different hiring pools. The approach is built around a simple intuition that UE and EE jobs have mechanically different ways of affecting unemployment. If a job hires only unemployed workers, simply hiring more is enough to affect unemployment. On the other extreme, job hiring exclusively job-to-job transitioners cannot affect unemployment directly. Instead, it can only affect it via the threat of poaching workers from jobs that do hire from unemployment. To incorporate this intuition and compare the two effects, I develop a labor search model with on-the-job search and separation of search: workers search in different locations depending on their current employment status, with locations following a job ladder structure. This is closely related to directed search models, with the key difference being the discrete and finite number of submarkets. More precisely, each submarket has a set of firms tied to it, meaning that these firms can only post vacancies in that submarket. Workers choose in which location to search, and then workers and firms randomly match within a submarket. Submarkets take the interpretation of the rungs of the job ladder, where higher rungs pay more and attract workers with better outside options. This in turn implies that the higher rungs hire more job-to-job transitioners and fewer out-of-unemployment workers. I let each location have its own wage determination mechanism, defined by its wage level and wage rigidity. Wage rigidity is assumed to be exogenously imposed on the firms. Since the rigidity is allowed to be different across rungs of the ladder, it is possible to study the effects of wage rigidity separately for each of the rungs.

I restrict attention to two submarkets, differentiated by both the wage level and wage rigidity. One of the submarkets has a higher wage level, and the employed workers (optimally) search in that submarket, while the unemployed workers mostly search in the other location. All the equilibria in the model are Block Recursive (following Menzio-Shi 2011): value and policy functions are independent of the labor distribution. Thus, it is possible simulate the economy even out of the steady-state, by first finding value and policy functions for each of the values of the aggregate productivity, and then letting the economy run given the optimal policy functions. Model simulation reveals that rigid wages in the location that hires employed workers have a 3 times smaller effect on the standard deviation of unemployment than rigidity in the location that hires unemployed workers. Thus, the direct effect of UE jobs is stronger than the indirect effect generated by rigidity in EE jobs. Moreover, this indirect effect gets weaker the higher up the job ladder the submarket is. The more steps the submarket has to go through to affect the hiring incentives of UE jobs, the smaller the impact on the volatility of unemployment. To quantify the value of accounting for wage rigidity heterogeneity across different submarkets, I compare my standard calibration with the baseline, where wages are equally rigid in all the jobs. The volatility of unemployment turns out to be $14\% - 34\%$ higher in the calibration accounting for the heterogeneity in wage rigidity.

1.1 Related Literature

The unemployment volatility puzzle was first introduced by Shimer (2005), who showed that standard labor search models (Mortensen and Pissarides 1994), when calibrated to business cycle frequency, are unable to produce unemployment volatility observed in the data. To solve the puzzle, Shimer (2005) suggests introducing real wage rigidity into the models. This suggestion has been theoretically supported by, e.g., Hall (2005), Hall and Milgrom 2008. Empirically, the idea came from classic wage estimation exercises (citations here) suggesting that wages do not co-move with business cycles.

However, this notion has been challenged by Pissarides (2009), who notes that the only important wages for the hiring incentives, which are the primary driver of unemployment fluctuations (Shimer 2012), are new hire wages, and that wage cyclicality for new hires is larger than that of incumbent workers (Bils 1985). He uses this evidence to conclude that real wage rigidity is not an appropriate solution to the volatility puzzle. Fukui (2021) challenges Pissarides' conclusion, showing that, with the addition of on-the-job search and wage posting (instead of Nash Bargaining) to the Mortensen-Pissarides model, the results flip, and only the incumbent wage rigidity now matters for unemployment volatility. Overall, there is no consensus yet on which wages matter more for unemployment volatility. In this paper, I do not need to take a stand on the debate, as my results of jobs hiring unemployed workers having rigid wages are there for both new hires and continuing workers.

Recent wage cyclicality studies have been motivated by this discussion, putting their focus on separately estimating and comparing new hire and incumbent wage cyclicalities. Though earlier studies usually find that new hire wages are significantly more cyclical than incumbent wages (Bils 1985, Shin 1994, Devereux and Hart 2006), Gertler and Trigari (2009) argue that the observed procyclicality may come from workers upgrading jobs or finding better matches in booms rather than true wage flexibility. Thus, they argue that not controlling for job composition and match up and downgrading results in procyclical biases. In line with this suggestion, several approaches have been attempted. Hagedorn and Manovskii (2013) use an indirect measure of match quality to directly control for it in their regressions. They find that new hire wages are similarly cyclical to those of incumbents. Grigsby, Hurst, and Yildirmaz (2021) use high-quality payroll data and compare wages of job changers to similar workers who decided to stay in their current jobs, to correct for the composition bias. They find weak new hire procyclicality. Hazell and Taska (2020) and Choi, Figueroa, and Villena-Roldan (2020) use online job vacancies to track posted wages of jobs across vacancies before matches have a chance to form and affect the observation. This lets them avoid both job composition and procyclical match upgrading biases, at the cost of only estimating new hire wages. Hazell and Taska find strong downward rigidity, while Choi, Figueroa, and Villena-Roldan find strong procyclicality. Lastly, Martins, Solon, and Thomas (2012), Carneiro, Guimaraes and Portugal (2012), Stuber (2017), and Dapi (2020) use matched employer-employee data to account for worker, firm, and job heterogeneity to control for job composition bias as well as (to some extent) procyclical match upgrading bias. My paper falls into this category, using both worker- and job-level fixed effects to control for the biases. Unlike the papers above, focusing on the comparison between the new hire and incumbent wages, my paper is less interested in controlling for these biases as they primarily affect new hire estimation, while this paper's empirical result result holds for both new hires and incumbents.

The papers closest in their empirical methodology to mine are those by Haefke, Sonntag, and Van Rens (2013) and Gertler, Huckfeldt, and Trigari (2020), which both directly estimate the wage cyclicality of new hires from unemployment to control for the procyclical match upgrading bias. Both use employee-side data: CPS and SIPP, respectively. Haefke, Sonntag, and Van Rens (2013) find strong procyclicality in UE new hire wages, while Gertler, Huckfeldt, and Trigari find that UE new hire wages are as rigid as wages of incumbent workers and indicate that the observed new hire wage procyclicality in other papers is entirely due to high cyclicality in EE new hire wages. They interpret high new hire EE cyclicality as an artifact of cyclical movements in match quality rather than true wage flexibility. My paper differs in two aspects. First, I compare wage rigidity across jobs for both new hires and incumbents and find that, even for incumbents, wages are more rigid in jobs hiring from unemployment. This supports the focus on the cross-firm rather than cross-worker heterogeneity and also implies that disaggregating only at the new hire level is not enough. Second, rather than comparing cyclicality across workers, I compare it across jobs, which reinterprets differences in wage rigidity as heterogeneous firm wage practices rather than match upgrading bias or workers leveraging their outside options in the bargaining stage. To argue for this interpretation, in Table 6 I use the worker dimension (whether a worker is UE or EE) in my main regression instead of the job dimention and find that the increment for the worker being a job-to-job transitioner is almost zero for incumbent workers. Thus, unlike Haefke et al and Gertler et al, who focus on previously unemployed workers as a way to deal with match upgrading bias, I treat this distinction at the job-level as a show of heterogeneous wage rigidity.

Another notable comparison is a paper by Lagakos and Ordonez (2011), who compare wage rigidity, measured as the responsiveness of wages to the marginal productivity of labor, across the sectors of the US economy based on the education level of their workers. They find that sectors with higher educated workers have more rigid wages, and suggest that the result comes from firms having higher displacement costs of higher educated workers: these workers are harder to substitute, so firms insure them more in order to keep them. It is not easy to reconcile their finding with mine, as one would expect firms that poach workers to have them be of higher education. I confirm this in my data. However, I fail to replicate their result using my data. Instead, I find the opposite of their result in that higher educated sectors have less rigid wages. I suspect that the difference comes from our measures of wage rigidity, as my measure of unemployment and their measure of marginal productivity of labor often do not co-move.

Theoretically, this paper relates to the labor search and job ladder literature. There are several approaches to modeling on-the-job search in the literature. The first approach assumes random search with either wage bargaining (Lise and Robin 2017, Moscarini and Postel-Vinay 2018,2019) or wage posting (Moscarini and Postel-Vinay 2013,2016, Morales-Jimenez 2019, Fukui 2020). All workers search in the same job pool and, over time, find better and better jobs, moving up the job ladder. Notably, Morales-Jimenez (2019) and Fukui (2020) model wage rigidity in their papers with applications to unemployment fluctuations. However, as is the case with the entire random search literature, these papers cannot account for the separation of search: in random search, all workers search in the same pool, and thus all jobs have the same probability to hire an unemployed worker. Instead, I require heterogeneity of jobs' hiring pools not just in the relative sense, but also in the absolute one: jobs at the top should be less likely to hire unemployed workers.

For this purpose, directed search (Moen 1997, Menzio and Shi 2011, Schaal 2017) is a more appropriate framework as separation of search is embedded into it. Recently, Balke and Lamadon (2022), Souchier (2022), and Rudanko (2023) attempted incorporating wage rigidity into a directed search framework. The former two use a dynamic contracting model between a risk-neutral firm and a risk-averse worker where risk-aversion incentivized firms to smooth wages. This, however, does not have any implications for unemployment fluctuations. Rudanko (2023) instead models wage rigidity via within-firm wage equity contraints. My paper takes the effect of wage rigidity on unemployment fluctuations as given and instead focuses on modeling the implications of heterogeneous wage cyclicality.

The rest of the paper is organized as follows. Section 2 presents the procedure of estimating wage rigidity across jobs based on their hiring pools. Section 3 describes the model and its theoretical properties. Section 4 explains the calibration of the model and gives the simulation results. Section 5 concludes.

2 Estimating wage cyclicaclity

This section presents novel evidence on the heterogeneity of wage rigidity across different jobs. I distinguish jobs based on the proportion of job-to-job transitioners that they hire. I then proceed to show that wages are significantly more rigid in the jobs that hire the fewest job-to-job transitioners. This heterogeneity is there for both incumbents and new hires. Lastly, I show that the result cannot be explained by procyclical match upgrading, suggesting actual differences in job-side wage rigidity.

2.1 Data

Data from this study come from a French matched employer-employee dataset - Déclarations Annuelles de Données Sociales (DADS), built by the French Statistical Institute (INSEE) from the social contributions declarations of firms. The dataset covers about 85% of all French workers and spans the years 1976-2019. On an annual basis, it provides employment information (salaries, hours worked, occupation, and, importantly, precise start and end dates of every employment spell), worker information (age, experience), and firm information (sector, industry, size). The dataset is available in a panel form for a subsample of workers. The key advantage of the dataset is the precise information on workers' employment spells combined with its matched nature. The former allows tracking workers' histories to determine whether, each time they find a new job, they transitioned to that job from employment or unemployment. The latter then allows relating that information to jobs, thus giving information on the kinds of workers that each job hires.

2.2 Identifying job hiring pool

To classify jobs based on who they hire, I first need to classify workers. An observation (worker·firm·year) is classified as a new hire if the worker is not observed at the same firm the year prior. A newly hired worker is considered a job-to-job transitioner if she is observed at a full-time job at most 4 weeks prior to having started this one. Thus, a worker is allowed to have a 4 week-long break between jobs before being considered an out-of-unemployment hire. Finally, for each job(firm·occupation) I calculate the share of new hires that are job-to-job transitioners and then pool jobs into equally sized brackets based on this share. For calculating this share and all further computations I restrict attention to the years 2003-2019, the Ile-de-France region, and 25-55yo workers in full-time jobs in the private sector. Appendix 1 provides more information on sampling choices. The majority of jobs poach about 60% of their workers from other jobs, but with significant heterogeneity: for the case of 10 brackets, the jobs in the highest bracket on average hire twice as many job-to-job transitioners as jobs in the lowest bracket. Those jobs also pay higher wages and hire more educated workers.

2.3 Estimation strategy

The empirical strategy is based on the approach initially suggested by Abowd and Kramarz (1999) and used extensively in matched employer-employee datasets since then (e.g. Carneiro et al 2012, Stuber 2017, Dapi 2020). It tests for real wage cyclicality using a level wage equation with controls for worker observed and unobserved (constant) heterogeneity, unobserved (constant) job heterogeneity, and business cycle conditions.

The baseline specification is

$$ln(w_{ijt}) = \mu(t) + \alpha U_t + new_{ijt} * (\tilde{\mu}(t) + \tilde{\alpha} U_t) + \gamma x_{it} + FE_i + FE_j + \epsilon_{ijt}$$

Brackets	Share of EE workers	Average wage	Share of college-educated workers
1	0.41	21.5	0.14
2	0.53	20.9	0.15
3	0.58	23.8	0.18
4	0.62	24.1	0.17
5	0.65	26.8	0.21
6	0.68	27.2	0.24
7	0.72	36.5	0.30
8	0.76	33.1	0.31
9	0.81	34.9	0.36
10	0.89	34.4	0.33

Table 1: Descriptives for the case of 10 brackets

where w_{ijt} is the real hourly wage of worker i, in a job j, in the year t; U_t, unemployment at time t, acts as a business cycle indicator; $\mu(t) = \mu_0 + \mu_1 t + \mu_2 t^2$ is a quadratic time trend, x_{it} is a vector of time-varying worker characteristics such as age and experience; FE_i, FE_j are worker and job fixed effects. To compare the behavior of wages over the business cycle between incumbents and new hires, these regressions include a dummy variable new_{ijt} for whether a worker is a new hire, as well as the interaction term between the dummy and the cycle indicator. A quadratic trend interacted with the dummy variable is not necessary but is added for consistency with the main regression.

Job fixed effects allow controlling for job up- and down-grading, which would have resulted in a procyclical bias. Worker fixed effects help control for worker heterogeneity, which, as shown by other studies (e.g. Keane, Moffitt, and Runkle 1988), leads to a countercyclical bias.

To estimate wage rigidity separately for jobs based on the proportion of job-to-job transitioners they hire, I add the corresponding brackets as an additional dimension of heterogeneity into the regression:

$$ln(w_{ijt}) = \mu(t) + \alpha U_t + New_{ijt} \cdot (\tilde{\mu}(t) + \tilde{\alpha}^{new} U_t) + \gamma x_{it} + FE_i + FE_j + \epsilon_{ijt}$$

$$+EE_j \cdot (\tilde{\mu}(t) + \tilde{\alpha}^{EE} U_t) + new_{ijt} EE_j \cdot (\tilde{\mu}(t) + \tilde{\alpha}^{new \cdot EE} U_t)$$

, where EE_j is a dummy variable corresponding to a particular job bracket. The coefficients of interests are α and $\tilde{\alpha}^{EE}$. Used as a wage cyclicality measure, α measures the semi-elasticity of real wages with respect to the unemployment rate for incumbents in the jobs

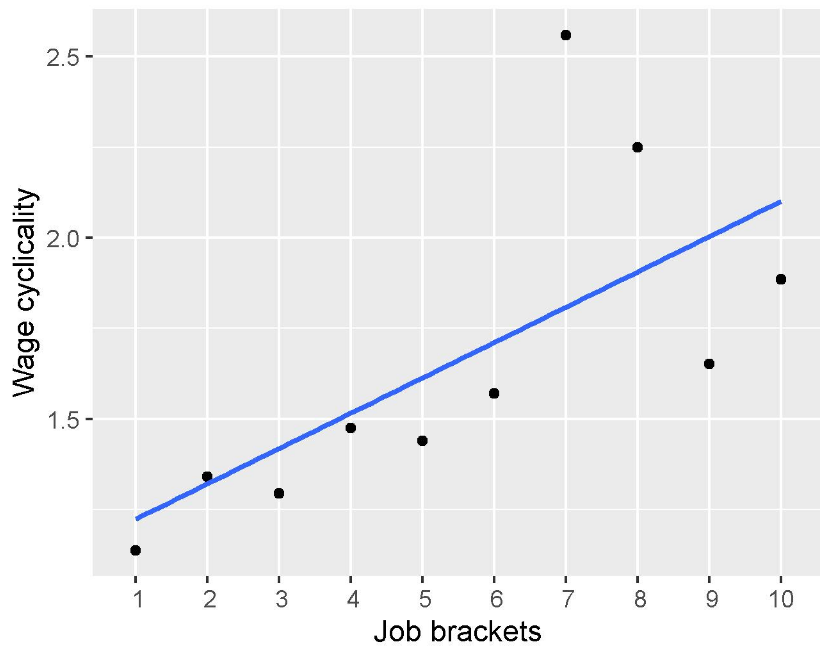

Figure 1: (Absolute) Wage cyclicality across jobs

hiring the lowest share of job-to-job transitioners. The coefficient vector $\tilde{\alpha}^{EE}$ then measures the differentials in semi-elasticity of wages of incumbents between jobs of the lowest bracket and the others. For example, in the case of 3 brackets, $\tilde{\alpha}^{EE}$ would be 2-dimensional, with the first dimension corresponding to cyclicality differences between the lowest and the middle bracket, and the second dimension measuring the differential between the lowest and the highest bracket.

2.4 Main result and robustness

Running the regression reveals strong cyclicality differences across jobs. Figure 1 plots absolute values of cyclicality coefficients for incumbent workers for each job ($\alpha + \tilde{\alpha}^{EE}$ for each job bracket past the first one), with the cyclicality values ranging from 1.3 in bracket 1 to above 2.5 in bracket 7. Overall the pattern, though nonmonotonic, clearly suggests that wages are more cyclical in jobs that higher fewer unemployed workers. The only question now is whether this result is susceptible to procyclical match upgrading bias.

Procyclical match upgrading bias comes from a hypothesis that, not only do workers find better jobs during booms, but they also find better matches, resulting in higher productivity. This brings about a wage increase that may appear even in jobs where wages are completely unresponsive to aggregate conditions, thus leading econometricians to mischaracterize the wages as cyclical. Since this bias relates not just to the job, but to the match upgrading, con-

Figure 2: Wage cyclicality comparison for long-term incumbents

trolling for the distribution of jobs is unlikely to help. This bias affects the wages for workers across jobs, rather than within jobs, thus primarily relating to the new hires. Moreover, as Gertler, Huckfeldt, and Trigari (2020) suggest, the bias works mainly through job-to-job transitioners who are more likely to upgrade their matches in booms than in recessions.

There are two points to consider in regard to this bias: the overall effect of it and the differential effect on job-to-job transitioners. Overall, both incumbent and new hire wage cyclicalities are likely over-estimated. Incumbent estimations are naturally less affected by the bias, but it may still be there nonetheless: if workers upgrade their matches during the boom in 2017, in 2018 they will be considered incumbents with higher wages. One potential way to deal with that would be to consider long-term incumbents, workers who have been at their jobs for at least two years. Figure 2 shows cyclicality for two versions of this regression (for incumbents that stayed for at least 2 years and at least 3 years). There are negligible differences between the average wage cyclicalities of the standard regression and that for >=2 years incumbents, and some differences, but going the opposite direction, for >=3 years incumbents. For the latter, since the cyclicality for >=3 years incumbents seems to actually be higher at points, and also significantly more non-monotone, likely implying issues with the sample size.

As for the differential effect, though the key comparison is across jobs, rather than workers, the two variables (job poaching workers and worker being a job-to-job transitioner) are correlated, so, even at the incumbent level, we may see a higher number of job-to-job transitioners in higher brackets leading to an upward bias. For that, first note that the differences in cyclicalities of >= 2 years incumbents across different brackets are similar to the differences in cyclicalities in the main regression, implying at most a minor effect of the bias on the differences in cyclicalities across job brackets. Secondly, for the case of >= 3 years incumbents, the largest differences in cyclicality occur in job brackets 3-6, where the number of job-to-job transitioners is not as high as in the higher brackets, moreover, the cyclicality is higher than the baseline rather than lower, again providing no evidence of the effect of procyclical match upgrading on my empirical result. In Appendix A I go into procyclical match upgrading in more detail, replicating the main empirical result of GHT and showing that the job-side heterogeneity does not absorb any of the procyclical match upgrading bias when it is introduced into their regression.

3 Model

I consider a labor search model with a new notion of search - separation of search: workers choose a location to search in, with each location having its own wage determination mechanism. Unlike random search, different workers may search in different locations, and, unlike directed search, wages in these locations are not tied to the location itself. It offers the same benefits as directed search in allowing workers in different positions to search in different submarkets, while also letting wages in different locations be chosen differently. Both of these conditions are necessary for my paper, as I am interested in the heterogeneous effects of wage rigidity across different steps of the job ladder.

3.1 Physical Environment

The economy is populated by a continuum of workers with measure 1 and a continuum of firms with positive measure. Each worker is endowed with an indivisible unit of labor and maximizes the expected sum of periodical consumption discounted at the factor $\beta \in (0, 1)$. Each firm operates a technology that turns one unit of labor into y units of output, common to all firms. Output values y lie in a set $Y = \{y_1, y_2, ..., y_{N(y)}\}$, where $N(y) \geq 2$ is an integer. Each firm maximizes the expected sum of profits discounted at the rate β.

Time is discrete and continues forever. At the beginning of each period, the state of the economy can be summarized by the tuple $\psi = (y, u)$, where y denotes aggregate productivity and $u \in [0, 1]$ denotes the proportion of unemployed workers.

Each period is divided into four stages: separation, search, matching, and production. At the separation stage, match is destroyed with probability $\delta \in (0, 1)$.

At the search stage, workers and firms search for matches across different locations. Specifically, a firm chooses how many vacancies to open in each location, and a worker chooses which location to visit if she has an opportunity to search. The cost of maintaining a vacancy for one period is $k > 0$. The worker has the opportunity to search with a probability that depends on her employment status. If the worker was unemployed at the beginning of the period, she can search with probability $\lambda_u(m) : M \to [0, 1]$, where M, $||M|| \geq 2$, is the set of locations workers can search in. If the worker was employed at the beginning of the period and did not lose her job during the separation stage, she can search with probability $\lambda_e(m) : M \to [0, 1]$. Finally, if the worker lost her job during the separation stage, she cannot search.

At the matching stage, the workers and the vacancies that are searching in the same location are brought into contact by a meeting technology with constant returns to scale that can be described in terms of the vacancy-to-worker ratio θ (i.e., the tightness). Specifically, the probability that a worker meets a vacancy is $p(\theta)$, where $p : R_+ \to [0, 1]$ is a twice continuously differentiable, strictly increasing, and strictly concave function that satisfies the boundary conditions $p(0) = 0$ and $p(\infty) = 1$.

At the production stage, an unemployed worker produces $b > 0$ units of output. A worker employed in a match produces y units of output. At the end of this stage, nature draws the next period's aggregate component of productivity, $\hat{y}$, from the probability distribution $\phi(\hat{y}|y), \phi : Y \times Y \to [0, 1]$.

3.2 Labor Market

The market consists of M submarkets. They take interpretation of job ladder rungs, and are ex-ante heterogeneous only in their wages. Within each submarket, matches are either new hires or incumbents. A match is a new hire in period t, if this is its first period of existence. Otherwise, the match is an incumbent. I take a reduced-form approach to wage determination:

$$w^k_{m,t} = \bar{w}_m \cdot y_t^{1-\alpha^k_m}$$

, where $k \in \{new, inc\}$, and $\bar{w}_m$ is the wage level at submarket k. Without loss of generality I assume that $\bar{w}_m$ increases with m, thus jobs higher up the ladder pay more. Rigidity in wages is modeled as degree of comovement between wages and aggregate productivity, $\alpha \in [0, 1]$ serves as a measure of wage rigidity.The new hire and incumbent wages follow the same pattern except that I allow for the difference in rigidity: $\alpha^{new}_m \neq \alpha^{inc}_m$. Wages being exactly the same otherwise means that the worker earns on average the same wage no matter her status. The difference in rigidity between the two jobs reflects both the difference in empirical findings and the difference in theoretical incentives: new hire wages are rigid for pay equity reasons, while incumbent wages are rigid due to workers' loss aversion (Bewley 1999).

This specification allows submarkets to be different in levels of both wage flexibility and worker bargaining power, and new hire and incumbent wages to be different in terms of wage flexibility. The exact formula for the determination mechanism is not crucial, as long as it exhibits comovement between wages and aggregate productivity, the comovement is dampened in more rigid submarkets, and the comovement is separate from the wage level. For example, Nash Bargaining wages with submarkets being heterogeneous only in their bargaining power would not work, since, though wages comove with productivity more under higher bargaining power, bargaining power also affects the average wages, and thus on its own the bargaining power has no effect on volatility of unemployment (Hagedorn and Manovskii 2008). Instead, if we also introduce a degree of wage rigidity into Nash Bargaining mechanism (say, wages are a convex combination of bargained wages and some fixed value, with rigidity parameter determining the weights), all the conditions are satisfied, and wage rigidity actually has an impact on unemployment volatility. I stick to the reduced-form wages as they most clearly show what kind of mechanism I am after.
I introduce the wage vector w_m^k alongside the distribution of employed labor $g : M \to [0,1]$ into an aggregate description of the economy $\psi(y,u,g,w)$. I denote double (u,g) as the labor distribution of the economy. The vacancy-to-worker ratio of submarket m is denoted as $\theta(m,\psi)$. In equilibrium, $\theta(m,\psi)$ will be consistent with the firms' and workers' search decisions.

At the separation stage, an employed worker moves into unemployment with probability $\delta \in (0,1)$. At the search stage, each firm chooses how many vacancies to create and in which submarkets to locate them. On the other side of the market, each worker who has the opportunity to search chooses which submarket to visit. At the matching stage, each worker searching in submarket m meets a vacancy with probability $p(\theta(m,\psi))$. Similarly, each vacancy located in submarket m meets a worker with probability $q((m,\psi))$. At the production stage, an unemployed worker produces b units of output, and an employed worker produces y units of output.

3.3 Worker and Firm problems

Consider an unemployed worker at the beginning of the production stage and denote $V_u(\psi)$ as her lifetime utility. In the current period, she produces and consumes b. In the next period, she may be able to search with probability λ_u and match with a firm in a submarket m with probability $p(\theta(m,\psi))$. If the worker indeed matches, her continuation utility is $V_e(\psi,m) - V_{ef}(\psi,m)$, where V_e, V_{ef}^{new} are the total value of a match and the firm's share of the value of a new match (both specific to submarket m), respectively. Thus,

$$V_u(\psi) = b + \beta E_{\hat{\psi}|\psi} \max_m \Big[V_u(\hat{\psi}) + \lambda_u(m) D(m, V_u(\hat{\psi}), \hat{\psi})\Big], \tag{1}$$

where D represents the (net) value of search in the submarket m:

$$D(m, V_u(\hat{\psi}), \hat{\psi}) = p(\theta(m, \hat{\psi}))(V_e(\hat{\psi}, m) - V_{ef}^{new}(\hat{\psi}, m) - V_u(\hat{\psi})) \tag{2}$$

Now, consider a worker-firm match at the beginning of the production stage. In the current period sum of worker's utility and firm's profit is the productivity of the match y. Next period, pair may separate with probability d, or worker match with another firm (if not fired) in submarket m with probability $\lambda_e(m) \cdot p(\theta(m, \hat{\psi}))$, and in both cases firm's continuation profit is zero. With probability $(1-d)(1-\lambda_e(m) \cdot p(\theta(m, \hat{\psi}))$ worker and firm stay together until the next production stage. Thus, the value of the match is

$$V_e(\psi, m) = y + \beta E_{\hat{\psi}|\psi} \Big[d V_u(\hat{\psi}) + (1-d)[V_e(\hat{\psi}, m) + \lambda_e(m^*) D(m^*, V_e(\hat{\psi}), \hat{\psi})]\Big], \tag{3}$$

where

$$m^* = arg \max_{m'} E_{\hat{\psi}|\psi} \Big[V_e(\hat{\psi}) - V_{ef}^{inc}(\hat{\psi}) + \lambda_e(m') D(m', V_e(\hat{\psi}, m) - V_{ef}^{inc}(\hat{\psi}, m), \hat{\psi})\Big] \tag{4}$$

is the submarket, where worker currently employed in submarket m chooses to search, maximizing own expected utility.

Firm's share of the value depends on whether the match is new or incumbent ($k = \{new, inc\}$):

$$V_{ef}^k(\psi, m) = y - w_m^k + \beta E_{\hat{\psi}|\psi} \Big[(1-d)(1-\lambda_e(m^*) p(\theta(m^*, \hat{\psi}))) \cdot V_{ef}^{inc}(\hat{\psi}, m)\Big] \tag{5}$$

At the search stage, a firm chooses how many vacancies to create and where to locate them. The firm's cost of creating a vacancy in submarket m is k. The firm's benefit from creating a vacancy in submarket m is

$$q(\theta(m, \psi))[\tilde{V}_{ef}^{new}(\psi, m)] \tag{6}$$

Where $\tilde{V}_{ef}(\psi, m) = \tilde{p}(m, \psi) \cdot V_{ef}^{new}(\psi, m)$ is the benefit of finding a worker, which is equal to (probability worker accepts the job offer)·(firm's share of a match value)[1] When the cost of the vacancy is strictly larger than the benefit, the firm creates no vacancies. Vice-versa, when the cost is strictly smaller, infinitely many vacancies are created. And when the cost and the benefit are equal, the firm's profit is independent of the number of created vacancies in submarket m.

[1]Note that I calibrate the model so that worker always accepts the job.

In any market visited by a positive number of workers, the tightness is consistent with the firm's incentives to create vacancies if and only if

$$k \geq q(\theta(m,\psi))[\tilde{V}_{ef}^{new}(\psi,m)] \tag{7}$$

and $\theta(i,\psi) \geq 0$ with complementary slackness.
Since $\eta(m)$ increases with m, V_{ef}^{new} decreases with m, and thus workers face a trade-off when choosing a submarket: it is easier to find a job in lower submarkets, while higher submarkets pay more. This trade-off is what ultimately leads workers in different positions to search in different locations.

4 Quantitative model

I calibrate and simulate the model in order to assess the importance of introducing heterogeneous wage rigidity on unemployment volatility. More precisely, I intend to capture the search separation and wage rigidity properties of different jobs found in the data and see just how much is gained by introducing the heterogeneity in wage rigidity into the model. I start by explaining the basics of the simulation, along with the extra ingredients specific to the quantitative version. I then explain the calibration procedure and, lastly, do several comparative statics exercises: compare the importance of wage rigidity across the different submarkets, and compare the volatility of unemployment of the model with heterogeneous wage rigidity to the one with the same level of rigidity across all submarkets.

4.1 From analytical to quantitative

To mirror my empirical results, I focus on the case of just two submarkets, one primarily occupied by the unemployed and the other by the employed workers. Connecting to the empirical section, the submarkets should be interpreted as the hiring pool brackets $EE_j = 0, EE_j = 1$ from the Section 2. Unlike in the theoretical model, in a real economy unemployed and employed workers do not search completely separately: some unemployed workers search for the highest paying jobs, and employed workers do often move horizontally. To account for that, I introduce taste shocks into the workers' search preferences. Essentially, each worker's net value of search $D(m, V_u(\hat{\psi}), \hat{\psi}) = p(\theta(m,\hat{\psi}))(V_e(\hat{\psi},m) - V_{ef}^{new}(\hat{\psi},m) - V_u(\hat{\psi})) + \epsilon_{imt}$ now has the shock variable ϵ_{imt} augmenting it. That way, even if on average unemployed workers prefer the lowest submarket, some unemployed workers will still be incentivized to search in better submarkets. I let the taste shock ϵ_{imt} be extreme value distributed, with location

0, scale 1 and shape 0, iid across workers, submarkets, and time. That way, the probability that, say, an unemployed worker decides to search in submarket m' given the aggregate conditions $\hat{\psi}$ is equal to $\frac{V_u(\hat{\psi})+\lambda_u(m')D(m',V_u(\hat{\psi}),\hat{\psi})}{\sum_{m\in M} V_u(\hat{\psi})+\lambda_u(m)D(m,V_u(\hat{\psi}),\hat{\psi})}$. Since we have a continuous measure of workers, this personal probability translates into the ratio of unemployed workers searching in the submarket m.

Since the model is Block Recursive, it is easily numerically solvable as both the value functions and the allocations do not depend on the distribution of labor. Once solved, I simulate the stochastic economy version of the model, with aggregate productivity as the only exogenously time changing variable, for 10068 periods and drop the first 10000 from the main analysis. The number of the remaining periods has been chosen to match the number of quarters in the data (68 from 2003 to 2019). I track unemployment over the simulation and use the ratio of the standard deviation of unemployment to the standard deviation of productivity (both log HP-filtered with the parameter 10^5) as the key statistic of the simulation.

4.2 Calibration

I calibrate the model to the quarterly frequency and take the standard literature values for the time discounting rate β and the flow utility of the unemployed workers b. I calibrate the cost of posting a vacancy κ to match the average French unemployment over the years 2003-2019.

β	Time discounting	Standard quarterly value	0.988
b	Flow utility of unemployment	Standard lower bound	0.4
y_ρ	Productivity persistence	Taken from M-R	0.938
y_σ	Volatility of productivity	Taken from M-R	0.02
δ	Separation rate	Taken from M-R	0.023
ϕ	Matching function constant	Taken from M-R	1.268
γ	Matching function elasticity	Taken from M-R	0.5
$\bar{w}$	Wage level	Ratio from the data	[0.65,0.95]
$\bar{y}_m$	Submarket productivity	Set profitability to 0.05	[0.7,1.0]
λ	Search efficiency	Match distribution of UE	
α	Wage rigidity	Match regression estimates	[0.79,0.65]
κ	Cost of posting a vacancy	Match average unemployment	3.2

In many ways, this model is still comparable to the standard DMP model, in that each particular submarket does employ random search with the classic free-entry condition. This results in many of the parameters of the model being comparable to the parameters of the standard DMP models. I take the productivity y low of motion, the job-finding probability function $p(\theta) = \phi\theta^{\gamma}$, and the separation rate δ from Mughir-Robin (2018), who used the OECD data applied to France to estimate these parameters.

Past this, there are several significant departures from the literature that my model takes, with no parameters easily comparable to the literature. Summarizing these departures:

1. Several submarkets: need wage-related parameters for each submarket

2. Mixed search: need to account for the separation of search properties of the data

3. Wage rigidity: need to match the rigidity estimates of Section 2

Several submarkets: it is necessary to establish both the general wage level $\bar{w}_1$ and the difference between the wage levels $\bar{w}_2/\bar{w}_1$. These values matter in that they define the relative (time-independent) profitability of the job to the firm, compared to the worker and also across submarkets. The heterogeneous profitability across submarkets would imply heterogeneous sensitivity of the firms themselves to wage rigidity. As I have no measure of how differently firms across the job ladder benefit from their workers, I shut down this channel and assume homogeneous profitability. For that, to keep heterogeneity in wages, I introduce submarket-specific productivity scale $\bar{y}_m$. Thus, the productivity of a job at time t in submarket m is $\bar{y}_m y_t$. I set the profitability $\bar{y}_m - \bar{w}_m = 0.05$, which is the conservative upper bound: Hall 2005 has 0.035, Shimer 2005 has ≈ 0.02, Hagedorn and Manovskii 2008 have it even lower. I normalize $\bar{y}_m$ to 1 and set $\bar{w}_2/\bar{w}_1$ to match the ratio of average wages across the two brackets of jobs. Wages in the jobs mostly hiring employed workers are about 50% higher than in the jobs hiring unemployed workers, so I set $\bar{w}_2/\bar{w}_1 \approx 1.5$. This results in submarket-specific productivity levels being equal to (0.7,1.0) and wage levels equal to (0.65,0.95).

Mixed search: I adjust the probability to search matrices $\lambda_u(m'), \lambda_e(m, m')$ to match the distribution of out-of-unemployment workers observed in the data. In particular, I target the proportion of UE new hires in each submarket as well as the ratio of the total number of unemployed new hires between the two submarkets. Since this is just 3 moments while the two matrices provide 6 degrees of freedom, I normalize $\lambda_u(1)$ to 1 and make a few additional restricting assumptions:

- Workers can only search in their own submarket or above: $\lambda_e(2, 1) = 0$

- Job-to-job transitioners search less: $\lambda_{EE}(= \lambda(1, 1) = \lambda(1, 2) = \lambda(2, 2)) < 1$

- Searching higher than 1 step above the current position is penalized: $\lambda_u(2) = \lambda_{jump} < 1$

$$\lambda = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} U \\ 1 \\ 2 \end{array} & \begin{pmatrix} 1.0 & \lambda_{jump} \\ \lambda_{EE} & \lambda_{EE} \\ 0.0 & \lambda_{EE} \end{pmatrix} \end{array} = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} U \\ 1 \\ 2 \end{array} & \begin{pmatrix} 1.0 & 0.32 \\ 0.08 & 0.08 \\ 0.0 & 0.08 \end{pmatrix} \end{array}$$

Wage rigidity: the only remaining parameters are the rigidity parameters α_m, determining the wage rigidity in each of the submarkets. I calibrate these parameters to match the estimates from Table 2. More precisely, in the simulation, I track wages in each submarket across time, aggregate them to the annual frequency, and regress log wages on unemployment. That is, for each submarket, I run the following regression:

$$ln(w^a_{m,t} + 1) = \beta_m u^a_t + \epsilon,$$

with the 1 being added to the logarithm in order to keep the percentage change interpretation. I calibrate the theoretical rigidity parameters α_m so that these regression estimates $\hat{\beta}_m, m \in M$ match the empirical estimates of Table 2.

4.3 Results

Under this calibration, the model achieves the ratio $std(u)/std(y)$ of 1.43, notably below 4.5 in the data, which may be explained by the conservatively high job profitability. I focus on two comparisons: the relative importance of wage rigidity between the two submarkets, and the change in wage rigidity from using heterogeneous wage rigidity estimates rather than homogeneous ones (from Table 1).

Relative importance of submarkets

I take the standard calibration and increase the wage rigidity parameter α by 0.05 in one of the submarkets. I then compare the increase in volatility of unemployment achieved by this change between the cases of increasing rigidity in the 1st submarket and in the 2nd. I find that rigidity in the entry-level submarket has a 3 times larger effect on the volatility of unemployment than rigidity in the 2nd submarket. Therefore, I confirm the original intuition that it is the wage rigidity of the jobs that actually do hire unemployed workers that matters the most for the unemployment volatility.

Value of heterogeneous wage rigidity

I recalibrate the model using the homogeneous wage rigidity estimates by applying the classic regression used commonly in the literature. Since my new hire increment is both large and insignificant, I consider two cases: with or without accounting for the increment. Ignoring the increment, the calibration gives the ratio of 1.25, while with it the ratio is 1.06. Thus, moving from the homogeneous wage rigidity calibration to the heterogeneous one increases the volatility of unemployment by $14\% - 34\%$. These percentage changes persist even for the lower profitability values, where the unemployment volatility of the homogeneous model is notably larger. Thus, taking into account the heterogeneity of wage rigidity across different jobs significantly increases the effect of rigidity on the volatility of unemployment.

5 Conclusion

I estimate wage rigidity separately for different jobs, based on their hiring pools. I use the matched employer-employee data to find that wages, both for new and incumbent workers, are twice as rigid in jobs hiring from unemployment than in jobs poaching workers. I then simulate a model incorporating the separation of search and heterogeneous wage rigidity across different submarkets to find that wage rigidity in the entry-level submarket matters significantly more for the volatility of unemployment than in the higher submarkets. Together, the empirical and the theoretical results imply that, for unemployment volatility purposes, wages are highly rigid. I confirm this by comparing the heterogeneous wage rigidity calibration of the model to the homogeneous one and find that accounting for the heterogeneity across jobs increases the unemployment volatility by $14\% - 34\%$.

Potential interesting application of the empirical result could be explaining some of the business cycle-related job ladder facts. As one example, Moscarini and Postel-Vinay (2018) find that the job-to-job transition rate is procyclical, and especially so at the bottom of the job ladder. This might be because wages at the bottom of the ladder are too high during recessions, thus disincentivizing workers from transitioning to a different job. It would also be interesting to understand where the observed heterogeneity in wage rigidity stems from.

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A Data appendix

A.1 Sample selection

I restrict attention to years 2002-2019 for tracking workers' histories due to the sample size of workers being doubled in 2002. Once each worker's history is accounted for (new hires and job-to-job transitioners are identified), I drop the year 2002. There are no restrictions on workers besides their age: for both job ratio calculations and further regressions, only workers aged 25-55 are selected. On the job side, I restrict attention to firms located in Ile-de-France, in the private sector, and full-time jobs only. For each job I calculate the number of new hires that were successfully identified as either previously unemployed or job-to-job transitioners, and, if there are fewer than 10 of those for a given job, I do not calculate the ratio of job-to-job transitioners for that job. Once each appropriate job has its ratio of job-to-job transitioners determined, I pool jobs into brackets based on the number of observations in each bracket. For example, for the case of just two brackets, I pick the ratio of job-to-job transitioners such that half of the observations have jobs with fewer job-to-job transitioners and half the observations have jobs with more. Once each job is assigned to a bracket, I add additional restrictions on observations for all the regressions: wages have to be above national minimum wage for that year and below 1000000, working days must be above 180, and working hours above 600. This way, the wage level regression accounts only for prolonged job spells with appropriately reported wages.

A.2 Classic results

	Total cyclicality	New hire	No interaction	Occupation time trend
U	-1.65^{***}	-1.61^{***}	-1.59^{***}	-1.53^{***}
	(0.22)	(0.20)	(0.19)	(0.18)
$U \cdot new_{ijt}$		-0.41	-0.55	-0.60
		(0.42)	(0.43)	(0.40)
$t \cdot new_{ijt}$		-96.15		
		(59.51)		
$t^2 \cdot new_{ijt}$		0.02		
		(0.01)		
Num. obs.	1498096	1484887	1484887	1484887
R^2 (full model)	0.93	0.94	0.94	0.94
R^2 (proj model)	0.15	0.19	0.19	0.20
Adj. R^2 (full model)	0.91	0.91	0.91	0.91
Adj. R^2 (proj model)	-0.15	-0.10	-0.10	-0.08

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 2: Baseline regressions with different time trends

A.3 Main result

	2 brackets	3 brackets	5 brackets	10 brackets
U	-1.34^{***}	-1.23^{***}	-1.25^{***}	-1.14^{***}
	(0.23)	(0.25)	(0.25)	(0.30)
$U \cdot new_{ijt}$	-0.49	-0.38	-0.27	-0.05
	(0.49)	(0.68)	(0.85)	(1.06)
$U \cdot (EE_j = 2)$	-0.64^{**}	-0.69^{***}	-0.13	-0.20
	(0.25)	(0.19)	(0.22)	(0.28)
$U \cdot (EE_j = 3)$		-0.67^{**}	-0.27^{**}	-0.16
		(0.34)	(0.13)	(0.30)
$U \cdot (EE_j = 4)$			-1.14^{***}	-0.34
			(0.23)	(0.30)
$U \cdot (EE_j = 5)$			-0.50	-0.30
			(0.40)	(0.22)
$U \cdot (EE_j = 6)$				-0.43^{**}
				(0.19)
$U \cdot (EE_j = 7)$				-1.42^{***}
				(0.36)
$U \cdot (EE_j = 8)$				-1.11^{***}
				(0.18)
$U \cdot (EE_j = 9)$				-0.51
				(0.36)
$U \cdot (EE_j = 10)$				-0.75^{*}
				(0.41)
$U \cdot new_{ijt} \cdot (EE_j = 2)$	0.04	-0.07	-0.10	-0.54
	(0.96)	(1.06)	(0.88)	(0.74)
$U \cdot new_{ijt} \cdot (EE_j = 3)$		-0.05	-0.45	-1.03
		(1.25)	(1.15)	(0.95)
$U \cdot new_{ijt} \cdot (EE_j = 4)$			-0.84	0.51
			(1.30)	(1.50)
$U \cdot new_{ijt} \cdot (EE_j = 5)$			0.43	-0.98
			(1.60)	(1.11)
$U \cdot new_{ijt} \cdot (EE_j = 6)$				-0.48
				(1.58)
$U \cdot new_{ijt} \cdot (EE_j = 7)$				0.23
				(1.31)
$U \cdot new_{ijt} \cdot (EE_j = 8)$				-2.18
				(1.90)
$U \cdot new_{ijt} \cdot (EE_j = 9)$				-0.55
				(1.67)
$U \cdot new_{ijt} \cdot (EE_j = 10)$				0.93
				(2.02)
Num. obs.	511026	511026	511026	511026
R^2 (full model)	0.93	0.93	0.94	0.94
R^2 (proj model)	0.22	0.22	0.22	0.22
Adj. R^2 (full model)	0.92	0.92	0.92	0.92
Adj. R^2 (proj model)	0.01	0.01	0.01	0.01

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 3: Main regression: all the bracket distinctions

	2 brackets	3 brackets	5 brackets	10 brackets
U	-1.35^{***}	-1.20^{***}	-1.17^{***}	-1.01^{***}
	(0.27)	(0.28)	(0.28)	(0.32)
$U \cdot (EE_j = 2)$	-0.64^{**}	-0.81^{***}	-0.28	-0.29
	(0.27)	(0.23)	(0.27)	(0.28)
$U \cdot (EE_j = 3)$		-0.70^{*}	-0.47^{***}	-0.33
		(0.38)	(0.16)	(0.38)
$U \cdot (EE_j = 4)$			-1.22^{***}	-0.56^{*}
			(0.24)	(0.33)
$U \cdot (EE_j = 5)$			-0.53	-0.48^{*}
			(0.46)	(0.27)
$U \cdot (EE_j = 6)$				-0.73^{***}
				(0.22)
$U \cdot (EE_j = 7)$				-1.42^{***}
				(0.38)
$U \cdot (EE_j = 8)$				-1.34^{***}
				(0.18)
$U \cdot (EE_j = 9)$				-0.71^{*}
				(0.41)
$U \cdot (EE_j = 10)$				-0.69
				(0.46)
Num. obs.	441694	441694	441694	441694
R^2 (full model)	0.94	0.94	0.94	0.94
R^2 (proj model)	0.20	0.20	0.20	0.20
Adj. R^2 (full model)	0.92	0.92	0.92	0.92
Adj. R^2 (proj model)	-0.02	-0.02	-0.02	-0.02

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 4: Focus on long incumbents (observed in the same firm 2 years ago)

A.4 Replicating Gertler, Huckfeldt, and Trigari 2020

I rerun the fixed effect version of the Gertler et al regression in the first two columns, first their baseline version, showing significant difference in cyclicality between wages of new hires and incumbents, and then their main regression, showing that these differences are contained entirely within the excess cyclicality of newly hired job-to-job transitioners, which GHT argue to be caused by procyclical match upgrading bias.

I add my dummy for job brackets (here it is only two brackets, thus just one dummy) into this regression and find that wage cyclicality is still heterogeneous across different jobs and, notably, that the coefficient for newly hired job-to-job transitioners is unchanged in size, but now only shows up in interaction with the higher job bracket, implying that procyclical match upgrading is only an issue higher up on the job ladder, but also that this is not an issue for the estimation of heterogeneous wage cyclicality of incumbents.

	Baseline	New hire distinction	Job distinction	+Job FE
U	-1.83^{***}	-2.49^{***}	-1.80^{***}	-1.64^{***}
	(0.03)	(0.04)	(0.10)	(0.09)
$U \cdot new_{ijt}$	-0.88^{***}			
	(0.10)			
$U \cdot new_{ijt} \cdot EE_{ijt}$		-0.74^{***}	0.02	0.10
		(0.12)	(0.31)	(0.30)
$U \cdot new_{ijt} \cdot (1 - EE_{ijt})$		-0.11	-0.11	-0.00
		(0.18)	(0.37)	(0.35)
$U \cdot EE_j$			-0.67^{***}	-0.73^{***}
			(0.13)	(0.12)
$U \cdot new_{ijt} \cdot (1 - EE_{ijt}) \cdot EE_j$			-0.13	-0.04
			(0.66)	(0.62)
$U \cdot new_{ijt} \cdot EE_{ijt} \cdot EE_j$			-0.72^{*}	-0.28
			(0.42)	(0.40)
Num. obs.	1692795	921477	351502	351502
R^2 (full model)	0.90	0.90	0.93	0.94
R^2 (proj model)	0.23	0.26	0.29	0.24
Adj. R^2 (full model)	0.88	0.87	0.90	0.91
Adj. R^2 (proj model)	0.09	0.08	0.06	-0.04

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 5: Replicating GHS. Standard errors clustered at individual level

U	-2.08^{***}
	(0.27)
$t \cdot new_{ijt}$	-161.31^{***}
	(56.71)
$t^2 \cdot new_{ijt}$	0.04^{***}
	(0.01)
$U \cdot new_{ijt}$	0.35
	(0.33)
$U \cdot EE_{ijt}$	0.11
	(0.20)
$t \cdot new_{ijt} \cdot EE_{ijt}$	57.52
	(53.27)
$t^2 \cdot new_{ijt} \cdot EE_{ijt}$	-0.01
	(0.01)
$U \cdot new_{ijt} \cdot EE_{ijt}$	-0.69
	(0.43)
Num. obs.	825344
R^2 (full model)	0.94
R^2 (proj model)	0.21
Adj. R^2 (full model)	0.91
Adj. R^2 (proj model)	-0.17

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 6: Wage cyclicality across workers, rather than jobs

B Model appendix

B.1 Block Recursivity

In this section, I define Block Recursive equilibrium and prove that every equilibrium of this model is Block Recursive. Intuitively, equilibrium is considered Block Recursive if none of the value and policy functions depend on the distribution of labor. Thus, if all the equilibria are Block Recursive, as I prove below for the case of equal new hire and incumbent wages, the model can be solved outside of the steady-state. The proof extends to the case of unequal new hire and incumbent wages if workers still always search with an intent to accept the job. This can be achieved either by making sure that new hire wages are always higher than incumbent, or by allowing workers uninterested in searching to not search.

Definition 1. *A Block Recursive Equilibrium (BRE) consists of the market tightness function $\theta : M \times W \times Y \to R_+$,value function of the unemployed worker $V_u : W \times Y \to R$, policy function for the unemployed worker $m_u : W \times Y \to M$, a joint value function for a firm-worker match $V_e : M \times Y \to R$, policy function for the worker-firm match $m^* : M \times W \times Y \to M$, and firm's share of the match value $V_{ef} : M \times W \times Y \to R$. These functions satisfy the following conditions: (i) $\theta(m, w, y)$ satisfies condition (7) for all $(m, w, y) \in M \times W \times Y$; (ii) $V_u(w, y)$ satisfies (1) for all $(w, y) \in W \times Y$ and $m_u(w, y)$ is the associated policy function; (iii) $V_e(m, w, y)$ satisfies (3) for all $(m, w, y) \in M \times W \times Y$, and $m^*(m, w, y)$ and $d(m, w, y)$ are the associated policy functions; (iv) $V_{ef}(m, w, y)$ satisfies condition (5) for all $(m, w, y) \in M \times W \times Y$.*

Proposition 1. *Let new hire wages be equal to incumbent wages. Then all equilibria are Block Recursive*

Proof. Let $\alpha_m^{new} = \alpha_m^{inc}\ \forall m$ so that $V_{ef}^{new} = V_{ef}^{inc}$. Let $(\theta, V_u, V_e, V_{ef}, m_u, m^*)$ be an equilibrium. I take 4 steps to show that it is Block Recursive.

1. Show that workers searching optimally are at worst indifferent between the new job offer and the current position and,hence, may always search with an intent to accept the new job offer. By definition of m^*, $E\{V_e(\hat{\psi}, m^*) - V_{ef}(\hat{\psi}, m^*) + \lambda_e(m^*)D(m^*, V_e(\hat{\psi}, m^*) - V_{ef}(\hat{\psi}, m^*), \hat{\psi})\} \geq$
   $E\{V_e(\hat{\psi}, m') - V_{ef}(\hat{\psi}, m') + \lambda_e(m')D(m', V_e(\hat{\psi}, m') - V_{ef}(\hat{\psi}, m'), \hat{\psi})\}$
   $\forall m' \in M$
   Letting $m' = m$ we get that $E\{V_e(\hat{\psi}, m^*) - V_{ef}(\hat{\psi}, m^*) + \lambda_e(m^*)D(m^*, V_e(\hat{\psi}, m^*) - V_{ef}(\hat{\psi}, m^*), \hat{\psi})\} \geq 0$
   For unemployed workers, it is assumed that $\forall \psi \in \Psi, \exists m \in M s.t. V_e(m, \psi) - V_{ef}(m, \psi) \geq V_u(\psi)$, since otherwise Nash Bargaining formulation would break down (it would require either $y < b$ or $\eta < 0$).

   Thus, workers always accept new job offers and, hence, $\tilde{V}_{ef} = V_{ef}$.

2. Express number of a submarket m as function of market tightness θ and aggregate economy ψ using market clearing condition (7): $m = [q(\theta) * V_{ef}(\psi)]^{-1}/k$. If m is not an integer, set m=0, and $\lambda_u(0) = \lambda_e(0) = 0$

3. Now worker's search problem can be expressed not as choosing m, but as choosing θ. This allows us to reexpress value functions (substitute m with $m(\theta, \psi)$ and $\theta(m, \psi)$ with θ). Thus, employed workers choose

$\theta^*(\theta) = argmax_{\theta'} E\{V_e(\hat{\psi}) - V_{ef}(\hat{\psi}) + \lambda_e(m(\theta,\hat{\psi}))p(\theta')(V_e(\hat{\psi}, m(\theta',\hat{\psi})) - V_{ef}(\hat{\psi}, m(\theta',\hat{\psi})) - (V_e(\hat{\psi}, m(\theta,\hat{\psi})) - V_{ef}(\hat{\psi}, m(\theta',\hat{\psi})))\}$

Then

$V_{ef}(\theta,\psi) = y - w_m + \beta E\{(1-d)(1-\lambda_e(m(\theta^*(\theta),\hat{\psi}))p(\theta^*(\theta))) * V_{ef}(\hat{\psi},\theta)\}$

Once $m(\theta,\psi)$ is substituted into the equation, one can notice that, if $V_{ef}(\hat{\psi},\theta)$ is independent of (u,g), then $V_{ef}(\psi,\theta)$ is independent of labor distribution as well. Thus, the fixed point of the value function is independent of (u,g) and,hence,V_{ef} is indeed independent of the labor distribution and, given this result and using similar arguments, V_e and V_u become independent as well.

4. Finally, all the policy functions are independent of (u, g) as well.

   Market tightness θ is pinned down by condition (7) and, since V_{ef} is independent of labor distribution, θ is as well. Since both θ and all the value functions are independent of (u,g), all the other policy functions are independent, too, for the same reasons as above: if next period's policy functions are independent of (u,g), then current period's policy functions are independent as well.

□