3D problems of underwater sound propagation

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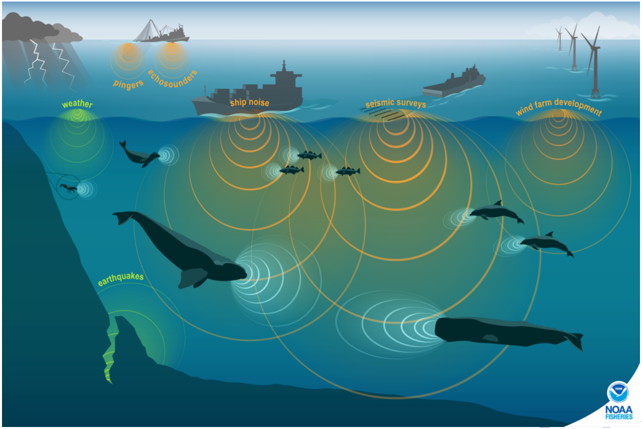
Sources of noise

Anthropogenic: shipping noise, seismic exploration, sonars, acoustic harassment devices, explosions, industrial activities and construction.
Natural sound: Wind, currents and turbulence, seismic activity, ice melting and marine life.



What animals are affected?

Sea hare or marine slug

Humpback Whale

How to measure the impact

Sound exposure level:

$$SEL(f_1, f_2, x, y, z) = 10 \log_{10} \left(\frac{\Delta t \int_{f_1}^{f_2} \hat{P}(\omega, x, y, z)^2 d\omega}{p_0^2 \Delta t_0} \right) .$$

Grey whales have "an assumed disturbance treshold" of $156 dB$ re $1 \mu Pa^2 \cdot s$ SEL

Gray whale

Sound in general

Frequency($f = 1/T$) or wavelength ($l = c/f$), speed (c), direction and amplitude or sound pressure ($p_{total} = p_{stat} + p$).

$$20 \cdot log_{10}(P/P_0) \text{ re } 1\mu Pa$$

Sounds and Noises	Average Sound Level[dB]
Normal breathing	10
Normal conversation	60
Washing machine, dishwasher	70
Motorcycle	95
Shouting or barking in the ear	110
Firecrackers	140–150

What equations?

Boundary value problem with the Helmholtz equation for the sound pressure $P = P(x, y, z)$:

$$P_{xx} + P_{yy} + P_{zz} + \frac{\omega^2}{c^2} P = -\delta(x, y, z - z_s)\,,$$

where $\omega = 2\pi f$ is angular frequency.
Boundary: at the ocean surface $P|_{z=0} = 0$, at the sea bottom $P_z|_{z=h} = 0\,.$
Radiation boundary condition at infinity.

What equations?

Separation of variables/Fourier method: $P = \mathcal{A}(x)\phi(z)$,
Horizontal refraction equation (HRE):

$$\mathcal{A}_{xx}\phi(z) + \mathcal{A}\phi_{zz} + \frac{\omega^2}{c^2}\mathcal{A}\phi = 0\,,$$

$$-\frac{\mathcal{A}_{xx}}{\mathcal{A}}(x) = \frac{\phi_{zz}}{\phi}(z) + \frac{\omega^2}{c^2}(z) = K^2\,.$$

$$\mathcal{A}_{xx} + K^2\mathcal{A} = 0, \quad \mathcal{A} = \mathrm{e}^{\pm\mathrm{i}Kx}\,,$$

$$\frac{d^2\phi_j}{dz^2} + \frac{\omega^2}{c^2}\phi_j = K_j^2\phi_j\,.$$

What equations?

ϕ is a solution of the following Sturm-Liouville problem:

$$\begin{cases} \dfrac{d^2\phi_j}{dz^2} + \dfrac{\omega^2}{c^2}\phi_j = K_j^2\phi_j\,, \quad z \in (0,h)\,, \\ \phi_j|_{z=0} = 0\,, \\ \phi_j|_{z=h(x,y)} = 0 \end{cases}$$

and K_j^2, $1 \le j \le M$ are the corresponding real-valued eigenvalues $\phi_j(z)$.

$$K_j(x,y) = \sqrt{\frac{\omega^2}{c^2} - \left(\frac{\pi j}{h(x,y)}\right)^2}\,,$$

$$\phi_j(z,x,y) = \sqrt{\frac{2}{h(x,y)}}\sin\left(\frac{\pi j z}{h(x,y)}\right)\,,$$

$\omega = 2\pi f$, $h(x,y)$ describes the bottom, $\{\phi\}_j$ form orthogonal basis.

What equations?

Pressure field:

$$P(x,y,z) = \sum_{j=1}^{M} \mathcal{A}_j \phi_j(z,x,y)\,.$$

$$\mathcal{A}_j xx + \mathcal{A}_j yy + K_j^2 \mathcal{A}_j = 0\,,$$

Fourier transformation:

$$\mathcal{A}_j = A_j \mathrm{e}^{\mathrm{i}K_j^0 x}\,,$$

$$A_{xx}\mathrm{e}^{\mathrm{i}K_j^0 x} + 2A_x \mathrm{e}^{\mathrm{i}K_j^0 x}\mathrm{i}K_j^0 + A\mathrm{e}^{\mathrm{i}K_j^0 x}(\mathrm{i}K_j^0)^2 + A_{yy}\mathrm{e}^{\mathrm{i}K_j^0 x} + K^2 A \mathrm{e}^{\mathrm{i}K_j^0 x} = 0\,,$$

AMPE:

$$2\mathrm{i}K_j^0 A_x + A_{yy} + (K_j^2 - (K_j^0)^2)A = 0\,,$$

Gaussian initial condition:

$$A_j(x,y)|_{x=0} = A_{j,0}(y) = \frac{1}{2\sqrt{\pi}}\mathrm{e}^{-y^2 (K_j^0)^2}\,,$$

Pressure field:

$$P(x,y,z) \approx \sum^{M} \mathrm{e}^{\mathrm{i}xK_j^0} A_j(x,y)\phi_j(z)\,.$$

Summary of algorithm

1. Calculate modes: $K_j = K_j(x, y)\,,\ \phi_j = \phi_j(z, x, y)\,.$
2. Apply Crank-Nicholson algorithm:

$$2\mathrm{i}K_j^0 A_x + A_{yy} + (K_j^2 - (K_j^0)^2)A = 0\,.$$

3. Solve tridiagonal system: $Qa_{n+1} = Pa_n\,.$
4. Repeat for all j.
5. Sum:

$$P(x, y, z) = \sum_{j=1}^{M} \mathrm{e}^{\mathrm{i}xK_j^0} A_j(x, y)\phi_j(z)\,.$$

6. Apply Perfectly matched layer (PML).

Crank-Nicolson method

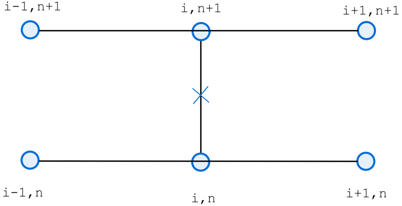
PDE:

$$2\mathrm{i}K^0 A_x + A_{yy} + (K^2 - (K^0)^2)A = 0\,.$$

Approximation of the partial derivatives:

$$A = \frac{a_{i,n} + a_{i,n+1}}{2}\,,$$

$$A_x = \frac{a_{i,n+1} - a_{i,n}}{\Delta x}\,,$$



$$A_{yy} = \frac{a_{i+1,n} - 2a_{i,n} + a_{i-1,n} + a_{i+1,n+1} - 2a_{i,n+1} + a_{i-1,n+1}}{2\Delta y^2}\,.$$

Insert into our PDE and solve tridiagonal problem.

Perfectly matched layer - PML

$$\tilde{y} = y + \mathrm{i}\int_0^y \sigma(s)ds\,,$$

$\sigma(y)$ positive monotonically increasing smooth function,
$y \to \tilde{y}$, $\tilde{A}(x,y) := A(x,\tilde{y})\,,$

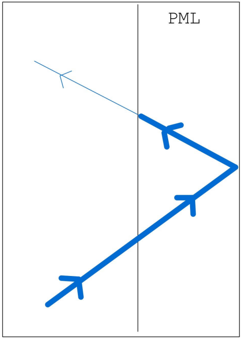
$$\left.\frac{\partial A}{\partial x}\right|_{y=\tilde{y}} = \frac{\partial \tilde{A}}{\partial x}\,,$$

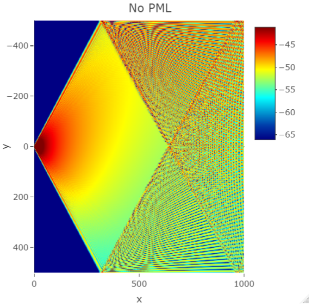
$$\left.\frac{\partial A}{\partial y}\right|_{y=\tilde{y}} = \frac{\partial (A|_{y=\tilde{y}})}{\partial \tilde{y}} = \frac{\partial A(x,\tilde{y})}{\partial \tilde{y}} = \frac{\partial \tilde{A}(x,y)}{\partial \tilde{y}} = \frac{\partial \tilde{A}(x,y)}{\partial y}\frac{\partial y}{\partial \tilde{y}} = \left(\frac{\partial \tilde{y}}{\partial y}\right)^{-1}\frac{\partial \tilde{A}(x,y)}{\partial y} = \frac{1}{1+\mathrm{i}\sigma(y)}\frac{\partial}{\partial y}\tilde{A}(x,y)\,.$$

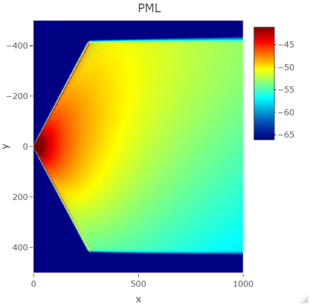
$$\frac{\partial}{\partial y} \to \frac{1}{1+\mathrm{i}\sigma(y)}\frac{\partial}{\partial y}\,.$$

Perfectly matched layer - PML

$$2\mathrm{i}K\frac{\partial\tilde{A}}{\partial x}+\frac{1}{1+\mathrm{i}\sigma(y)}\frac{\partial}{\partial y}\left(\frac{1}{1+\mathrm{i}\sigma(y)}\frac{\partial\tilde{A}}{\partial y}\right)+\left(K_j^2-(K_j^0)^2\right)\tilde{A}=0\,.$$

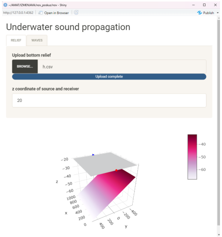


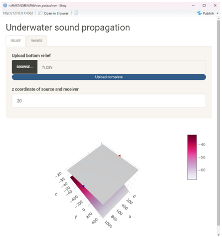


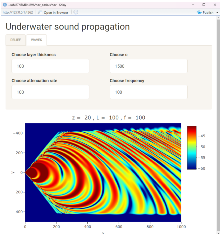


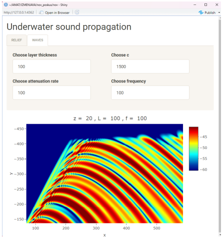
Software

Language: R. Interface: shiny. Plots: plotly.
Input: bottom_relief.csv, z, layer thickness, speed of the sound, attenuation rate, frequency.
Output: relief plot, sound propagation plot.



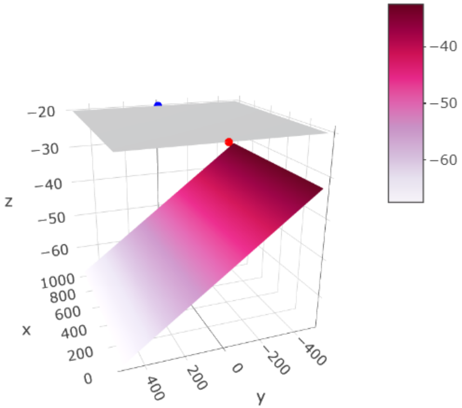




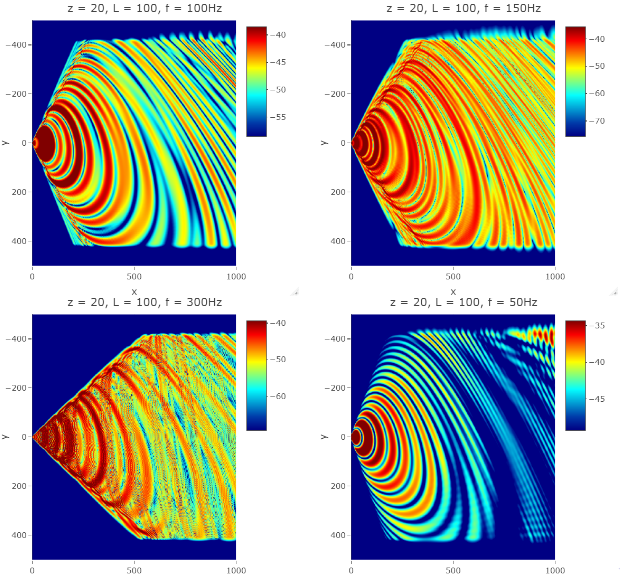


Numerical example

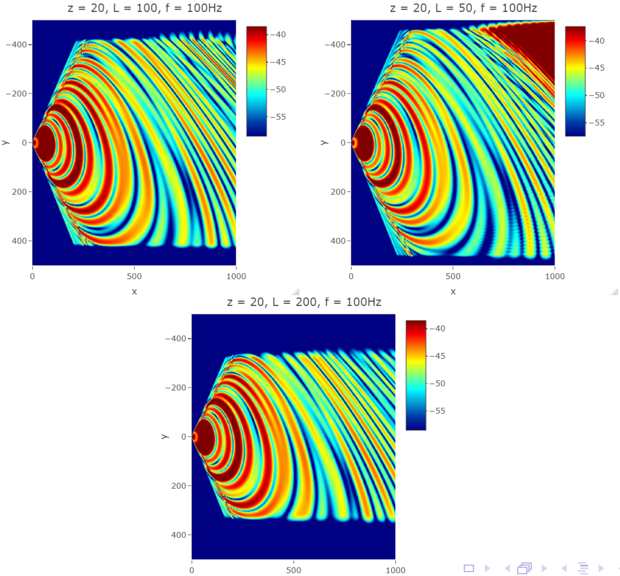
$$x \in [1, 2, \ldots 1000]\,,$$
$$y \in [-500, -499, \ldots, 500]\,,$$
$$L = 100\,,\ \sigma = 100\,,$$
$$c = 1500 m/s\,,\ f = 100 Hz\,,$$
$$h(x,y) = h_0 + \tan(\alpha) y\,,$$
$$h_0 = 50m\,,\ \alpha = 2^\circ\,.$$



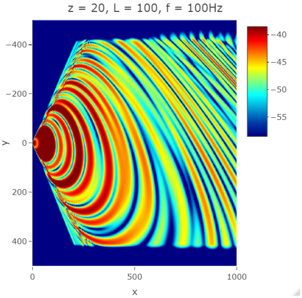
Changing frequency

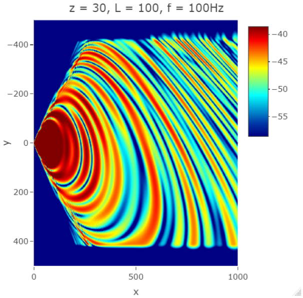


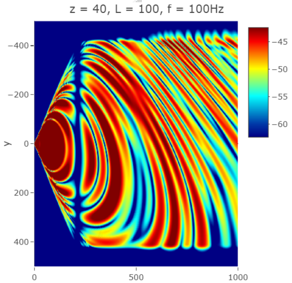
Changing L


z = 20, L = 100, f = 100Hz
z = 20, L = 50, f = 100Hz
z = 20, L = 200, f = 100Hz
x
y
−400
−200
0
200
400
0
500
1000
−40
−45
−50
−55

Changing z


z = 20, L = 100, f = 100Hz
−400
−200
0
200
400
y
0
500
1000
x
−40
−45
−50
−55


z = 30, L = 100, f = 100Hz
−400
−200
0
200
400
y
0
500
1000
x
−40
−45
−50
−55


z = 40, L = 100, f = 100Hz
−400
−200
0
200
400
y
0
500
1000
−45
−50
−55
−60

Thank you for your attention!

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