Chapter 6. Fluid Mechanics

Notes:

- *Most of the material in this chapter is taken from Young and Freedman, Chap. 12.*

6.1 Fluid Statics

Fluids, i.e., substances that can flow, are the subjects of this chapter. But before we can delve into this topic, we must first define a few fundamental quantities.

6.1.1 Mass Density and Specific Gravity

We have already encountered the **mass density** (often abbreviated to **density**) in previous chapters. Namely, the mass density is simply the ratio of the mass m of an object to its volume V

$$\rho = \frac{m}{V} \tag{6.1}$$

with units of $\mathrm{kg/m^3}$. Evidently, the density of objects can vary greatly depending of the materials composing them. For example, the density of water is $1{,}000\ \mathrm{kg/m^3}$ at $4^\circ\mathrm{C}$, that of iron is $7{,}800\ \mathrm{kg/m^3}$, while a neutron star has a mean density of approximately $10^{18}\,\mathrm{kg/m^3}$!

The **specific gravity** of a substance is defined as *the ratio of the mass density to that of water at 4^oC* (i.e., $1{,}000\ \mathrm{kg/m^3}$). It would probably be more precise to use the term relative density instead of specific gravity, but such is not the custom…

6.1.2 Pressure and Buoyance

A fluid is composed at the microscopic level by molecules and/or atoms that are constantly wiggling around. When the fluid is contained in a vessel these particles will collide with the walls of the container, a process that will then change their individual momenta. The change of momentum that a particle experiences will impart an impulse over the time interval during which the collision takes place, as a result the walls of the vessel will "feel" a force. The **pressure** p at a given point on a wall is defined as *the force component perpendicular to the wall at that point per unit area*. That is, if $dF_\perp$ is this elemental perpendicular force applied to an infinitesimal area dA on a wall, then the pressure on that area is

$$p \equiv \frac{dF_\perp}{dA}. \tag{6.2}$$

When the pressure is the same at all points of a macroscopic, plane surface of area A, then the perpendicular force $F_\perp$ must also be the same everywhere on that surface and

$$p = \frac{F_{\perp}}{A}. \tag{6.3}$$

The **pascal** (Pa) is the unit of pressure with

$$1 \text{ Pa} = 1 \text{ N/m}^2. \tag{6.4}$$

Related to the pascal is the **bar**, which equals 10^5Pa, and, accordingly, the **millibar**, which equals 100 Pa. The **atmospheric pressure** p_{a}, i.e., the average atmospheric pressure at sea level, is 1 **atmosphere** (atm) with

$$\begin{aligned} 1 \text{ atm} &= 101{,}325 \text{ Pa} \\ &= 1{,}103.25 \text{ millibar.} \end{aligned} \tag{6.5}$$

It is important to note that motion of the particles that cause the pressure is random in orientation and pressure is therefore isotropic. That is, pressure at one point is the same in all directions. Also, since the pressure at a point is directly proportional to the force effected at that point, it should be clear that weight can be a source of pressure. For example, the pressure in the earth's atmosphere decreases as one goes to higher altitude as the weight of the, or the amount of, fluid above is reduced. Similarly, an increase in pressure is felt by a diver who descends to greater depths in a body of water.

We can quantify this effect by studying how pressure varies within a fluid contained in a vessel. Accordingly, referring to Figure 1, we consider a fluid of uniform density ρ under the effect of gravity g and consider a fluid element of thickness dy and area A. We assume that the bottom of the vessel is located at $y = 0$ and the position of the fluid element is at $y\,(>0$; y thus increases upwards). If the pressure at the bottom of the element is p, then the pressure immediately on top of it will be $p + dp$. If we further assume that the fluid is in equilibrium, then this fluid element must be static and the different forces, say, at the bottom of the element must cancel each other out. That is,

$$pA - \left[\left(p + dp \right) A + dw \right] = 0, \tag{6.6}$$

where dw is the weight of the fluid element

$$dw = \left(\rho A dy \right) g. \tag{6.7}$$

The quantity between parentheses in equation (6.7) is simply the mass of the fluid element. Equation (6.6) then becomes

$$dp\, A = -\rho g\, A dy, \tag{6.8}$$

or

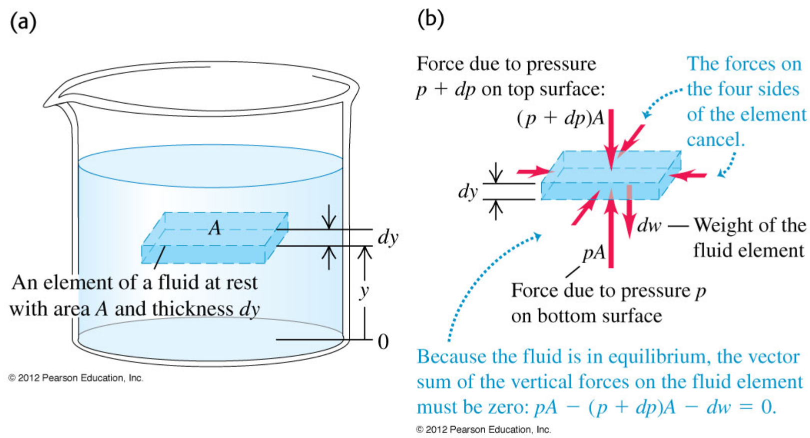

Figure 1 – The pressure as function of height in a fluid.

$$\frac{dp}{dy} = -\rho g. \tag{6.9}$$

Equation (6.9) is often called the equation of **hydrostatic equilibrium**. This result shows that pressure decreases as one moves upward in the vessel, as expected. We can integrate this equation to find the difference in pressure between two points y_1 and y_2 ($y_2 > y_1$) with

$$\begin{aligned} p_2 - p_1 &= \int_1^2 dp \\ &= -\rho g \int_1^2 dy \\ &= -\rho g \left(y_2 - y_1 \right), \end{aligned} \tag{6.10}$$

which we rewrite as (with $\Delta y = y_2 - y_1 > 0$)

$$p_1 = p_2 + \rho g \Delta y. \tag{6.11}$$

If we set y_2 at the top surface of the fluid (i.e., near the opening of the vessel), then $p_2 \equiv p_0$, where '0' means 'zero depth', equals the pressure at the exterior of the fluid. For example, if the vessel is located at sea level, then

$$p_0 = 1 \text{ atm}, \tag{6.12}$$

and

$$p_1 = p_0 + \rho g \Delta y. \tag{6.13}$$

It is then convenient to think of $\Delta y > 0$ as the depth in the fluid where the pressure p_1 is encountered. Equation (6.13) also implies that increasing p_0 by some amount will increase the pressure at any point within the fluid by the same amount. This is the so-called **Pascal's Law**

Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel.

We can use equation (6.11) to explain the behavior of objects submerged (sometimes not completely) in a fluid, such as water. Let us consider Figure 2 where an objet of mass m, horizontal area A, and height h is immersed in a fluid of density ρ; the whole apparatus is subject to gravity. We denote by p_1 and p_2 the pressures at the bottom and top surfaces of the object, respectively, likewise the force components perpendicular to those surfaces are F_1 and F_2. But we know from equation (6.11) that

$$\begin{aligned} p_1 A - p_2 A &= F_1 - F_2 \\ &= \rho g h A, \end{aligned} \tag{6.14}$$

or, while defining the volume of the object with $V = hA$, we have

$$F_1 - F_2 = \rho V g. \tag{6.15}$$

Since the $F_1 - F_2$ is net **buoyancy force** acting on the body and ρV is the mass of fluid displaced by the presence of the body, we are then led to **Archimedes' Principle**

The net upward, buoyancy force acting on a partially or completely immersed body equals the weight of fluid displaced by the body.

It is important to note that the buoyancy force is *independent of the weight of the object*. Also, although we derived this result for an object of rectangular volume, it should be clear that it applies to any possible shape since only the net perpendicular forces on the areas spanned by the top and bottom surfaces of the object come into play.

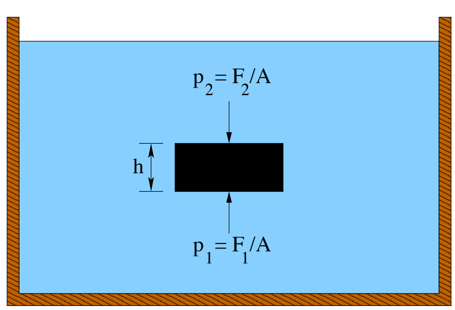

Figure 2 - An object immersed in a fluid.

6.1.3 Exercises

1. **Hydraulic lift.** Consider the representation of a hydraulic lift shown in Figure 3. Use Pascal's Law to explain the lift's functioning.

Solution.

According to equation (6.11), two points in fluid located at the same height are subjected to the same pressure. If we consider points 1 and 2 in Figure 3 both located at the surface of the fluid, then we can write

$$\begin{aligned} p_1 &= p_2 \\ \frac{F_1}{A_1} &= \frac{F_2}{A_2} \end{aligned} \tag{6.16}$$

and therefore

$$F_2 = \frac{A_2}{A_1} F_1. \tag{6.17}$$

It follows that we can multiply at point 2 the effect of the force F_1 applied at point 1 over an area A_1 (with a piston, for example) if we use a surface $A_2 > A_1$ at point 2. The resulting force is multiplied by the ratio of the areas, as shown in equation (6.17).

2. **Archimedes' Principle and Buoyancy.** We drop a rectangular piece of cork of mass m, volume Ah, and density $\rho_m = 240\ \mathrm{kg/m^3}$ in a vessel containing water (of density $\rho = 1000\ \mathrm{kg/m^3}$). Determine the depth at which the object's bottom surface settles once equilibrium is reached.

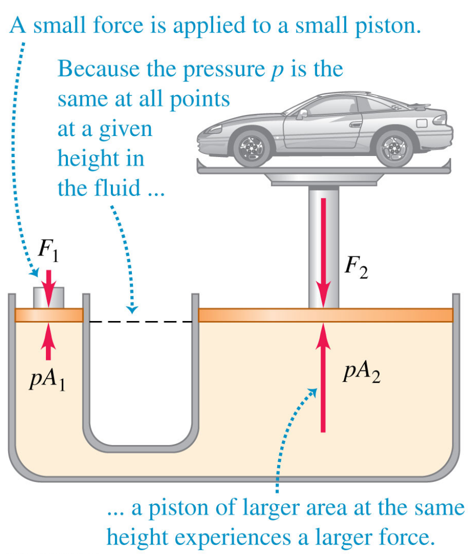

Figure 3 – A hydraulic lift.

Assume that an atmospheric pressure $p_0 = 1$ atm is present at the surface of the water and that $\rho_{\text{air}} = 1.2\ \text{kg/m}^3$.

Solution.

If denser than water, an object would sink to the bottom of the vessel. We can understand this by considering Newton's Second Law for the forces acting on the object

$$p_1 A - p_2 A - mg = ma. \tag{6.18}$$

But by replacing $p_1 A - p_2 A$ with the right-hand side of equation (6.14) and setting $m = \rho_m A h$ we get

$$(\rho - \rho_m) g A h = \rho_m A h a, \tag{6.19}$$

or

$$a = g\left(\frac{\rho}{\rho_m} - 1\right) < 0 \tag{6.20}$$

and the object sinks to the bottom.

If on the other hand the object is less dense than water, as is the case here, then the object will float, as shown in Figure 4. We denote by $h_1 > 0$ the depth at which the object's bottom surface is located relative to the surface of the water and by $h_2 > 0$ the portion above the water (i.e., $h = h_1 + h_2$). Using Newton's Second Law we write

$$p_1 A - \left(p_0 + \frac{dp_0}{dy} h_2\right) A - mg = 0 \tag{6.21}$$

since we assume equilibrium (i.e., $a = 0$), and where p_1 is the pressure at the object's bottom surface, $m = \rho_m A h$ (as before), and $h_2 \cdot dp_0 / dy$ is the change in atmospheric pressure from the water surface to the top of the object.

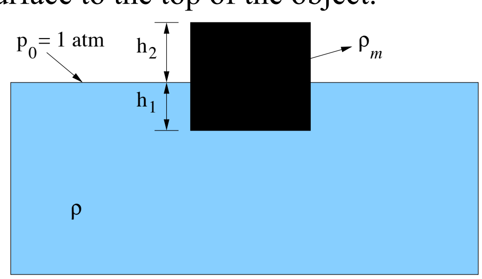

Figure 4 – An object partially immersed in water.

From Archimedes' buoyancy relation we have

$$p_0 A - \left(p_0 + \frac{dp_0}{dy} h_2 \right) A = \rho_{\text{air}} A h_2 g, \tag{6.22}$$

or, as expected for hydrostatic equilibrium,

$$\frac{dp_0}{dy} = -\rho_{\text{air}} g. \tag{6.23}$$

Also from the equation of hydrostatic equilibrium (i.e., equation (6.13)), we can write

$$p_1 = p_0 + \rho g h_1. \tag{6.24}$$

Inserting equations (6.23) and (6.24) into equation (6.21), while setting $h_2 = h - h_1$, we find that

$$(p_0 + \rho g h_1) - \left[p_0 - \rho_{\text{air}} g (h - h_1) \right] - \rho_m g h = 0, \tag{6.25}$$

and

$$\begin{aligned} h_1 &= \left(\frac{\rho_m - \rho_{\text{air}}}{\rho - \rho_{\text{air}}} \right) h \\ &\simeq \frac{\rho_m}{\rho} h \\ &\simeq 0.24 h, \end{aligned} \tag{6.26}$$

since the density of air is completely negligible compared to those of water and cork. It should be clear from this discussion that an object that is completely immersed in a fluid, and at equilibrium (as in Figure 2), must have the same density of the fluid.

3. (Prob. 12.33 in Young and Freedman.) A rock is suspended by a light string. When the rock is in the air, the tension in the string is 39.2 N. When the rock is totally immersed in water, the tension in the string is 28.4 N. When the rock is totally immersed in an unknown liquid, the tension is 18.6 N. What is the density of the unknown liquid?

Solution.

According to Archimedes' Principle the buoyancy force acting on the rock equals the weight of the displaced volume of liquid. That is,

$$F_{\text{b}} = \rho V g, \tag{6.27}$$

where V is the volume of the rock. According to Newton's Second Law

$$T + F_{\mathrm{b}} - mg = 0, \tag{6.28}$$

with T the tension in the string and m the mass of the rock. Applying equation (6.28) to the cases of air and water and then equating them, we have

$$T_{\mathrm{air}} + \rho_{\mathrm{air}} Vg = T_{\mathrm{water}} + \rho_{\mathrm{water}} Vg \tag{6.29}$$

and

$$\begin{aligned} V &= \frac{T_{\mathrm{air}} - T_{\mathrm{water}}}{\left(\rho_{\mathrm{water}} - \rho_{\mathrm{air}}\right) g} \\ &= \frac{(39.2 - 28.4)\,\mathrm{N}}{(1000 - 1.2)\,\mathrm{kg/m^3} \cdot 9.81\ \mathrm{m/s^2}} \\ &= 1.10 \times 10^{-3}\,\mathrm{m^3}. \end{aligned} \tag{6.30}$$

One way to proceed is to insert this result in equation (6.28) to find the mass of the rock (let us choose the case where it is immersed in water)

$$\begin{aligned} m &= \frac{T_{\mathrm{water}}}{g} + \rho_{\mathrm{water}} V \\ &= \frac{28.4\ \mathrm{N}}{9.81\ \mathrm{m/s^2}} + 1000\ \mathrm{kg/m^3} \cdot 1.10 \times 10^{-3}\,\mathrm{m^3} \\ &= 4.00\ \mathrm{kg}. \end{aligned} \tag{6.31}$$

Equation (6.28) for the unknown liquid then yields

$$\begin{aligned} \rho_{\mathrm{u}} &= \frac{mg - T_{\mathrm{u}}}{Vg} \\ &= \frac{4.00\ \mathrm{kg} \cdot 9.81\ \mathrm{m/s^2} - 18.6\ \mathrm{N}}{1.10 \times 10^{-3}\,\mathrm{m^{-3}} \cdot 9.81\ \mathrm{m/s^2}} \\ &= 1907\ \mathrm{kg/m^3}. \end{aligned} \tag{6.32}$$

Alternatively, we could have written

$$T_{\mathrm{air}} + \rho_{\mathrm{air}} Vg = T_{\mathrm{u}} + \rho_{\mathrm{u}} Vg, \tag{6.33}$$

and, with a similar outcome,

$$\rho_{\mathrm{u}} = \rho_{\mathrm{air}} + \frac{T_{\mathrm{air}} - T_{\mathrm{u}}}{Vg}. \tag{6.34}$$

6.2 Fluid Flows

The kinematics and motions of fluids can be a very complicated affair. In what follows, we will limit our studies to **ideal fluids**, i.e., those that are *incompressible* and *non-viscous* (that have no internal friction). We will also concentrate on **steady flows**, when motions have taken place for a long enough period that no transient behavior remains. Finally, we will not consider **turbulence** (random and chaotic flow patterns) and limit ourselves to **laminar flows** where layers of fluid can smoothly flow next to one another without the creation of turbulence.

6.2.1 The Continuity Equation

As a fluid is in steady motion and its flow progresses with time, it must be that *the amount of mass flowing per unit time across to areas perpendicular to the flow is conserved*. That is, if we consider the flow configuration shown in Figure 5, then we can write the following

$$\begin{aligned} \frac{dm}{dt} &= \rho A_1 \frac{dx_1}{dt} \\ &= \rho A_1 v_1 \end{aligned} \tag{6.35}$$

for the amount of mass dm passing through area A_1 in the time interval dt (ρ is the density of the fluid). If the flow is steady, then the amount of mass flowing through some other area A_2 during the same interval dt. More specifically, we can write

$$\rho A_1 v_1 = \rho A_2 v_2, \tag{6.36}$$

or simply

$$A_1 v_1 = A_2 v_2. \tag{6.37}$$

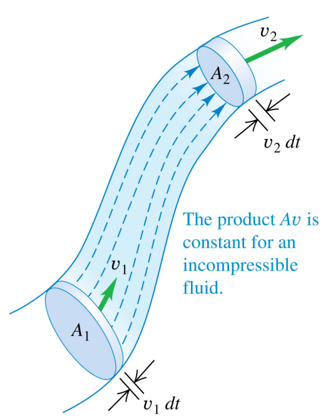

Figure 5 – Continuity and mass conservation in a flow.

Equation (6.37) is the so-called **continuity equation** for steady flows (of incompressible fluids). Generally, we can state that the **volume flow rate** is conserved

$$\begin{aligned}\frac{dV}{dt} &= Av \\ &= \text{constant}.\end{aligned} \tag{6.38}$$

6.2.2 Bernoulli's Equation

We now seek to apply the mass continuity equation while taking into account any changes of pressure that can accompany the flow of fluids. Such changes in pressure are to be expected whenever the cross-sectional area A changes along a flow. This is because as the area varies, the velocity must also change according to the continuity equation (6.37); if the flow speed changes, then there must be forces acting on the flow to cause this acceleration. Finally, pressure variations must also occur since pressure is defined as the force per cross-sectional area.

To derive the equation that relates these quantities, we will use the work-energy theorem defined in Chapter 2, which we write her for convenience

$$W_{\text{other}} = \Delta K + \Delta U_{\text{grav}}, \tag{6.39}$$

where W_{other} is the work done by all forces other than gravity, ΔK is the change in kinetic energy, and ΔU_{grav} is the change in gravitational potential energy. In our case we will substitute $W_{\text{other}} \to W_{\text{pressure}} \equiv W_{\text{p}}$. Let us consider the tube of changing cross-section shown in Figure 6. We first concentrate on the section on the left of width $v_1 dt$ and cross-section A_1, through which the flow speed is v_1 and the pressure p_1. We can ask what amount of work dW_1 was done by the pressure on a fluid that has traveled from the entrance to the exit of that section? The important fact to remember is that pressure is isotropic, meaning that the force $p_1 A_1$ at the entrance has the same magnitude as the force at the exit but of opposite direction. It therefore follows that

$$\begin{aligned}dW_{\text{p},1} &= \mathbf{F}_{\text{net},1} \cdot d\mathbf{r}_1 \\ &= \underbrace{\left(p_1 A_1 v_1 dt\right)}_{\text{entrance}} + \underbrace{\left(-p_1 A_1 v_1 dt\right)}_{\text{exit}} \\ &= 0.\end{aligned} \tag{6.40}$$

The pressure does no work when cross-section of the flow is constant. The same result $dW_{\text{p},2} = 0$ would be found for the section, of width $v_2 dt$ and cross-section A_2, through which the flow speed is v_2 and the pressure p_2, on the right of the tube.

The same cannot be said for the middle section of the tube, where the cross-section changes from A_1 to A_2. In this case we find

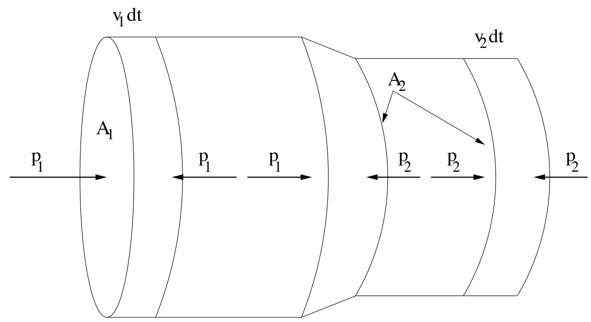

Figure 6 – A tube of changing cross-section, through which an incompressible and non-viscous fluid is flowing.

$$\begin{aligned} dW_{\mathrm{p},12} &= \mathbf{F}_{\mathrm{net},12} \cdot d\mathbf{r}_{12} \\ &= \underbrace{\left(p_1 A_1 v_1 dt\right)}_{\text{entrance}} + \underbrace{\left(-p_2 A_2 v_2 dt\right)}_{\text{exit}}, \end{aligned} \tag{6.41}$$

where dt is an infinitesimal time interval such that $v_1 dt$ and $v_2 dt$ are much smaller than the width of the section. We now use equation (6.37) for mass continuity and transform equation (6.41) to

$$dW_{\mathrm{p},12} = \left(p_1 - p_2\right) dV, \tag{6.42}$$

with dV the volume element spanned in the interval dt (see equation (6.38)). This equation can be integrated over between any two points along a tube, and generalized to (for an incompressible and non-viscous fluid)

$$W_{\mathrm{p}} = \left(p_1 - p_2\right) dV. \tag{6.43}$$

We can now write down the corresponding changes in kinetic and gravitational potential energies (if there is a change in vertical position y between point 1 and 2)

$$\begin{aligned} \Delta K &= \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) dV \\ \Delta U_{\mathrm{grav}} &= \rho g\left(y_2 - y_1\right) dV. \end{aligned} \tag{6.44}$$

Combining equations (6.39), (6.43), and (6.44) we get the so-called **Bernoulli's Equation**

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2. \tag{6.45}$$

Since equation (6.45) applies for any two points along the flow, we can write

$$p+\frac{1}{2}\rho v^2+\rho g y=\text{constant}. \tag{6.46}$$

It is interesting to note that, although it is defined as an applied force per unit area, *pressure can also be thought of as energy per unit volume*. Finally, we note that when the fluid is not moving $v_1 = v_2 = 0$ in equation (6.45) we recover

$$p_1 = p_2 + \rho g\left(y_2 - y_1\right), \tag{6.47}$$

which is the same as equation (6.11), previously derived for cases of hydrostatic equilibrium.

6.3 Exercises

4. (Prob. 12.40 in Young and Freedman.) **Artery blockage.** A medical technician is trying to determine what percentage of a patient's artery is blocked by plaque. To do this, she measures the blood pressure just before the region of blockage and finds that it is $1.20\times10^4\,\text{Pa}$, while in the region of blockage it is $1.15\times10^4\,\text{Pa}$. Furthermore, she knows that the blood flowing through the normal artery just before the point of blockage is traveling at 30.0 cm/s, and that the specific gravity of this patient's blood is 1.06. What percentage of cross-sectional area of the patient's artery is blocked by the plaque?

Solution.

We use Bernoulli's equation (i.e., equation (6.45)) while assuming that $y_1 = y_2$, where '1' and '2' correspond to regions before and at the blockage, respectively. We first determine the speed of the blood in the blockage with

$$\begin{aligned} v_2 &= \sqrt{\frac{2\left(p_1-p_2\right)}{\rho}+v_1^2} \\ &= \sqrt{\frac{2\left(1.20\times10^4-1.15\times10^4\right)\text{Pa}}{1{,}060\ \text{kg/m}^3}+\left(0.300\ \text{m/s}\right)^2} \\ &= 1\ \text{m/s}. \end{aligned} \tag{6.48}$$

We then use the continuity equation to find

$$\begin{aligned} \frac{A_2}{A_1} &= \frac{v_1}{v_2} \\ &= 0.30. \end{aligned} \tag{6.49}$$

The artery is therefore 70% blocked by the plaque.

5. (Prob. 12.55 in Young and Freedman.) A dam has the shape of a rectangular solid. The side facing the lake has an area A and a height h. The surface of the fresh water lake behind the dam is at the top of the dam. (a) Show that the net horizontal force exerted by the water on the dam is $\rho gAh/2$ – that is, the average gauge pressure across the face of the dam times the area. (b) Show that the torque exerted by the water about an axis along the bottom of the dam is $\rho gAh^2/6$.

Solution.

(a) The pressure at a given depth $h-y$ ($y=0$ is at the bottom of the dam) is given by

$$p = p_0 + \rho g(h-y) \tag{6.50}$$

with p_0 the atmospheric pressure at the top of the lake (and dam). The force exerted by the water on a horizontal strip of the dam of width dy at that depth is

$$\begin{aligned} dF_\perp &= p\frac{A}{h}dy \\ &= \frac{A}{h}\left[p_0 + \rho g(h-y)\right]dy, \end{aligned} \tag{6.51}$$

with A/h the width of the dam since it is rectangular. The total force exerted by the water on the dam will then be

$$\begin{aligned} F_\perp &= \int_0^h dF_\perp \\ &= \frac{A}{h}\left[(p_0 + \rho gh)\int_0^h dy - \rho g\int_0^h y\,dy\right] \\ &= \frac{A}{h}\left[(p_0 + \rho gh)h - \frac{1}{2}\rho gh^2\right] \\ &= p_0 A + \frac{1}{2}\rho gAh. \end{aligned} \tag{6.52}$$

However, the dam "feels" a force equal to $p_0 A$ on its side opposing the lake from the atmosphere. The total force on the dam is therefore

$$F_{\text{total}} = \frac{1}{2}\rho gAh. \tag{6.53}$$

(b) The torque about the axis at $y=0$ acting on a horizontal strip of the wall at depth $h-y$ is

$$
\begin{aligned}
d\tau &= \left(dF_{\perp} - p_0 \frac{A}{h} dy \right) \cdot y \\
&= \frac{A}{h} \rho g y (h - y) dy.
\end{aligned}
\tag{6.54}
$$

The total torque on the dam is thus

$$
\begin{aligned}
\tau &= \int_0^h d\tau \\
&= \frac{A}{h} \rho g \left(h \int_0^h y\, dy - \int_0^h y^2\, dy \right) \\
&= \frac{A}{h} \rho g \left(\frac{h^3}{2} - \frac{h^3}{3} \right) \\
&= \frac{1}{6} \rho g A h^2.
\end{aligned}
\tag{6.55}
$$

6. (Prob. 12.98 in Young and Freedman.) A *siphon*, as shown in Figure 7, is a convenient device for removing liquids from containers. To establish the flow, the tube must be initially filled with fluid. Let the fluid have a density ρ, and let the atmospheric pressure be p_{atm}. Assume that the cross-sectional area of the tube is the same at all points along it. (a) If the lower end of the siphon is at a distance h below the surface of the liquid in the container, what is the speed of the fluid as it flows out of the lower end of the siphon? (Assume that the container has a very large diameter, and ignore any effect of viscosity.) (b) A curious feature of a siphon is that the fluid initially flows "uphill." What is the greatest height H that the high point of the tube can have if flow is still to occur?

Solution.

(a) The pressure at the top of the liquid in the container is $p_0 = p_{\text{atm}}$, the same as it is at the lower end of the tube. Applying Bernoulli's equation with points "1" and "2" at the top of the liquid in the container and the lower end of the tube, respectively, we have

$$
p_0 + \frac{1}{2} \rho v_1^2 + \rho g h = p_0 + \frac{1}{2} \rho v_2^2,
\tag{6.56}
$$

or

$$
v_2^2 = v_1^2 + 2gh.
\tag{6.57}
$$

But with a very large container we can assume that $v_1 \approx 0$ and

$$
v_2 = \sqrt{2gh}.
\tag{6.58}
$$

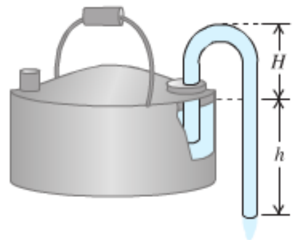

Figure 7 – A siphon, for removing liquids from a container.

(b) The pressure at the high point of the tube can be related to that at its low end with

$$p = p_0 - \rho g\left(H + h\right). \tag{6.59}$$

The liquid will not flow if the *absolute pressure* is negative anywhere in the tube, since a zero absolute pressure would imply that a perfect vacuum be present at that point. We then write

$$p_0 - \rho g\left(H + h\right) > 0, \tag{6.60}$$

or

$$H < \frac{p_0}{\rho g} - h. \tag{6.61}$$

We note that equation (6.61) also implies a limitation on $H + h$, which for water and normal atmospheric pressure yields

$$\begin{aligned} H + h &< \frac{p_{\text{atm}}}{\rho g} \\ &< 10.3 \text{ m}. \end{aligned} \tag{6.62}$$