Assignment #3

[redacted]

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1. Use Lipton's DNA algorithms to determine whether the following formula in the language of propositional calculus is satisfiable.

$$(C \vee \neg G) \wedge (A \vee C \vee G) \wedge (G \vee \neg A) \wedge (A \vee \neg C) \wedge (\neg A \vee \neg C \vee \neg G)$$

Show, in details the contents of all the intermediate test tubes.

Since there are 3 different variables, the number of possible outcomes is $2^3 = 8$. Furthermore, denote the elements of the conjunctive formula by their order in the formula, {1, 2, 3, …, etc.}

Test tube 1 contains all the possible outcomes in the order ACG.

First minter,
$t_1 = \{000, 001, 010, 011, 100, 101, 110, 111\}$
$t_2 = (t_1, 1, 1) = \{010, 011, 110, 111\}$
$t_1{}' = (t_1 - t_2) = \{000, 001, 100, 101\}$
$t_3 = (t_1{}', 2, 0) = \{000, 100\}$
$t_4 = (t_2 \cup t_3) = \{000{,}010, 011, 100, 110, 111\}$

Second minter,
$t_4 = \{000{,}010, 011, 100, 110, 111\}$
$t_5 = (t_4, 3, 1) = \{\ 100, 110, 111\}$
$t_5{}' = (t_4 - t_5) = \{000, 010, 011\}$
$t_6 = (t_5{}', 4, 1) = \{\ 010, 011\}$
$t_6{}' = (t_5{}' - t_6) = \{000\}$
$t_7 = (t_6{}', 5, 1) = \{\emptyset\}$
$t_8 = (t_5 \cup t_6 \cup t_7) = \{010, 011, 100, 110, 111\}$

Third minter,
$t_8 = \{010, 011, 100, 110, 111\}$
$t_9 = (t_8, 6, 1) = \{011, 111\}$
$t_9{}' = (t_8 - t_9) = \{010, 100, 110\}$
$t_{10} = (t_9{}', 7, 0) = \{010\}$
$t_{11} = (t_9 \cup t_{10}) = \{010, 011, 111\}$

Fourth minter,
$t_{11} = \{010, 011, 111\}$
$t_{12} = (t_{11}, 8, 1) = \{111\}$
$t_{12}' = (t_{11} - t_{12}) = \{010, 011\}$
$t_{13} = (t_{12}', 9, 0) = \{\emptyset\}$
$t_{14} = (t_{12} \cup t_{13}) = \{111\}$

Fifth minter,
$t_{14} = \{111\}$
$t_{15} = (t_{14}, 10, 0) = \{\emptyset\}$
$t_{15}' = (t_{14} - t_{15}) = \{111\}$
$t_{16} = (t_{15}', 11, 0) = \{\emptyset\}$
$t_{16}' = (t_{15}' - t_{16}) = \{111\}$
$t_{17} = (t_{16}', 12, 0) = \{\emptyset\}$
$t_8 = (t_{15} \cup t_{16} \cup t_{17}) = \{\emptyset\}$

Therefore, the formula is not satisfiable because there were no values left in the final test tube.

2. Design a combinatorial circuit for a room with four doors, one light and a switch near each door that controls the light. If the position of one switch is changed, the state of the light will change; i.e. if the light is on, it will go off and if it is off, it will go on. Assume that if all switches are closed, the light is on. Use the methodology described in class. Justify your answers.

   Firstly, denote a closed circuit with a 1, and an open circuit with a 0.

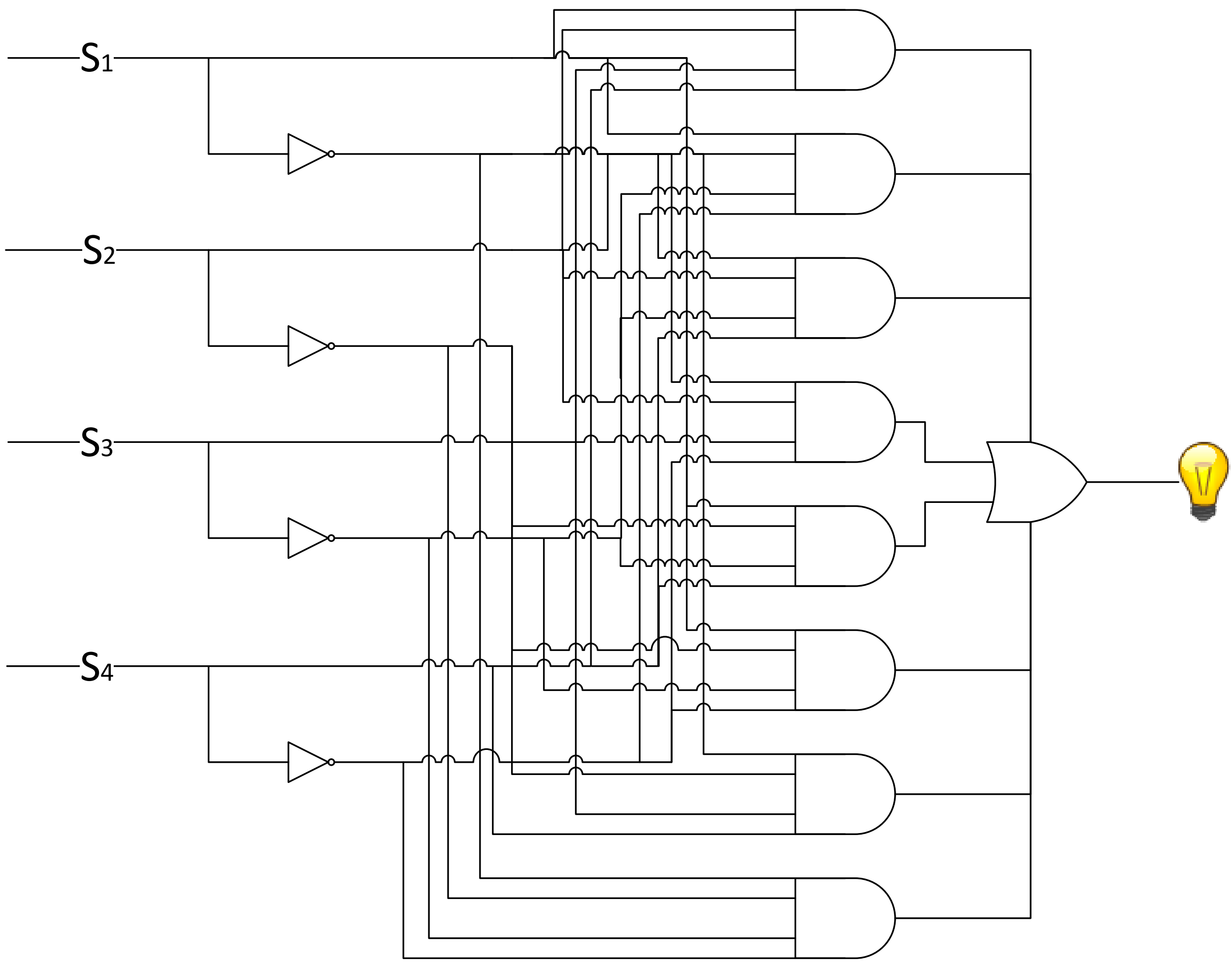
   Therefore, we need a truth table with all the possible closed and open circuits' combinations. Hence,

#	Switch 1	Switch 2	Switch 3	Switch 4	Off/On	Reasoning
1	**1**	**1**	**1**	**1**	On	Since all the circuits are closed, the light is on.
2	1	1	1	0	Off	Position of switch 4 changes; relative to #1
3	1	1	0	1	Off	Position of switch 3 changes; relative to #1
4	**1**	**1**	**0**	**0**	On	Position of switch 3 changes; relative to #2
5	1	0	1	1	Off	Position of switch 2 changes; relative to #1
6	**1**	**0**	**1**	**0**	On	Position of switch 2 changes; relative to #2
7	**1**	**0**	**0**	**1**	On	Position of switch 3 changes; relative to #5
8	1	0	0	0	Off	Position of switch 2 changes; relative to #4
9	0	1	1	1	Off	Position of switch 1 changes; relative to #1
10	**0**	**1**	**1**	**0**	On	Position of switch 4 changes; relative to #9
11	**0**	**1**	**0**	**1**	On	Position of switch 3 changes; relative to #9
12	0	1	0	0	Off	Position of switch 3 changes; relative to #10
13	**0**	**0**	**1**	**1**	On	Position of switch 2 changes; relative to #9
14	0	0	1	0	Off	Position of switch 4 changes; relative to #13
15	0	0	0	1	Off	Position of switch 3 changes; relative to #13
16	**0**	**0**	**0**	**0**	On	Position of switch 4 changes; relative to #15

$$S_1S_2S_3S_4 \vee S_1S_2\overline{S_3S_4} \vee \overline{S_1}S_2\overline{S_3}S_4 \vee \overline{S_1}S_2S_3\overline{S_4} \vee S_1\overline{S_2S_3}S_4 \vee S_1\overline{S_2}S_3\overline{S_4} \vee \overline{S_1S_2}S_3S_4 \vee \overline{S_1}\,\overline{S_2S_3S_4}$$

	$\boldsymbol{S_3S_4}$	$\boldsymbol{S_3\overline{S_4}}$	$\boldsymbol{\overline{S_3S_4}}$	$\boldsymbol{\overline{S_3}S_4}$
$\boldsymbol{S_1S_2}$	1	0	1	0
$\boldsymbol{S_1\overline{S_2}}$	0	1	0	1
$\boldsymbol{\overline{S_1S_2}}$	1	0	1	0
$\boldsymbol{\overline{S_1}S_2}$	0	1	0	1

Therefore there are no further minimizations.



3. A vending machine is to be designed that will dispense one product that costs 15 cents. The machine will have a sliding bar with three coin positions on it for one quarter (25 cents), one dime (10 cents), and one nickel (5 cents). When the bar is pushed in, a logic network inside the machine will make decisions to dispense the product, alert the change-making device to make change, and/or inform the user that he/she did not insert enough money. (Assume that the change machine inside the vending machine computes and dispenses the correct amount of change.)

   Design minimal circuits that will do the following after the bar is pushed in:

   a. If there is no money, turn the insufficient funds light (I).
   b. If there is exactly the right amount of money, dispense the product (P).
   c. If there is too much money, dispense the product (P) and single the change machine (C).
   d. If there is some money, but not enough, turn on the insufficient funds light (I) and signal change the change machine (C).

   Use the methodology described in class. Justify your answers.

   Firstly, the output table. Let Q = quarter, D = dime, and N = Nickel

Q	D	N	Action
0	0	0	I
0	0	1	I, C
0	1	0	I, C
0	1	1	P
1	0	0	P, C
1	0	1	P, C
1	1	0	P, C
1	1	1	P, C

Therefore,

$$I = \bar{Q}\bar{D}\bar{N} \vee \bar{Q}\bar{D}N \vee \bar{Q}D\bar{N}$$

	QD	$Q\bar{D}$	$\bar{Q}\bar{D}$	$\bar{Q}D$
N	0	0	1	0
$\bar{N}$	0	0	1	1

$$I = \bar{Q}\bar{D} \vee \bar{Q}\bar{N}$$
$$I = \bar{Q}(\bar{D} \vee \bar{N})$$

$$C = \bar{Q}\bar{D}N \vee \bar{Q}D\bar{N} \vee Q\bar{D}\bar{N} \vee Q\bar{D}N \vee QD\bar{N} \vee QDN$$

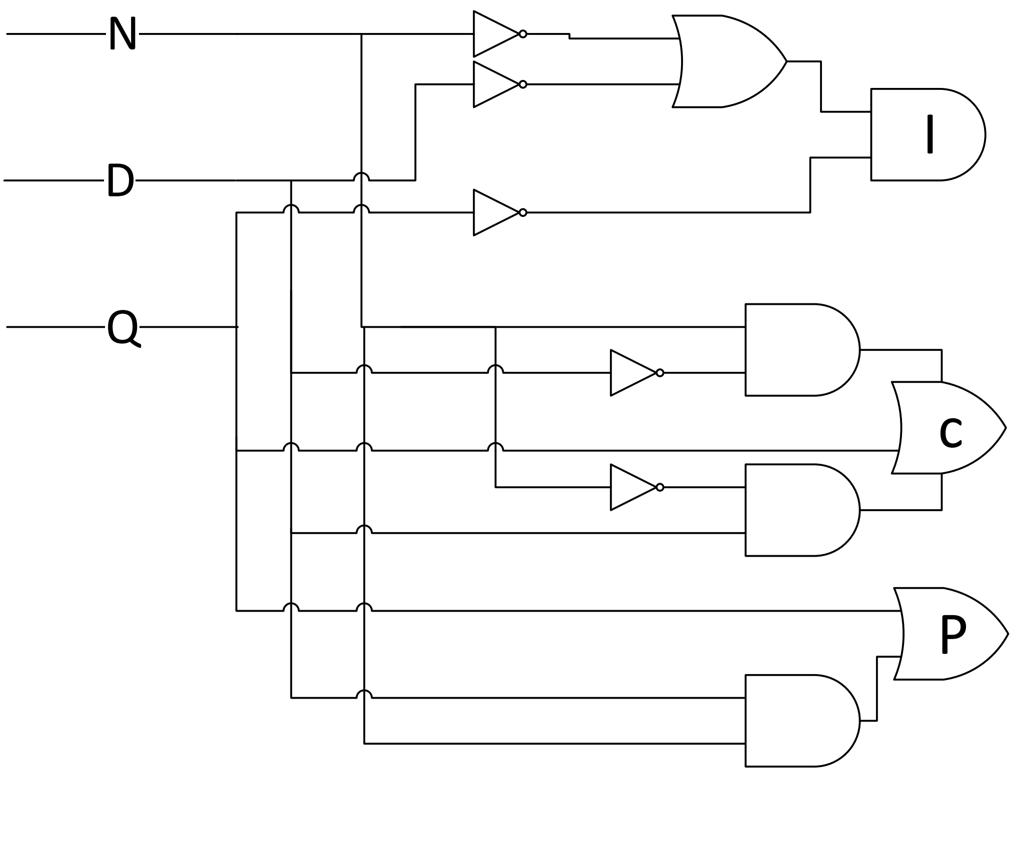
	QD	$Q\overline{D}$	$\overline{Q}\overline{D}$	$\overline{Q}D$
N	1	1	1	0
$\overline{N}$	1	1	0	1

$C = Q \vee \overline{D}N \vee D\overline{N}$

$P = \overline{Q}DN \vee Q\overline{D}\overline{N} \vee Q\overline{D}N \vee QD\overline{N} \vee QDN$

	QD	$Q\overline{D}$	$\overline{Q}\overline{D}$	$\overline{Q}D$
N	1	1	0	1
$\overline{N}$	1	1	0	0

$P = Q \vee DN$



4. Simplify each of the following formulas using Karnaugh maps. Clearly circle the blocks that you use for simplification in the Karnaugh maps.

   a. $ABCD \vee \bar{B}CD \vee A\bar{B}\bar{C}\bar{D} \vee AC\bar{D}$
   $ABCD \vee \bar{B}CD(A \vee \bar{A}) \vee A\bar{B}\bar{C}\bar{D} \vee AC\bar{D}(B \vee \bar{B})$
   $ABCD \vee A\bar{B}CD \vee \bar{A}\bar{B}CD \vee A\bar{B}\bar{C}\bar{D} \vee ABC\bar{D} \vee A\bar{B}C\bar{D}$

	$\boldsymbol{AB}$	$\boldsymbol{A\bar{B}}$	$\boldsymbol{\bar{A}\bar{B}}$	$\boldsymbol{\bar{A}B}$
$\boldsymbol{CD}$	1	1	1	0
$\boldsymbol{C\bar{D}}$	1	1	0	0
$\boldsymbol{\bar{C}\bar{D}}$	0	1	0	0
$\boldsymbol{\bar{C}D}$	0	0	0	0

$AC \vee \bar{B}CD \vee A\bar{B}\bar{D}$

   b. $ABCD \vee A\bar{B}CD \vee ABC\bar{D} \vee A\bar{B}C\bar{D} \vee \bar{A}B\bar{C}\bar{D} \vee \bar{A}B\bar{C}D$

	$\boldsymbol{AB}$	$\boldsymbol{A\bar{B}}$	$\boldsymbol{\bar{A}\bar{B}}$	$\boldsymbol{\bar{A}B}$
$\boldsymbol{CD}$	1	1	0	0
$\boldsymbol{C\bar{D}}$	1	1	0	0
$\boldsymbol{\bar{C}\bar{D}}$	0	0	0	1
$\boldsymbol{\bar{C}D}$	0	0	0	1

$AC \vee \bar{A}B\bar{C}$

   c. $(A \vee B)(B \vee C)(\bar{A} \vee C)$
   $[A \wedge (B \vee C) \vee B \wedge (B \vee C)](\bar{A} \vee C)$
   $(AB \vee AC \vee B \vee BC)(\bar{A} \vee C)$
   $\bar{A} \wedge (AB \vee AC \vee B \vee BC) \vee C \wedge (AB \vee AC \vee B \vee BC)$
   $\cancel{\bar{A}AB} \vee \cancel{\bar{A}AC} \vee \bar{A}B \vee \bar{A}BC \vee ABC \vee AC \vee BC$
   $\bar{A}B(C \vee \bar{C}) \vee \bar{A}BC \vee ABC \vee AC(B \vee \bar{B}) \vee BC(A \vee \bar{A})$
   $\bar{A}B\bar{C} \vee ABC \vee A\bar{B}C \vee \bar{A}BC$

	$\boldsymbol{AB}$	$\boldsymbol{A\bar{B}}$	$\boldsymbol{\bar{A}\bar{B}}$	$\boldsymbol{\bar{A}B}$
$\boldsymbol{C}$	1	1	0	1
$\boldsymbol{\bar{C}}$	0	0	0	1

   $AC \vee \bar{A}B$

d. $ABCD \vee \overline{(C \vee D)(B \vee C \vee \bar{D})(A \vee C \vee \bar{D})}$

$ABCD \vee \overline{(C \vee D)} \vee \overline{(B \vee C \vee \bar{D})} \vee \overline{(A \vee C \vee \bar{D})}$

$ABCD \vee \bar{C}\bar{D} \vee \bar{B}\bar{C}D \vee \bar{A}\bar{C}D$

$ABCD \vee \bar{C}\bar{D}(A \vee \bar{A})(B \vee \bar{B}) \vee \bar{B}\bar{C}D(A \vee \bar{A}) \vee \bar{A}\bar{C}D(B \vee \bar{B})$

$ABCD \vee AB\bar{C}\bar{D} \vee A\bar{B}\bar{C}\bar{D} \vee \bar{A}B\bar{C}\bar{D} \vee \bar{A}\bar{B}\bar{C}\bar{D} \vee A\bar{B}\bar{C}D \vee \bar{A}\bar{B}\bar{C}D \vee \bar{A}B\bar{C}D \vee \bar{A}\bar{B}\bar{C}D$

$ABCD \vee AB\bar{C}\bar{D} \vee A\bar{B}\bar{C}\bar{D} \vee \bar{A}B\bar{C}\bar{D} \vee \bar{A}\bar{B}\bar{C}\bar{D} \vee A\bar{B}\bar{C}D \vee \bar{A}\bar{B}\bar{C}D \vee \bar{A}B\bar{C}D$

	$\boldsymbol{AB}$	$\boldsymbol{A\bar{B}}$	$\boldsymbol{\bar{A}\bar{B}}$	$\boldsymbol{\bar{A}B}$
$\boldsymbol{CD}$	1	0	0	0
$\boldsymbol{C\bar{D}}$	0	0	0	0
$\boldsymbol{\bar{C}\bar{D}}$	1	1	1	1
$\boldsymbol{\bar{C}D}$	0	1	1	1

$\bar{C}\bar{D} \vee \bar{B}\bar{C} \vee \bar{A}\bar{C} \vee ABCD$

e. $A\bar{B}CD \vee AB\bar{C}D \vee A\overline{(B \vee C)} \vee \overline{(B \vee C \vee D)} \vee \overline{(A \vee B \vee C)}$

$A\bar{B}CD \vee AB\bar{C}D \vee A\bar{B}\bar{C} \vee \bar{B}\bar{C}\bar{D} \vee \bar{A}\bar{B}\bar{C}$

$A\bar{B}CD \vee AB\bar{C}D \vee A\bar{B}\bar{C}(D \vee \bar{D}) \vee \bar{B}\bar{C}\bar{D}(A \vee \bar{A}) \vee \bar{A}\bar{B}\bar{C}(D \vee \bar{D})$

$A\bar{B}CD \vee AB\bar{C}D \vee A\bar{B}\bar{C}D \vee A\bar{B}\bar{C}\bar{D} \vee A\bar{B}\bar{C}\bar{D} \vee \bar{A}\bar{B}\bar{C}\bar{D} \vee \bar{A}\bar{B}\bar{C}D \vee \bar{A}\bar{B}\bar{C}\bar{D}$

$A\bar{B}CD \vee AB\bar{C}D \vee A\bar{B}\bar{C}D \vee A\bar{B}\bar{C}\bar{D} \vee \bar{A}\bar{B}\bar{C}\bar{D} \vee \bar{A}\bar{B}\bar{C}D$

	$\boldsymbol{AB}$	$\boldsymbol{A\bar{B}}$	$\boldsymbol{\bar{A}\bar{B}}$	$\boldsymbol{\bar{A}B}$
$\boldsymbol{CD}$	0	1	0	0
$\boldsymbol{C\bar{D}}$	0	0	0	0
$\boldsymbol{\bar{C}\bar{D}}$	0	1	1	0
$\boldsymbol{\bar{C}D}$	1	1	1	0

$\bar{B}\bar{C} \vee A\bar{C}D \vee A\bar{B}D$