Set 34 Suppose that you own a wind farm with five wind turbines of the same type, and suppose that you have kept the records of their first failure times ${t_1}^{obs}, \dots, {t_5}^{obs}$, which are 14, 6, 8, 11, and 9 months.

(a) Construct a 95% (asymptotic) confidence interval for the probability that the wind turbine of this type functions without a failure for $t = 10$ months after its installation.

$$\hat{p} = \mathbb{P}(T \geq 10) = \frac{2}{5}$$

Then,

$$\hat{p} - 1.96\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} \leq p \leq \hat{p} + 1.96\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$$

$$\frac{2}{5} - 1.96\frac{\sqrt{\frac{2}{5}\left(1-\frac{2}{5}\right)}}{\sqrt{5}} \leq p \leq \frac{2}{5} + 1.96\frac{\sqrt{\frac{2}{5}\left(1-\frac{2}{5}\right)}}{\sqrt{5}}$$

$$\frac{2}{5} - 1.96\frac{\sqrt{\frac{6}{25}}}{\sqrt{5}} \leq p \leq \frac{2}{5} + 1.96\frac{\sqrt{\frac{6}{25}}}{\sqrt{5}}$$

$$\frac{2}{5} - 1.96\frac{\sqrt{6}}{5\sqrt{5}} \leq p \leq \frac{2}{5} + 1.96\frac{\sqrt{6}}{5\sqrt{5}}$$

Therefore, the 95% confidence interval is

$$\left[\frac{2\sqrt{5} - 1.96\sqrt{6}}{5\sqrt{5}}, \frac{2\sqrt{5} + 1.96\sqrt{6}}{5\sqrt{5}}\right]$$

(b) Construct a *conservative* 95% (asymptotic) confidence interval for the probability that the wind turbine of this type functions without a failure for $t = 10$ months after its installation.

The conservative assumption is

$$\hat{p} = 0.5$$

Then,

$$\hat{p} - 1.96\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} \leq p \leq \hat{p} + 1.96\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$$

$$\frac{1}{2} - 1.96\frac{\sqrt{\frac{1}{2}\left(1-\frac{1}{2}\right)}}{\sqrt{5}} \leq p \leq \frac{1}{2} + 1.96\frac{\sqrt{\frac{1}{2}\left(1-\frac{1}{2}\right)}}{\sqrt{5}}$$

$$\frac{1}{2} - 1.96\frac{\sqrt{\frac{1}{4}}}{\sqrt{5}} \leq p \leq \frac{1}{2} + 1.96\frac{\sqrt{\frac{1}{4}}}{\sqrt{5}}$$

$$\frac{1}{2} - 1.96\frac{1}{2\sqrt{5}} \leq p \leq \frac{1}{2} + 1.96\frac{1}{2\sqrt{5}}$$

Therefore, the 95% conservative confidence interval is

$$\left[\frac{\sqrt{5} - 1.96}{2\sqrt{5}}, \frac{\sqrt{5} + 1.96}{2\sqrt{5}}\right]$$

(c) Construct a 95% (asymptotic) confidence interval for the probability that the wind turbine of this type fails within $t = 10$ months after its installation.

$$\hat{p} = \mathbb{P}(T < 10) = 1 - \mathbb{P}(T \geq 10) = 1 - \frac{2}{5} = \frac{3}{5}$$

Then,

$$\hat{p} - 1.96\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} \leq p \leq \hat{p} + 1.96\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$$

$$\frac{3}{5} - 1.96\frac{\sqrt{\frac{3}{5}\left(1-\frac{3}{5}\right)}}{\sqrt{5}} \leq p \leq \frac{3}{5} + 1.96\frac{\sqrt{\frac{3}{5}\left(1-\frac{3}{5}\right)}}{\sqrt{5}}$$

$$\frac{3}{5} - 1.96\frac{\sqrt{\frac{6}{25}}}{\sqrt{5}} \leq p \leq \frac{3}{5} + 1.96\frac{\sqrt{\frac{6}{25}}}{\sqrt{5}}$$

$$\frac{3}{5} - 1.96\frac{\sqrt{6}}{5\sqrt{5}} \leq p \leq \frac{3}{5} + 1.96\frac{\sqrt{6}}{5\sqrt{5}}$$

Therefore, the 95% confidence interval is

$$\left[\frac{3\sqrt{5} - 1.96\sqrt{6}}{5\sqrt{5}}, \frac{3\sqrt{5} + 1.96\sqrt{6}}{5\sqrt{5}}\right]$$

(d) Construct a *conservative* 95% (asymptotic) confidence interval for the probability that the wind turbine of this type fails within for $t = 10$ months after its installation.

The conservative assumption is

$$\hat{p} = 0.5$$

Then,

$$\hat{p} - 1.96\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} \leq p \leq \hat{p} + 1.96\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$$

$$\frac{1}{2} - 1.96\frac{\sqrt{\frac{1}{2}\left(1-\frac{1}{2}\right)}}{\sqrt{5}} \leq p \leq \frac{1}{2} + 1.96\frac{\sqrt{\frac{1}{2}\left(1-\frac{1}{2}\right)}}{\sqrt{5}}$$

$$\frac{1}{2} - 1.96\frac{\sqrt{\frac{1}{4}}}{\sqrt{5}} \leq p \leq \frac{1}{2} + 1.96\frac{\sqrt{\frac{1}{4}}}{\sqrt{5}}$$

$$\frac{1}{2} - 1.96\frac{1}{2\sqrt{5}} \leq p \leq \frac{1}{2} + 1.96\frac{1}{2\sqrt{5}}$$

Therefore, the 95% conservative confidence interval is

$$\left[\frac{\sqrt{5} - 1.96}{2\sqrt{5}}, \frac{\sqrt{5} + 1.96}{2\sqrt{5}}\right]$$