Physics 1501A – 2nd Assignment

<u>Your solution to problems 4, 6, and 7 has to be handed in, in class, on Friday, November 9, 2012.</u>

1. A 2.00-kg object moving at 6.00 m/s collides with a 4.00-kg object that is initially at rest. After the collision, the 2.00-kg object moves backward at 1.00 m/s. This is a one-dimensional collision, and no external forces are acting.

a) Calculate the velocity of the 4.00-kg object after the collision.
b) Calculate the velocity of the centre of mass of the two objects (i) before the collision and (ii) after the collision.
c) Determine whether or not the collision was elastic. If not, determine the kinetic energy gained or lost as a result of the collision.
d) If the duration of the collision was 1.50 ms, calculate (i) the average force exerted by the 2.00-kg object on the 4.00-kg object, and (ii) the average force exerted by the 4.00-kg object on the 2.00-kg object, for the time period of the collision.

Solution.

a) We must have conservation of linear momentum. If $m_1 = 2.00\ \text{kg}$ and $m_2 = 4.00\ \text{kg}$, then

$$m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}, \tag{1.1}$$

but since $v_{B1} = 0$ we have

$$\begin{aligned} v_{B2} &= \frac{m_A}{m_B}\left(v_{A1} - v_{A2}\right) \\ &= \frac{2.00\ \text{kg}}{4.00\ \text{kg}}\left(6.00 + 1.00\right)\ \text{m/s} \\ &= 3.50\ \text{m/s}. \end{aligned} \tag{1.2}$$

b) Before the collision we have

$$\begin{aligned} v_{\text{cm}} &= \frac{m_A v_{A1}}{m_A + m_B} \\ &= \frac{2.00\ \text{kg} \cdot 6.00\ \text{m/s}}{6.00\ \text{kg}} \\ &= 2.00\ \text{m/s}, \end{aligned} \tag{1.3}$$

which must be unchanged after the collision because of the principle of conservation of linear momentum.

c) We must now calculate the kinetic energy before and after the collision

$$\begin{aligned} K_1 &= \frac{1}{2} m_A v_{A1}^2 \\ &= \frac{1}{2} 2.00 \text{ kg} \cdot 36.00 \text{ m}^2/\text{s}^2 \\ &= 36.0 \text{ J} \end{aligned} \tag{1.4}$$

and

$$\begin{aligned} K_2 &= \frac{1}{2} m_A v_{A2}^2 + \frac{1}{2} m_B v_{B2}^2 \\ &= \frac{1}{2} 2.00 \text{ kg} \cdot 1.00 \text{ m}^2/\text{s}^2 + \frac{1}{2} 4.00 \text{ kg} \cdot 12.25 \text{ m}^2/\text{s}^2 \\ &= 25.5 \text{ J}. \end{aligned} \tag{1.5}$$

The collision is *not elastic* since we find that $K_1 > K_2$. More precisely, 10.5 J of kinetic energy is *lost* during the collision.

d) The 4.00-kg mass sees its momentum change by an amount of

$$\begin{aligned} J_B &= p_{B2} - p_{B1} \\ &= 4.00 \text{ kg} \cdot 3.50 \text{ m/s} \\ &= 14.0 \text{ N} \cdot \text{s}. \end{aligned} \tag{1.6}$$

The force exerted on it be the other mass is therefore

$$\begin{aligned} F_{12} &= \frac{J_B}{\Delta t} \\ &= \frac{14.0 \text{ N} \cdot \text{s}}{1.50 \times 10^{-3} \text{ s}} \\ &= 9.33 \times 10^3 \text{ N}. \end{aligned} \tag{1.7}$$

From Newton's Third Law we know that

$$\begin{aligned} F_{21} &= -F_{12} \\ &= -9.33 \times 10^3 \text{ N}. \end{aligned} \tag{1.8}$$

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2. (Prob. 8.12 in Young and Freedman.) A bat strikes a 0.145-kg baseball. Just before impact, the ball is travelling horizontally to the right at 50.0 m/s, and it leaves the bat travelling to the left at an angle of 30° above horizontal with a speed of 65.0 m/s. If the ball and bat are in contact for 1.75 ms, find the horizontal and vertical components of the average force on the ball.

Solution.

We use the equation relating the impulse to the change in linear momentum in both the x (i.e., horizontal; positive to the right) and y (vertical; positive upward) directions

$$\begin{aligned} J_x &= \Delta p_x \\ &= 0.145\ \text{kg} \cdot \left[-65.0\cos\left(30^\circ\right) - 50.0\right]\ \text{m/s} \\ &= -15.4\ \text{kg} \cdot \text{m/s} \\ J_y &= \Delta p_y \\ &= 0.145\ \text{kg} \cdot \left[65.0\sin\left(30^\circ\right) - 0\right]\ \text{m/s} \\ &= 4.71\ \text{kg} \cdot \text{m/s}. \end{aligned} \tag{2.1}$$

The average force components are therefore

$$\begin{aligned} F_{\text{ave},x} &= \frac{\Delta p_x}{\Delta t} \\ &= \frac{-15.4\ \text{kg} \cdot \text{m/s}}{1.75\ \text{ms}} = -8800\ \text{N} \\ F_{\text{ave},y} &= \frac{\Delta p_y}{\Delta t} \\ &= \frac{4.71\ \text{kg} \cdot \text{m/s}}{1.75\ \text{ms}} = 2690\ \text{N}. \end{aligned} \tag{2.2}$$

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3. (Prob. 8.17 in Young and Freedman.) The expanding gas that leaves the muzzle of a rifle also contributes to the recoil. A 0.30-caliber bullet has a mass of 0.00720 kg and a speed of 601 m/s relative to the muzzle when fired from a rifle of mass 2.80 kg. The loosely held rifle recoils at a speed of 1.85 m/s relative to the earth. Find the momentum of the propellant gases in a coordinate system attached to the earth as they leave the muzzle of the rifle.

Solution.

We must have conservation of linear momentum. We therefore write for the momentum of the propellant gases, and final velocities of the bullet and rifle relative to the earth

$$
\begin{aligned}
&m_{\mathrm{b}}\left(v_{\mathrm{b}}+v_{r}\right)+p_{\mathrm{g}}=m_{\mathrm{r}}v_{\mathrm{r}}\\
&p_{\mathrm{g}}+0.00720\ \mathrm{kg}\cdot(601-1.85)\,\mathrm{m/s}-2.80\ \mathrm{kg}\cdot 1.85\ \mathrm{m/s}=0
\end{aligned}
\tag{3.1}
$$

or

$$
\begin{aligned}
p_{\mathrm{g}}&=(-4.31+5.18)\,\mathrm{kg}\cdot\mathrm{m/s}\\
&=0.87\ \mathrm{kg}\cdot\mathrm{m/s}.
\end{aligned}
\tag{3.2}
$$

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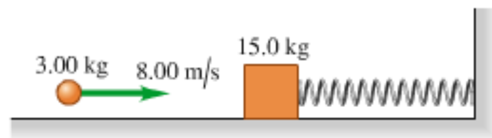
4. (Prob. 8.44 in Young and Freedman.) A 15.0-kg block is attached to a very light horizontal spring of force constant 500.0 N/m and is resting on a frictionless horizontal table. Suddenly it is struck by a 3.00-kg stone travelling horizontally at 8.00 m/s to the right, whereupon the stone rebounds at 2.00 m/s horizontally to the left (see the figure on the right). Find the maximum distance that the block will compress the spring after the collision.



Solution.

We define the positive direction (i.e., $x>0$) to the right. Since we must have conservation of linear momentum we write

$$
\begin{aligned}
m_{\mathrm{s}}v_{\mathrm{s},1}&=m_{\mathrm{s}}v_{\mathrm{s},2}+m_{\mathrm{b}}v_{\mathrm{b},2}\\
3.00\ \mathrm{kg}\cdot 8.00\ \mathrm{m/s}&=-3.00\ \mathrm{kg}\cdot 2.00\ \mathrm{m/s}+15.00\ \mathrm{kg}\cdot v_{\mathrm{b},2},
\end{aligned}
\tag{4.1}
$$

where '1' and '2' stand for conditions before and immediately after the collision. We then find that $v_{\mathrm{b},2}=2.00\ \mathrm{m/s}$. After the collision the kinetic energy store in the block will be transferred to the spring as it compresses. We therefore find

$$
\frac{1}{2}m_{\mathrm{b}}v_{\mathrm{b},2}^{2}=\frac{1}{2}kx^{2},
\tag{4.2}
$$

or

$$
x=v_{\mathrm{b},2}\sqrt{\frac{m_{\mathrm{b}}}{k}}=0.346\ \mathrm{m}.
\tag{4.3}
$$

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5. (Prob. 8.78 in Young and Freedman.) A small wooden block with mass 0.800 kg is suspended from the lower end of a light cord that is 1.60 m long. The block is initially at rest. A bullet with a mass of 12.0 g is fired at the block and becomes embedded in it. After the collision the combined object swings on the end of the cord. When the block has risen a vertical height of 0.800 m, the tension in the cord is 4.80 N. What was the initial speed v_0 of the bullet?

Solution.

The angle θ made by the string from the vertical when the block-bullet system has risen by 0.800 m is given by $\cos\theta = 0.8/1.6 = 0.5$, or $\theta = 60^\circ$. A consideration of the forces acting on the block-bullet at that time yields

$$T = mg\cos\theta + m\frac{v^2}{R}, \tag{5.1}$$

where T is the tension in the string, m is the combined mass of the block and the bullet, and R is the length of the string. We transform equation (5.1) to

$$v^2 = \frac{R}{m}\left(T - mg\cos\theta\right). \tag{5.2}$$

We are dealing with an inelastic collision. We must therefore first apply the principle of conservation of energy for the block bullet system for times just after the collision and later on. We therefore write, with V the speed of the combined block-bullet right after the collision,

$$\frac{1}{2}mV^2 + mgy_0 = \frac{1}{2}mv^2 + mgy \tag{5.3}$$

or

$$\begin{aligned} V^2 &= v^2 + 2g\left(y - y_0\right) \\ &= \frac{R}{m}\left(T - mg\cos\theta\right) + 2g\left(y - y_0\right) \\ &= \frac{RT}{m} + 2g\left(y - y_0 - \frac{R}{2}\cos\theta\right) \\ &= \frac{1.60\text{ m}\cdot 4.80\text{ N}}{0.812\text{ kg}} + 2\cdot 9.80\text{ m/s}^2\left(0.8 - \frac{1.60}{4}\right)\text{m} \\ &= 17.3\text{ m}^2/\text{s}^2. \end{aligned} \tag{5.4}$$

We finally apply the principle of conservation of linear momentum to times just before and right after the collision to get

$$
\begin{aligned}
v_0 &= \frac{mV}{m_{\text{bullet}}} \\
&= \frac{0.812\ \text{kg} \cdot 4.16\ \text{m/s}}{0.012\ \text{kg}} \\
&= 281\ \text{m/s}.
\end{aligned} \tag{5.5}
$$

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6. (Prob. 8.88 in Young and Freedman.) A 20.0-kg lead sphere is hanging from a hook by a thin wire 3.50 m long and is free to swing in a complete circle. Suddenly it is struck horizontally by 5.00-kg steel dart that embeds itself in the lead sphere. What must be the minimum initial speed of the dart so that the combination makes a complete circular loop after the collision?

Solution.

For the sphere-dart object to go on a complete circle it must have a high enough radial velocity v_3 such that the centrifugal acceleration is at least as strong in magnitude as gravity. That is, $v_3^2/r \geq g$, with r is the length of the thin wire. From the conservation of energy *after the collision* between the bottom and top of the circle we write

$$
\begin{aligned}
\frac{1}{2}\left(m_{\text{d}} + m_{\text{s}}\right)v_2^2 &= \frac{1}{2}\left(m_{\text{d}} + m_{\text{s}}\right)v_3^2 + 2\left(m_{\text{d}} + m_{\text{s}}\right)gr \\
&\geq \frac{5}{2}\left(m_{\text{d}} + m_{\text{s}}\right)gr,
\end{aligned} \tag{6.1}
$$

or

$$
v_2 \geq \sqrt{5gr} = 13.1\ \text{m/s}. \tag{6.2}
$$

Finally, we apply the conservation of linear momentum before and after the collision, at the bottom of the circle

$$
\begin{aligned}
m_{\text{d}} v_1 &= \left(m_{\text{s}} + m_{\text{d}}\right)v_2 \\
&\geq \left(m_{\text{s}} + m_{\text{d}}\right)\sqrt{5gr},
\end{aligned} \tag{6.3}
$$

or

$$
\begin{aligned}
v_1 &\geq \left(1 + \frac{m_{\text{s}}}{m_{\text{d}}}\right)\sqrt{5gr} \\
&\geq 65.5\ \text{m/s}.
\end{aligned} \tag{6.4}
$$

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7. (Prob. 9.16 in Young and Freedman.) At $t=0$ a grinding wheel has an angular velocity of 24.0 rad/s. It has a constant angular acceleration of $30.0\ \text{rad/s}^2$ until a circuit breaker trips at $t=2.00\ \text{s}$. From then on, it turns through 432 rad as it coasts to a stop at constant angular acceleration. (a) Through what total angle did the wheel turn between $t=0$ and the time it stopped? (b) At what time did it stop? (c) What was its acceleration as it slowed down?

Solution.

a) The angular speed and displacement at a given time by

$$\begin{aligned} \omega_1 &= \omega_0 + \alpha_0 t_1 \\ \theta_1 &= \theta_0 + \omega_0 t_1 + \frac{1}{2}\alpha_0 t_1^2, \end{aligned} \tag{7.1}$$

which we can apply at $t_1 = 2.00\ \text{s}$ with $\theta_0 = 0$, $\omega_0 = 24.0\ \text{rad/s}$, and $\alpha_0 = 30.0\ \text{rad/s}^2$; this yields $\theta_1 = 108\ \text{rad}$. The total angle travelled by the wheel is therefore 540 rad. We can also calculate that $\omega_1 = 84.0\ \text{rad/s}$.

b) Equations (7.1) still apply afterwards but then with a deceleration $\alpha_1 < 0$ and the initial angular displacement and speed being θ_1 and ω_1. We then write

$$\begin{aligned} \omega_2 &= \omega_1 + \alpha_1 t_2 \\ &= 0 \\ \theta_2 &= \omega_1 t_2 + \frac{1}{2}\alpha_1 t_2^2 \\ &= 432\ \text{rad}. \end{aligned} \tag{7.2}$$

The first of these equations can be inserted in the second to yield

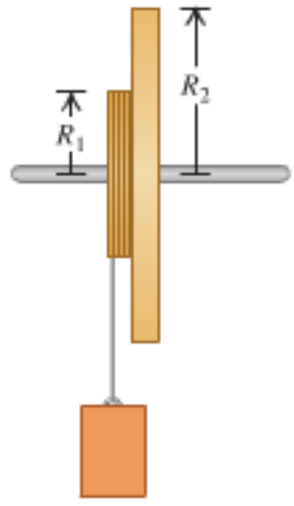
$$\begin{aligned} \theta_2 &= \omega_1 t_2 - \frac{1}{2}\omega_1 t_2 \\ &= \frac{1}{2}\omega_1 t_2, \end{aligned} \tag{7.3}$$

or $t_2 = 10.3\ \text{s}$.

c) Again we use the first of equations (7.2) to calculate

$$\alpha_1 = -\frac{\omega_1}{t_2} = -8.16\ \text{rad/s}^2. \tag{7.4}$$

8. (Prob. 9.87 in Young and Freedman.) Two metal disks, one with radius $R_1 = 2.50$ cm and mass $M_1 = 0.80$ kg and the other with radius $R_2 = 5.00$ cm and mass $M_2 = 1.60$ kg, are welded together and mounted on a frictionless axis through their common centre, as shown in the figure. (a) What is the total moment of inertia of the two disks? (b) A light string is wrapped around the edge of the smaller of the two disks, and a 1.50-kg block is suspended from the free end of the string. If the block is released from rest at a distance of 2.00 m above the floor, what is its speed just before it strikes the floor? (c) Repeat the calculation of part (b), this time with the string wrapped around the edge of the larger disk. In which case is the final speed of the block greater? Explain why this is so.



Solution.

(a) The total moment of inertia of the assembly consists of the sum of the moments of inertia of the disks. That is,

$$\begin{aligned} I &= I_1 + I_2 \\ &= \frac{1}{2}M_1R_1^2 + \frac{1}{2}M_2R_2^2 \\ &= 2.25\times10^{-3}\,\text{kg}\cdot\text{m}^2. \end{aligned} \tag{8.1}$$

(b) We must have conservation of energy with

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2, \tag{8.2}$$

where h is distance the block falls and m is its mass. Because $v = \omega R_1$ we can also write

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{R_1^2}, \tag{8.3}$$

which yields

$$\begin{aligned} v &= \sqrt{\frac{2gh}{1 + I/mR_1^2}} \\ &= 3.40 \text{ m/s}. \end{aligned} \tag{8.4}$$

(c) When the string is attached to the bigger disk we have $v = \omega R_2$, and with the same calculations as in (b) we get

$$v = \sqrt{\frac{2gh}{1 + I/mR_2^2}} \tag{8.5}$$
$$= 4.95 \text{ m/s.}$$

Since the total kinetic energy is the same in (b) and (c), i.e., $K = mgh$, the block will move faster in (c) since when $v = \omega R_2 > \omega R_1$ the portion of kinetic energy stored in the block in the first term on the right-hand side of equation (8.3) is higher.

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9. (Prob. 9.96 in Young and Freedman.) A thin, uniform rod is bent into a square of side length a. If the total mass is M, find the moment of inertia about an axis through the centre and perpendicular to the plane of the square. (*Hint:* Use the parallel-axis theorem.)

Solution.

Each side has length a and mass $M/4$, and the moment of inertia of each side about an axis perpendicular to the side and through its center is

$$\begin{aligned} I_0 &= \frac{1}{12}\left(\frac{1}{4}Ma^2\right) \\ &= \frac{1}{48}Ma^2. \end{aligned} \tag{9.1}$$

Using the parallel-axis theorem we find the moment of inertia of each side about an axis through the centre and perpendicular to the plane of the square is

$$\begin{aligned} I_1 &= I_0 + \frac{M}{4}\left(\frac{a}{2}\right)^2 \\ &= \frac{Ma^2}{12}, \end{aligned} \tag{9.2}$$

such that the moment of the square is four times that value with

$$I = \frac{1}{3}Ma^2. \tag{9.3}$$
