Set 36 Suppose that you own a fleet of taxis consisting of hybrid cars of the same type, and suppose that you have decided to re-invest some of your profits by adding solar roofs to the taxis. Hence, you buy the best solar cells available on the market and, being an innovative engineer, decide to arrange them in a most efficient way. You quickly realize that the shape of the car roof is one of the most important factors, and you can easily shape the roof into any form of your liking (the drag coefficient is not an issue for your taxis because they cannot drive fast in the city). Your computer modeling suggests a certain shape, which is somewhat different from the original roof-shape of the taxis. Should you keep the original roof-shape or modify it? To answer this question, you randomly select ten taxis from your fleet, add solar roofs to all of them, drive for a week, and record their gas-free mileage. After that, you modify the roofs of the same ten taxis, add solar roofs, drive them for a week, and record their gas-free mileage. Hence, you have ten pairs of data. Construct a 95% (asymptotic) confidence interval for the difference between the (population) average gas-free mileage under the original and modified roofs.

Let X be an RV that determines the average gas free mileage under the original roof. Let Y be an RV that determines the average gas free mileage under the modified roof. Let Z be an RV that determines the difference between gas free mileage under original vs. modified roofs, where $z_i = x_i - y_i$

$$\bar{Z} = \mathbb{E}[Z]$$

$$\hat{\sigma}_Z = \sqrt{\mathbb{E}[(\bar{Z} - Z)^2]}$$

Then,

$$\bar{Z} - 1.96\frac{\hat{\sigma}_Z}{\sqrt{n}} \leq \mu \leq \bar{Z} + 1.96\frac{\hat{\sigma}_Z}{\sqrt{n}}$$

$$\bar{Z} - 1.96\frac{\hat{\sigma}_Z}{\sqrt{10}} \leq \mu \leq \bar{Z} + 1.96\frac{\hat{\sigma}_Z}{\sqrt{10}}$$

Therefore, the mean difference confidence interval is

$$\left[\bar{Z} - 1.96\frac{\hat{\sigma}_Z}{\sqrt{10}}, \bar{Z} + 1.96\frac{\hat{\sigma}_Z}{\sqrt{10}}\right]$$