Set 35 Suppose that you have designed a product and want to assess its reliability in the form of its functionality during the required, say, 10 day mission. For this, you want to have an estimate of the reliability function $R(t)$ at $t = 10$, and you also want to have a 95% (asymptotic) confidence interval for $R(10)$ whose margin of error does not exceed 0.01. How many prototypes should you manufacture and then wait for their failure times in order to construct the aforementioned confidence interval when:

(a) there is no additional information available to you?

Since there is no $\hat{p}$ provided, choose a conservative $\hat{p} = 0.5$

Next, since the margin of error is,

$$Margin\ of\ error = z * SE$$

And,

$$SE(standard\ error) = \frac{\sigma}{\sqrt{n}}$$

$$z = 1.96$$

Then,

$$\frac{1}{100} = 1.96\frac{\sigma}{\sqrt{n}}$$

$$\sqrt{n} = 100 * 1.96 * \sqrt{\hat{p}(1-\hat{p})}$$

$$\sqrt{n} = 100 * 1.96 * \sqrt{\frac{1}{2}\left(1-\frac{1}{2}\right)}$$

$$\sqrt{n} = 100 * 1.96 * \frac{1}{2}$$

$$\sqrt{n} = 25 * 1.96$$

$$n = 49^2$$

Therefore, $n = 2401$

(b) your expertise in physics, chemistry, and of course engineering tells you that $R(10)$ should be somewhere between 0.85 and 0.98?

Since $0.85 \leq R(10) \leq 0.98$ then select a $\hat{p}$ value that is closest to 0.5, $\hat{p} = 0.85$,

$$\frac{1}{100} = 1.96\frac{\sigma}{\sqrt{n}}$$

$$\sqrt{n} = 100 * 1.96 * \sqrt{\hat{p}(1-\hat{p})}$$

$$\sqrt{n} = 100 * 1.96 * \sqrt{\frac{85}{100}\left(1 - \frac{85}{100}\right)}$$

$$\sqrt{n} = 100 * 1.96 * \frac{85}{100} * \frac{15}{100}$$

$$\sqrt{n} = 12.75 * 1.96$$

$$n = 24.99^2$$

Therefore, $n \approx 624.50$

(c) your expertise tells you that $R(10)$ should be somewhere between 0.45 and 0.70?

Since $0.45 \leq R(10) \leq 0.70$ then select a $\hat{p}$ value that is closest to 0.5, $\hat{p} = 0.5$,

$$\frac{1}{100} = 1.96\frac{\sigma}{\sqrt{n}}$$

$$\sqrt{n} = 100 * 1.96 * \sqrt{\hat{p}(1-\hat{p})}$$

$$\sqrt{n} = 100 * 1.96 * \sqrt{\frac{1}{2}\left(1 - \frac{1}{2}\right)}$$

$$\sqrt{n} = 100 * 1.96 * \frac{1}{2}$$

$$\sqrt{n} = 25 * 1.96$$

$$n = 49^2$$

Therefore, $n = 2401$