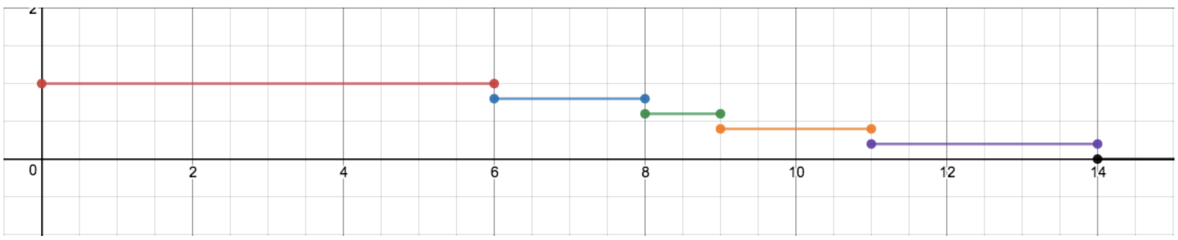
Set 32 Suppose that you own a wind farm with five wind turbines of the same type, and suppose that you have kept the records of their first failure times $t_1^{obs}, \dots, t_5^{obs}$, which are 14, 6, 8, 11, and 9 months.

(a) Draw the sample (empirical) reliability function.




(b) Calculate the area under the empirical reliability function.

$$A = 6 * 1 + 2 * \frac{4}{5} + 1 * \frac{3}{5} + 2 * \frac{2}{5} + 3 * \frac{1}{5}$$

$$A = 9 + \frac{3}{5} = 9.6$$

(c) Calculate the sample mean $\bar{t}$ of the data.

$$\bar{t} = \frac{(6 + 8 + 9 + 11 + 14)}{5} = 9.6$$

(d) Construct a 95% (asymptotic) confidence interval for the population MTTF (mean time to failure) when someone has told you that, based on historical records, the population variance $\sigma^2$ is 9.

$$\bar{t} - 1.96\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{t} + 1.96\frac{\sigma}{\sqrt{n}}$$

$$9.6 - 1.96\frac{\sqrt{9}}{\sqrt{5}} \le \mu \le 9.6 + 1.96\frac{\sqrt{9}}{\sqrt{5}}$$

$$9.6 - 1.96\frac{3}{\sqrt{5}} \le \mu \le 9.6 + 1.96\frac{3}{\sqrt{5}}$$

Therefore, the 95% confidence interval is

$$\left[9.6 - 1.96\frac{3}{\sqrt{5}}, 9.6 + 1.96\frac{3}{\sqrt{5}}\right]$$

(e) Calculate the sample standard deviation $\hat{\sigma}$ of the data.

$$\hat{\sigma} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(t_i - \bar{t})}$$

$$\hat{\sigma} = \sqrt{\frac{1}{5}((6-9.6)^2 + (8-9.6)^2 + (9-9.6)^2 + (11-9.6)^2 + (14-9.6)^2)}$$

$$\hat{\sigma} = \sqrt{\frac{1}{5}(3.6^2 + 1.6^2 + 0.6^2 + 1.4^2 + 4.4^2)}$$

$$\hat{\sigma} = \sqrt{\frac{1}{5}(12.96 + 2.56 + 0.36 + 1.96 + 19.36)}$$

$$\hat{\sigma} = \sqrt{\frac{37.2}{5}}$$

(f) Construct a 95% (asymptotic) confidence interval for the population MTTF when no information about the population variance $\sigma^2$ is available.

$$\bar{t} - 1.96\frac{\hat{\sigma}}{\sqrt{n}} \leq \mu \leq \bar{t} + 1.96\frac{\hat{\sigma}}{\sqrt{n}}$$

$$9.6 - 1.96\frac{\sqrt{\frac{37.2}{5}}}{\sqrt{5}} \leq \mu \leq 9.6 + 1.96\frac{\sqrt{\frac{37.2}{5}}}{\sqrt{5}}$$

$$9.6 - 1.96\frac{\sqrt{37.2}}{\sqrt{5}}\frac{1}{\sqrt{5}} \leq \mu \leq 9.6 + 1.96\frac{\sqrt{37.2}}{\sqrt{5}}\frac{1}{\sqrt{5}}$$

$$9.6 - 1.96\frac{\sqrt{37.2}}{5} \leq \mu \leq 9.6 + 1.96\frac{\sqrt{37.2}}{5}$$

Therefore, the 95% interval is

$$\left[9.6 - 1.96\frac{\sqrt{37.2}}{5}, 9.6 + 1.96\frac{\sqrt{37.2}}{5}\right]$$