Physics 1501A – 3rd Assignment

<u>Your solution to problems 1, 4, and 5 has to be handed in, in class, on Wednesday, December 5, 2012.</u>

1. (Prob. 10.26 in Young and Freedman.) A bowling ball rolls without slipping up a ramp that slopes upward at an angle β to the horizontal. Treat the ball as uniform solid sphere, ignoring the finger holes.

a) Draw the free-body diagram for the ball. Explain why the friction force must be directed uphill.
b) What is the acceleration of the centre of mass of the ball?
c) What minimum coefficient of static friction is needed to prevent slipping?

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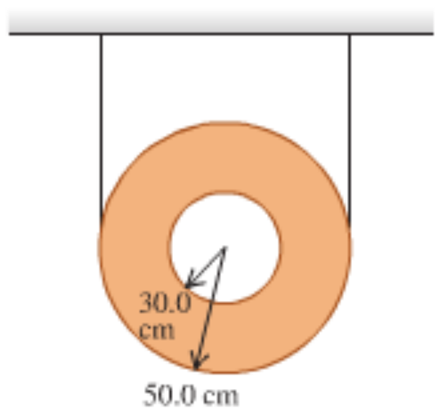
2. (Prob. 10.62 in Young and Freedman.) A uniform hollow disk has two pieces of thin, light wire wrapped around its outer rim and is supported from the ceiling, as shown in the figure. Suddenly one of the wires breaks, and the remaining wire does not slip as the disk rolls down. Use energy conservation to find the speed of the centre of this disk after it has fallen a distance of 2.20 m.



Solution.

We must have conservation of energy as the disk falls after the wire breaks. That is,

$$mgh = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2, \tag{2.1}$$

where v_{cm} is the speed of the centre of mass, which is located at the centre of disk, and

$$\begin{aligned} I &= \frac{1}{2}m\left(R_1^2 + R_2^2\right) \\ v_{\text{cm}} &= \omega R_2. \end{aligned} \tag{2.2}$$

We then insert equations (2.2) in equation (2.1) to get (h is the distance the disk falls)

$$
\begin{aligned}
mgh &= \frac{1}{2}mv_{\mathrm{cm}}^2 + \frac{1}{2}I\frac{v_{\mathrm{cm}}^2}{R_2^2} \\
&= \frac{1}{2}mv_{\mathrm{cm}}^2\left(1+\frac{I}{mR_2^2}\right) \\
&= \frac{1}{4}mv_{\mathrm{cm}}^2\left(3+\frac{R_1^2}{R_2^2}\right),
\end{aligned}
\tag{2.3}
$$

and

$$
\begin{aligned}
v_{\mathrm{cm}} &= 2\sqrt{\frac{gh}{3+R_1^2/R_2^2}} \\
&= 5.07\ \mathrm{m/s}.
\end{aligned}
\tag{2.4}
$$

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3. (Prob. 10.48 in Young and Freedman.) Suppose that an asteroid traveling straight toward the centre of the earth were to collide with our planet at the equator and bury itself just below the surface. What would have to be the mass of the asteroid, in term of the earth's mass M, for the day to become 25.0% longer than it presently is as a result of the collision? Assume that the asteroid is very small compared to the earth and that the earth is uniform throughout.

Solution.

We can apply the principle of conservation of angular momentum since the asteroid will not produce a torque on the earth because it is heading for the centre of the earth. Furthermore, for the same reason, it will not contribute any angular momentum. We therefore write

$$
I_{\mathrm{E}}\omega_1 = \left(I_{\mathrm{E}} + I_{\mathrm{A}}\right)\omega_2, \tag{3.1}
$$

where the subscripts '1' and '2' denote the condition before and after the collision, respectively, while I_{E} and I_{A} are the moments of inertia of the earth and the asteroid. Using the information provided in the lecture notes for the moments of inertia for a uniform sphere and a point mass we modify equation (3.1) to

$$
\left(\frac{2}{5}MR^2\right)\omega_1 = \left(\frac{2}{5}MR^2 + mR^2\right)\omega_2, \tag{3.2}
$$

and

$$
\frac{\omega_1 - \omega_2}{\omega_2} = \frac{5m}{2M}. \tag{3.3}
$$

Since the period is increased by 25.0%, the frequency must be reduced such that $1/\omega_2 = 1.25/\omega_1$. We therefore have that

$$\frac{\omega_1 - \omega_2}{\omega_2} = 0.250 \tag{3.4}$$

and

$$\begin{aligned} m &= \frac{2}{5}(0.250)M \\ &= 0.100M. \end{aligned} \tag{3.5}$$

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4. An asteroid traveling in orbit about the earth has a perigee (i.e., the point at which it makes its closest approach to the earth) that is exactly equal to the radius of the earth R. Suppose that as the asteroid collides with our planet at the equator, it buries itself just below the surface. Just before the collision the asteroid's speed was $v = a\omega_1 R$, where a is a constant and ω_1 the angular speed of the earth about its rotation axis (also before the collision). Assume that the asteroid is very small compared to the earth and that the earth is uniform throughout.

a) What would have to be the mass m of the asteroid, in term of the earth's mass M and a, for the day to become 25.0% *shorter* than it presently is as a result of the collision?

b) Your answer in a) should imply that $a > 4/3$ for the day to become 25.0% shorter after the collision. Show this, and explain, with words only, why should one expect a lower limit for a for this to happen?

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5. (Prob. 10.88 in Young and Freedman.) A uniform, 0.0300-kg rod of length 0.400 m rotates in a horizontal plane about a fixed axis through its centre and perpendicular to the rod. Two small rings, each with mass 0.0200 kg, are mounted so that they can slide along the rod. They are initially held by catches located at 0.0500 m on each side of the centre of the rod, and the system is rotating at 30 rev/min. With no other changes in the system (i.e., the 'system' consists of the rod and the rings), the catches are released, and the rings slide outward along the rod and fly off at the ends.

a) What is the angular speed of the system at the moment the rings reach the ends of the rod?

b) What is the angular speed of the rod after the rings leave it?
