STUDENT NAME: ________________________________ STUDENT NUMBER: ______________

PHYSICS 1501A

Term Test No. 2

29 October 2012 P&AB 148 9:30 – 10:20 AM

INSTRUCTIONS:

- This is a CLOSED BOOK exam.
- You may use a calculator of any type (except on a smartphone).
- Round all numerical answers to **three significant figures**. (Round intermediate answers to no fewer than four significant figures, or keep them in storage registers, to avoid loss of accuracy.)
- Answer **BOTH** questions. Use **both** sides of the paper. Marks are shown in brackets.
- **EXPLAIN YOUR REASONING!** State the physical principles you are using. A correct final answer without showing the method will not be given credit.
- A numerical answer is not correct without **units** (unless it's a dimensionless quantity).
- You may request extra *scrap* paper if you need it, but write anything you want to hand in on the exam paper itself.
- If you don't understand the wording of a question, ask for clarification.
- **EXPLAIN YOUR REASONING!**
- Good luck!

USEFUL INFORMATION IS ON THE ***LAST*** PAGE!

1	2	Total

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1. A small, hard metal block with mass 0.800 kg is suspended from the lower end of a light cord that is 1.60 m long. The block is initially at rest. A bullet with a mass of 12.0 g is fired at the block, strikes it, and then rebounds with a speed of 100 m/s in a direction opposite to that of its initial velocity. After the collision the block swings on the end of the cord. When the block has risen a vertical height of 0.800 m, the tension in the cord is 4.80 N.

a) What was the initial speed v_0 of the bullet? [4 marks]

b) Show whether the collision between the bullet and the block was elastic or inelastic. [2 mark]

Solution.

a) The angle θ made by the string from the vertical when the block-bullet system has risen by 0.800 m is given by $\cos\theta = 0.8/1.6 = 0.5$, or $\theta = 60^\circ$. A consideration of the forces acting on the block at that time yields

$$T = m_\mathrm{b} g\cos\theta + m_\mathrm{b}\frac{v_\mathrm{b}^2}{R}, \tag{1.1}$$

where T is the tension in the string, m_b is the mass of the block, v_b its speed at that time, and R the length of the string. We transform equation (1.1) to

$$v_\mathrm{b}^2 = \frac{R}{m_\mathrm{b}}\left(T - m_\mathrm{b} g\cos\theta\right). \tag{1.2}$$

Although we haven't established that yet, we could be dealing with an inelastic collision. We should therefore first apply the principle of conservation of energy for the block for times just after the collision and later on. We therefore write, with V the speed of the block right after the collision,

$$\frac{1}{2}m_\mathrm{b}V^2 + m_\mathrm{b}gy_0 = \frac{1}{2}m_\mathrm{b}v_\mathrm{b}^2 + m_\mathrm{b}gy \tag{1.3}$$

or

$$
\begin{aligned}
V^2 &= v_b^2 + 2g\left(y - y_0\right) \\
&= \frac{R}{m_b}\left(T - m_b g\cos\theta\right) + 2g\left(y - y_0\right) \\
&= \frac{RT}{m_b} + 2g\left(y - y_0 - \frac{R}{2}\cos\theta\right) \\
&= \frac{1.60\ \text{m}\cdot 4.80\ \text{N}}{0.800\ \text{kg}} + 2\cdot 9.81\ \text{m/s}^2\left(0.800 - \frac{1.60}{4}\right)\text{m} \\
&= 17.4\ \text{m}^2/\text{s}^2.
\end{aligned}
\tag{1.4}
$$

We now apply the principle of conservation of linear momentum to times just before and right after the collision to get (with v_2 the speed of the block after the collision)

$$
m_{\text{bullet}} v_0 = m_{\text{bullet}} v_2 + m_b V,
\tag{1.5}
$$

or

$$
\begin{aligned}
v_0 &= v_2 + \frac{m_b V}{m_{\text{bullet}}} \\
&= -100\ \text{m/s} + \frac{0.800}{0.012} 4.17\text{m/s} \\
&= 178\ \text{m/s}.
\end{aligned}
\tag{1.6}
$$

b) The kinetic energy of the system K_1 before the collision is that of the bullet alone, since the block is then at rest

$$
\begin{aligned}
K_1 &= \frac{1}{2} m_{\text{bullet}} v_0^2 \\
&= 190\ \text{J}.
\end{aligned}
\tag{1.7}
$$

The kinetic energy of the system right after of the collision is

$$
\begin{aligned}
K_2 &= \frac{1}{2} m_{\text{bullet}} v_2^2 + \frac{1}{2} m_b V^2 \\
&= \frac{1}{2} 0.012\ \text{kg}\cdot 10{,}000\ \text{m}^2/\text{s}^2 + \frac{1}{2} 0.800\ \text{kg}\cdot 17.4\ \text{m}^2/\text{s}^2 \\
&= 67.0\ \text{J}.
\end{aligned}
\tag{1.8}
$$

The collision is clearly inelastic since $K_2 < K_1$.

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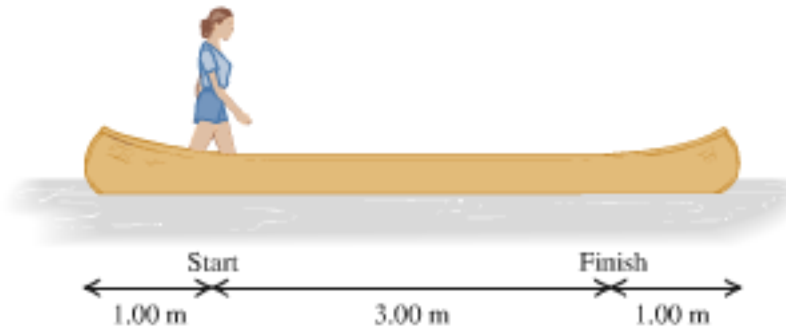
2. A 45.0-kg woman stands up in a 60.0-kg and 5.00-m long canoe. She walks from a point 1.00 m from the left end of the canoe to a point 1.00 m from the right end (see the figure below). If you ignore resistance to motion of the canoe in the water, then how far does the canoe move relative to someone stationary on the shore during this process and in what direction? [4 marks]



Solution.

The first thing to realize is that there is no net force acting on the woman-canoe system, which implies that the velocity of centre of mass of this system is zero ($v_{\mathrm{cm}} = 0$). We set x_{w} for the position of the woman and x_{c} for that of the centre (of mass) of the canoe, relative to some origin stationary relative to someone standing at rest on the shore. The centre of mass of the system is then initially given by

$$x_{\mathrm{cm},1} = \frac{m_{\mathrm{w}} x_{\mathrm{w1}} + m_{\mathrm{c}} x_{\mathrm{c1}}}{m_{\mathrm{w}} + m_{\mathrm{c}}}, \tag{2.1}$$

while after the woman has moved from the left to the right it becomes

$$x_{\mathrm{cm},2} = \frac{m_{\mathrm{w}} x_{\mathrm{w2}} + m_{\mathrm{c}} x_{\mathrm{c2}}}{m_{\mathrm{w}} + m_{\mathrm{c}}}. \tag{2.2}$$

But since the velocity of the centre of mass is zero at all times, equations (2.1) and (2.2) must equal and

$$m_{\mathrm{w}} x_{\mathrm{w1}} + m_{\mathrm{c}} x_{\mathrm{c1}} = m_{\mathrm{w}} x_{\mathrm{w2}} + m_{\mathrm{c}} x_{\mathrm{c2}}. \tag{2.3}$$

But we know that initially

$$x_{\mathrm{w1}} = x_{\mathrm{c1}} - 1.50 \text{ m}, \tag{2.4}$$

since the woman is then located 1.50 m to the left of the centre of the canoe, which measures a total of 5.00 m. Likewise, when she moves to the point 1.00 m from the other end of the canoe she finds herself 1.50 m to right of the centre of canoe. We therefore write

$$x_{\mathrm{w2}} = x_{\mathrm{c2}} + 1.50 \text{ m}, \tag{2.5}$$

and modify equation (2.3) to

$$\begin{aligned} x_{c2} - x_{c1} &= (-3.00\ \text{m}) \cdot \frac{m_w}{m_w + m_c} \\ &= -1.29\ \text{m}. \end{aligned} \tag{2.6}$$

The canoe therefore moves 1.29 m to the left.

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Useful Information

Feel free to tear this sheet off. ***You need not hand it in***.

$A_x = A\cos\theta$ $A_y = A\sin\theta$ $A = \sqrt{A_x^2 + A_y^2}$ $\theta = \tan^{-1}\left(A_y / A_x\right)$

$ax^2 + bx + c = 0$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$ $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$ $v^2 - v_0^2 = 2\mathbf{a}\cdot\left(\mathbf{r} - \mathbf{r}_0\right)$ (when $\mathbf{a} = \text{constant}$)

$\mathbf{F}_{\text{el}} = -kx\,\mathbf{e}_x$ $\mathbf{F}_{\text{grav}} = mg\,\mathbf{e}_y$ $f_k = \mu_k n$ $f_s \le \mu_s n$

$W = \mathbf{F}\cdot\left(\mathbf{r}_2 - \mathbf{r}_1\right) = K_2 - K_1$ $K = \frac{1}{2}mv^2$ $U_{\text{grav}} = mgy$ $U_{\text{el}} = \frac{1}{2}kx^2$

$K_1 + U_{\text{grav},1} + U_{\text{el},1} + W_{\text{other}} = K_2 + U_{\text{grav},2} + U_{\text{el},2}$ $W_{\text{other}} = \Delta K + \Delta U_{\text{grav}} + \Delta U_{\text{el}}$

$\mathbf{p} = m\mathbf{v}$ $\mathbf{p}_{A1} + \mathbf{p}_{B1} = \mathbf{p}_{A2} + \mathbf{p}_{B2}$ $\mathbf{r}_{\text{cm}} = \sum_{i=1}^{N} m_i \mathbf{r}_i \Big/ \sum_{i=1}^{N} m_i$ $\mathbf{F}_{\text{cm}} = \sum_{i=1}^{N} \mathbf{F}_{\text{ext},i} = \sum_{i=1}^{N} \dfrac{d\mathbf{p}_i}{dt}$

$\mathbf{J} = \mathbf{F}_{\text{net}}\,\Delta t = \mathbf{p}_2 - \mathbf{p}_1$ $\mathbf{v}_{\text{cm}} = \dfrac{d\mathbf{r}_{\text{cm}}}{dt}$ $\mathbf{a}_{\text{cm}} = \dfrac{d\mathbf{v}_{\text{cm}}}{dt} = \dfrac{d^2\mathbf{r}_{\text{cm}}}{dt^2}$

General constants:

$g = 9.81\ \text{m/s}^2$ $G = 6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$

$\text{m (milli)} = 10^{-3}$ $\mu\ \text{(micro)} = 10^{-6}$ $\text{n (nano)} = 10^{-9}$ $\text{p (pico)} = 10^{-12}$

$\text{k (kilo)} = 10^{3}$ $\text{M (mega)} = 10^{6}$ $\text{G (giga)} = 10^{9}$ $\text{T (tera)} = 10^{12}$

$1\ \text{cm} = 10^{-2}\ \text{m}$ $1\ \text{g} = 10^{-3}\ \text{kg}$