Set 44 Let $X$ be a discrete random variable, and let $A$ and $B$ be two disjoint (that is, $A \cap B = \emptyset$) and exhaustive (that is, $A \cup B = \Omega$) subsets of the sample space $\Omega$. Prove

$$\mathbb{E}[X] = \mathbb{E}[X|A]\mathbb{P}(A) + \mathbb{E}[X|B]\mathbb{P}(B)$$

The law of total expectation states that,

$$\mathbb{E}[X] = \sum_{i}^{n} \mathbb{E}[X|Y_i] * \mathbb{P}(Y_i)$$

Where $Y_1, Y_2, \dots, Y_n$ are portions of the whole space $\Omega$. Next, we know that $A$ and $B$ are the partitions that form $\Omega$, therefore,

$$\mathbb{E}[X] = \sum_{i}^{n} \mathbb{E}[X|Y_i] * \mathbb{P}(Y_i)$$

$$\mathbb{E}[X] = \mathbb{E}[X|Y_0] * \mathbb{P}(Y_0) + \mathbb{E}[X|Y_1] * \mathbb{P}(Y_1)$$

$$\mathbb{E}[X] = \mathbb{E}[X|A] * \mathbb{P}(A) + \mathbb{E}[X|B] * \mathbb{P}(B)$$