Set 45 Let X and Y be two discrete random variables. Prove that the expectation of the conditional expectation is the unconditional expectation, that is,

$$\mathbb{E}\big[\mathbb{E}[X|Y]\big] = \mathbb{E}[X]$$

$$\mathbb{E}\left[\sum_x x * \mathbb{P}(X = x|Y)\right]$$

$$\sum_y \left[\sum_x x * \mathbb{P}(X = x|Y = y)\right] * \mathbb{P}(Y = y)$$

$$\sum_y \sum_x x * \mathbb{P}(X = x|Y = y) * \mathbb{P}(Y = y)$$

$$\sum_x x \sum_y \mathbb{P}(X = x|Y = y) * \mathbb{P}(Y = y)$$

$$\sum_x x \sum_y \mathbb{P}(X = x|Y = y)$$

$$\sum_x x * \mathbb{P}(X = x)$$

$$\mathbb{E}[X]$$