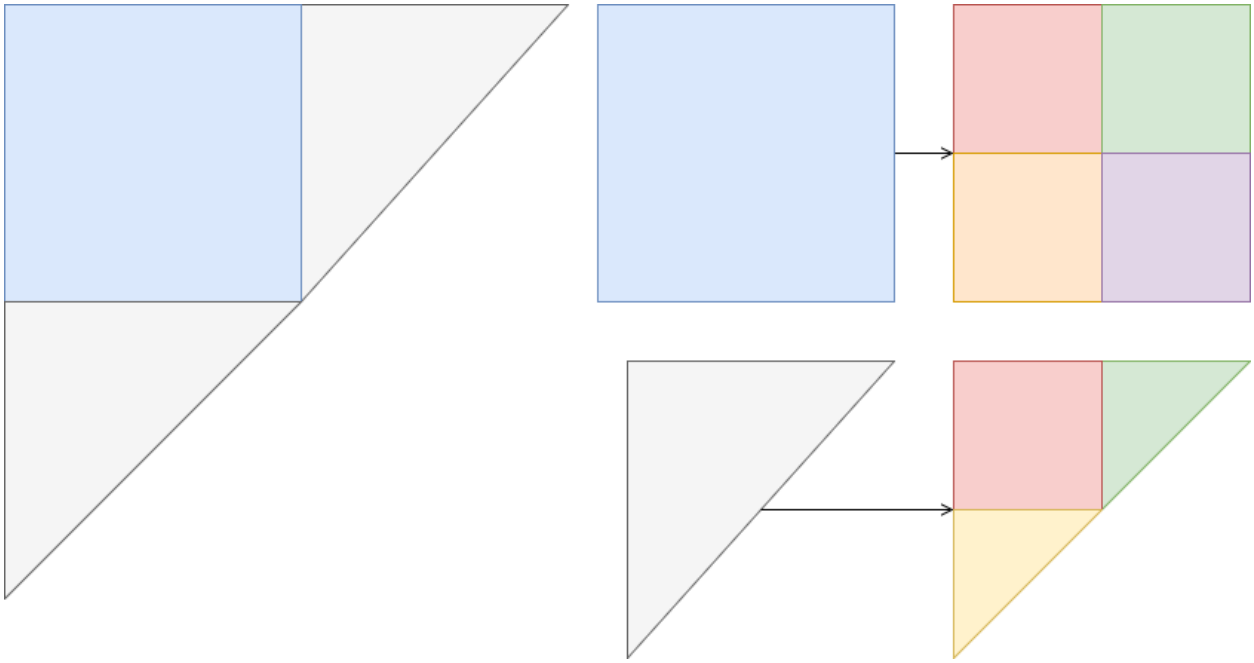
PROBLEM 2. In the chapter *Analysis of Multithreaded Algorithms*, we studied the 2-way and 3-way construction of a tableau.

Question 1. Describe in plain words, how to construct a tableau in a *k*-way fashion, for an arbitrary integer $k \geq 2$, using the same stencil (the one of the Pascal triangle construction) as in the lectures.

The Pascal stencil tableau can be constructed by dividing and conquering the elements of the Pascal Triangle. The structure of the initial case of the triangle is a half k by k table split diagonally through the middle and including the diagonal cells, from the locations $[0, n]$ to $[n, 0]$, where n is the order of the table. Each of the k^2 elements is solved recursively until a base case is reached. The edge elements use the same structure as the base case while the non-edge elements use a square tableau structure.

The following is a example of a Pascal Triangle with $k = 2$. With one recursive iteration.



Question 2. Determine the work and the span for an input square array of order n.

Work:

$$T_1 = k^2 \times T\left(\frac{n}{k}\right) + \theta(1)$$

Using the master theorem,

$$a = k^2, b = k, c = 1$$

Therefore,

$$T_1 = \theta\left(n^{\log_{k^{k^2}}}\right)$$

$$T_1 = \theta\left(n^{2 \times \log_{k^k}}\right)$$

$$T_1 = \theta(n^{2 \times 1})$$

$$T_1 = \theta(n^2)$$

Span:

$$T_\infty = (2 \times n - 1) \times T\left(\frac{n}{k}\right) + \theta(1)$$

Using the master theorem,

$$a = (2 \times n - 1), b = k, c = 1$$

Therefore,

$$T_\infty = \theta\left(n^{\log_{k^{2 \times n-1}}}\right)$$

$$T_\infty = \theta\left(n^{\log_{k^n}}\right)$$

Question 3. Realize a Julia or CilkPlus a multithreaded implementation off that algorithm. Collect running times (both serial and parallel) for increasing values of n (say consecutive powers of 2) and different values of k (at least 2 and 3).

Program could be found under src/pascal
Make command: make
Run command: ./pascal n k, where n = k^x

K	N		TIME
2	32	Serial	0m0.006s
		Parallel	0m0.013s
	256	Serial	0m0.009s
		Parallel	0m0.013s
	2048	Serial	0m0.050s
		Parallel	0m0.084s
	4096	Serial	0m0.174s
		Parallel	0m0.294s
	8192	Serial	0m0.681s
		Parallel	0m1.111s
	16384	Serial	0m2.646s
		Parallel	0m4.588s
	32768	Serial	0m11.206s
		Parallel	0m18.049s
3	9	Serial	0m0.008s
		Parallel	0m0.012s
	81	Serial	0m0.007s
		Parallel	0m0.011s
	729	Serial	0m0.012s
		Parallel	0m0.020s
	6561	Serial	0m0.278s
		Parallel	0m0.611s
8	64	Serial	0m0.007s
		Parallel	0m0.012s
	4096	Serial	0m0.118s
		Parallel	0m0.266s
	32768	Serial	0m7.468s
		Parallel	0m15.083s
32	1024	Serial	0m0.050s
		Parallel	0m0.036s
	32768	Serial	0m7.606s
		Parallel	0m14.105s