Assignment #5

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1. Use **formal deduction** to prove the validity of the following argument:

Premise 1: $\forall x(P(x) \to Q(x))$
Premise 2: $\exists x(R(x) \land \neg Q(x))$
Premise 3: $\forall x(R(x) \to P(x) \lor S(x))$
Conclusion: $\exists x(R(x) \land S(x))$

Let $\sum = \forall x\big(P(x) \to Q(x)\big), \exists x\big(R(x) \land \neg Q(x)\big), \forall x(R(x) \to P(x) \lor S(x))$

Add $R(u), \neg Q(u)$ to the set of premises.

1. $\sum, R(u), \neg Q(u) \vdash \forall x(P(x) \to Q(x))$	$(\in)$
2. $\sum, R(u), \neg Q(u) \vdash P(u) \to Q(u)$	$(-\forall, 1)$
3. $\sum, R(u), \neg Q(u) \vdash \neg Q(u)$	$(\in)$
4. $\sum, R(u), \neg Q(u) \vdash \neg Q(u) \to \neg P(u)$	$(contra.pos., 3)$
5. $\sum, R(u), \neg Q(u) \vdash \neg P(u)$	$(-\to, 4)$
6. $\sum, R(u), \neg Q(u) \vdash \forall x(R(x) \to P(x) \lor S(x))$	$(\in)$
7. $\sum, R(u), \neg Q(u) \vdash R(u) \to P(u) \lor S(u)$	$(-\forall, 6)$
8. $\sum, R(u), \neg Q(u) \vdash P(u) \lor S(u)$	$(-\to, 7)$
9. $\sum, R(u), \neg Q(u) \vdash S(u)$	$(Dis.Solly., 8)$
10. $\sum, R(u), \neg Q(u) \vdash R(u)$	$(\in)$
11. $\sum, R(u), \neg Q(u) \vdash R(u) \land S(u)$	$(+\land, 9, 10)$
12. $\sum, R(u), \neg Q(u) \vdash \exists x(R(x) \land S(x))$	$(+\exists, 11)$
13. $\sum \vdash R(u) \land \neg Q(u)$	$(+\lor, 3, 10)$
14. $\sum \vdash \exists x(R(x) \land \neg Q(x))$	$(+\exists, 13)$

Therefore, $\sum \vdash \exists x(R(x) \land S(x))$.

2. Use **resolution for propositional calculus with the set of support** strategy to show that the following argument is valid:

Premise 1: $\neg R \rightarrow S \equiv R \vee S$
Premise 2: $R \rightarrow P \equiv \neg R \vee P$
Premise 3: $P \vee S \rightarrow Q \equiv (\neg P \wedge \neg S) \vee Q \equiv (\neg P \vee Q) \wedge (\neg S \vee Q)$
Conclusion: Q

1. $R \vee S$
2. $\neg R \vee P$
3. $\neg P \vee Q$
4. $\neg S \vee Q$
5. $\neg Q$
6. $S \vee P$ — Resolve R: $(1, 2)$
7. $S \vee Q$ — Resolve P: $(6, 3)$
8. Q — Resolve S: $(7, 4)$
9. $\emptyset$ — Resolve Q: $(8, 5)$

Contradiction reached by resolution, the argument is valid.

3. Consider that by translating an argument into the language of propositional calculus and by adding the negation of the conclusion to the set of premises we obtained the set S of clauses.

$\{Q, \neg S\}$	$\{\neg Q, T\}$	$\{\neg Q, P\}$	$\{Q, R, T\}$
$\{\neg Q, \neg S\}$	$\{\neg R, \neg P\}$	$\{\neg R, S, P\}$	$\{Q, S, \neg T\}$
$\{S, \neg T, \neg P\}$			

Apply the **Davis-Putnam procedure** to find or whether or not the original argument was valid i.e. whether or not the set S is satisfiable. Show in detail all the intermediary steps. In particular, for each elimination of a variable, show which are the sets S_i, S_i', T_i and U_i. For each resolvent indicate what the parent clauses are. Eliminate the variables in the order Q, R, S, T, P.

$S = \{Q, \neg S\}^1, \{\neg Q, T\}^2, \{\neg Q, P\}^3, \{Q, R, T\}^4, \{\neg Q, \neg S\}^5, \{\neg R, \neg P\}^6, \{\neg R, S, P\}^7,$
$\{Q, S, \neg T\}^8, \{S, \neg T, \neg P\}^9$

$S_1 =$ all clauses 1-9
$S_1' = S_1$
$T_1 = \{Q, \neg S\}^1, \{\neg Q, T\}^2, \{\neg Q, P\}^3, \{Q, R, T\}^4, \{\neg Q, \neg S\}^5, \{Q, S, \neg T\}^8$

Eliminate Q:

$U_1 = \{T, \neg S\}^{1,2}, \{P, \neg S\}^{1,3}, \{\neg S\}^{1,5}, \{R, T\}^{2,4}, \cancel{\{S, T, \neg T\}^{2,8}}, \{R, T, P\}^{3,4}, \{S, P, \neg T\}^{3,8},$
$\{\neg S, R, T\}^{4,5}, \cancel{\{\neg S, \neg S, \neg T\}^{5}}$

$S_2 = (S_1 - T_1) \vee U_1$

$$S_2{}' = \{\neg R, \neg P\}^1,\ \{\neg R, S, P\}^2, \{S, \neg T, \neg P\}^3, \{T, \neg S\}^4, \{P, \neg S\}^5, \{\neg S\}^6, \{R, T\}^7,$$
$$\{R, T, P\}^8, \{S, P, \neg T\}^9, \{\neg S, R, T\}^{10}$$
$$T_2 = \{\neg R, \neg P\}^1,\ \{\neg R, S, P\}^2, \{R, T\}^7, \{R, T, P\}^8, \{\neg S, R, T\}^{10}$$

Eliminate R:

$$U_2 = \{T, \neg P\}^{1,7}, \cancel{\{T, P, \neg P\}^{1,8}}, \{T, \neg S, \neg P\}^{1,10},\ \{T, S, P\}^{2,7}, \cancel{\{T, S, P\}^{2,8}}, \cancel{\{\neg S, S, T, P\}^{2,10}}$$

$$S_3 = (S_2 - T_2) \vee U_2$$
$$S_3{}' = \{S, \neg T, \neg P\}^1, \{T, \neg S\}^2, \{P, \neg S\}^3, \{\neg S\}^4, \{S, P, \neg T\}^5, \{T, \neg P\}^6, \{T, \neg S, \neg P\}^7,\ \{T, S, P\}^8$$
$$T_3 = \{S, \neg T, \neg P\}^1, \{T, \neg S\}^2, \{P, \neg S\}^3, \{\neg S\}^4, \{S, P, \neg T\}^5, \{T, \neg S, \neg P\}^7,\ \{T, S, P\}^8$$

Eliminate S:

$$U_3 = \cancel{\{T, \neg T, \neg P\}^{1,2}}, \cancel{\{P, \neg T, \neg P\}^{1,3}}, \{\neg T, \neg P\}^{1,4}, \cancel{\{T, \neg T, \neg P\}^{1,7}}, \cancel{\{T, \neg T, P\}^{2,5}}, \{T, P\}^{2,8}$$
$$\{P, \neg T\}^{3,5}, \cancel{\{P, T\}^{3,8}}, \cancel{\{\neg T, P\}^{4,5}}, \cancel{\{T, P\}^{4,8}}, \cancel{\{\neg P, T, P, \neg T\}^{5,7}}, \cancel{\{T, P, \neg P\}^{7,8}}$$

$$S_4 = (S_3 - T_3) \vee U_3$$
$$S_4{}' = \{T, \neg P\}^1, \{\neg T, \neg P\}^2, \{T, P\}^3, \{P, \neg T\}^4$$
$$T_4 = \{T, \neg P\}^1, \{\neg T, \neg P\}^2, \{T, P\}^3, \{P, \neg T\}^4$$

Eliminate T:

$$U_4 = \{\neg P\}^{1,2}, \cancel{\{\neg P, P\}^{1,4}}, \cancel{\{P, \neg P\}^{2,3}}, \{\ P\}^{3,4}$$

$$S_5 = (S_4 - T_4) \vee U_4$$
$$S_5{}' = \{\neg P\}^1, \{P\}^2$$
$$T_5 = \{\neg P\}^1, \{P\}^2$$

Eliminate T:

$$U_5 = \{\emptyset\}^{1,2}$$

Contradiction, since $U_5 = \{\emptyset\}$, argument is valid.

4. Prove the validity of the following argument by using **resolution for predicate calculus**. Use the method described in class.

Premise 1: $\exists x[P(x) \wedge \forall y(Q(y) \rightarrow R(x,y))]$
Premise 2: $\forall x \forall y(P(x) \wedge R(x,y) \rightarrow S(y))$
Premise 3: $\forall x(S(x) \rightarrow Q(x))$
Conclusion: $\forall x(Q(x) \leftrightarrow S(x))$

Premise 1: $\exists x \forall y[P(x) \wedge (\neg Q(y) \vee R(x,y))] \equiv P(a) \wedge (\neg Q(y) \vee R(a,y))$
Premise 2: $\forall x \forall y(\neg P(x) \vee \neg R(x,y) \vee S(y)) \equiv \neg P(x) \vee \neg R(x,y) \vee S(y)$
Premise 3: $\forall x(\neg S(x) \vee Q(x)) \equiv \neg S(x) \vee Q(x)$
Conclusion: $\forall x(Q(x) \wedge S(x) \vee \neg S(x) \wedge \neg Q(x))$
$\neg$*Conclusion*: $\exists x(\neg Q(x) \vee \neg S(x) \wedge S(x) \vee Q(x)) \equiv \neg Q(b) \vee \neg S(b) \wedge S(b) \vee Q(b)$

1. $P(a)$	
2. $\neg Q(y) \vee R(a,y)$	
3. $\neg P(x) \vee \neg R(x,y) \vee S(y)$	
4. $\neg S(x) \vee Q(x)$	
5. $\neg Q(b) \vee \neg S(b)$	
6. $S(b) \vee Q(b)$	
7. $\neg P(a) \vee \neg R(a,y) \vee S(y)$	$x = b$ (3)
8. $\neg P(a) \vee S(y) \vee \neg Q(y)$	Resolve $R(a,y)$: $(7,2)$
9. $S(y) \vee \neg Q(y)$	Resolve $P(a)$: $(8,1)$
10. $S(b) \vee \neg Q(b)$	$y = b$ (9)
11. $S(b)$	Resolve $Q(b)$: $(10,6)$
12. $\neg S(b) \vee Q(b)$	$x = b$ (4)
13. $\neg S(b)$	Resolve $Q(b)$: $(12,5)$
14. $\emptyset$	Resolve $S(b)$: $(13,11)$

Contradiction, therefore, the argument is valid.