Set 41 Let X have the uniform on $[-1,1]$ density, and let Y be another random variable given by the equation $Y = X^2$. Hence, the value of Y is completely determined by the value of X. Are the two random variables X and Y correlated or uncorrelated? Prove your answer.

$$\operatorname{corr}(X,Y) = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y}$$

$$\operatorname{cov}(X,Y) = \operatorname{cov}(X,X^2)$$

$$\mathbb{E}[(X - \mathbb{E}[X])(X^2 - \mathbb{E}[X^2])]$$

$$\mathbb{E}\left[X^3 - X\mathbb{E}[X^2] - X^2\mathbb{E}[X] + \mathbb{E}[X]\mathbb{E}[X^2]\right]$$

$$\mathbb{E}[X^3] + \mathbb{E}\left[-X\mathbb{E}[X^2]\right] + \mathbb{E}\left[-X^2\mathbb{E}[X]\right] + \mathbb{E}\left[\mathbb{E}[X]\mathbb{E}[X^2]\right]$$

$$\mathbb{E}[X^3] - \mathbb{E}[X]\mathbb{E}[X^2] - \mathbb{E}[X]\mathbb{E}[X^2] + \mathbb{E}[X]\mathbb{E}[X^2]$$

$$\mathbb{E}[X^3] - \mathbb{E}[X]\mathbb{E}[X^2]$$

$$\int_{-1}^{1} x^3 dx - \int_{-1}^{1} x^2 dx * \int_{-1}^{1} x^3 dx = 0$$

Therefore, since the covariance is 0, then the correlation is also 0. Hence, uncorrelated.