STUDENT NAME: ________________________________ STUDENT NUMBER: ______________

PHYSICS 1501A

Term Test No. 1

5 October 2012 P&AB 148 9:30 – 10:20 AM

INSTRUCTIONS:

- This is a CLOSED BOOK exam.
- You may use a calculator of any type (except on a smartphone).
- Round all numerical answers to at most **three significant figures**. (Round intermediate answers to no fewer than four significant figures, or keep them in storage registers, to avoid loss of accuracy.)
- Answer **BOTH** questions. Use **both** sides of the paper. Marks are shown in brackets.
- **EXPLAIN YOUR REASONING!** State the physical principles you are using. A correct final answer without showing the method will not be given credit.
- A numerical answer is not correct without **units** (unless it's a dimensionless quantity).
- You may request extra *scrap* paper if you need it, but write anything you want to hand in on the exam paper itself.
- If you don't understand the wording of a question, ask for clarification.
- **EXPLAIN YOUR REASONING!**
- Good luck!

USEFUL INFORMATION IS ON THE ***LAST*** PAGE!

1	2	Total

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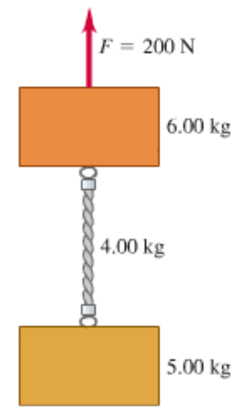
1. The two blocks in the figure on the right are connected by a heavy uniform rope with a mass of 4.00 kg. An upward force of 200 N is applied as shown.



a) Draw three free-body diagrams: one for the 6.00-kg block, one for the 4.00-kg rope, and another for the 5.00-kg block. For each force indicate what body exerts that force. [1.5 mark]

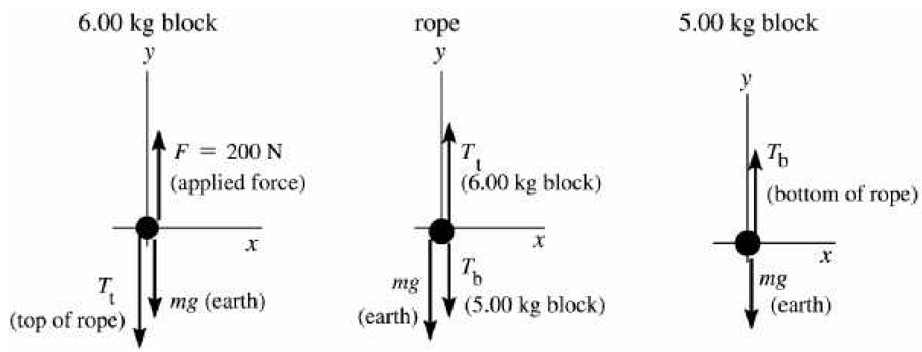
b) Show that the acceleration of the system equals $3.53\ \text{m/s}^2$. [1 mark]

c) Show that the tension at the top of the heavy rope equals 120 N. [1 mark]

d) What is the tension at the midpoint of the rope? [1.5 mark]

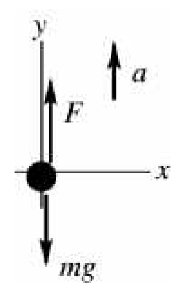
Solution.

a) The free-body diagram are as follows



with T_t and T_b the tensions at the top and bottom of the rope.

b) We treat the two blocks and the rope as a single system with a total mass of 15.0 kg. If we take $+y$ as upward, then the free-body diagram is shown on the right. Newton's Second Law then specifies that



$$\sum F_y = F - m_{\text{tot}} g = m_{\text{tot}} a, \tag{1.1}$$

or

$$\begin{aligned} a &= \frac{F}{m_{\text{tot}}} - g \\ &= \frac{200\ \text{N}}{15.0\ \text{kg}} - 9.81\ \text{m/s}^2 = 3.53\ \text{m/s}^2. \end{aligned} \tag{1.2}$$

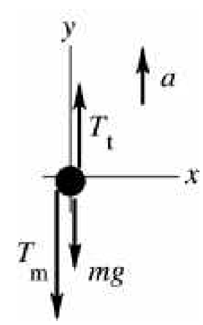
c) Using the free-body diagram for the 6.00 kg block we can write

$$\sum F_y = F - m_1 g - T_\mathrm{t} = m_1 a, \tag{1.3}$$

or

$$\begin{aligned} T_\mathrm{t} &= F - m_1(g+a) \\ &= 200\ \mathrm{N} - 6.00\ \mathrm{kg}\cdot(9.81+3.53)\ \mathrm{m/s^2} \\ &= 120\ \mathrm{N}. \end{aligned} \tag{1.4}$$

d) We can consider the top half of the rope (with a mass m_h of 2.00 kg) with T_m the tension at its midpoint. This gives us the free-body diagram on the right. We then write



$$\sum F_y = T_\mathrm{t} - m_\mathrm{h} g - T_m = m_\mathrm{h} a, \tag{1.5}$$

or

$$\begin{aligned} T_\mathrm{m} &= T_\mathrm{t} - m_\mathrm{h}(g+a) \\ &= 120\ \mathrm{N} - 2.00\ \mathrm{kg}\cdot(9.81+3.53)\ \mathrm{m/s^2} \\ &= 93.3\ \mathrm{N}. \end{aligned} \tag{1.6}$$

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2. We want to slide a 12.0-kg crate up a 2.50-m ramp inclined at 30°. A worker, ignoring friction, calculated that he could do this by giving it an initial speed of 5.00 m/s at the bottom and letting go. But friction is *not* negligible; the crate slides only 1.6 m up the ramp, stops, and slides back down.

a) Use the work-energy theorem and show that the magnitude of the friction force acting on the crate, assuming that it is constant, is 34.9 N. [2.5 marks]
b) How fast is the crate moving when it reached the bottom of the ramp? [2.5 marks]

Solution.

a) If we denote by 1 and 2 the points at the bottom of the ramp when the worker gives the initial push and the top of the ramp after the crate stops, respectively, then we can write the following

$$\begin{aligned} K_1 &= \frac{1}{2}mv_1^2 = \frac{1}{2}(12.0\text{ kg})\left(5.0\text{ m/s}^2\right)^2 = 150\text{ J} \\ \mathrm{U}_{\text{grav},1} &= 0 \\ K_2 &= \frac{1}{2}mv_2^2 = 0 \\ U_{\text{grav},2} &= mgy_2 = (12.0\text{ kg})\left(9.81\text{ m/s}^2\right)(0.80\text{ m}) = 94.2\text{ J} \\ \mathrm{W}_{\text{other}} &= -fs, \end{aligned} \tag{2.1}$$

where f is the constant friction force and $s = 1.6\text{ m}$. The work-energy theorem then states that

$$\begin{aligned} K_1 + U_{\text{grav},1} + W_{\text{other}} &= K_1 + U_{\text{grav},1} \\ W_{\text{other}} &= -fs \\ &= \left(K_2 + U_{\text{grav},2}\right) - \left(K_1 + U_{\text{grav},1}\right) \\ &= -55.8\text{ J}. \end{aligned} \tag{2.2}$$

We therefore find that

$$f = -\frac{W_{\text{other}}}{s} = \frac{55.8\text{ J}}{1.60\text{ m}} = 34.9\text{ N}. \tag{2.3}$$

b) We denote by 3 the point at the bottom of the ramp after the crate has slid back. The friction force does the same amount of negative work between point 2 and 3 as it did between points 1 and 2 (i.e., -55.8 J). We then write

$$
\begin{aligned}
K_2 + U_{\text{grav},2} + W_{\text{other}} &= K_3 + U_{\text{grav},3} \\
W_{\text{other}} &= -fs \\
&= \left(K_3 + U_{\text{grav},3}\right) - \left(K_2 + U_{\text{grav},2}\right) \\
&= K_3 + U_{\text{grav},3} - U_{\text{grav},2} \\
&= \frac{1}{2} m v_3^2 + mg\left(y_3 - y_2\right) \\
&= -55.8\ \text{J}.
\end{aligned} \tag{2.4}
$$

Since $y_3 = 0$ and $y_2 = s \cdot \sin\left(30^\circ\right) = 0.80\ \text{m}$, we can write

$$
\begin{aligned}
v_3 &= \sqrt{2\left(g y_2 - \frac{fs}{m}\right)} \\
&= \sqrt{2\left[\left(9.81\ \text{m/s}^2\right)\left(0.80\ \text{m}\right) - \frac{\left(34.9\ \text{N}\right)\left(1.60\ \text{m}\right)}{12.0\ \text{kg}}\right]} \\
&= 2.52\ \text{m/s}.
\end{aligned} \tag{2.5}
$$

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Useful Information

Feel free to tear this sheet off. ***You need not hand it in***.

Vector components in two dimensions, magnitude, and orientation

$A_x = A\cos\theta \qquad A_y = A\sin\theta \qquad A = \sqrt{A_x^2 + A_y^2} \qquad \theta = \tan^{-1}\left(A_y / A_x\right)$

Quadratic equation and its roots

$ax^2 + bx + c = 0 \qquad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Constant acceleration in one dimension (along x; x_0 and v_0 are the initial position and velocity)

$v_x = v_{0x} + a_x t \qquad x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \qquad v_x^2 - v_{0x}^2 = 2a_x\left(x - x_0\right)$

Spring restoring force, gravity, and kinetic and static friction forces

$\mathbf{F}_{\text{el}} = -kx\,\mathbf{e}_x \qquad \mathbf{F}_{\text{grav}} = mg\,\mathbf{e}_y \qquad f_k = \mu_k n \qquad f_s \le \mu_s n$

Work done by a constant force and the work-energy theorem

$W_{\text{other}} = \mathbf{F}_{\text{other}} \cdot \left(\mathbf{r}_2 - \mathbf{r}_1\right) = \left(K_2 - K_1\right) + \left(U_{\text{grav},2} - U_{\text{grav},1}\right)$

General constants:

$g = 9.81 \text{ m/s}^2 \qquad G = 6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$

$\text{m (milli)} = 10^{-3} \qquad \mu \text{ (micro)} = 10^{-6} \qquad \text{n (nano)} = 10^{-9} \qquad \text{p (pico)} = 10^{-12}$

$\text{k (kilo)} = 10^{3} \qquad \text{M (mega)} = 10^{6} \qquad \text{G (giga)} = 10^{9} \qquad \text{T (tera)} = 10^{12}$

$1 \text{ cm} = 10^{-2} \text{ m} \qquad 1 \text{ g} = 10^{-3} \text{ kg}$