27 c) $R_n(t) = \dfrac{\#(t_i > t)}{n}$ Bumperical reliability function $= \bar{X}$

$$= \frac{\sum_{i=1}^{n} I\{t_i > t\}}{n} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$R(10) = P(T > 10)$

$= E[I_{T>10}]$

$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} I[t_i > 10\}$

Confidence interval

$R(10) = R_n(10) \pm 1.96\frac{\sigma}{\sqrt{n}}$

$\mu_x \qquad \bar{x}$

$\sigma = \sqrt{p(1-p)}$

$\downarrow$

$P(T>10)$

36) $\dfrac{\bar{x} - E[x]}{\sqrt{\dfrac{\sigma_x^2}{n}}} \approx G \quad 1.96 \quad 0.95$

$P[-1.96 \leq \Downarrow \leq 1.96]$

1	X_1	Y_1
2	X_2	Y_2
$\vdots$	$\vdots$	$\vdots$
n	X_n	Y_n

X and Y dependent

$\mu_{X-Y} = E[X-Y]$

$= E[X] - E[Y]$

$Z := X - Y$

$\bar{z} = \frac{1}{n}\sum_i^n z_i$

$\sigma_z^2 = \frac{1}{n}\sum_{i=1}^n (z_i - \bar{z})^2 \leftarrow \hat{\sigma}_z$

$\bar{z} \pm 1.96 \dfrac{\hat{\sigma}_z}{\sqrt{n}}$

$E[Z] = \quad E(X] - E[Y]) + \quad \bar{x} - \bar{y} -$

$\sqrt{\dfrac{\sigma_x^2}{n_1} + \dfrac{\sigma_y^2}{n_2}}$

$\sigma_y^2 = 2\sum_{x_i - x_i}^2$

$$\frac{(\bar{x} - \bar{y}) - (E[X] - E[Y])}{\sqrt{\dfrac{\sigma_x^2}{n_1}}\sqrt{\dfrac{\sigma_x^2}{n_2}}}$$

CI for $E[X] - E[Y]$ is $\quad \bar{x} - \bar{y} \pm 1.96\sqrt{\dfrac{\hat{\sigma}_x^2}{n_1} + \dfrac{\hat{\sigma}_y^2}{n_2}}$

3b continue

$$\sigma_{X-Y}^2 = E\left[\left((X-Y) - (E[X] - E[Y])\right)^2\right]$$

$$= E\left[\{(X - E[X]) - (Y - E(Y)\}^2\right]$$

$$= \sigma_X^2 - 2\,\underbrace{cov(X,Y)}_{=0} + \sigma_Y^2$$

when independent/uncorelated

Stats

42) $E[X] = E[X \cdot 1]$ $\quad 1 = I_A + I_c$ $\quad I_A(w) = \begin{cases} 1 & w \in A \\ 0 & w \notin A \end{cases}$

$$E[X] = E[X I_A] + E[X I_c]$$

$$= \boxed{\frac{E[X I_A]}{P(A)}} P(A) + \frac{E[X I_c]}{P(c)} P(c)$$

$E[X|A]$

43) $S_N = \sum_{i=1}^{N} X_i$ $\quad E[X] = \sum_i E[X|A_i] P(A_i)$

$$E[S_N] = E\left[\sum_{i=1}^{N} X_i\right] = \sum E[S_N | N=i] P(N=i)$$

$$= \sum_i E[S_i | N=i] P(N=i)$$

$S_i = X_1 + X_2 + \dots + X_i$

$$= \sum_i \left(\underset{\uparrow\, 500}{E[X_1]} + \dots \underset{\uparrow\, 500}{E[X_i]} \right) P(N=i)$$

$500 \Rightarrow 200$

$$= \sum (500\, x_i) P(N=i)$$

$$= 500 \sum i\, P(N=i)$$

$$= 500\, \underbrace{E[N]}_{\Rightarrow 500}$$

Stats Lec

30 a) $\mu = E[X]$

$\hat{u} = \frac{1}{n}\sum^{n} x_i$

$P\left(|\bar{X} - \mu| > \sqrt{20}\frac{\sigma}{\sqrt{n}}\right) \leq 0.05$

$z \qquad \delta$

$P(|z| > \delta) \leq \frac{E[Z^2]}{\delta^2}$

$\frac{E[|\bar{x} - \mu|^2]}{\frac{20\sigma}{n}} = \frac{\frac{\sigma^2}{n}}{\frac{20\sigma^2}{n}} = \frac{1}{20}$

$\sigma^2_{\bar{x}}$

b) $P(A) \leq 0.05$

$P(\bar{A}) \geq 0.95$

$P\left(|\bar{X} - \mu| \leq \sqrt{20}\frac{\sigma}{\sqrt{n}}\right) \geq 0.95$

margin of error

$\mu - ME \leq \bar{X} \leq \mu + ME$

$\bar{X} - ME \leq \mu \leq \bar{X} + ME$

31) $P\left(|\bar{X} - \mu| > ?\frac{\sigma}{n}\right) = 0.05$

$P\left(\frac{|\bar{X} - \mu|}{\frac{\sigma}{\sqrt{n}}} > ?\right) \approx 0.05 \approx P(|G| > ?)$

$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \approx G$

$P(|G| \leq ?) = 0.95$

$P(-? \leq G \leq ?) = 0.95$

$= \int_{-?}^{?} f(x)\,dx = 0.95$

$a = 1.96$

42) $E[T] = E[T|A]\,\frac{5}{20} + E[T|B]\,\frac{15}{20}$

$\qquad\qquad\qquad 6 \qquad\qquad\qquad\qquad 4$