Set 37 Suppose that instead of using ten same cars as in Set 36, you are now dealing with a taxi fleet that is sufficiently large to allow you to select a number of cars, say 10, whose unmodified roofs are fitted with solar panels, and also select a number of cars, say 15, whose modified roofs are fitted with solar panels. Hence, the whole experiment can now be run within one week, instead of two as in Set 36, and so you obtain two sets of data at the same time: one has 10 and another 15 observations (gas-free mileage recordings). Construct a 95% (asymptotic) confidence interval for the difference between the (population) average gas-free mileage under the original and modified roofs.

Let X be an RV that determines the average gas free mileage under the original roof. Let Y be an RV that determines the average gas free mileage under the modified roof.

$$\bar{x} = \mathbb{E}[X], \bar{y} = \mathbb{E}[Y]$$

$$\hat{\sigma}_x = \sqrt{\mathbb{E}[(\bar{x} - x)^2]}, \hat{\sigma}_y = \sqrt{\mathbb{E}[(\bar{y} - y)^2]}$$

Then,

$$\bar{x} - \bar{y} - 1.96\left(\frac{\hat{\sigma}_x}{\sqrt{n_x}} + \frac{\hat{\sigma}_y}{\sqrt{n_y}}\right) \le \mu \le \bar{x} - \bar{y} + 1.96\left(\frac{\hat{\sigma}_x}{\sqrt{n_x}} + \frac{\hat{\sigma}_y}{\sqrt{n_y}}\right)$$

$$\bar{x} - \bar{y} - 1.96\left(\frac{\hat{\sigma}_x}{\sqrt{10}} + \frac{\hat{\sigma}_y}{\sqrt{15}}\right)\mu \le \bar{x} - \bar{y} + 1.96\left(\frac{\hat{\sigma}_x}{\sqrt{10}} + \frac{\hat{\sigma}_y}{\sqrt{15}}\right)$$

Therefore, the mean difference confidence interval is

$$\left[\bar{x} - \bar{y} - 1.96\left(\frac{\hat{\sigma}_x}{\sqrt{10}} + \frac{\hat{\sigma}_y}{\sqrt{15}}\right), \bar{x} - \bar{y} + 1.96\left(\frac{\hat{\sigma}_x}{\sqrt{10}} + \frac{\hat{\sigma}_y}{\sqrt{15}}\right)\right]$$