Set 43 Suppose that you own a grocery store. Let N denote the number of shoppers who visit your grocery store, and let X_n (where $n = 1, 2 \ldots$) denote the amount of money that the n-th shopper spends during one week. Based on historical data, you know that each shopper spends, in average, \$200 in your store during one week, and the average number of shoppers who visit your store during one week is 500. Let S_N be the total sum of money spent by all the N shoppers during one week. Assume that the number of shoppers N and the amounts X_n, $n \geq 1$, of money that they spend are independent. What is the average $\mathbb{E}[S_N]$ of the total amount of money S_N spent during one week at your grocery store?

$$S_N = \sum_{i=1}^{n} X_i$$

$$\mathbb{E}[S_N] = \mathbb{E}\left[\sum_{i=1}^{n} X_i\right]$$

$$\mathbb{E}[S_N] = \sum_{i=1}^{n} \mathbb{E}[X_i]$$

Since,

$$\mathbb{E}[X_i] = \mathbb{E}[S_i | N = i]\mathbb{P}(N = i)$$

Then,

$$\mathbb{E}[S_N] = \sum_{i=1}^{n} \mathbb{E}[S_i | N = i] * \mathbb{P}(N = i)$$

We know that the average customer spends \$200,

$$\mathbb{E}[S_N] = \sum_{i=1}^{n} 200 * \mathbb{P}(N = i)$$

$$\mathbb{E}[S_N] = 200 \sum_{i=1}^{n} \mathbb{P}(N = i)$$

$$\mathbb{E}[S_N] = 200 * 500$$

$$\mathbb{E}[S_N] = 100{,}000$$