Physics 1501A – 3rd Assignment

<u>Your solution to problems 1, 4, and 5 has to be handed in, in class, on Wednesday, December 5, 2012.</u>

1. (Prob. 10.26 in Young and Freedman.) A bowling ball rolls without slipping up a ramp that slopes upward at an angle β to the horizontal. Treat the ball as uniform solid sphere, ignoring the finger holes.

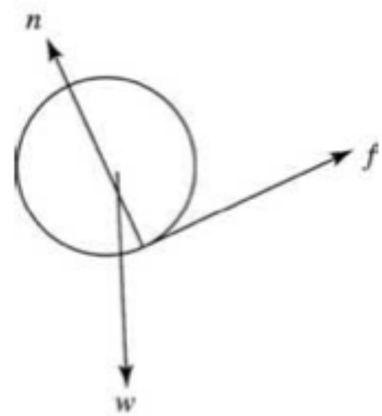
a) Draw the free-body diagram for the ball. Explain why the friction force must be directed uphill.

b) What is the acceleration of the centre of mass of the ball?

c) What minimum coefficient of static friction is needed to prevent slipping?

Solution

a) The free-body diagram is as shown in the figure on the right. Since the angular speed of the ball must decrease as it rolls up the ramp (i.e., to the right on the figure), the torque provided by the friction force must act in a counter-clockwise manner (the axis of rotation is defined as coming out of the page). This can only be so if the friction force is directed uphill.



b) The net force acting on the centre of mass of the sphere along an axis directed uphill along the ramp is given by

$$f - mg\sin(\beta) = ma_{\text{cm}}. \tag{1.1}$$

But the acceleration of the centre of mass is related to the angular acceleration through $a_{\text{cm}} = -\alpha R$, with R the radius of the sphere (the negative sign is due to the fact that a_{cm} is positive uphill and α positive counter-clockwise), and then to the friction force via the torque with

$$\begin{aligned}\tau &= fR \\ &= I\alpha \\ &= -\left(\frac{2}{5}mR^2\right)\left(\frac{a_{\text{cm}}}{R}\right).\end{aligned} \tag{1.2}$$

Combining equations (1.1) and (1.2) yields

$$-\frac{2}{5}ma_{\text{cm}} - mg\sin(\beta) = ma_{\text{cm}} \tag{1.3}$$

or

$$a_{\text{cm}} = -\frac{5}{7} g \sin(\beta). \tag{1.4}$$

The negative sign verifies that the ball is slowing down as it proceeds up the ramp.

c) From equation (1.2) the friction force is

$$\begin{aligned} f &= -\frac{2}{5} m a_{\text{CM}} \\ &= \frac{2}{7} mg \sin(\beta), \end{aligned} \tag{1.5}$$

which must be smaller or equal to the product of the coefficient of static friction and the normal force. That is,

$$\begin{aligned} \mu_{\text{s}} &\geq \frac{f}{n} \\ &\geq \frac{2}{7} \tan(\beta). \end{aligned} \tag{1.6}$$

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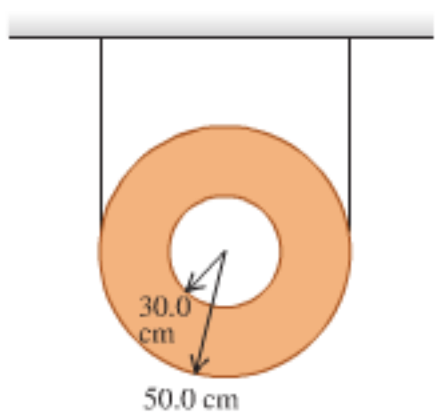
2. (Prob. 10.62 in Young and Freedman.) A uniform hollow disk has two pieces of thin, light wire wrapped around its outer rim and is supported from the ceiling, as shown in the figure. Suddenly one of the wires breaks, and the remaining wire does not slip as the disk rolls down. Use energy conservation to find the speed of the centre of this disk after it has fallen a distance of 2.20 m.



Solution.

We must have conservation of energy as the disk falls after the wire breaks. That is,

$$mgh = \frac{1}{2} m v_{\text{cm}}^2 + \frac{1}{2} I \omega^2, \tag{2.1}$$

where v_{cm} is the speed of the centre of mass, which is located at the centre of disk, and

$$\begin{aligned} I &= \frac{1}{2} m \left(R_1^2 + R_2^2 \right) \\ v_{\text{cm}} &= \omega R_2. \end{aligned} \tag{2.2}$$

We then insert equations (2.2) in equation (2.1) to get (h is the distance the disk falls)

$$\begin{aligned} mgh &= \frac{1}{2}mv_{\mathrm{cm}}^2 + \frac{1}{2}I\frac{v_{\mathrm{cm}}^2}{R_2^2} \\ &= \frac{1}{2}mv_{\mathrm{cm}}^2\left(1+\frac{I}{mR_2^2}\right) \\ &= \frac{1}{4}mv_{\mathrm{cm}}^2\left(3+\frac{R_1^2}{R_2^2}\right), \end{aligned} \tag{2.3}$$

and

$$\begin{aligned} v_{\mathrm{cm}} &= 2\sqrt{\frac{gh}{3+R_1^2/R_2^2}} \\ &= 5.07\ \mathrm{m/s}. \end{aligned} \tag{2.4}$$

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3. (Prob. 10.48 in Young and Freedman.) Suppose that an asteroid traveling straight toward the centre of the earth were to collide with our planet at the equator and bury itself just below the surface. What would have to be the mass of the asteroid, in term of the earth's mass M, for the day to become 25.0% longer than it presently is as a result of the collision? Assume that the asteroid is very small compared to the earth and that the earth is uniform throughout.

Solution.

We can apply the principle of conservation of angular momentum since the asteroid will not produce a torque on the earth because it is heading for the centre of the earth. Furthermore, for the same reason, it will not contribute any angular momentum. We therefore write

$$I_{\mathrm{E}}\omega_1 = \left(I_{\mathrm{E}}+I_{\mathrm{A}}\right)\omega_2, \tag{3.1}$$

where the subscripts '1' and '2' denote the condition before and after the collision, respectively, while I_{E} and I_{A} are the moments of inertia of the earth and the asteroid. Using the information provided in the lecture notes for the moments of inertia for a uniform sphere and a point mass we modify equation (3.1) to

$$\left(\frac{2}{5}MR^2\right)\omega_1 = \left(\frac{2}{5}MR^2+mR^2\right)\omega_2, \tag{3.2}$$

and

$$\frac{\omega_1 - \omega_2}{\omega_2} = \frac{5m}{2M}. \tag{3.3}$$

Since the period is increased by 25.0%, the frequency must be reduced such that $1/\omega_2 = 1.25/\omega_1$. We therefore have that

$$\frac{\omega_1 - \omega_2}{\omega_2} = 0.250 \tag{3.4}$$

and

$$\begin{aligned} m &= \frac{2}{5}(0.250)M \\ &= 0.100M. \end{aligned} \tag{3.5}$$

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4. An asteroid traveling in orbit about the earth has a perigee (i.e., the point at which it makes its closest approach to the earth) that is exactly equal to the radius of the earth R. Suppose that as the asteroid collides with our planet at the equator, it buries itself just below the surface. Just before the collision the asteroid's speed was $v = a\omega_1 R$, where a is a constant and ω_1 the angular speed of the earth about its rotation axis (also before the collision). Assume that the asteroid is very small compared to the earth and that the earth is uniform throughout.

a) What would have to be the mass m of the asteroid, in term of the earth's mass M and a, for the day to become 25.0% *shorter* than it presently is as a result of the collision?

b) Your answer in a) should imply that $a > 4/3$ for the day to become 25.0% shorter after the collision. Show this, and explain, with words only, why should one expect a lower limit for a for this to happen?

Solution.

a) Since the asteroid's perigee equals the radius of the earth, its angular momentum relative to axis of rotation of the earth before the collision is

$$\begin{aligned} L_{\text{ast}} &= |\mathbf{R} \times \mathbf{p}| \\ &= R \cdot mv \\ &= am\omega_1 R^2. \end{aligned} \tag{4.1}$$

We now apply the principle of conservation of angular momentum. We therefore write

$$I_{\mathrm{E}}\omega_1 + a\omega_1 mR^2 = (I_{\mathrm{E}} + I_{\mathrm{A}})\omega_2, \tag{4.2}$$

where the subscripts '1' and '2' denote the condition before and after the collision, respectively, while I_{E} and I_{A} are the moments of inertia of the earth and the asteroid. Using the information provided in the lecture notes for the moments of inertia for a uniform sphere and a point mass we modify equation (4.2) to

$$\begin{aligned}\left(\frac{2}{5}MR^2\right)\omega_1 + a\omega_1 mR^2 &= \left(\frac{2}{5}MR^2 + mR^2\right)\omega_2 \\ \frac{\omega_1}{\omega_2} &= \frac{2M/5+m}{2M/5+am}.\end{aligned} \tag{4.3}$$

Since the period is decreased by 25.0%, the frequency must be increased such that $1/\omega_2 = 0.75/\omega_1$, with

$$\frac{\omega_1}{\omega_2} = 0.75 \tag{4.4}$$

and

$$\begin{aligned}m &= \frac{2M}{5}\left(\frac{1-\omega_1/\omega_2}{a\omega_1/\omega_2 - 1}\right) \\ &= \frac{0.1M}{0.75a-1}.\end{aligned} \tag{4.5}$$

b) Since it is impossible for the asteroid to have a mass that is smaller than zero, we must have that

$$\frac{0.1M}{0.75a-1} > 0, \tag{4.6}$$

or

$$\begin{aligned}a &> \frac{1}{0.75} \\ &> \frac{4}{3}.\end{aligned} \tag{4.7}$$

In order to *spin up* the earth the asteroid must contribute some increase in angular momentum to the total angular momentum of the earth/asteroid system after the collision to (more than) offset the increase in moment of inertia (because the asteroid buries itself below the surface of the equator). For example, if the asteroid did not contribute any angular momentum after the collision (as in a head-on collision, for example), then the earth would *spin down* and the day would become longer, not shorter, after the collision.

5. (Prob. 10.88 in Young and Freedman.) A uniform, 0.0300-kg rod of length 0.400 m rotates in a horizontal plane about a fixed axis through its centre and perpendicular to the rod. Two small rings, each with mass 0.0200 kg, are mounted so that they can slide along the rod. They are initially held by catches located at 0.0500 m on each side of the centre of the rod, and the system is rotating at 30 rev/min. With no other changes in the system (i.e., the 'system' consists of the rod and the rings), the catches are released, and the rings slide outward along the rod and fly off at the ends.

a) What is the angular speed of the system at the moment the rings reach the ends of the rod?
b) What is the angular speed of the rod after the rings leave it?

Solution.

a) Although the rod and the rings exert forces on each other (through the catches), there is no net force acting on the system they compose. It follows that the angular momentum is constant as long as the rings stay on the rod. The moment of inertia of the rod is $I_{\text{rod}} = ML^2/12$, with M and L its mass and length, and that of each ring $I_{\text{ring}} = mr^2$, with m the mass of a ring and r its position relative to the centre of the rod. If ω_1 and ω_2 are, respectively, the angular speeds when the rings are located 0.0500 m from the centre of the rod and at its ends, then we can write

$$\left(\frac{1}{12}ML^2 + 2mr_1^2\right)\omega_1 = \left(\frac{1}{12}ML^2 + 2mr_2^2\right)\omega_2, \tag{5.1}$$

with $r_1 = L/8$ and $r_2 = L/2$. We can therefore transform equation (5.1) to

$$\begin{aligned}\omega_2 &= \omega_1\left(\frac{M/12 + 2m/64}{M/12 + 2m/4}\right)\\ &= \frac{\omega_1}{4}\\ &= 7.5 \text{ rev/min.}\end{aligned} \tag{5.2}$$

b) Although the (internal) forces that rod and the rings exert on each other will vanish as the rings escape from the rod, there is no change in the status of the external forces acting on the system. That is, there is still no net force acting on the system after the rings leave the rod. The angular speed therefore remains unchanged at 7.5 rev/min.
