Set 39 Suppose you have paired data $({x_1}^{obs}, {y_1}^{obs}), \ldots, ({x_n}^{obs}, {y_n}^{obs})$ which are outcomes of two random variables X and Y. You construct a 95% confidence interval for the difference of the means of X and Y.

(a) Assuming that the correlation between X and Y is positive, is your constructed confidence interval shorter or longer than the 95% confidence interval that you would have in the case of independent X and Y? Prove your answer.

A positive correlation will mean that the confidence interval range will be shorter when compared to a confidence interval of independent variables because the positive correlation means that both random variables correlate in the same direction which means that the difference of those random variables will be lower, hence, tighter confidence interval.

(b) If the 95% confidence interval does not cover 0, would you retain or reject someone's claim that the means of X and Y are equal? Justify your answer.

If the confidence interval covers 0 then the equivalence claim for the means of X and Y cannot be rejected because the possibility of a difference mean of 0, no difference, will still hold.

(c) If the entire 95% confidence interval is to the right of 0, would you conclude or not that the mean of X is larger than the mean of Y? Justify your answer.

With a 95% confidence, yes. This is with the assumption that the means difference confidence interval is computed as $X - Y$.