Physics 1501A - 1st Assignment

<u>Your solution to problems 4, 8, and 14 has to be handed in, in class, on Friday, October 19, 2012.</u>

1. Using the definition of the derivative of a function $g(t)$

$$\frac{dg(t)}{dt} \equiv \lim_{\Delta t \to 0} \frac{g(t+\Delta t)-g(t)}{\Delta t}, \tag{1.1}$$

prove the following relations

$$\begin{aligned} &\frac{dt^n}{dt} = nt^{n-1} \\ &\frac{d}{dt}\cos(\omega t) = -\omega \sin(\omega t) \\ &\frac{d}{dt}e^{at} = ae^{at} \\ &\frac{d}{dt}\ln(bt) = \frac{1}{t}, \end{aligned} \tag{1.2}$$

where $n, \omega, a,$ and b are constants. You may need the following relations, which are valid when $\Delta t \ll t$

$$\begin{aligned} (t+\Delta t)^n &\simeq t^n + nt^{n-1}\Delta t \\ \cos[\omega(t+\Delta t)] &\simeq \cos(\omega t) - \omega\,\Delta t \sin(\omega t) \\ e^{a(t+\Delta t)} &\simeq e^{at}(1+a\Delta t) \\ \ln[b(t+\Delta t)] &\simeq \ln(bt) + \Delta t/t. \end{aligned} \tag{1.3}$$

It would also be a good exercise to try to derive equations (1.3) on your own.

Solution.

The derivation of equations (1.2) is straightforward by the insertion of equations (1.3) into (1.1). That is,

$$\begin{aligned} \lim_{\Delta t \to 0} \frac{(t+\Delta t)^n - t^n}{\Delta t} &= \lim_{\Delta t \to 0} \frac{t^n + nt^{n-1}\Delta t - t^n}{\Delta t} = nt^{n-1} \\ \lim_{\Delta t \to 0} \frac{\cos[\omega(t+\Delta t)] - \cos(\omega t)}{\Delta t} &= \lim_{\Delta t \to 0} \frac{\cos(\omega t) - \omega\,\Delta t \sin(\omega t) - \cos(\omega t)}{\Delta t} \\ &= -\omega \sin(\omega t) \end{aligned} \tag{1.4}$$

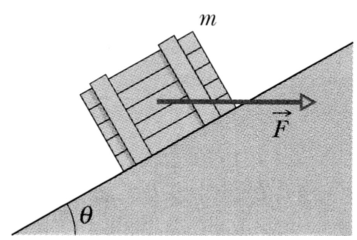
and

$$\lim_{\Delta t\to 0}\frac{e^{a(t+\Delta t)}-e^{at}}{\Delta t}=\lim_{\Delta t\to 0}\frac{e^{at}\left(1+a\Delta t\right)-e^{at}}{\Delta t}=ae^{at}$$

$$\begin{aligned}\lim_{\Delta t\to 0}\frac{\ln\left[b\left(t+\Delta t\right)\right]-\ln\left(bt\right)}{\Delta t}&=\lim_{\Delta t\to 0}\frac{\ln\left(bt\right)+\Delta t/t-\ln\left(bt\right)}{\Delta t}\\&=1/t\,.\end{aligned}\tag{1.5}$$

---

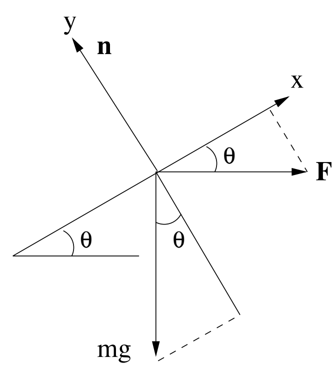
2. A crate of mass $m = 100\text{ kg}$ is pushed at constant speed up a frictionless ramp $(\theta = 30°)$ by a *horizontal* force $\mathbf{F}$, as shown in the diagram on the right.



a) Draw and label a free-body diagram for the crate.
b) Write Newton's Second Law in component form for the crate, with separate terms for the individual forces acting.
c) Calculate (i) the magnitude of the force exerted by the ramp on the crate and (ii) the magnitude of the force $\mathbf{F}$.

Solution.

a) The free-body diagram for the crate is shown here (on the right).



b) Newton's Second Law for the crate the net force components along the x and y directions

$$\begin{aligned}F_{\text{net},x}&=F\cos\theta-mg\sin\theta\\&=ma_x\\&=0\\F_{\text{net},y}&=n-mg\cos\theta-F\sin\theta\\&=ma_y\\&=0,\end{aligned}\tag{2.1}$$

since the crate is moving at constant velocity in the x direction, and not at all along the y-axis .

c) From equations (2.1) we have

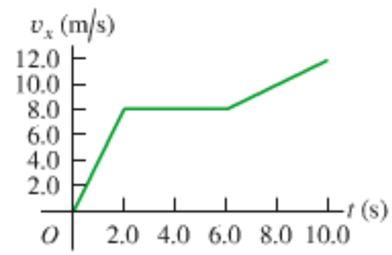
$$
\begin{aligned}
F &= \frac{mg\sin\theta}{\cos\theta} \\
&= mg\tan\theta \\
&= 100\ \text{kg}\cdot 9.80\ \text{m/s}^2\cdot 0.58 \\
&= 566\ \text{N},
\end{aligned}
\tag{2.2}
$$

and

$$
\begin{aligned}
n &= mg\cos\theta + F\sin\theta \\
&= mg\left(\cos\theta + \tan\theta\sin\theta\right) \\
&= \text{mg}\left(\cos\theta + \sin^2\theta/\cos\theta\right) \\
&= mg/\cos\theta \\
&= \frac{100\ \text{kg}\cdot 9.80\ \text{m/s}^2}{0.87} \\
&= 1130\ \text{N}.
\end{aligned}
\tag{2.3}
$$

---

3. (Prob. 4.14 in Young and Freedman.) A 2.75-kg cat moves in a straight line (i.e., along the x-axis). The figure on the right shows a graph of the x-component of that cat's velocity as a function of time.



a) Find the maximum net force on this cat. When does this force occur?
b) When is the net force on that cat equal to zero?
c) What is the net force at time 8.5 s?

Solution.

a) Since the force is proportional to the acceleration it will be maximum when the acceleration is also maximum. However, we know that the acceleration is the time derivative of the velocity, which is effectively the slope of the x-component of that cat's velocity as a function of time. It is clear from the figure that the slope is maximum on the first section of the curve where $0\ \text{s} < t < 2.0\ \text{s}$. We then find

$$
a_{x,\max} = \frac{\Delta v}{\Delta t} = \frac{8.0\ \text{m/s}}{2.0\ \text{s}} = 4.0\ \text{m/s}^2, \tag{3.1}
$$

and

$$
F_{x,\max} = ma_{x,\max} = 2.75\ \text{kg}\cdot 4.0\ \text{m/s}^2 = 11\ \text{N}. \tag{3.2}
$$

b) The net force is zero when the acceleration, or the slope, is zero. This happens when $2.0\ \mathrm{s} < t < 6.0\ \mathrm{s}$.

c) In the interval where $6.0\ \mathrm{s} < t < 10.0\ \mathrm{s}$ the acceleration is

$$a_x = \frac{\Delta v}{\Delta t} = \frac{(12.0 - 8.0)\ \mathrm{m/s}}{(10.0 - 6.0)\ \mathrm{s}} = 1.0\ \mathrm{m/s^2}, \tag{3.3}$$

and $F_x = 2.75\ \mathrm{N}$.

---

4. (Prob. 4.30 in Young and Freedman.) A .22 riffle bullet, travelling at 350 m/s, strikes a large tree, which it penetrates to a depth of 0.130 m. The mass of the bullet is 1.80 g. Assume a constant friction force.

a) How much time is required for the bullet to stop?

b) What force, in Newton, does the tree exert on the bullet?

Solution.

a) Since the force of friction, which slows down and eventually stops the bullet, is constant, then acceleration (or deceleration, in this case) is also constant with

$$a = \frac{v_f - v_i}{\Delta t}, \tag{4.1}$$

where v_i, v_f, and Δt are the initial and final velocities, and the time interval required for the bullet to stop. We can now use equation (2.16) of the Lecture Notes to relate this acceleration to the distance travelled by the bullet

$$\begin{aligned} x_f - x_i &= \frac{1}{2a}(v_f - v_i)(v_f + v_i) \\ &= \frac{1}{2}(v_f + v_i)\Delta t, \end{aligned} \tag{4.2}$$

or

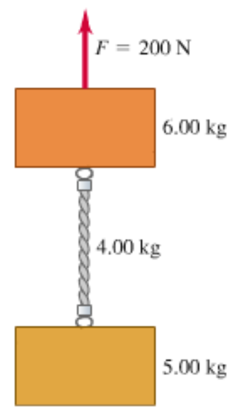
$$\Delta t = 2\left(\frac{x_f - x_i}{v_f + v_i}\right) = \frac{2 \cdot 0.130\ \mathrm{m}}{350\ \mathrm{m/s}} = 7.43 \times 10^{-4}\,\mathrm{s}. \tag{4.3}$$

b) The force exerted by the three on the bullet is calculated from Newton's Second Law and equation (4.1) with

$$\begin{aligned} F &= ma \\ &= \frac{m\left(v_f - v_i\right)}{\Delta t} = \frac{0.0018\ \text{kg} \cdot (-350\ \text{m/s})}{7.43 \times 10^{-4}\,\text{s}} = 848\ \text{N}. \end{aligned} \tag{4.4}$$

---

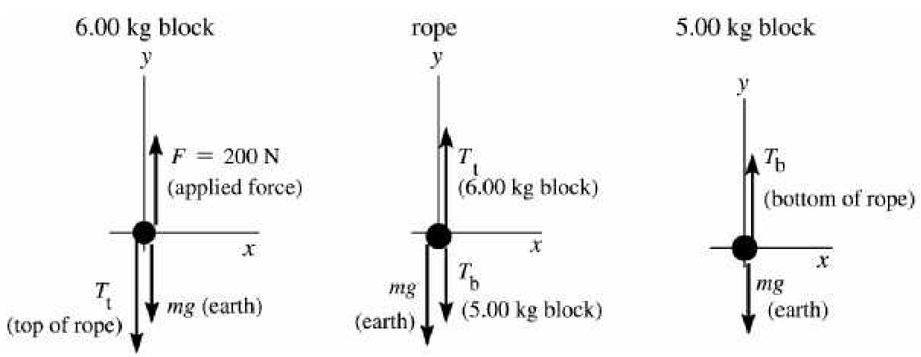
5. (Prob. 4.54 in Young and Freedman.) The two blocks in the figure on the right are connected by a heavy uniform rope of with a mass of 4.00 kg. An upward force of 200 N is applied as shown.



a) Draw three free-body diagrams: one for the 6.00-kg block, one for the 4.00-kg rope, and another for the 5.00-kg block. For each force indicate what body exerts that force.
b) What is the acceleration of the system?
c) What is the tension at the top of the heavy rope?
d) What is the tension at the midpoint of the rope?

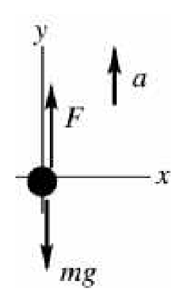
Solution.

a) The free-body diagram are as follows



with T_t and T_b the tensions at the top and bottom of the rope.

b) We treat the two blocks and the rope as a single system with a total mass of 15.0 kg. If we take $+y$ as upward, then the free-body diagram is shown on the right. Newton's Second Law then specifies that



$$\sum F_y = F - m_\text{tot} g = m_\text{tot} a, \tag{5.1}$$

or

$$
\begin{aligned}
a &= \frac{F}{m_{\text{tot}}} - g \\
&= \frac{200\ \text{N}}{15.0\ \text{kg}} - 9.81\ \text{m/s}^2 = 3.53\ \text{m/s}^2 .
\end{aligned} \tag{5.2}
$$

c) Using the free-body diagram for the 6.00 kg block we can write

$$
\sum F_y = F - m_1 g - T_{\text{t}} = m_1 a, \tag{5.3}
$$

or

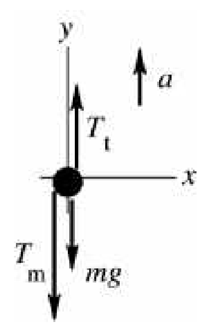
$$
\begin{aligned}
T_{\text{t}} &= F - m_1 (g + a) \\
&= 200\ \text{N} - 6.00\ \text{kg} \cdot (9.81 + 3.53)\ \text{m/s}^2 \\
&= 120\ \text{N}.
\end{aligned} \tag{5.4}
$$

d) We can consider the top half of the rope (with a mass m_{h} of 2.00 kg) with T_{m} the tension at its midpoint. This gives us the free-body diagram on the right. We then write

$$
\sum F_y = T_{\text{t}} - m_{\text{h}} g - T_m = m_{\text{h}} a, \tag{5.5}
$$

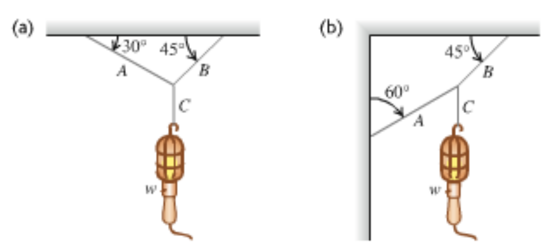
or

$$
\begin{aligned}
T_{\text{m}} &= T_{\text{t}} - m_{\text{h}} (g + a) \\
&= 120\ \text{N} - 2.00\ \text{kg} \cdot (9.81 + 3.53)\ \text{m/s}^2 \\
&= 93.3\ \text{N}.
\end{aligned} \tag{5.6}
$$



---

6. (Prob. 5.7 in Young and Freedman.) Find the tension in each cord in the figure on the right assuming the weight of the suspended object is w.



Solution.

We define $+x$ and $+y$ as right and upward, respectively, and we apply Newton's Second Law for both directions to the knot where all the cords are joined. For (a) we find

$$\begin{aligned} \text{For } x: &\quad -T_A \cos\left(30^\circ\right) + T_B \cos\left(45^\circ\right) = 0 \\ \text{For } y: &\quad T_A \sin\left(30^\circ\right) + T_B \sin\left(45^\circ\right) - w = 0, \end{aligned} \tag{6.1}$$

since $T_C = w$. Taking into account that $\sin\left(45^\circ\right) = \cos\left(45^\circ\right)$, we subtract the first to the second equations to get

$$\begin{aligned} T_A &= \frac{w}{\sin\left(30^\circ\right) + \cos\left(30^\circ\right)} = 0.732\,w \\ T_B &= T_A \frac{\cos\left(30^\circ\right)}{\cos\left(45^\circ\right)} = 0.897\,w. \end{aligned} \tag{6.2}$$

For (b) we have (once again $T_C = w$)

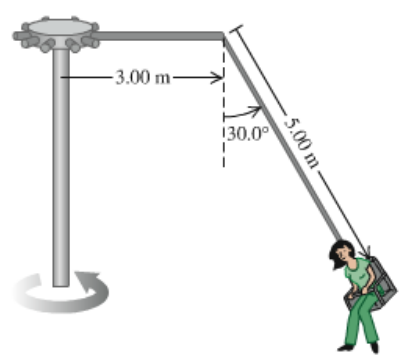
$$\begin{aligned} \text{For } x: &\quad -T_A \sin\left(60^\circ\right) + T_B \cos\left(45^\circ\right) = 0 \\ \text{For } y: &\quad -T_A \cos\left(60^\circ\right) + T_B \sin\left(45^\circ\right) - w = 0, \end{aligned} \tag{6.3}$$

and

$$\begin{aligned} T_A &= \frac{w}{\sin\left(60^\circ\right) - \cos\left(60^\circ\right)} = 2.73w \\ T_B &= T_A \frac{\sin\left(60^\circ\right)}{\cos\left(45^\circ\right)} = 3.35\,w. \end{aligned} \tag{6.4}$$

---

7. (Prob. 5.46 in Young and Freedman.) The "Giant Swing" at an amusement park consists of a vertical central shaft with a number of horizontal arms attached at its upper end, as shown in the figure. Each arm supports a seat suspended from a cable 5.00 m long. The upper end of the cable is fastened to the arm at a point 3.00 m from the axis of the central shaft. You may treat the person and the seat at the lower end of the cable as a single point mass. Calculate the time for one revolution of the swing if the cable supporting a seat makes an angle of 30.0° with the vertical.



Solution.

If we choose the x- and y-axes along the horizontal and vertical directions, respectively, with the origin at the instantaneous position of the person of mass m, then we write

$$\begin{aligned} ma_x &= T\sin(\theta) \\ ma_y &= T\cos(\theta) - mg, \end{aligned} \tag{7.1}$$

where T is the tension in the cable and $\theta = 30^\circ$. Since the circle traced by the motion of the person is in the horizontal plane we have

$$\begin{aligned} a_x &= \frac{v^2}{r} \\ a_y &= 0, \end{aligned} \tag{7.2}$$

with v the tangential speed of the person and $r = \left[3.00 + 5.00\sin(\theta)\right]\mathrm{m}$. From the second of equations (7.1) we then have

$$T = \frac{mg}{\cos(\theta)}, \tag{7.3}$$

and while inserting this relation in the first of equations (7.1)

$$\frac{v^2}{r} = g\tan(\theta). \tag{7.4}$$

We also know that $v = \omega r$ and therefore, from equation (7.4),

$$\omega^2 r = g\tan(\theta), \tag{7.5}$$

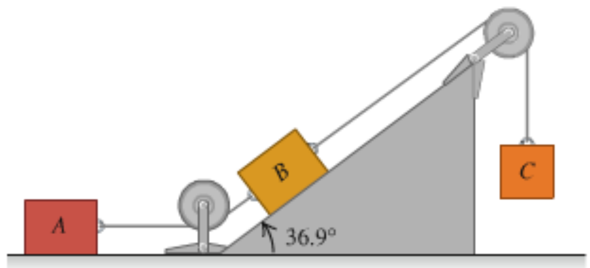
or solving for the period of revolution

$$\begin{aligned} \Delta t &= \frac{2\pi}{\omega} \\ &= 2\pi\sqrt{\frac{r}{g\tan(\theta)}} \\ &= 6.19\ \mathrm{s}. \end{aligned} \tag{7.6}$$

---

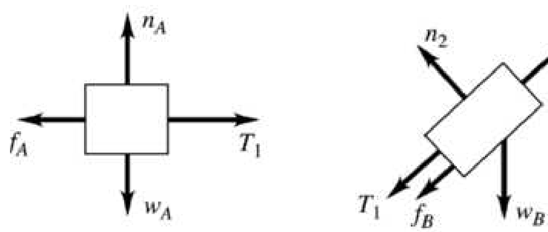
8. (Prob. 5.103 in Young and Freedman.) Blocks *A*, *B*, and *C* are placed as in the figure below and connected by ropes of negligible mass. Both *A* and *B* weigh 25.0 N each, and the coefficient of kinetic friction between each block and the surface is 0.35. Block *C* descends at a constant velocity.

a) Draw two separate free-body diagrams showing the forces acting on *A* and on *B*.
b) Find the tension in the rope connecting blocks *A* and *B*.
c) What is the weight of block *C*?
d) If the rope connecting *A* and *B* were cut, what would be the acceleration of *C*?



Solution.

a) The two free-body diagrams are shown below with the different weights, and normal and tension and friction forces indicated. For block *B* we use a coordinate system with axes parallel and perpendicular to the surface of the incline.



b) There is no net force applied on each block since they are moving with constant speed. It follows that the tension force on the rope connecting blocks *A* and *B* must equal the friction force on block *A*, with

$$T_1 = f_A = \mu_k n_A = 0.35 \cdot 25.0\ \text{N} = 8.8\ \text{N}. \tag{8.1}$$

c) The tension in the rope between blocks *B* and *C* equals the weight of block *C* w_C. Solving Newton's equation for the second block we have

$$\begin{aligned} w_C &= T_1 + f_B + w_B \sin\left(36.9^\circ\right) \\ &= T_1 + \mu_k w_B \cos\left(36.9^\circ\right) + w_B \sin\left(36.9^\circ\right) \\ &= 8.8\ \text{N} + 25.0\ \text{N} \cdot \left[0.35 \cos\left(36.9^\circ\right) + \sin\left(36.9^\circ\right)\right] = 30.8\ \text{N}. \end{aligned} \tag{8.2}$$

d) If the rope connecting A and B were cut, then $T_1 = 0$ and there would be a net force acting on blocks B and C. Equation (8.2) would be transformed to

$$m_B a = T_2 - w_B\left[\mu_k \cos\left(36.9^\circ\right) + \sin\left(36.9^\circ\right)\right]. \tag{8.3}$$

But T_2 does not equal the weight of block C anymore, rather we have

$$m_C a = w_C - T_2. \tag{8.4}$$

Inserting this relation in equation (8.3) we finally find that

$$\begin{aligned} a &= \left\{w_C - w_B\left[\mu_k \cos\left(36.9^\circ\right) + \sin\left(36.9^\circ\right)\right]\right\} \Big/ \left(m_B + m_C\right) \\ &= g\left\{w_C - w_B\left[\mu_k \cos\left(36.9^\circ\right) + \sin\left(36.9^\circ\right)\right]\right\} \Big/ \left(w_B + w_C\right). \end{aligned} \tag{8.5}$$

---

9. (Prob. 6.21 in Young and Freedman.) You are a member of an Alpine Rescue Team. You must slide a box up an incline of constant slope angle α so that it reaches a stranded skier who is a vertical distance h above the bottom of the incline. The incline is slippery, but there is some friction present, with the kinetic friction coefficient μ_k. Use the work-energy theorem to calculate the minimum speed you must give the box at the bottom of the incline so that it will reach the skier. You must express your answer in terms of g, h, μ_k, and α.

Solution.

Let point 1 be at the bottom of the incline and point 2 at the skier. We apply the work-energy theorem (equation (2.36) in the Lecture Notes) with work being done by gravity and friction. We then write (with m the mass of the box and v_0 its initial velocity at the bottom of the incline)

$$\begin{aligned} W_{\text{tot}} &= K_2 - K_1 \\ &= 0 - \frac{1}{2} m v_0^2 \end{aligned} \tag{9.1}$$

and

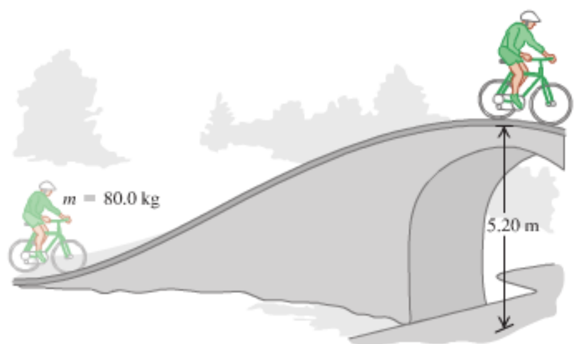
$$
\begin{aligned}
W_{\text{tot}} &= W_{\text{grav}} + W_f \\
&= -mgh - \mu_k mg\cos(\alpha)\cdot s \\
&= -mgh - \mu_k mg\cos(\alpha)\cdot\frac{h}{\sin(\alpha)} \\
&= -mgh\left[1+\mu_k/\tan(\alpha)\right],
\end{aligned}
\tag{9.2}
$$

since the force normal to the incline is $\mu_k mg\cos(\alpha)$ and the distance travelled along the incline is $h/\sin(\alpha)$. Equating equations (9.1) and (9.2) we find

$$
v_0 = \sqrt{2gh\left[1+\mu_k/\tan(\alpha)\right]}. \tag{9.3}
$$

---

10. (Prob. 6.78 in Young and Freedman.) You and your bicycle have a combined mass of 80.0 kg. When you reach the base of a bridge, you are travelling along the road at a speed of 5.00 m/s (see the figure on the right). At the top of the bridge you have climbed a vertical distance of 5.20 m and have slowed to 1.50 m/s. You can ignore work done by friction and any efficiency in the bike or your legs.



a) What is the total work done on you and your bicycle when you go from the base to the top of the bridge?
b) How much work have you done with the force you apply on the pedals?

Solution.

a) Let denote by "1" and "2" the base and the top of the bridge, respectively. The work-energy theorem then states that

$$
\begin{aligned}
W_{\text{tot}} &= K_2 - K_1 \\
&= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \\
&= \frac{1}{2}80.0\ \text{kg}\left[(1.50\ \text{m/s})^2 - (5.00\ \text{m/s})^2\right] \\
&= -910\ \text{J}.
\end{aligned}
\tag{10.1}
$$

b) Since we can neglect friction, work is only done by you and gravity. We therefore can write

$$
W_{\text{tot}} = W_{\text{you}} + W_{\text{grav}}, \tag{10.2}
$$

which also implies that

$$\begin{aligned} W_{\text{you}} &= W_{\text{tot}} - W_{\text{grav}} \\ &= \Delta K - (-mg\Delta y) \\ &= -910\ \text{J} + 80.0\ \text{kg} \cdot 9.80\ \text{m/s}^2 \cdot 5.20\ \text{m} \\ &= -910\ \text{J} + 4080\ \text{J} \\ &= 3170\ \text{J}. \end{aligned} \tag{10.3}$$

---

11. (Prob. 6.103 in Young and Freedman.) Consider a spring of mass M, equilibrium length L_0, and spring constant k. The work done to stretch or compress the spring to a length L is $kX^2/2$, where $X = L - L_0$. Consider a spring, as described above, that has one end fixed and the other end moving with a speed v. Assume that the speed of points along the length of the spring varies linearly with distance l from the fixed end. Assume also that the mass M of the spring is distributed uniformly along the length of the spring. (a) Calculate the kinetic energy of the spring in terms of M and v. (*Hint:* Divide the spring into pieces of length dl; find the speed of each piece in terms of l, v, and L; find the mass of each piece in terms of dl, M, and L; and integrate from 0 to L. The result is *not* $Mv^2/2$, since not all of the spring move with the same speed. Also note that $\int x^2\,dx = x^3/3$.) In a spring gun, a spring of mass 0.243 kg and force constant 3200 N/m is compressed 2.50 cm from its unstretched length. When the trigger is pulled, the spring pushes horizontally on a ball of mass $m = 0.053$ kg. The work done by friction is negligible. Calculate the ball's speed when the spring reaches its uncompressed length (b) ignoring the mass of the spring and (c) including, using the results of part (a), the mass of the spring. (d) In part (c), what is the ratio of the final kinetic energy of the ball to that of the spring?

Solution.

(a) Since the speed $u(l)$ of a spring element dl scales linearly with its position l, and that the speeds at both end must (obviously) be $u(0) = 0$ and $u(L) = v$, we have

$$u(l) = v\frac{l}{L}. \tag{11.1}$$

On the other hand the mass dm of an element dl is simply $dm = Mdl/L$, while the total kinetic energy of the spring is the sum (or integral) of kinetic energy of all the elements of length dl and mass dm

$$
\begin{aligned}
K &= \int_0^L dk \\
&= \frac{1}{2}\int_0^L u^2(l)\,dm \\
&= \frac{1}{2}\frac{Mv^2}{L^3}\int_0^L l^2\,dl \\
&= \frac{1}{6}Mv^2 .
\end{aligned}
\tag{11.2}
$$

(b) From the principle of conservation of energy we can write, when neglecting the mass of the spring,

$$
\begin{aligned}
U_{\mathrm{el}} &= K_{\mathrm{ball}} \\
\frac{1}{2}kx^2 &= \frac{1}{2}mv^2
\end{aligned}
\tag{11.3}
$$

or

$$
\begin{aligned}
v &= x\sqrt{\frac{3k}{M}} \\
&= 0.025\ \mathrm{m}\sqrt{\frac{3\cdot 3200\ \mathrm{N/m}}{0.243\ \mathrm{kg}}} \\
&= 6.1\ \mathrm{m/s}.
\end{aligned}
\tag{11.4}
$$

(c) If we now take the mass of the spring into account we have

$$
\begin{aligned}
U_{\mathrm{el}} &= K_{\mathrm{spring}} + K_{\mathrm{ball}} \\
\frac{1}{2}kx^2 &= \frac{1}{6}Mv^2 + \frac{1}{2}mv^2
\end{aligned}
\tag{11.5}
$$

or

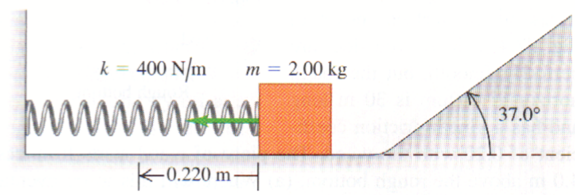
$$
\begin{aligned}
v &= x\sqrt{\frac{k}{m+M/3}} \\
&= 0.025\ \mathrm{m}\sqrt{\frac{3200\ \mathrm{N/m}}{(0.053+0.243/3)\ \mathrm{kg}}} \\
&= 3.9\ \mathrm{m/s}.
\end{aligned}
\tag{11.6}
$$

(d) The ratio of the kinetic energies is

$$\frac{K_{\text{ball}}}{K_{\text{spring}}} = \frac{3m}{M} \tag{11.7}$$
$$= 0.65.$$

---

12. (Prob. 7.42 in Young and Freedman.) A 2.0-kg block is pushed against a spring of negligible mass and force constant $k = 400$ N/m, compressing it 0.22 m. The block is then released, and moves along a frictionless, horizontal surface then up an incline with slope $\theta = 37.0^\circ$.



a) What is the speed of the block as it slides along the horizontal surface after having left the spring?
b) How far does the block travel up the inclined plane before starting to slide back down?

Solution.

There are two forces at play in this problem: the restoring force of the spring and gravity. But these two forces are linked to corresponding potential energies. We can therefore set $W_{\text{other}} = 0$ in the equation for the principle of conservation of energy to obtain a solution.

a) If we denote by "1" and "2" the physical conditions of the system before and after the mass is released, respectively, then we have

$$U_{\text{el},1} + U_{\text{grav},1} + K_1 = U_{\text{el},2} + U_{\text{grav},2} + K_2. \tag{12.1}$$

But it should be clear that

$$K_1 = U_{\text{el},2} = 0, \tag{12.2}$$

since $v_1 = x_2 = 0$, and that

$$U_{\text{grav},1} = U_{\text{grav},2}. \tag{12.3}$$

It therefore follows that

$$\frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2, \tag{12.4}$$

or

$$
\begin{aligned}
v_2 &= \sqrt{\frac{k}{m}}x_1 \\
&= \sqrt{\frac{400\ \text{N/m}}{2.0\ \text{kg}}}0.22\ \text{m} \\
&=3.1\ \text{m/s.}
\end{aligned} \tag{12.5}
$$

b) We now denote by "3" the conditions when the block reaches its highest elevation on the inclined plane. We therefore write

$$
U_{\text{grav},2} + K_2 = U_{\text{grav},3} + K_3, \tag{12.6}
$$

with or since $v_3 = 0$

$$
\Delta U_{\text{grav}} = K_2 \tag{12.7}
$$

where $\Delta U_{\text{grav}} = U_{\text{grav},3} - U_{\text{grav},2} = mg\Delta y$ and Δy the change in the block's position in the vertical direction. We thus write

$$
\begin{aligned}
\Delta y &= \frac{v_2^2}{2g} \\
&= \frac{9.68\ \text{m}^2/\text{s}^2}{2 \cdot 9.8\ \text{m/s}^2} \\
&= 0.494\ \text{m.}
\end{aligned} \tag{12.8}
$$

However, we seek to find out how far on the incline the block gets to before coming to a stop. If we define this distance as Δl, then

$$
\begin{aligned}
\Delta l &= \frac{\Delta y}{\sin(\theta)} \\
&= 0.821\ \text{m.}
\end{aligned} \tag{12.9}
$$

---

13. (Prob. 7.51 in Young and Freedman.) A bungee cord is 30.0 m long and, when stretched a distance x, it exerts a restoring force of magnitude kx. Your father-in-law (mass 95.0-kg) stands on a platform 45.0 m above the ground, and one end is tied securely to his ankle and the other end to the platform. You have promised him that when he steps off the platform he will fall a maximum distance of only 41.0 m before the cords stops him. You had several bungee cords to select from, and you tested them by stretching them out, tying one end to a tree, and pulling on the other end with a force of 380.0 N. When you do this, what distance will the bungee cord that you select have stretched?

Solution.

We denote by “1” and “2” the conditions on the platform before the jump and when the cord stops your father-in-law such that

$$mgy_1 = mgy_2 + \frac{1}{2}kx^2. \tag{13.1}$$

It follows from equation (13.1) that

$$\begin{aligned} k &= 2mg\frac{(y_1 - y_2)}{x^2} \\ &= 2 \cdot 95.0 \text{ kg} \cdot 9.80 \text{ m/s}^2 \frac{(45.0 - 4.0) \text{ m}}{(41.0 - 30.0)^2 \text{ m}^2} \\ &= 631 \text{ N/m}. \end{aligned} \tag{13.2}$$

The distance the selected cord stretches when tested is

$$\begin{aligned} x &= \frac{F}{k} \\ &= \frac{380.0 \text{ N}}{631 \text{ N/m}} \\ &= 0.602 \text{ m}. \end{aligned} \tag{13.3}$$

---

14. (Prob. 7.82 in Young and Freedman.) **Pendulum.** A small rock with mass 0.12 kg is fastened to a massless string with length 0.80 m to form a pendulum. The pendulum is swinging so as to make a maximum angle of 45° with the vertical. Air resistance is negligible.

a) What is the speed of the rock when the string passes through the vertical position?
b) What is the tension in the string when it makes an angle of 45°?
c) What is the tension in the string when it passes through the vertical?

Solution.

Let us set $y = 0$ at the bottom of the trajectory and define that as point 1, and point 2 when the string makes an angle of 45° with the vertical.

a) At point 2 the speed and kinetic energy are zero and the potential energy is

$$U_2 = mgl\left[1 - \cos\left(45^\circ\right)\right], \tag{14.1}$$

while at point 1 the potential energy is zero and the kinetic energy is $K_1 = mv^2/2$. Because of the conservation of energy we must equate these two quantities to find

$$v = \sqrt{2gl\left[1-\cos\left(45^\circ\right)\right]} = 2.1 \text{ m/s}. \tag{14.2}$$

b) At point 2 the speed is zero and thus the centripetal acceleration is also zero. The tension in the string equals the component of the weight along the string. That is,

$$T_2 = mg\cos\left(45^\circ\right) = 0.83 \text{ N}. \tag{14.3}$$

c) At point 1 the centripetal acceleration is maximum at $a_{\text{rad}} = v^2/l$ and the tension equals the sum of the weight and the mass times this acceleration

$$T_1 = mg + mv^2/l = 1.9 \text{ N}, \tag{14.4}$$

where equation (14.2) was used for the velocity.
