# Double integral with use of composite trapezoid method

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## 1 Task description

2. Numerical calcualtion of:

$$\int_a^b \int_c^d f(x,y)\,dydx$$

where D = $[a,b] \times [c,d]$. Use composite trapezoid rule with respect to each variable.

## 2 Formula derivation

To derive formula we are using composite trapezoid:

$$I_1 = \int_a^b f(x)dx \approx \frac{h}{2} \cdot (f(a) + f(b) + 2\sum_{i=2}^{n} f(x_i)) \tag{1}$$

We can apply composite trapezoid rule twice. Firstly for the inner integral [1]:

$$\int_a^b \int_c^d f(x,y)dydx \approx \int_a^b \frac{h_y}{2}\left[f(x,c) + f(x,d) + 2\sum_{i=2}^{n} f(x,y_i)\right]dx \tag{2}$$

Inner integral can be treated as a function g(x) Therefore we can apply composite trapezoid rule to the outer integral again:

$$\begin{aligned} I \approx \frac{h_x}{2}(\frac{h_y}{2}&\left[f(a,c) + f(a,d) + 2\sum_{i=2}^{n} f(a,y_i)\right] + \\ &\frac{h_y}{2}\left[f(b,c) + f(b,d) + 2\sum_{i=2}^{n} f(b,y_i)\right] + \\ 2\sum_{j=2}^{n}&\frac{h_y}{2}\left[f(x_j,c) + f(x_j,d) + 2\sum_{i=2}^{n} f(x_j,y_i)\right]) \end{aligned} \tag{3}$$

Finally formula is simplified to following form:

$$\begin{aligned} I \approx \frac{h_y h_x}{4}((a,c) + f(a,d) + f(b,c) + f(b,d) + 4\sum_{i=2}^{n}\sum_{j=2}^{n} f(x_j,y_i) + \\ 2\sum_{i=2}^{n}[f(a,y_i) + f(b,y_i)] + 2\sum_{j=2}^{n}[f(x_j,c) + f(x_j,d)) \end{aligned} \tag{4}$$

# 3 Results comparison

Finally, the table below, shows comparison of evaluation of different integrals. Function *integral* is built in matlab function which numerically integrate function fun from xmin to xmax using global adaptive quadrature. [?]. Moreover we compare both results to the Wolfram Alpha results [3]

| Integral | Double trapezoid integral | MATLAB integral | Wolfram Alpha |
|---|---|---|---|
| $\int_0^1 \int_0^8 x^3 - y^2 dydx$ | 173.1574 | 173.1488 | 173.1488 |
| $\int_0^1 \int_0^8 x^3 - y^2 dydx$ | -168.6750 | -168.6667 | -168.(6) |
| $\int_{-5}^5 \int_{-3}^3 x^7 - y^2 + cos(x-y) dydx$ | -180.5767 | -180.54 | -180.54 |

Furthermore below plot presents value of $\int_0^1 \int_0^8 x^3 - y^2 dydx$ for different values of n. As n increases the results are getting more accurate:

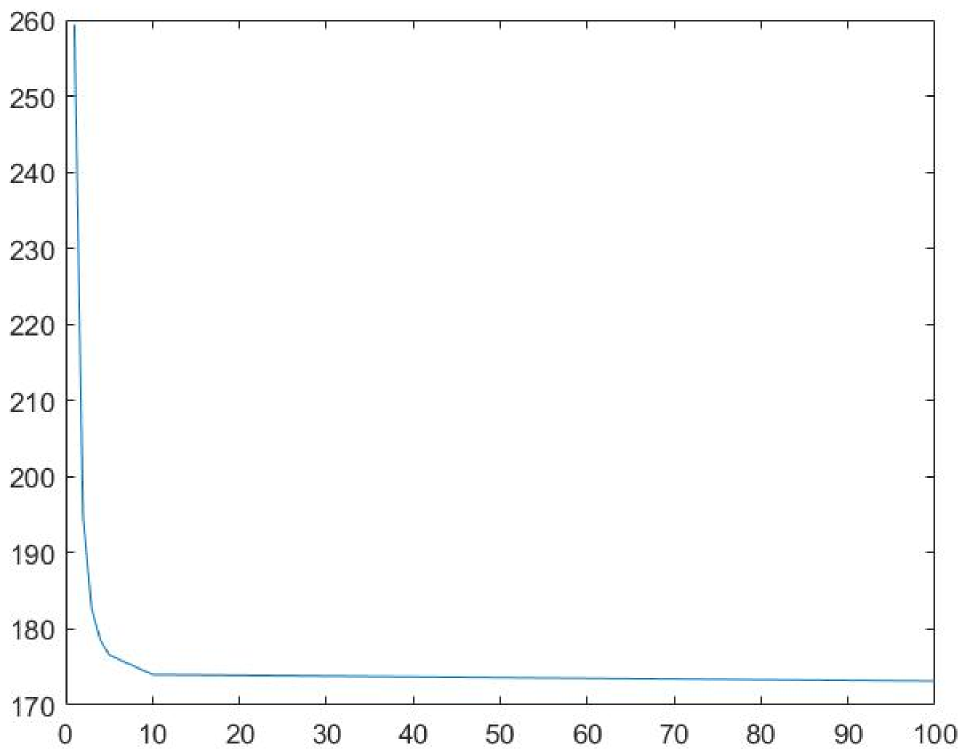


Figure 1: Convergence of quadrature

# References


[1] http://utkstair.org/clausius/docs/che505/pdf/IE_eval_N-Dints.pdf

[2] https://www.mathworks.com/help/matlab/ref/ode45.html

[3] https://www.wolframalpha.com/examples/mathematics/calculus-and-analysis/integrals/