Quantifying symmetry violations for optimising algorithmic designs

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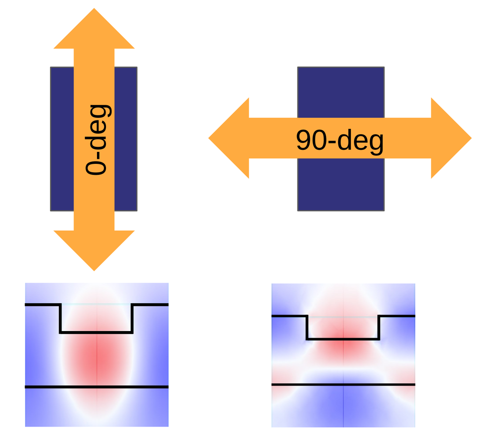
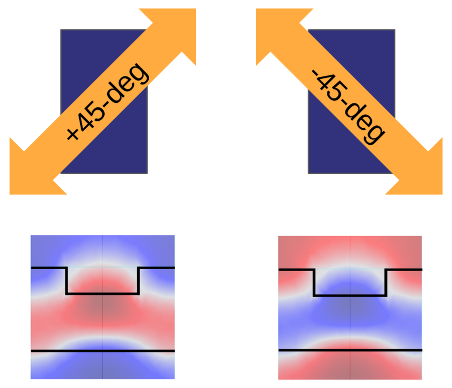
Introduction

Structures with sufficient **asymmetry can yield unique light-matter interactions** under differing polarisations of light (1,2).



Light polarised to the horizontal & vertical states *(top)* show differing electric field responses *(bottom)* in rectangular nanograting structures (outlined in black).

Axes of structures **lacking asymmetry result in identical light-matter responses** for varying polarisations of light (1,2).

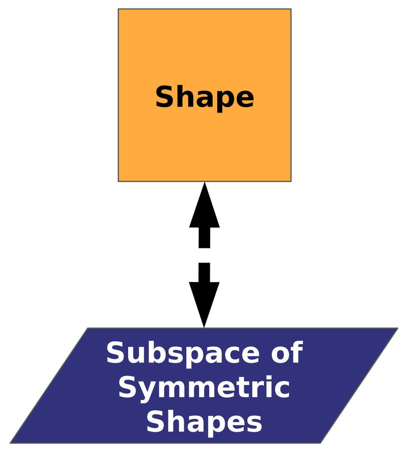


Light polarised to the diagonal states *(top)* show the same, but oppositely signed, electric field responses *(bottom)* in rectangular nanograting structures (outlined in black).

Designing sufficiently asymmetric structures is vital for creating selective light-matter interactions across a wide arrangement of polarisations of light.

Materials & Methods

The **degree of symmetry of a shape can be quantified** by the distance of its projection onto a subspace of symmetric shapes (3).



The distance from the shape to a subspace of symmetric shapes is inversely proportional the shape's degree of symmetry.

Averaging the distance across a range of angles provides rotational and reflective symmetry scores:

$$\frac{\sum_{\forall_i}^{\theta_i} (\mathrm{norm}(\mathrm{dot}(\text{Symmetry Operator (argument: } \theta), \text{Shape})))}{\text{Number of angles}}$$

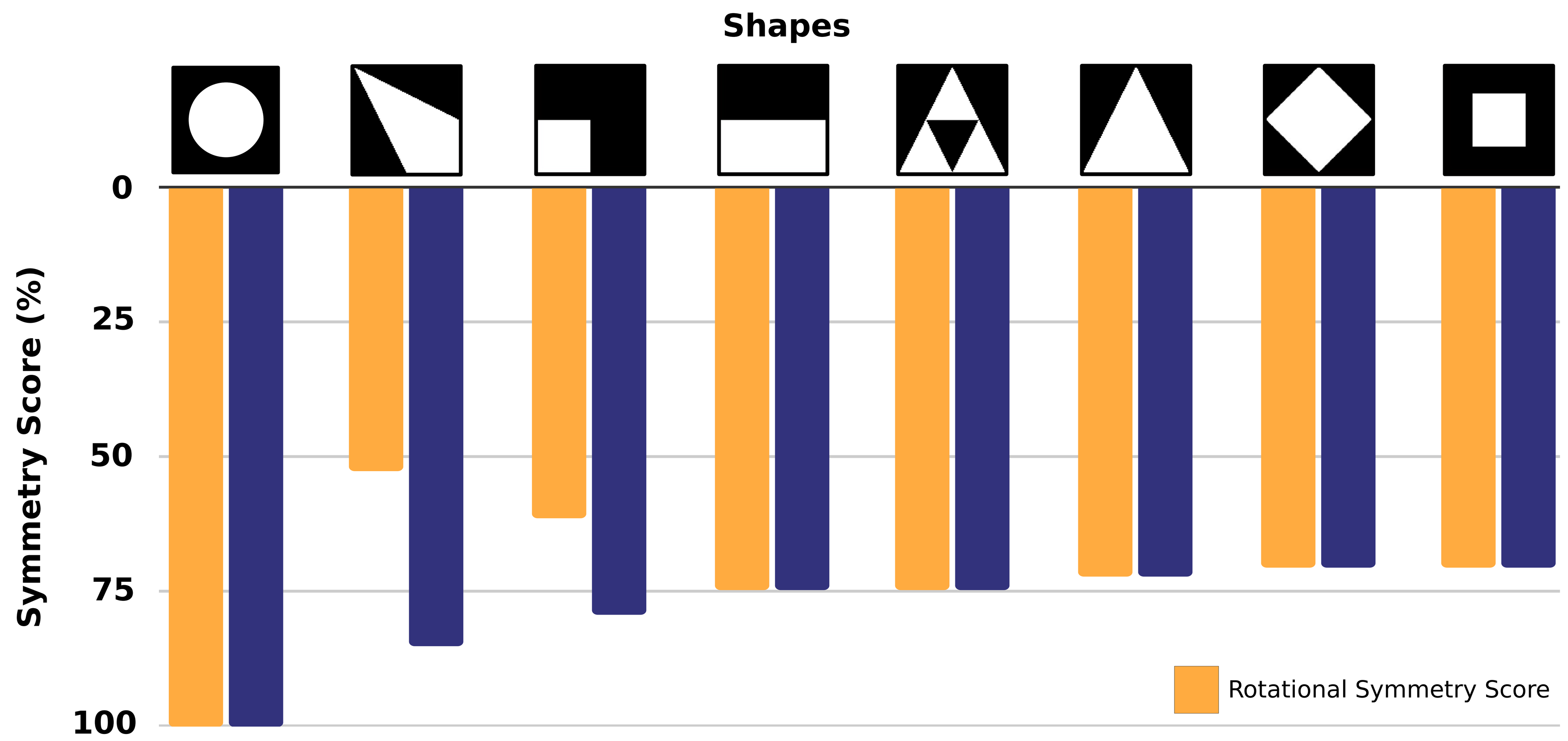
Manipulating the symmetry operator specifies the type of symmetry scored: rotational or reflective.

Results

The symmetry scores corresponding to the degree of rotational or reflective symmetries were calculated for various shapes. These calculations transform symmetry from a binary concept – a shape is either symmetrical or asymmetrical – to a continuous spectrum: a shape has a certain degree of symmetry. These shapes were taken from an informations security hash visualisation (4) known as identicons (5).

The symmetry scores below take into account the shape (in white) in the context of the empty space surrounding it (in black) as light-matter interactions are affected by the empty space around a structure along with the geometry of the structure. The shapes are presented in order of decreasing rotational symmetry score from the left to the right.

Rotational & Reflective Symmetry Scores



First Steps Towards a Genetic Algorithm

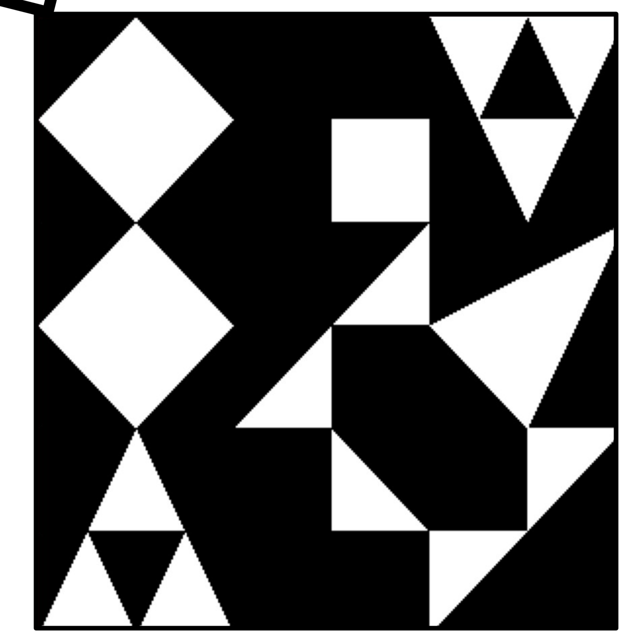
The ability to determine the degree of symmetry of a shape can provide future insight towards engineering selective light-matter interactions across differing polarisations of light. To this extent, **an evolution-inspired, genetic-based machine learning model is being constructed that takes advantage of symmetry scoring to construct an idealised asymmetric structure.** Such an algorithm is envisioned to use the aforementioned shapes (among others) as elementary building blocks.

Shape 4 rotated 180º	Shape 10 rotated 270º	Shape 6 rotated 270º
Shape 4 rotated 270º	Shape 9 rotated 270º	Shape 7 rotated 0º
Shape 6 rotated 90º	Shape 13 rotated 0º	Shape 9 rotated 90º

An M-by-N grid acts as a blueprint denoting the building block shape and a respective angle (0, 90, 180, or 270 degrees) for an asymmetric structure.

0100 10	1010 11	0110 11
0100 11	1001 11	0111 00
0110 01	1101 00	1001 01

A decimal-to-binary transformation to create a binary-based "DNA analogue" that can evolve within a machine learning model's parameters.



Each decimal-containing grid square will refer to a building block shape. When put together, the building block shapes will result in a new asymmetric structure.

Conclusions

The asymmetry inherent in a geometric shape affects the expected light-matter interactions. **Quantifying the degree of symmetry paves the path towards correlating the effects of changing specific symmetries to the observed light-matter interactions.** This can open a future where the idealised asymmetry of a structure can be engineered for an intended application.

The ability to select for light-matter interactions through a geometric-tuning of structures can have implications for sensing applications in several fields such as astrobiology (6), geology (7), and medicine (1,2). **Future steps of this work aim to realise the ideal combination of elementary building blocks to create maximal asymmetry.**

Literature Cited

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Further Information

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