

# Entropy and optimality in river deltas

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**The form and function of river deltas is intricately linked to the evolving structure of their channel networks, which controls how effectively deltas are nourished with sediments and nutrients. Understanding the coevolution of deltaic channels and their flux organization is crucial for guiding maintenance strategies of these highly stressed systems from a range of anthropogenic activities. To date, however, a unified theory explaining how deltas self-organize to distribute water and sediment up to the shoreline remains elusive. Here, we provide evidence for an optimality principle underlying the self-organized partition of fluxes in delta channel networks. By introducing a suitable nonlocal entropy rate ( $nER$ ) and by analyzing field and simulated deltas, we suggest that delta networks achieve configurations that maximize the diversity of water and sediment flux delivery to the shoreline. We thus suggest that prograding deltas attain dynamically accessible optima of flux distributions on their channel network topologies, thus effectively decoupling evolutionary time scales of geomorphology and hydrology. When interpreted in terms of delta resilience, high  $nER$  configurations reflect an increased ability to withstand perturbations. However, the distributive mechanism responsible for both diversifying flux delivery to the shoreline and dampening possible perturbations might lead to catastrophic events when those perturbations exceed certain intensity thresholds.**

spectral graph theory | information theory | self-organization | resilient deltas

**R**iver deltas are depositional landforms forming downstream of major rivers, often home to large populations and important natural resources (1–10). In the last decades, many deltas of the world have been under threat from a range of stressors, including sea-level rise, upstream dam development, and local exploration (2, 4, 5, 11–17). Deltas are nourished by channel networks whose connectivity constrains, if not drives, the evolution, functionality, and resilience of these systems. Remarkably, the properties of delta channel networks differ substantially from the tree-like topology of the rivers that feed them (18). Tree-like networks, defined by the absence of loops, are characteristic of tributary river networks and are found abundantly in nature across different systems and scales (e.g., botanical trees, veins of leaves, blood vessels, lightning, and river networks). The propensity of nature in choosing tree-like configurations has been grounded as an optimality principle. Specifically, tributary river channel networks achieve minimal total energy dissipation, that is, minimal loss of potential energy as water and sediment flow downstream, albeit often manifesting as feasible optimality, that is, a dynamically accessible local minimum due to initial conditions and other constraints (19–22). Similar to river networks, vascular networks in biological systems (e.g., animals, plants, insects, etc.), which transport materials through space filling fractal networks of branching tubes, achieve states of minimal energy dissipation (23, 24). Analogous optimality principles have also been suggested to constrain processes as diverse as root water uptake in plants (minimization of internal dissipation) (25) and

land surface energy and water balance (maximization of power) (26–28).

Despite the importance of deltaic systems and the recent advances in quantifying their connectivity properties (18, 29–34), an optimality principle for the organization of their distributary channel networks, akin to that existing for the tributary networks, remains elusive. A recent framework based on spectral graph theory (29, 30, 32) sheds light on the topologic and steady-state flux partitioning characteristics of delta channel networks and their relationship to underlying morphodynamic controls (e.g., sediment composition and tidal and wave energy), paving the way for quantitative delta classification and inference of process from form. However, the diversity of topologic structures of channel networks across a broad spectrum of deltas, and the fact that deltas are highly dynamic systems within which topology and flux partition coevolve, make it challenging to find a universal first-order optimality principle (e.g., minimization of energy, maximization of entropy, or minimization of free energy) governing their formation.

We are physically motivated by the foundational principle that deltas build land by spreading their fluxes on their delta top, as opposed to creating single pathways to the ocean which would diminish the formation of islands that retain sediment and nutrients and reduce land-building potential. This notion resonates with previous results applied to tidal deltas showing that tidal channels self-organize to uniformly distribute the tidal prism across the delta (35). Under this premise, we postulate the

## Significance

**River deltas are critically important Earthscapes at the land-water interface, supporting dense populations and diverse ecosystems while also providing disproportionately large food and energy resources. Deltas exhibit complex channel networks that dictate how water, sediment, and nutrients are spread over the delta surface. By adapting concepts from information theory, we show that a range of field and numerically generated deltas obey an optimality principle that suggests that deltas self-organize to increase the diversity of sediment transport pathways across the delta channels to the shoreline. We suggest that optimal delta configurations are also more resilient because the same mechanism that diversifies the delivery of fluxes to the shoreline also enhances the dampening of possible perturbations.**

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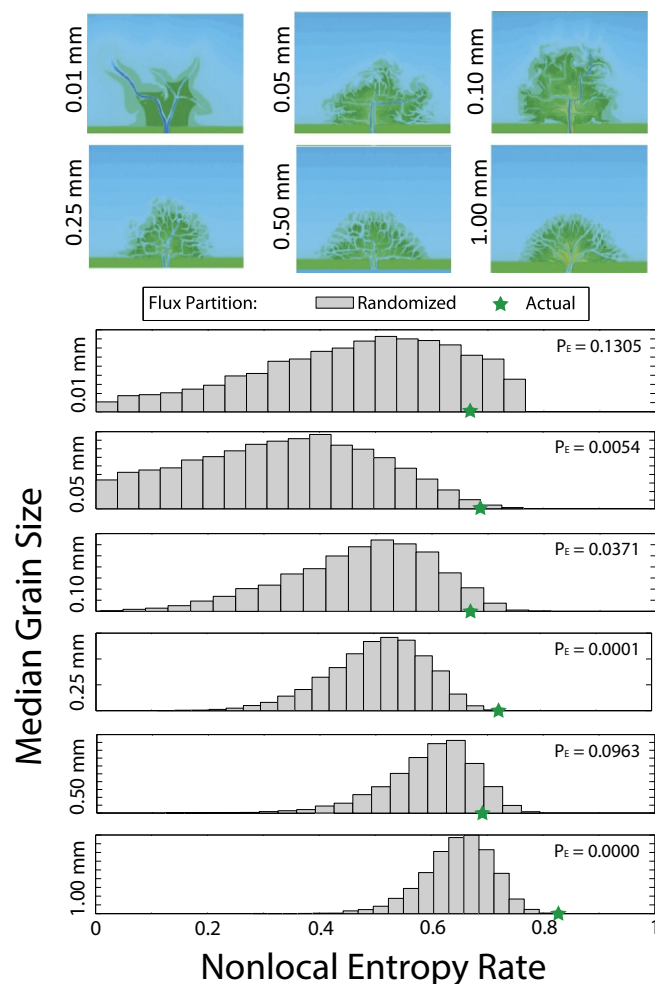
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**Fig. 3.**  $nER$  for simulated deltas. We examine the  $nER$  of numerically simulated deltas obtained by the Delft3D model. The simulated deltas are river-dominated, with no vegetation, and with a lognormal distribution of incoming sediment size with median grain size  $D_{50}$  varying from 0.01 to 1.0 mm and the same variance in the log space (for more details see [SI Appendix](#)). These deltas exhibit a wide variety of channel network topologies as shown in ref. 32. Similar to the analysis conducted for the field deltas, green stars represent the values of  $nER$  using channel width (extracted from simulations) as proxy for flux partition in each bifurcation. Compared with  $10^5$  randomizations of flux partition (histogram), five out of six deltas analyzed exhibit a maximal value of  $nER$ , defining maximal as a value where the probability of exceedance by a random realization is less than 0.1.

Our results suggest that the flux partitions at each bifurcation, which have evolved naturally, are not random but rather follow a rule that optimizes the delta system as a whole. In fact, an interesting paradox arises from our analysis. Although the entropy introduced locally by each bifurcation, considered as an insulated unit, is suboptimal [the maximum would correspond to a symmetric bifurcation which is not consistent with stability theory of delta bifurcations that requires asymmetric local flux partition (45)], the specific assemblage of those bifurcations forming the delta network as a system is optimal (in terms of  $nER$ ) and consistent with maximization of the diversity of fluxes delivered to the shoreline.

Turning attention to delta dynamics, we further hypothesize that during an avulsion the delta  $nER$  would decrease (see schematic in Fig. 4A). This is because during this phase of topologic reorganization the flux distribution inherited from the previous channel network structure is in general suboptimal with

respect to the incipient channel network reworked by the avulsion (i.e., the new channel structure created during the avulsion received a disproportionately small share of the flux, creating an asymmetry in the flux delivery to the shoreline and thus reducing the value of  $nER$ ). Testing this hypothesis in field deltas is challenging because avulsions occur infrequently. However, using numerical models we can observe that during an avulsion cycle the  $nER$  drops significantly at the onset of a new flow path and following the abandonment of the old channel (Fig. 4B). Since the time scale of the avulsion itself is negligible in comparison with the lifespan of the topologies before and following the avulsion, it is observed that the flux partition is able to self-organize to achieve a configuration that maximizes  $nER$ . This supports our assumption that the time scale of the flux reorganization is several orders of magnitude smaller than the characteristic time scale of topologic reorganization which is set by the time lapse between avulsion cycles.

An important implication of this optimality principle can be interpreted in terms of the resilience of deltas to withstand perturbations. Intuitively, if a perturbation (e.g., flux reduction) is applied to a delta during its high- $nER$  state, the perturbation will be damped as it will spread through the diverse pathways connecting the delta top to the shoreline. However, if the same perturbation is applied to a delta in a low- $nER$  state, the perturbation will be more confined to a localized part of the delta but will exert a more severe disturbance. As revealed by our analysis, river deltas operate in configurations characterized by high values of  $nER$ , supporting the idea that deltas self-organize to achieve resilient morphologies priming self-maintenance. As a word of caution, especially relevant in the current scenario where deltas are subjected to increasing anthropogenic stresses, this distributive mechanism that dampens the intensity of perturbations can also lead to delta-wide catastrophic disturbances and tipping points when those perturbations exceed certain thresholds.

## Conclusions

Deltas are highly productive regions supporting extensive agriculture and aquaculture and diverse ecosystems and containing natural resources such as hydrocarbon deposits. Climate change and human actions, both in the upstream basins and locally, act as stressors on these landscapes, calling for a thorough understanding of these complex systems and their response to perturbations. We examined the existence of an optimality principle that governs the self-organization of water and sediment fluxes on delta channel networks. Specifically, (i) we put forth the hypothesis that maximizing  $nER$ , which quantifies the diversity in flux delivery to the shoreline, is a selective criterion in the evolutionary dynamics of delta networks; (ii) we tested this hypothesis by analyzing 10 field deltas of diverse complexity, age, and environmental settings and showed that all but one, the Niger delta, exhibited maximum  $nER$ ; (iii) we further supported the existence of an optimality principle by analysis of Delft3D simulated deltas; (iv) we showed that during major reorganization, such as avulsions,  $nER$  exhibits suboptimal values and increases back to high rates (maximum values) when the flux distribution self-adjusts to the new delta channel network topology; and, finally, (v) we discussed the relation between entropy and resilience, arguing that delta flux configurations characterized by maximal  $nER$  are more resilient in the face of random perturbations. In the anthropocene where human activities have become a major agent of geomorphic change, understanding delta self-organization within an optimality perspective offers new ways of thinking about delta dynamics and disturbances that might hinder self-maintenance.

## Materials and Methods

**Deltas as Directed Graphs.** Tejedor et al. (29, 30) presented a rigorous framework based on graph theory within which a delta channel network is

represented by a directed graph, that is, a collection of vertices (bifurcations and junctions) and directed edges (channels in-between vertices, where the direction is given by the flow). All information about network connectivity and directionality of the flow can be stored in a sparse matrix called the adjacency matrix,  $A$ . Specifically,  $A$  is an  $N \times N$  matrix, where  $N$  is the number of vertices, and whose entry  $a_{ij}$  is unity if vertex  $i$  receives fluxes directly from vertex  $j$  (i.e., vertices  $i$  and  $j$  are connected by a link directed from  $j$  to  $i$ ) and zero otherwise. From  $A$  we can derive an important matrix called Laplacian, which is equivalent to a diffusivity operator in a graph. To construct the Laplacian we need first to introduce the degree matrices for directed graphs. The in-degree (out-degree) matrix  $D^{in}$  ( $D^{out}$ ) is an  $N \times N$  diagonal matrix whose entries  $d_{ii}$  depict the number of links entering (exiting) vertex  $i$  and are computed as the sum of the entries in the  $i$ -th row (column) of  $A$ . The Laplacian matrix,  $L^{in}$  ( $L^{out}$ ), is defined as  $D^{in} - A$  ( $D^{out} - A$ ). Tejedor et al. (29, 30) showed that certain eigenvectors of the Laplacian operator contain important topologic information of the deltaic network. Furthermore, information about flux propagation can be obtained if  $A$  is replaced with the weighted adjacency matrix  $W$ , where the weights  $w_{ij}$  correspond to the fraction of flux in link  $(ij)$  with respect to the flux in vertex  $j$ . Similar to the at-station hydraulic geometry relationship (49)—width to landscape-forming discharge—reported for tributary rivers (50) and tidal channels (51), we assume the flux partition at the bifurcation to be proportional to the width of the downstream channels (18). Note that even though we do not consider explicitly in the computation of steady-state fluxes relevant processes such as water–sediment interchange between channel and islands (31, 52, 53), vegetation (54), tides (35), and so on, all of these processes set the hydrogeomorphic attributes of the channel network. Therefore, the computation of steady-state fluxes in the channel network based on physical attributes such as channel widths can be interpreted to a certain degree as the result of the integrative effect of all of the main processes acting on a delta. For the purposes of this paper, there are two probability distributions that can be computed by simple algebraic manipulation in the above-mentioned operators, namely, the steady-state flux distribution and the node-to-outlet transition probability distribution.

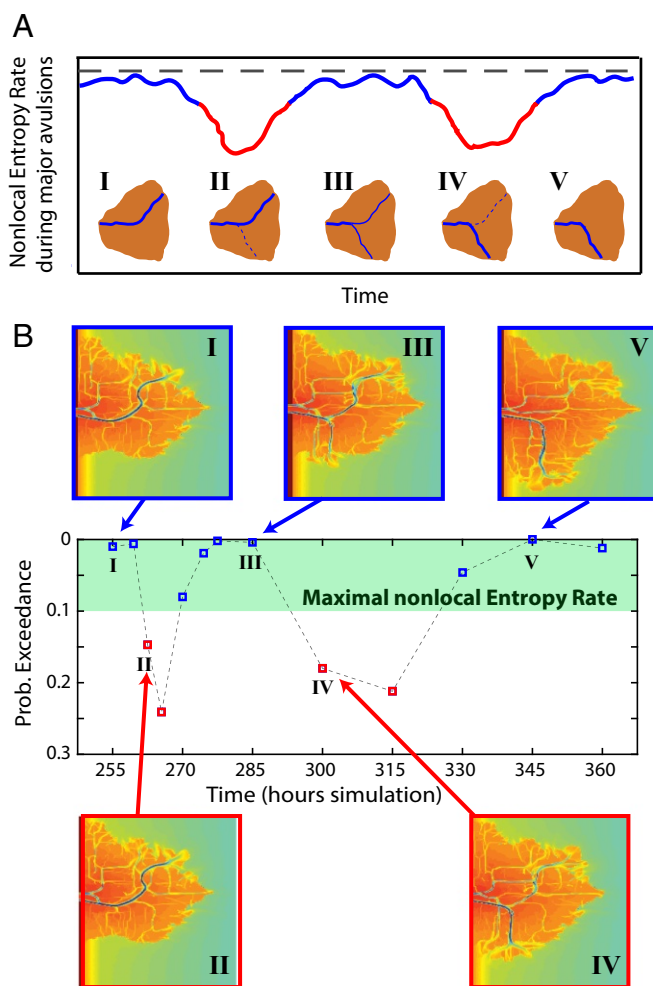
**Steady-State Flux Distribution.** Having a delta represented as a directed acyclic graph (DAG) allows us to compute the steady-state flux by assuming conservation of mass. For instance, Tejedor et al. (29) showed how the steady-state flux can be formulated as an eigenvalue–eigenvector problem of the Laplacian matrix of the graph. Here, we present the input/output model as a more intuitive way to compute the delta steady-state flux. Let us consider a DAG fed from the most upstream node (apex) with a constant unit flux. Without loss of generality, and for simplicity in the subsequent derivations, the apex is assumed to be labeled as node 1. Then, we can define the stationary distribution,  $F$ , as

$$F = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + W \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + W^2 \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \cdots = (I - W)^{-1} \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad [4]$$

where the column vector  $e_1 = (1 \ 0 \ 0 \ \cdots \ 0)^T$  corresponds to the initial state, and  $W$  corresponds to the weighted adjacency matrix of the graph. Alternatively,  $e_1$  represents a constant inflow at the apex normalized to 1, and  $W^k e_1$  the resulting downstream response at time/distance specified by  $k$ . For a DAG, there exists at least one indexing of the graph such that each offspring vertex has a higher index than its parent vertex. In this indexing, by construction, the matrix  $W$  is strictly upper triangular, and therefore the matrix  $W$  is nilpotent, guaranteeing the convergence of the sum as expressed in Eq. 4. In SI Appendix we prove that the stationary flux distribution  $F$  can also be obtained as the stationary probability of a Markov process  $\pi$ .

**Node–Outlet Transition Probability Distribution.** The node-to-outlet transition probability,  $p_{ik}$ , is defined as the probability that a package of flux at node  $i$  drains to outlet  $k$ . Thus, the transport from each node  $i$ , to the different outlets  $M$ , can be understood as a discrete stochastic process with probability distribution  $\{p_{ik}\}$ , with  $k = 1, \dots, M$ . Tejedor et al. (29) showed that when a delta channel network is represented by a directed acyclic graph  $G$  with a weighted adjacency matrix  $W$ ,

- i) The null space of the weighted in-degree Laplacian  $L_W^{in}(G^R)$  for the reverse graph  $G^R$  has dimension (multiplicity of the eigenvalue zero) equal to the number of outlets  $M$ ;



**Fig. 4.**  $nER$  during an avulsion cycle. (A) The results presented in Figs. 2 and 3 support the hypothesis that river deltas self-organize to maximize  $nER$ . However, deltas in dynamic equilibrium experience avulsions, periodically causing major reorganization of their channel networks. Our hypothesis is that during these periods of major reorganization delta  $nER$  drops significantly until the delta fluxes are self-organized back to a new structure as shown in the schematic of A. (Insets, from left to right) Schematics of the five stages within a single avulsion cycle (notice that for illustration purposes, only the main channel was drawn to better depict the avulsion cycle): (I) starting from a given channel network topology, (II) a new channel structure is created downstream of the avulsion node draining initially a small part of the flux, but progressively receiving larger proportions of fluxes until (III) the new and the old channel structures receive similar amounts of flux, and eventually transition to (IV) a configuration wherein the new channel structure has more flux, to finally lead to (V) the abandonment of the former channel structure downstream of the avulsion node. (B) We have tested this hypothesis using Delft3D simulations and analyzing subsequent instances of a delta evolution, wherein an avulsion cycle is observed. The probability (Prob.) of exceedance of the delta  $nER$  by a randomization of the flux configuration is displayed. The results agree with the posed hypothesis, showing that  $nER$  drops at the beginning and end of the avulsion cycle, keeping a maximal value otherwise.

- ii) There exists a unique basis  $\gamma_k$ ,  $k = 1, \dots, M$ , of this null space in  $\mathbb{R}^N$  (i.e., the basis consists of  $M$  vectors each having  $N$  entries) with the property

$$\gamma_k(i) = \begin{cases} 1, & i = k \\ 0, & i \neq k \end{cases} \quad \text{for } k = 1, \dots, M. \quad [5]$$

That is, the entry of the vector  $\gamma_k$  corresponding to outlet  $k$  is one, and zero at all other outlets.

iii) The value  $\gamma_k(i)$  represents the portion of flux at the vertex  $i$  that drains to the outlet  $k$ , that is,  $p_{ik}$ .

Thus, if we define a matrix  $T$ , whose columns form the basis of the null space of  $L_W^0(G^R)$ ,  $\{\gamma\}$ , then the  $i$ -th row corresponds to the probability distribution  $\{p_{ik}\}$ .

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