Solutions of HW3

1) Find the extremum of the fuctional

$$J[y] = \int_1^2 y'(1+x^2y')\,dx$$

with the boundary condition $y(1)=0$ and $y(2)=1$.

Solution: Euler's equation $\underbrace{\frac{\partial f}{\partial y}}_{0} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$

$$\Rightarrow \frac{\partial f}{\partial y'} = 1 + 2x^2y' \equiv \text{constant} = C$$

$$\Rightarrow y' = \frac{C-1}{2x^2} \Rightarrow \int dy = \int \frac{C-1}{2x^2}\,dx$$

$$\Rightarrow y = \frac{(C-1)}{2}\left(-\frac{1}{x}\right) + \underset{\uparrow \text{int. const.}}{d}$$

$$y(1) = \frac{(C-1)}{2}\left(-\frac{1}{1}\right) + d = 0 \Rightarrow d = \frac{C-1}{2}$$

$$y(2) = \frac{(C-1)}{2}\left(-\frac{1}{2}+1\right) = 1 \Rightarrow C = 5 \Rightarrow y(x) = \left(\frac{5-1}{2}\right)\left(1-\frac{1}{x}\right)$$

$$\boxed{y(x) = 2\left(1-\frac{1}{x}\right)}$$

Grading Policy / Homework 3

1) $f(y, y', x)$ 10

Euler's Equation 25

Integral 25

$y(x)$ 10

constants c & d 30