CSCI-GA.1170-003/004 Fundamental Algorithms March 13, 2020

Solutions to Problem 3 of Homework 6 (20 points)

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In parts (a), (b) you are given two statements pertaining to a binary search tree. Either prove the statement or disprove it by providing a counterexample[1].

(a) (4 points) For any binary search tree T and any element $x \notin T$, if one applies in sequence $\textsc{Insert}(x)$ followed by $\textsc{Delete}(x)$, then the result will be always T.

Solution:

True. For any binary search tree T and any element $x \notin T$, inserting x will always create a new leaf node, and subsequently deleting x will remove the new leaf node and always yield the original binary search tree T.

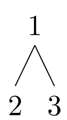
□

(b) (4 points) For any binary search tree T and any element $x \in T$, if one applies in sequence $\textsc{Delete}(x)$ followed by $\textsc{Insert}(x)$, then the result will be always T.

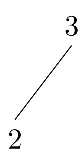
Solution:

False.

Consider the tree below and let $x = 1$



Now if we Delete(1) we can have



and then Insert(1) we have

Clearly now we have a different tree.

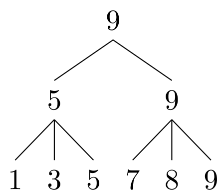
□

[1] If the statement is true, you need to give a general argument for any T. If false, you choose a specific T which illustrates the problem.

In parts (c), (d), (e) we will consider a 2-3 Tree and study the commutative properties of the operations. For purposes of this question, use *only* the algorithm as described in the handouts.

(c) (4 points) To an empty 2-3 Tree insert the following elements in the following order: 1, 5, 7, 9, 3, 8. Draw the resulting tree. Call this tree as T.

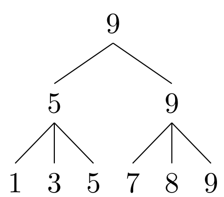
Solution:



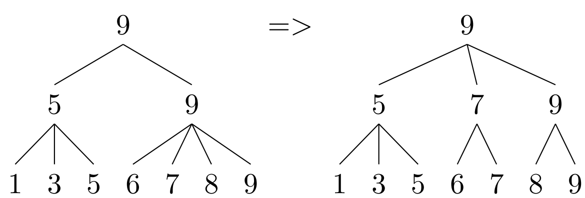
$\square$

(d) (4 points) Show an element $x \notin T$ such that applying in sequence $\textsc{Insert}(x)$ and $\textsc{Delete}(x)$ will result with a tree T' that is different from T.

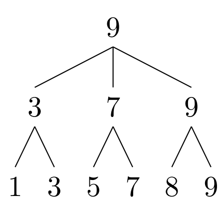
Solution: We start with



Now take $x \notin T$ where $x = 6$. Now if we Insert(6) we have



and if we Delete(6) we have

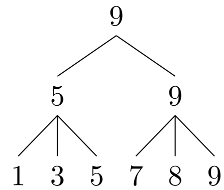


Clearly, we have a tree T' that is different from T.

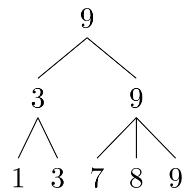
$\square$

(e) (4 points) Show an element $x \in T$ such that applying in sequence $\textsc{Delete}(x)$ and $\textsc{Insert}(x)$ will result with a tree T' that is different from T. Note that the tree T is as solved in Part (c).

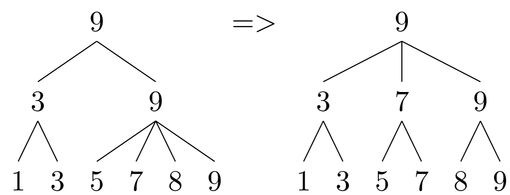
Solution: We start with



Now take $x \in T$ where $x = 5$. Now if we Delete(5) we have



and if we Insert(5) we have



Clearly, we have a tree T' that is different from T.

$\square$