

Curved, Warped, and Lumpy: Some Initial Results from the American Social Fabric Project

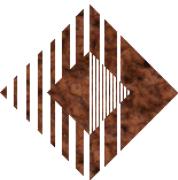
Carter T. Butts

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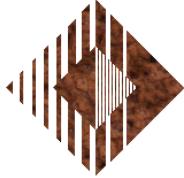
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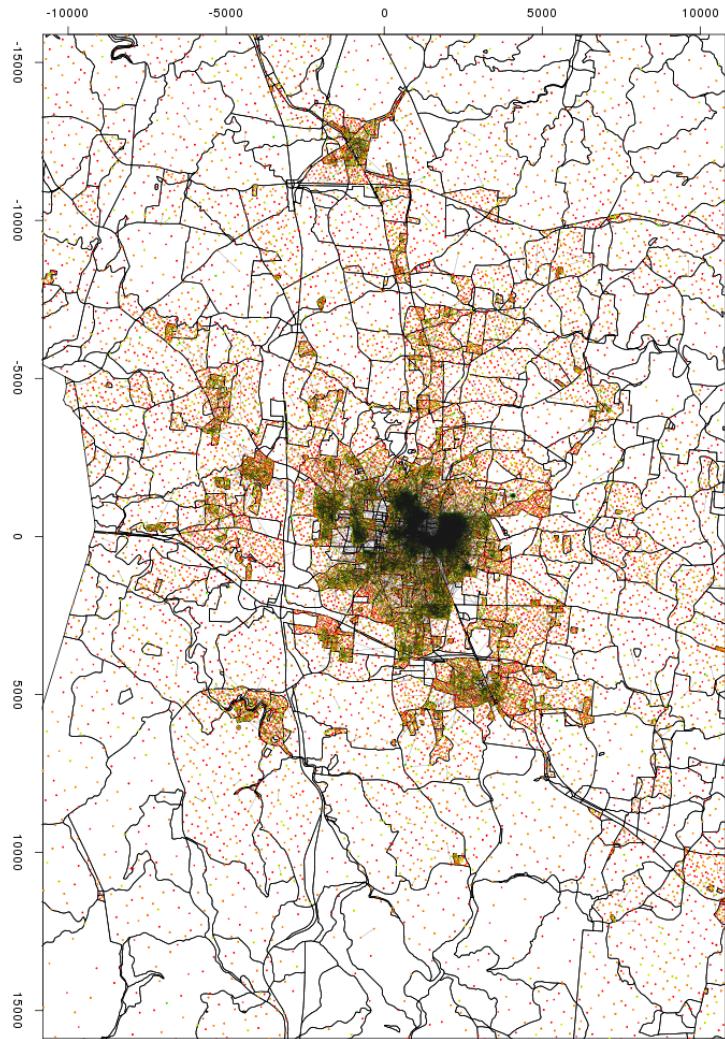


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Road Map

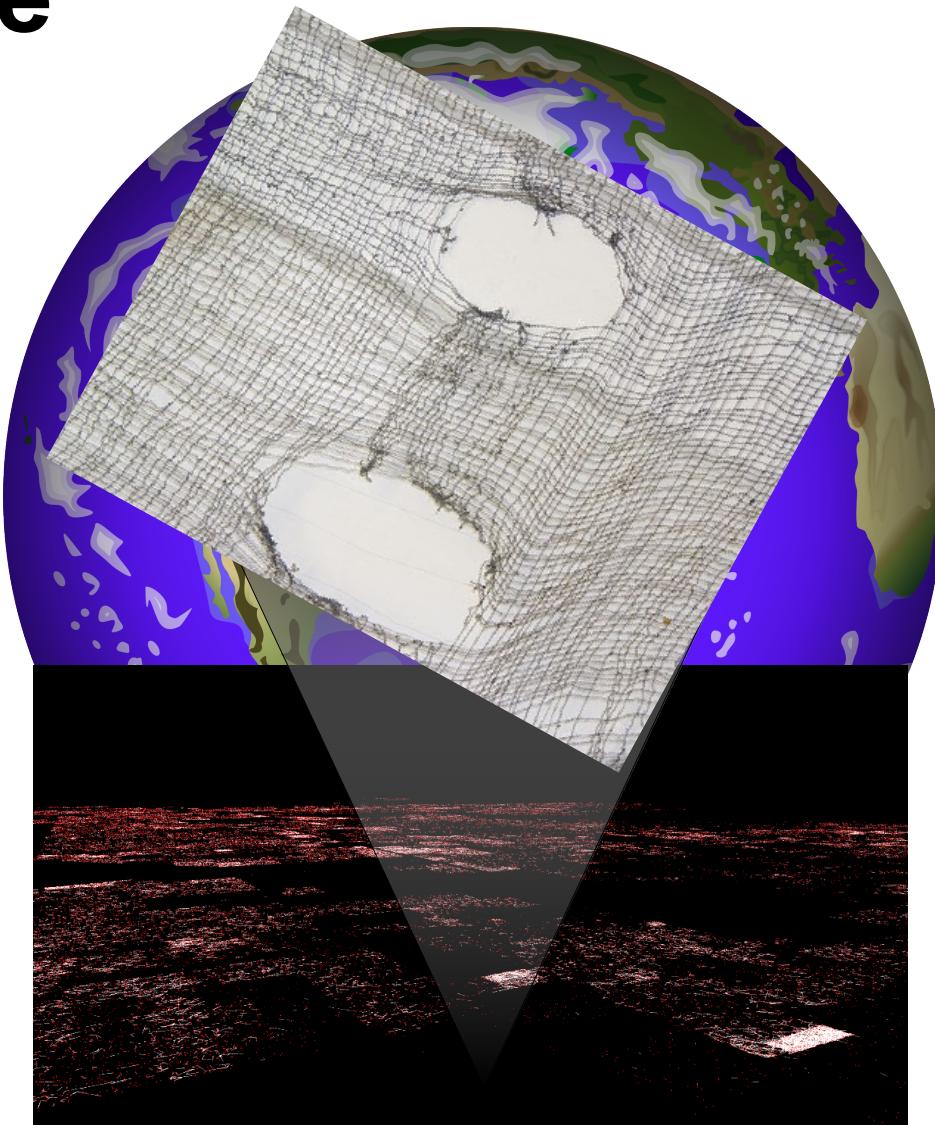
- **Introduction**
- **A “first-order” look at spatial network structure**
- **Measuring network “distortions” in the western US**
- **Conclusions and such**

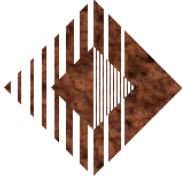




"Social Fabric" - Structure Across Space

- ***Little known fact:*** social systems exist in a physical world (!)
 - Even computer-mediated interactions have physical elements
- **Accounting for spatial influences on interaction**
 - Studies by Festinger, Hägarstrand, Latané, Zipf, Stewart, Freeman, Wellman, and many others suggest powerful effects of distance on interaction
 - Powerful predictor – or confounder
- **Measuring spatial macrostructure**
 - New (“fabric-like”) properties emerge at the macro level which demand to be treated directly

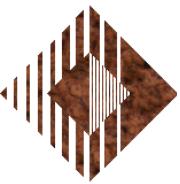




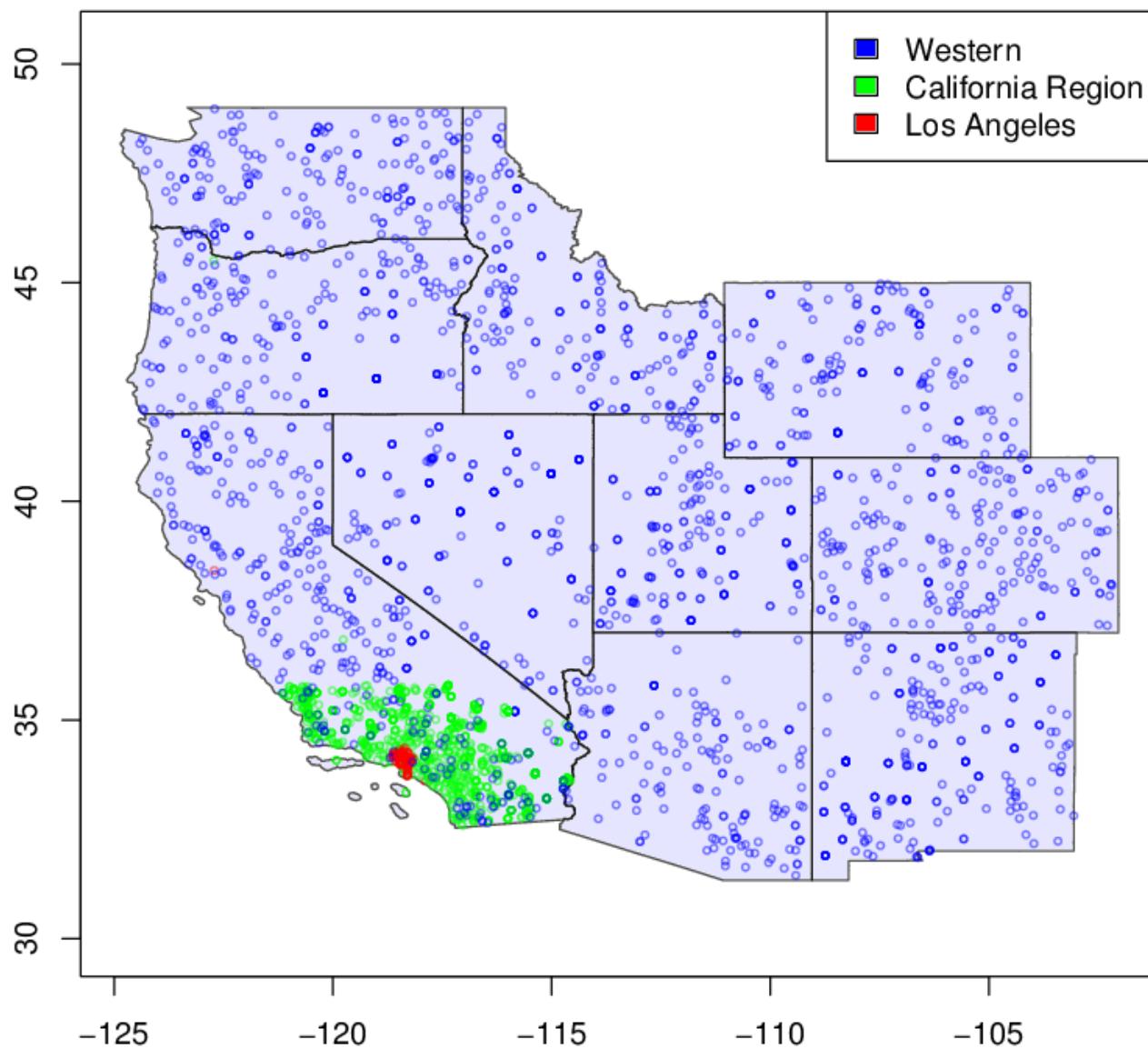
The American Social Fabric Project

- NSF-funded effort to examine the spatial structure of large-scale interpersonal networks
- Egocentric network survey
 - Spatially stratified (apx uniform) samples from SoCal, western US
 - Random sample of LA residents
 - Elicitation for 6 relational types; geographical information for ego/alter
 - Fairly large ($N=3370$), good spatial coverage

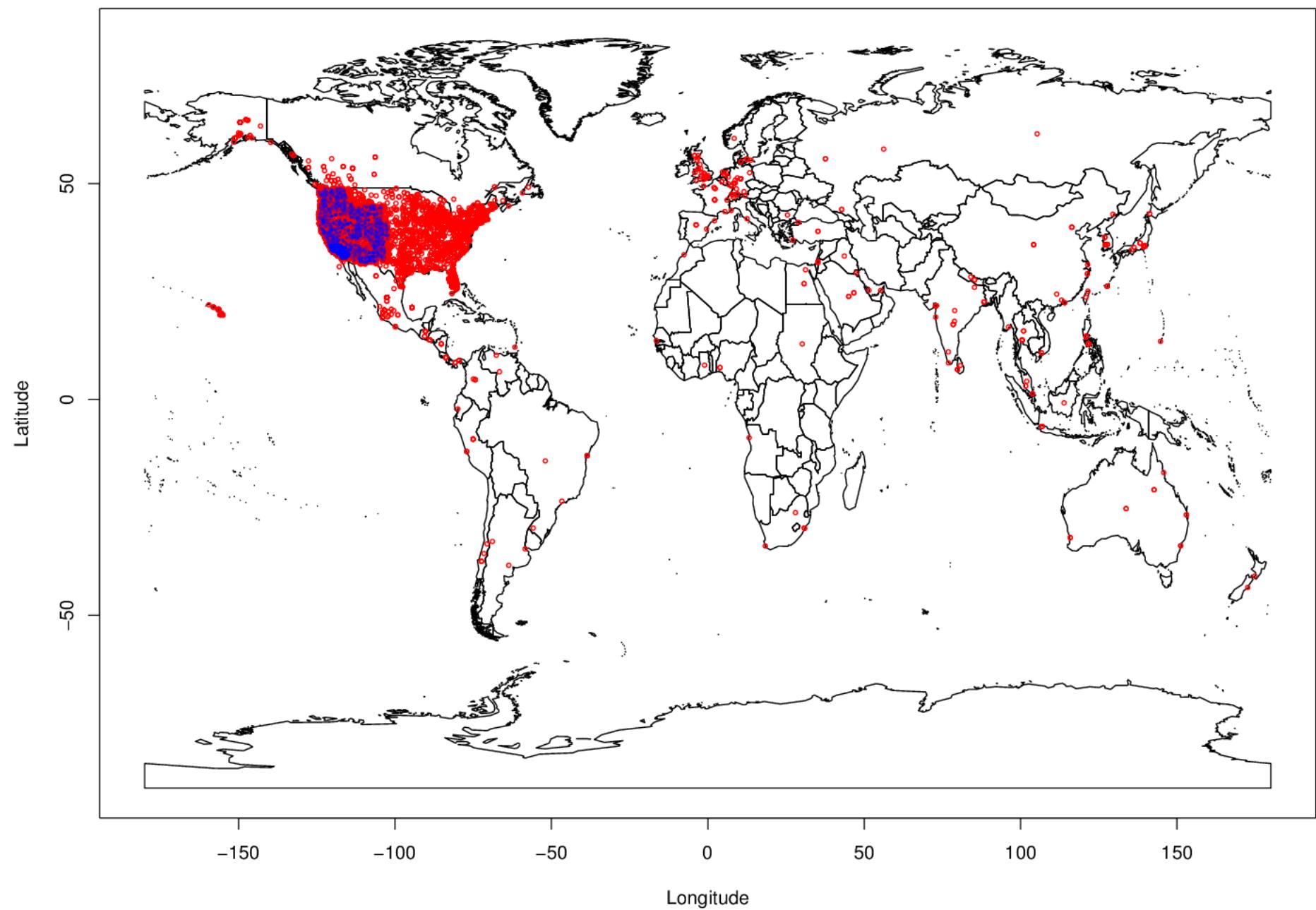


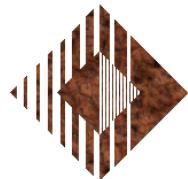


ASFP Ego Locations, by Sample



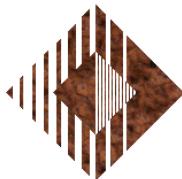
ASFP Ego and Alter Locations



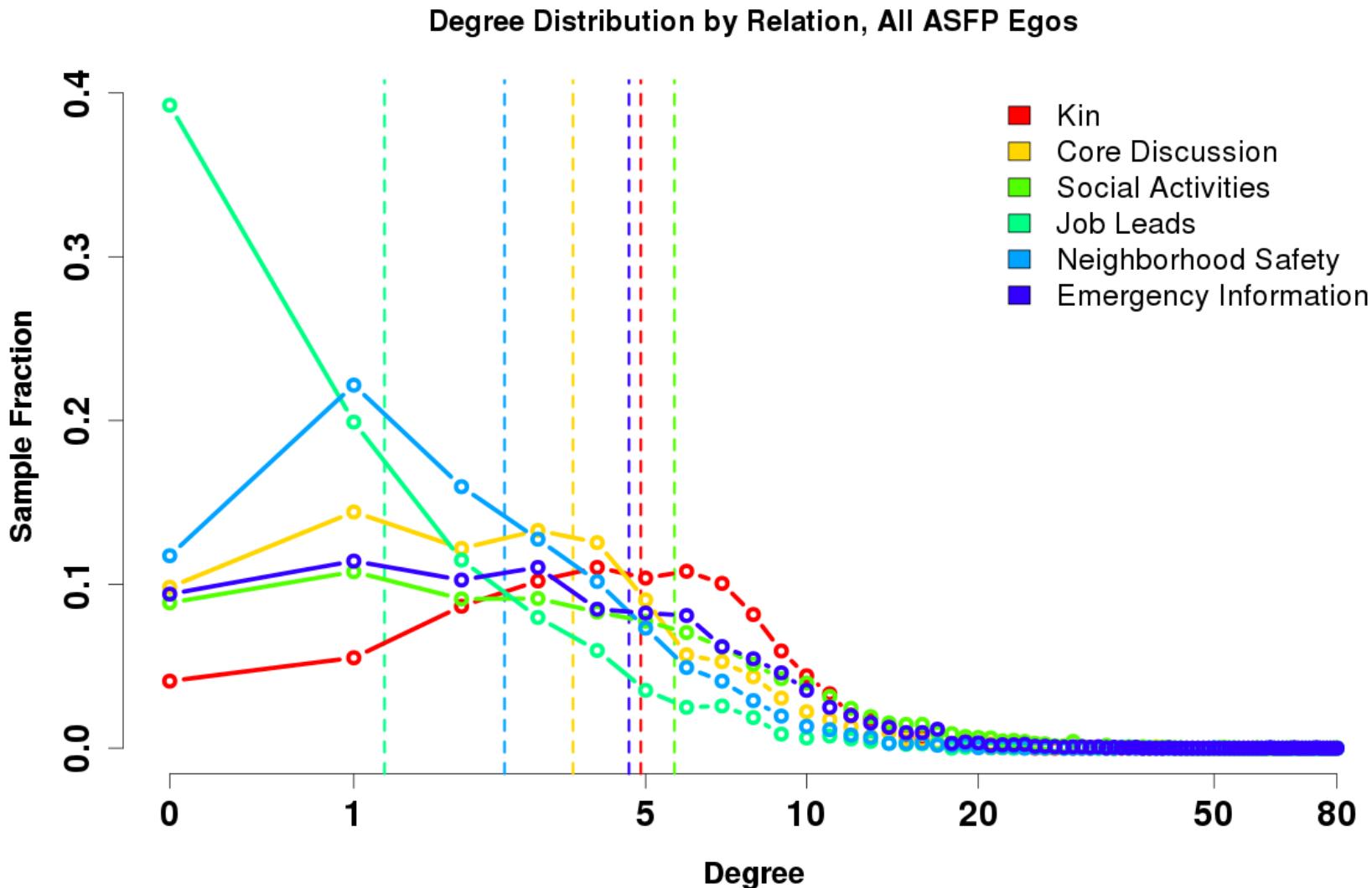


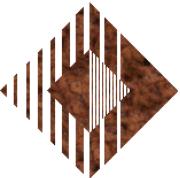
ASFP Egos and Alters



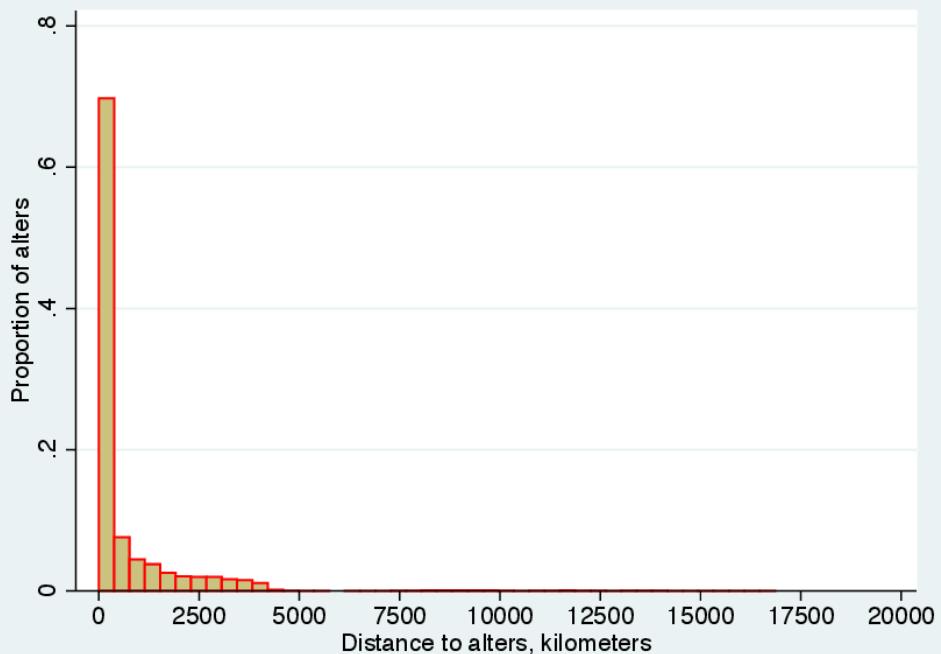


ASFP Degree Distribution

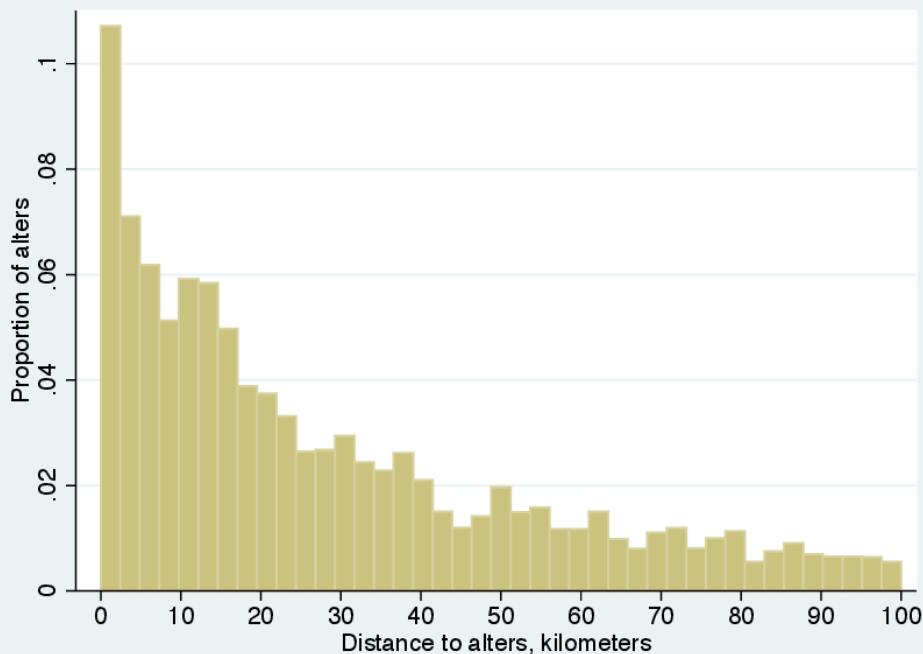




Ego/Alter Distances, all ASFP Relations



All Distances

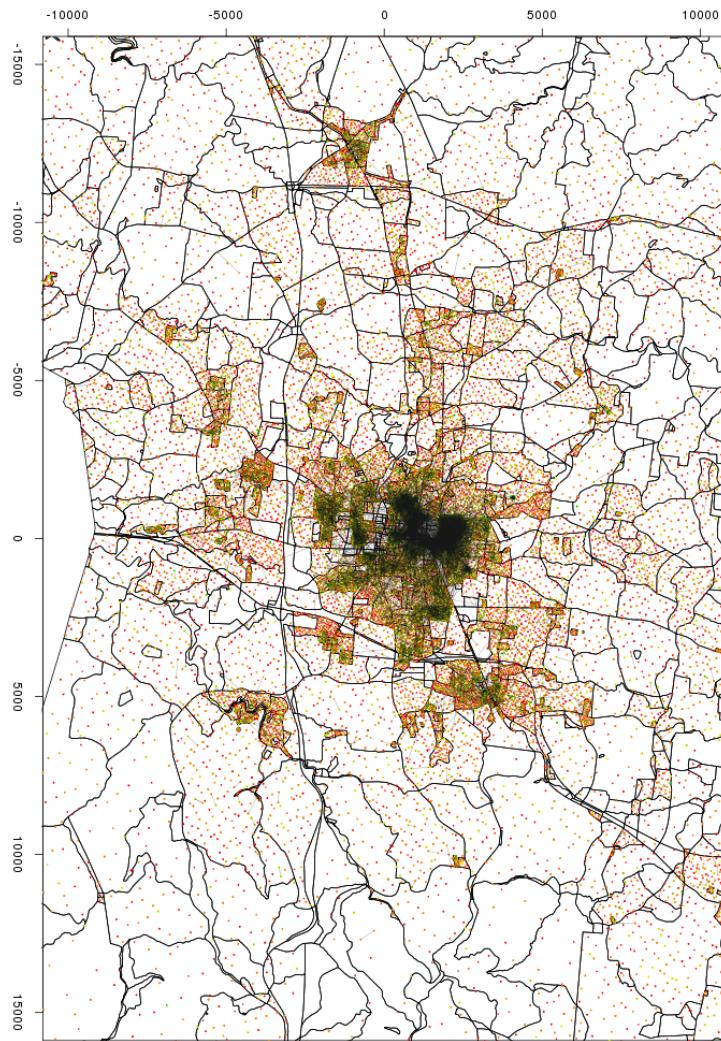


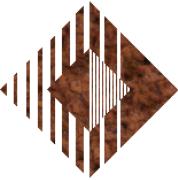
Distances<100km



Road Map

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- Conclusions and such



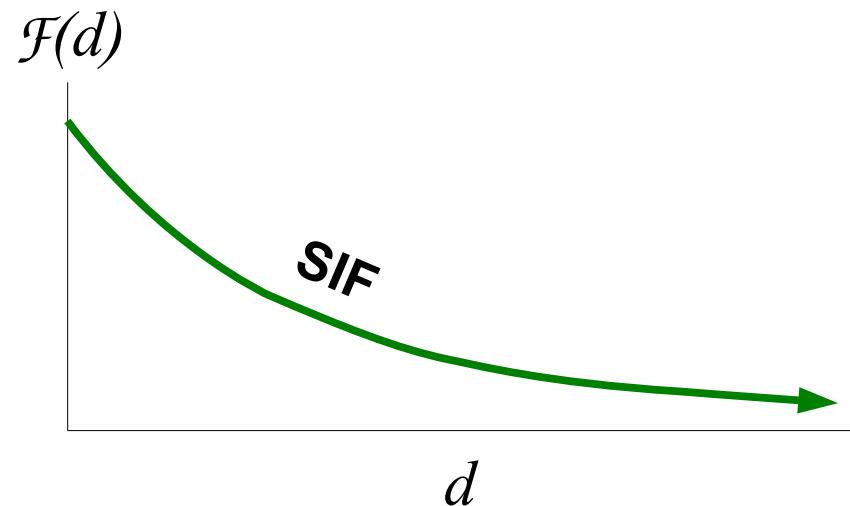
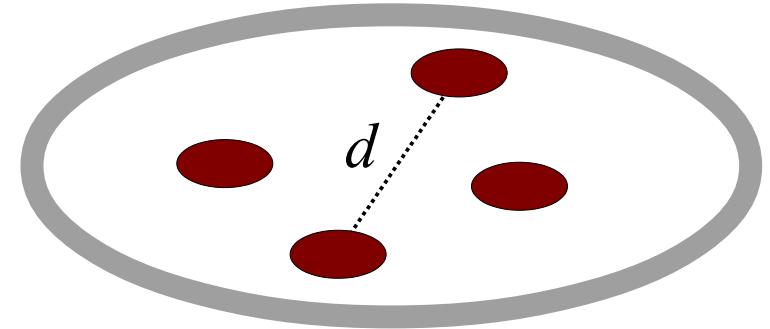


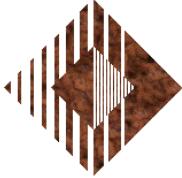
Spatial Structure in Networks

- a “First Order” View

- Many complexities: let's start simple

- Populations in space; each individual associated with a location
 - For two randomly selected individuals at distance d , marginal tie probability given by some function $\mathcal{F}(d)$ (the *spatial interaction function*, or SIF)
 - First-order approximation to network structure; summarizes expected degree of interaction through space





A Simple First Question

- If we choose a random individual, how many ties will he or she have between distance d and $d+\delta$?
 - Useful tool: Ripley's K function. $K(d)$ is the expected number of points within distance d of a randomly chosen point.
 - First derivative, $K'(d)$ gives expected number of points at distance d (proportional to *pair correlation function* in point process literature)
 - Expected number of ties given by
$$\mathbf{E}Y_{i,(d,d+\delta)} = \int_d^{d+\delta} \mathcal{F}(x) K'(x) dx$$
 - Can interpret as product of *baseline opportunities* for interaction (Mayhew, 1984) with conditional probability of interaction; $\mathcal{F}(d)$ depends on relation, $K'(d)$ on population
 - Two “main ingredients” of structure
- What should K' be like for a real population? Let's start with the most naive thing imaginable....

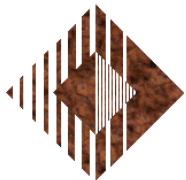
UPDATED AND EXPANDED



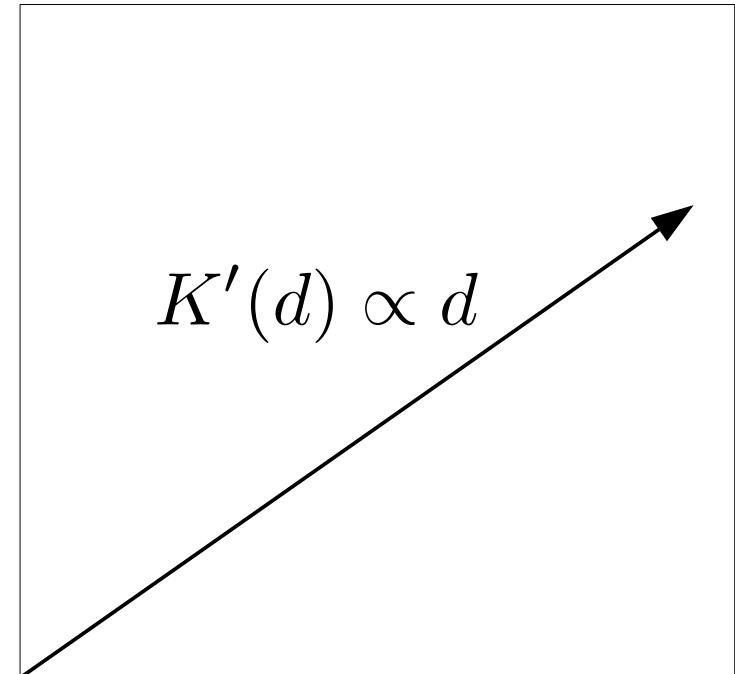
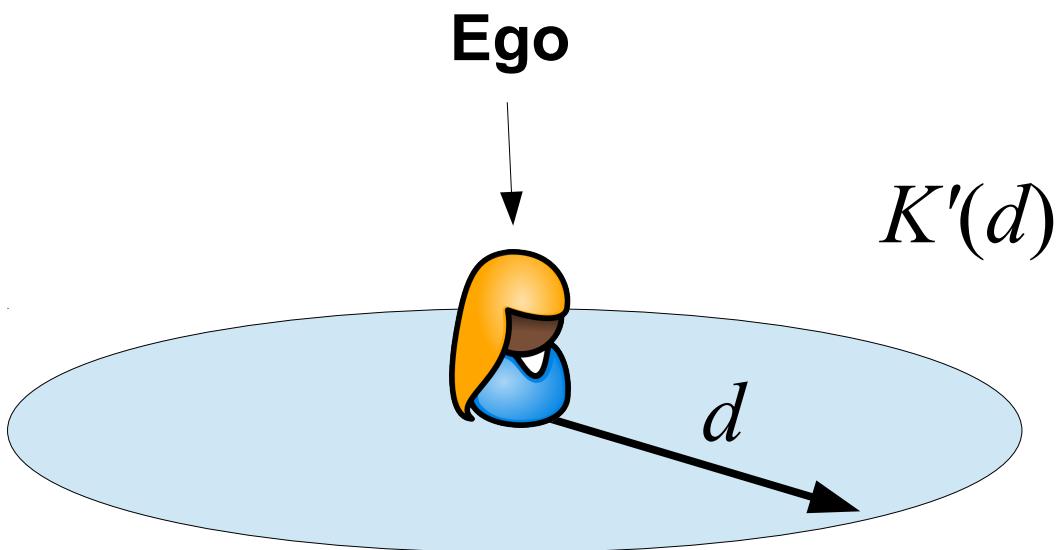
The World Is Flat

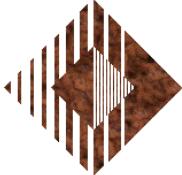
A BRIEF HISTORY OF
THE TWENTY-FIRST CENTURY

Thomas L. Friedman

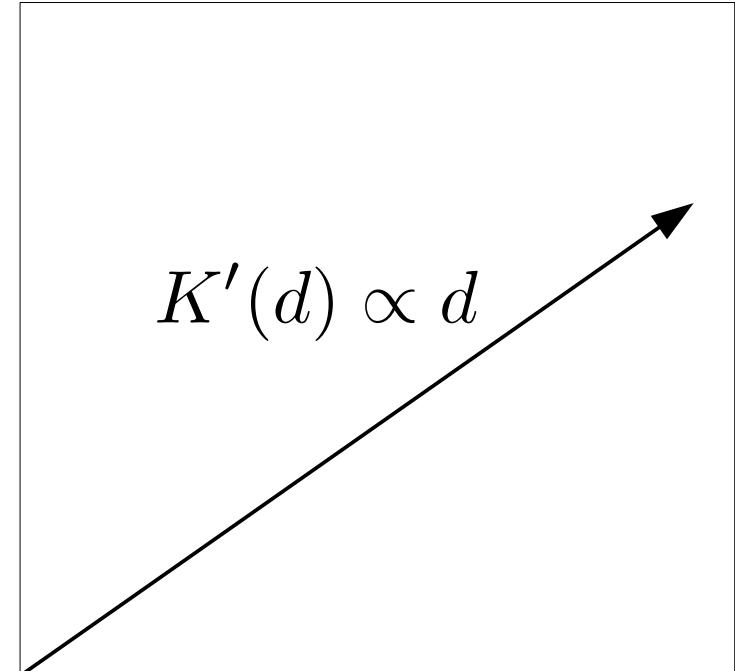
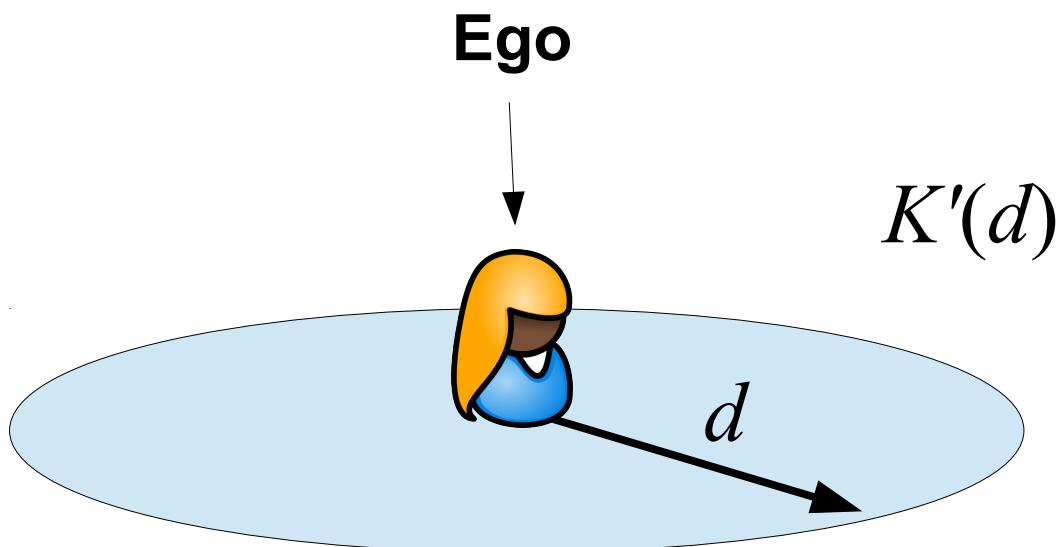


Interaction Opportunity in a Flat World





Interaction Opportunity in a Flat World



Interaction opportunities grow without bound, forever!

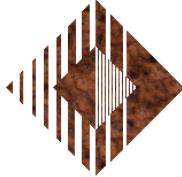


(NASA VIRS Image)

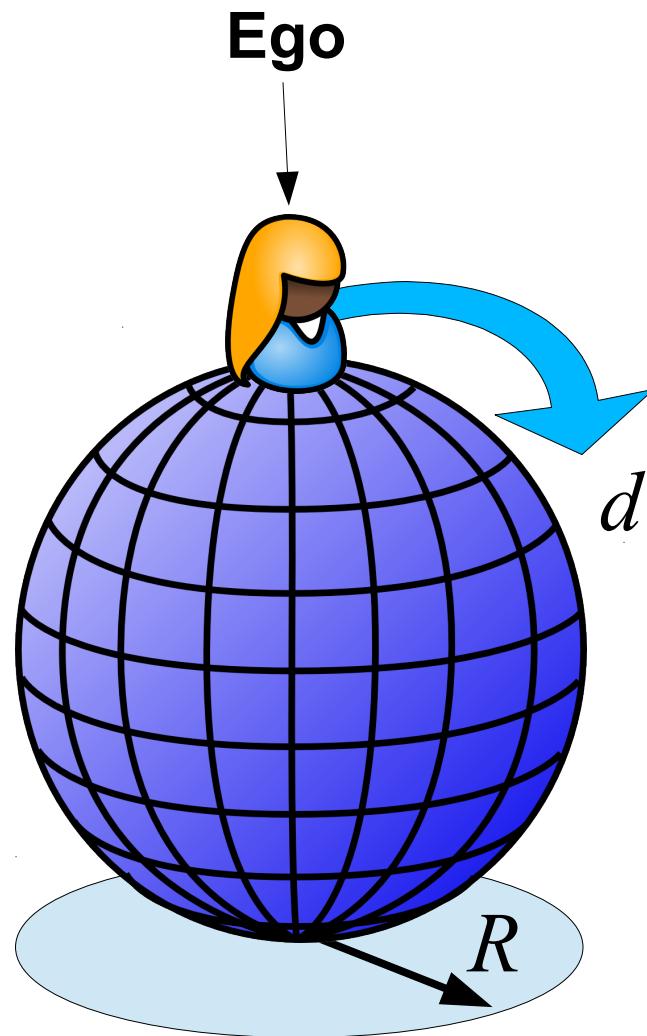


(Not flat.)

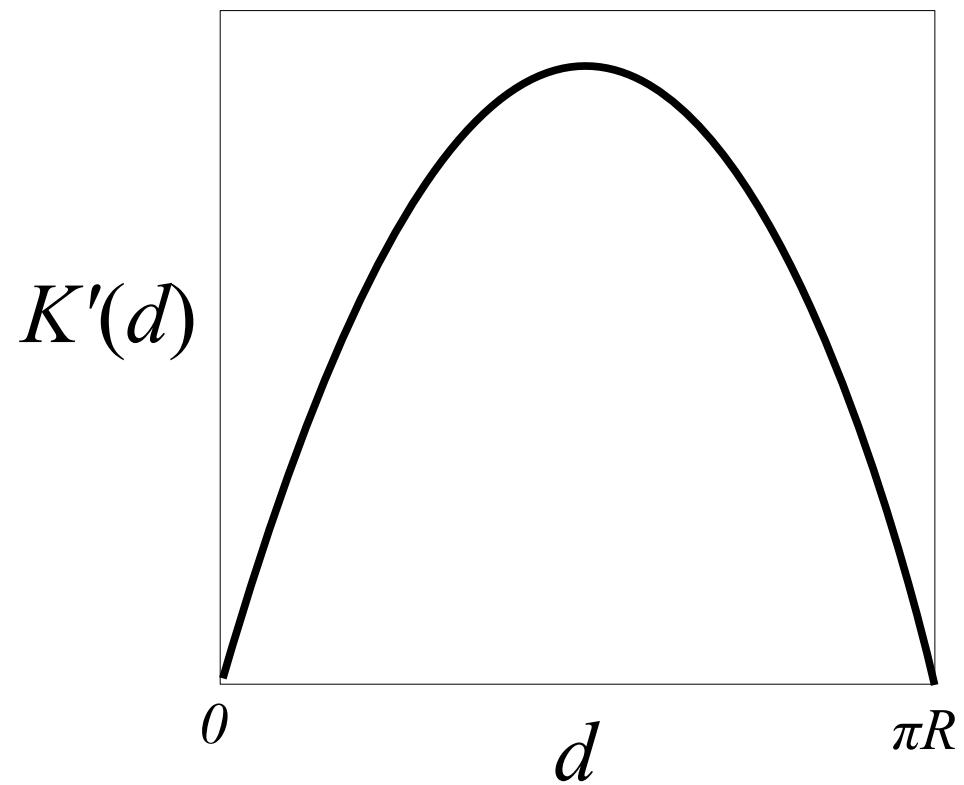
(NASA VIRS Image)

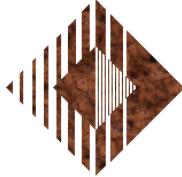


Interaction Opportunity in a Round World

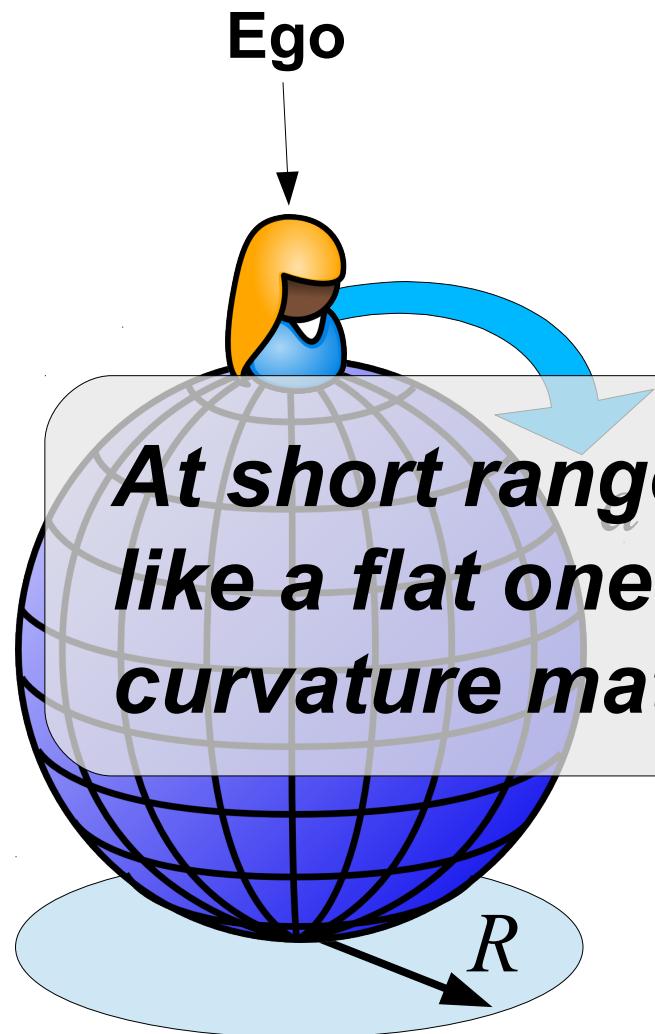


$$K'(d) \propto \sin\left(\frac{d}{\pi R}\right)$$



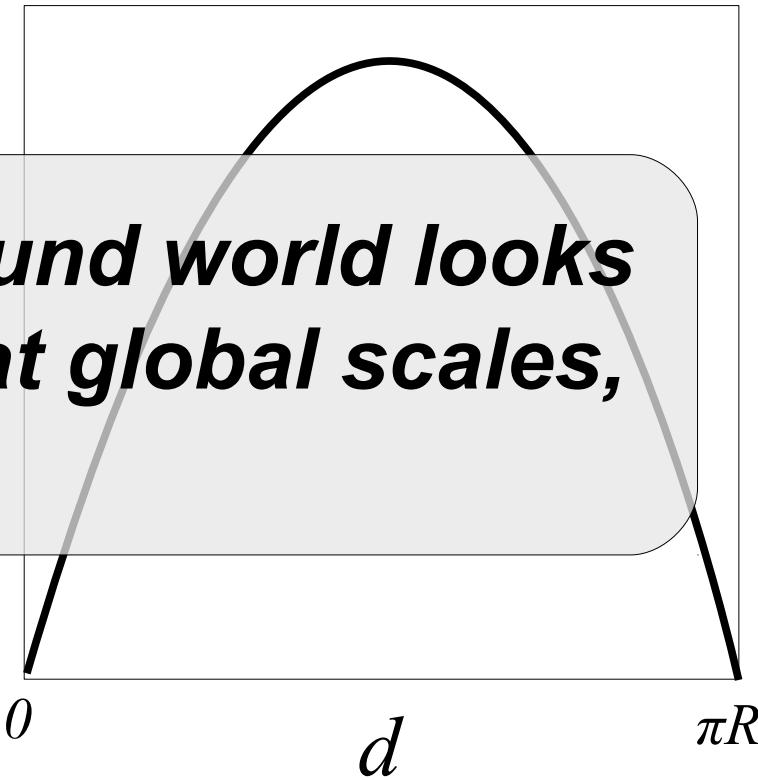


Interaction Opportunity in a Round World



At short range, a round world looks like a flat one - but at global scales, curvature matters!

$$K'(d) \propto \sin\left(\frac{d}{\pi R}\right)$$



UPDATED AND EXPANDED



Curved



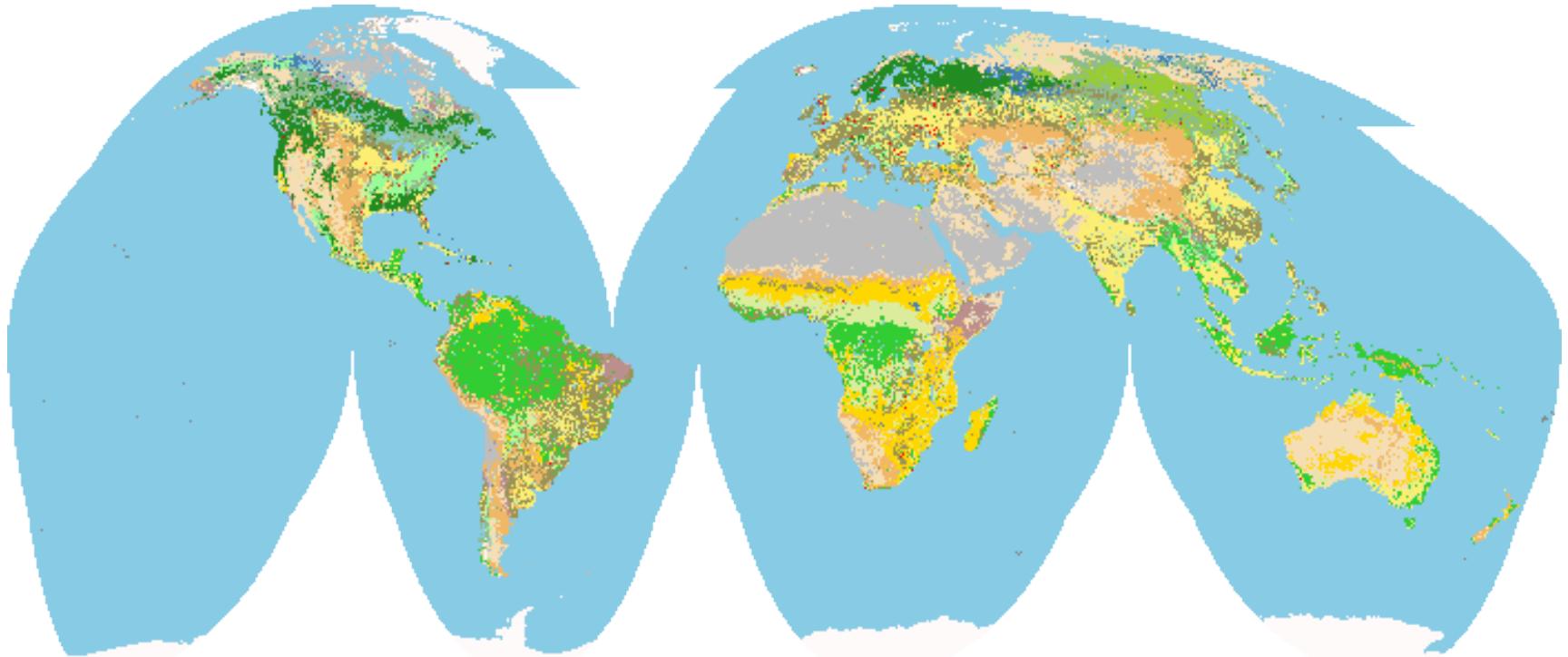
The World Is Flat



A BRIEF HISTORY OF
THE TWENTY-FIRST CENTURY

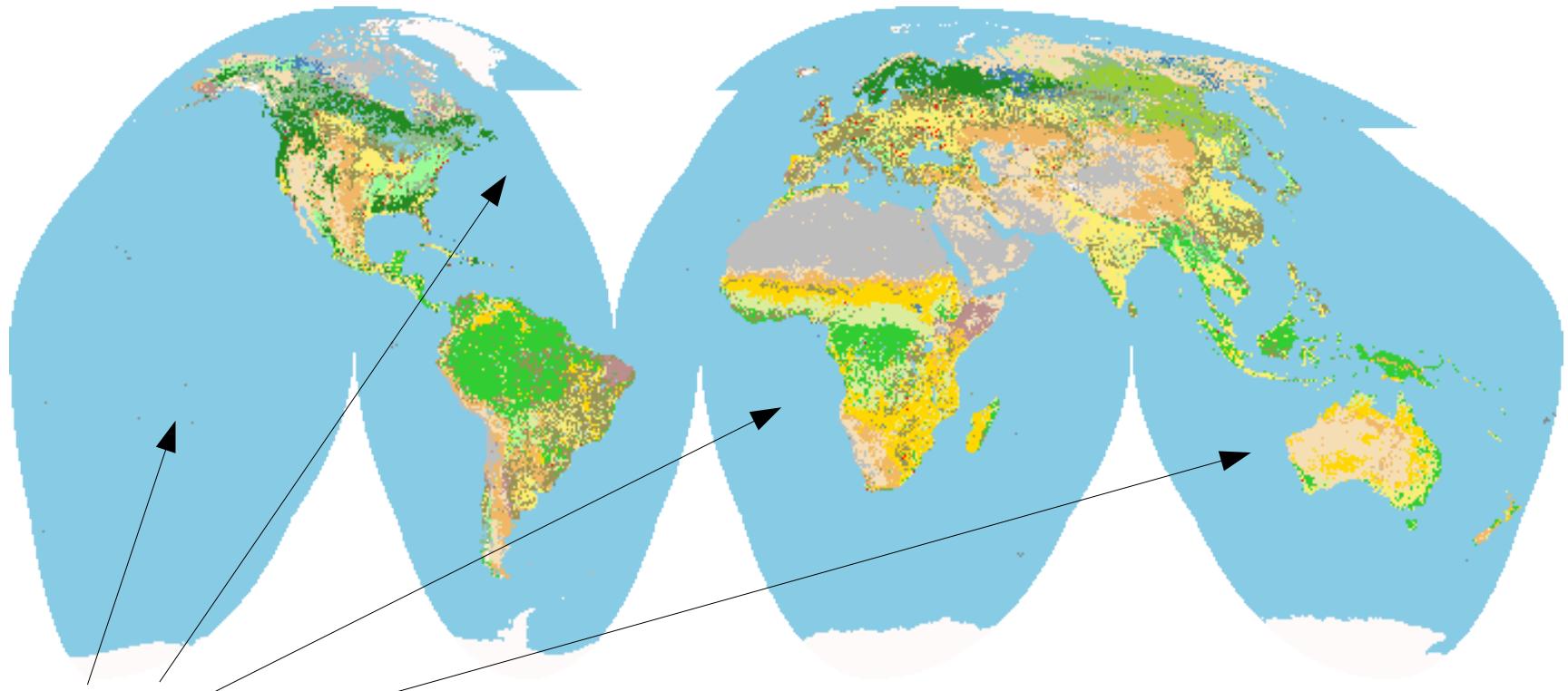
Thomas L. Friedman

The World



USGS Landcover Institute - Global 1km Landcover Map

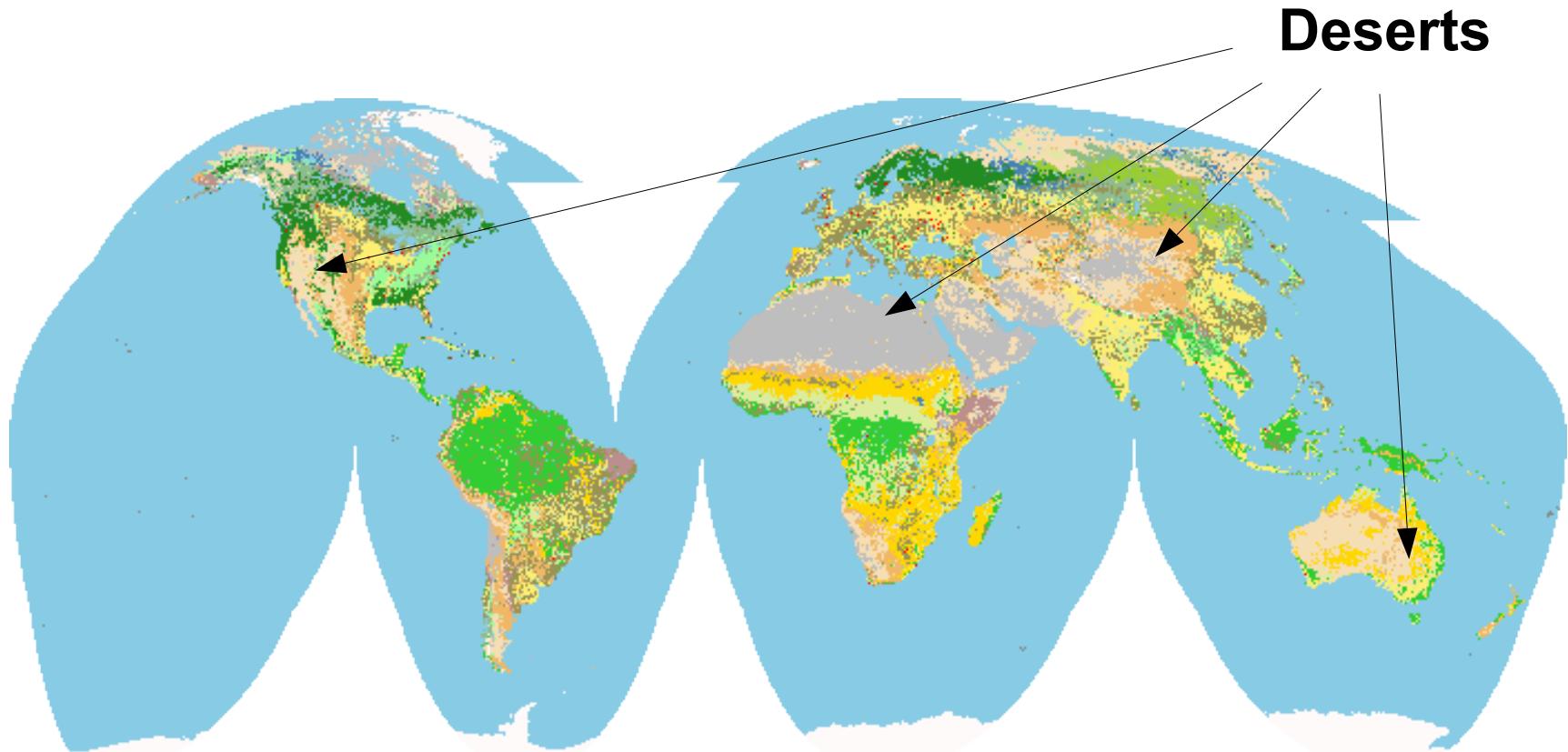
The World



Oceans

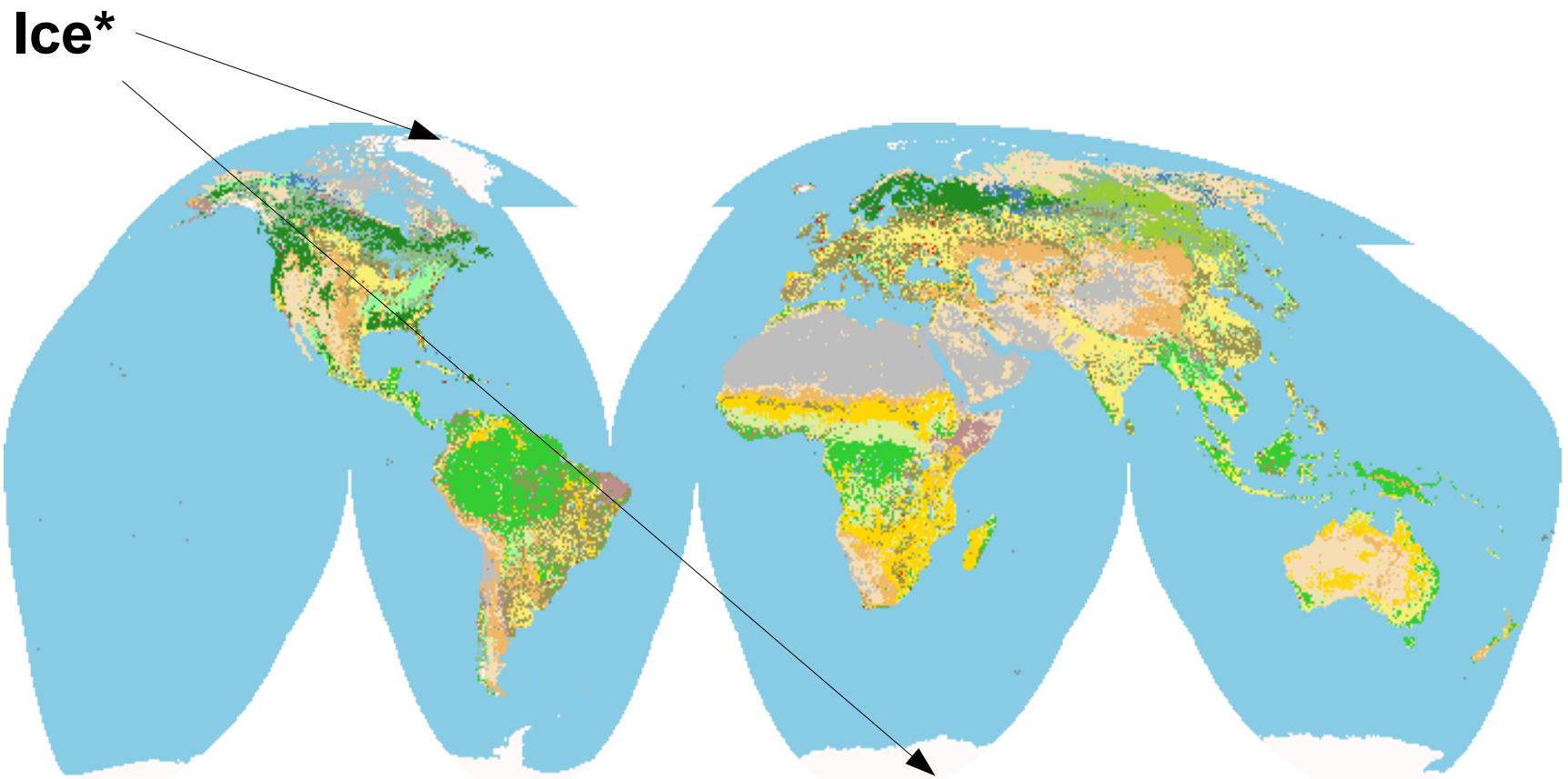
USGS Landcover Institute - Global 1km Landcover Map

The World



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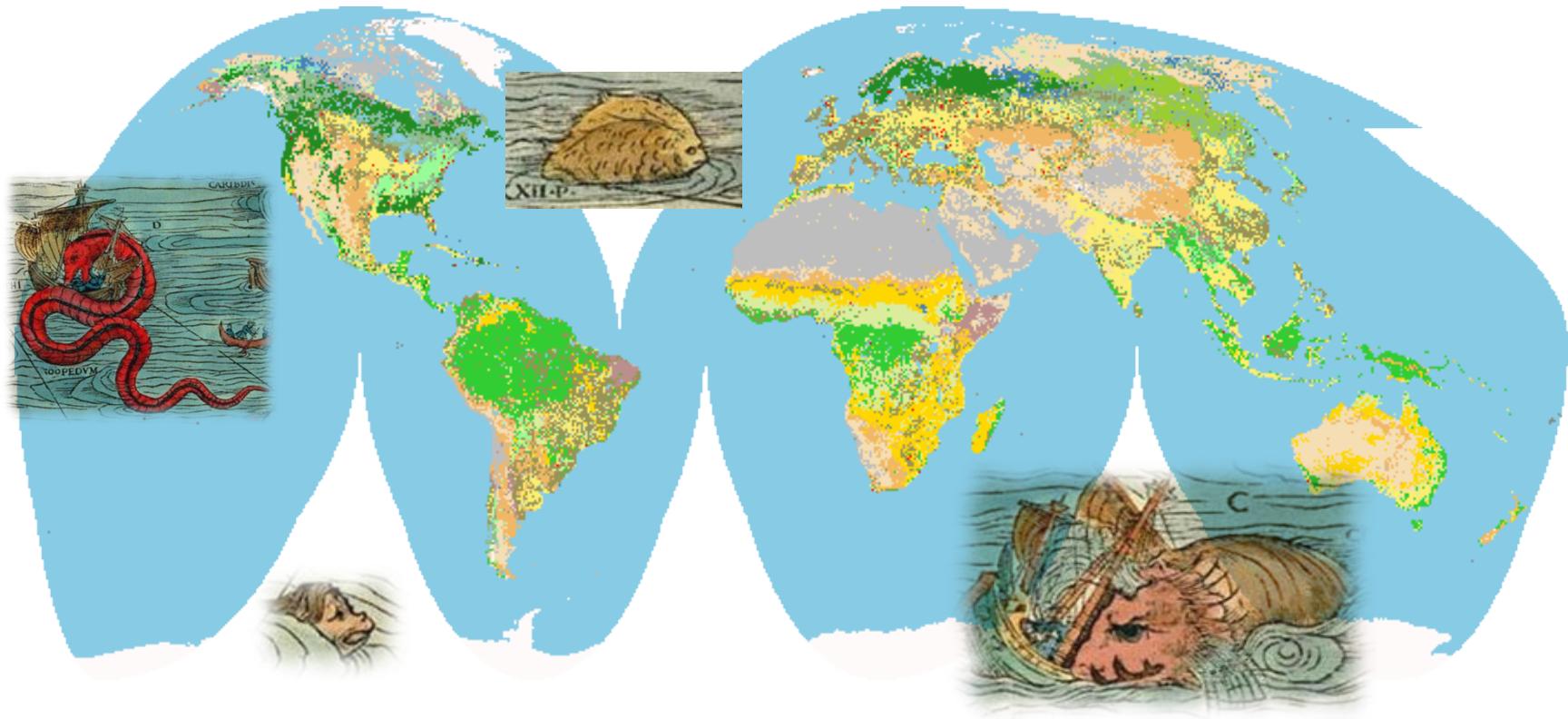
The World



(*For Now)

USGS Landcover Institute - Global 1km Landcover Map

The World



USGS Landcover Institute - Global 1km Landcover Map

UPDATED AND EXPANDED



(*And Lumpy)

Curved*

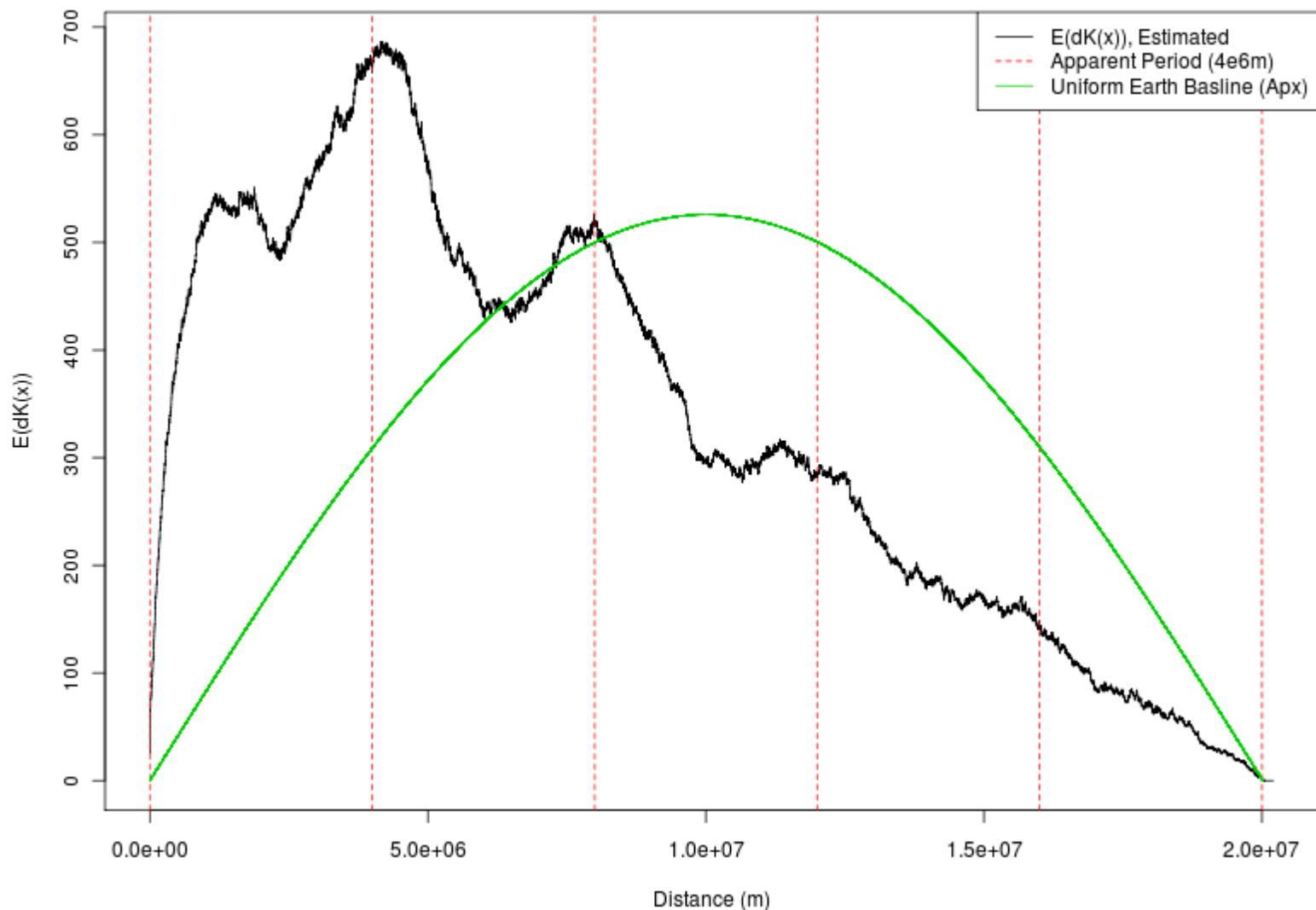


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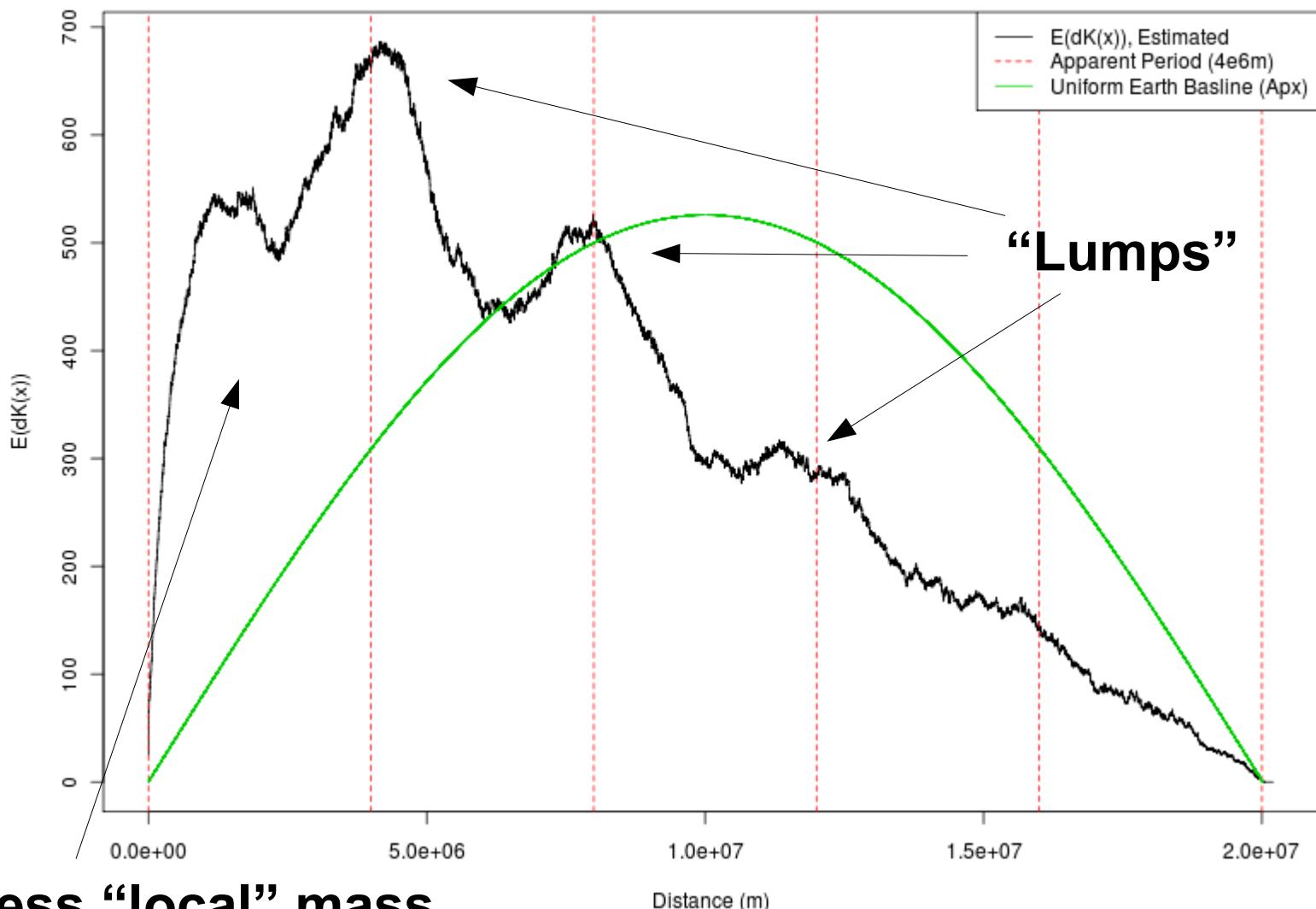
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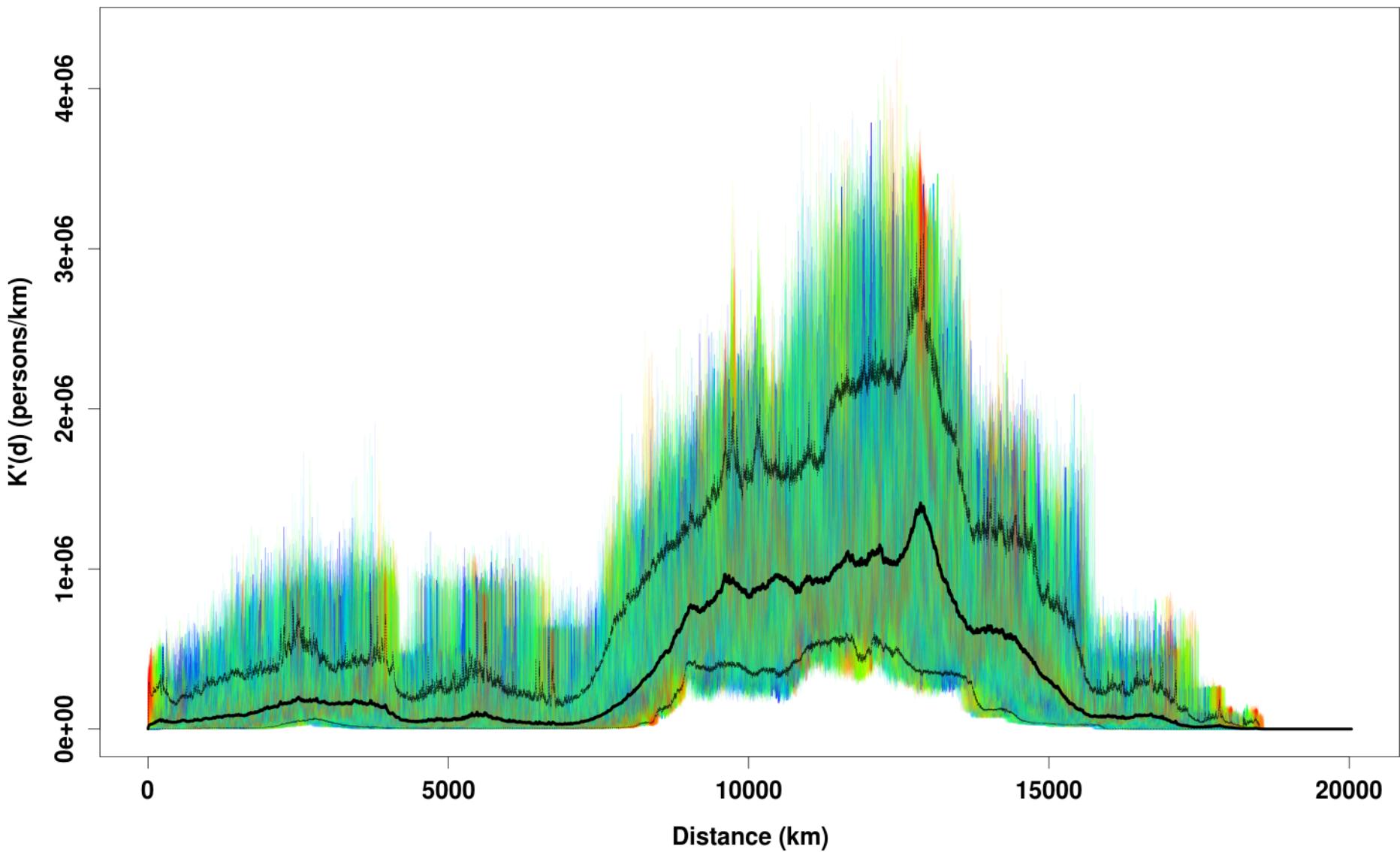
Expected dK by Distance, Global Average



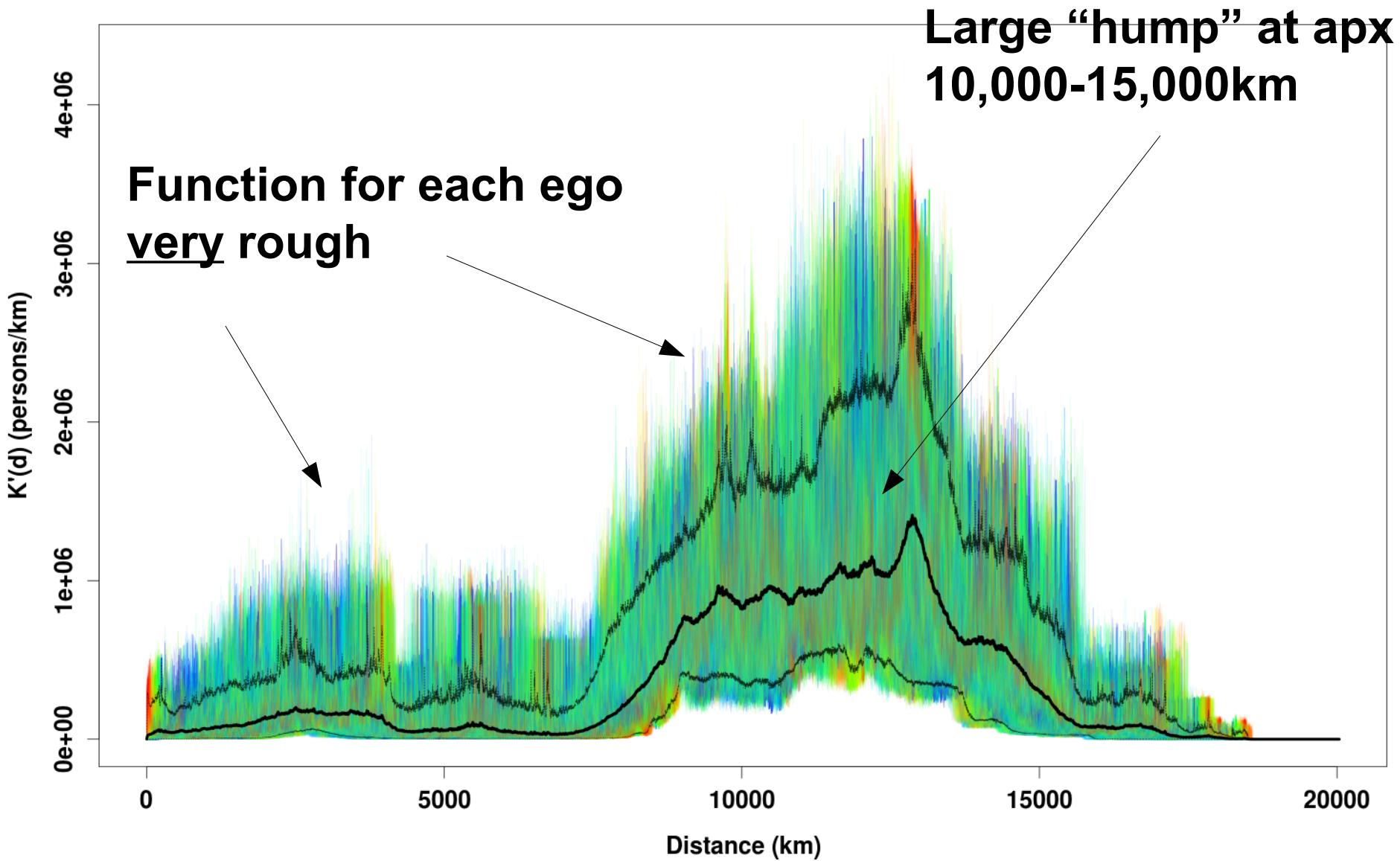
Expected dK by Distance, Global Average

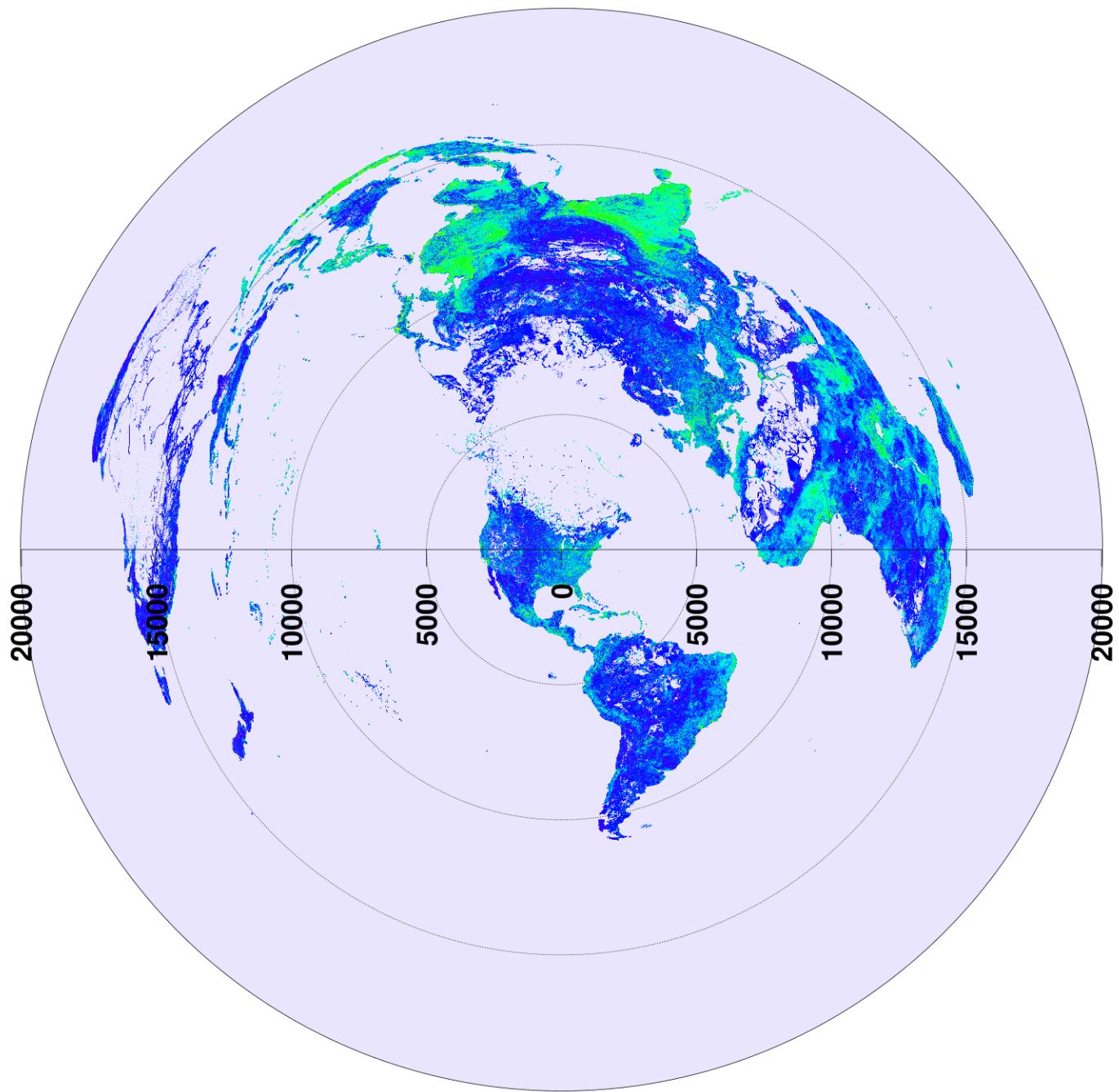


$K'(d)$ for ASFP Egos

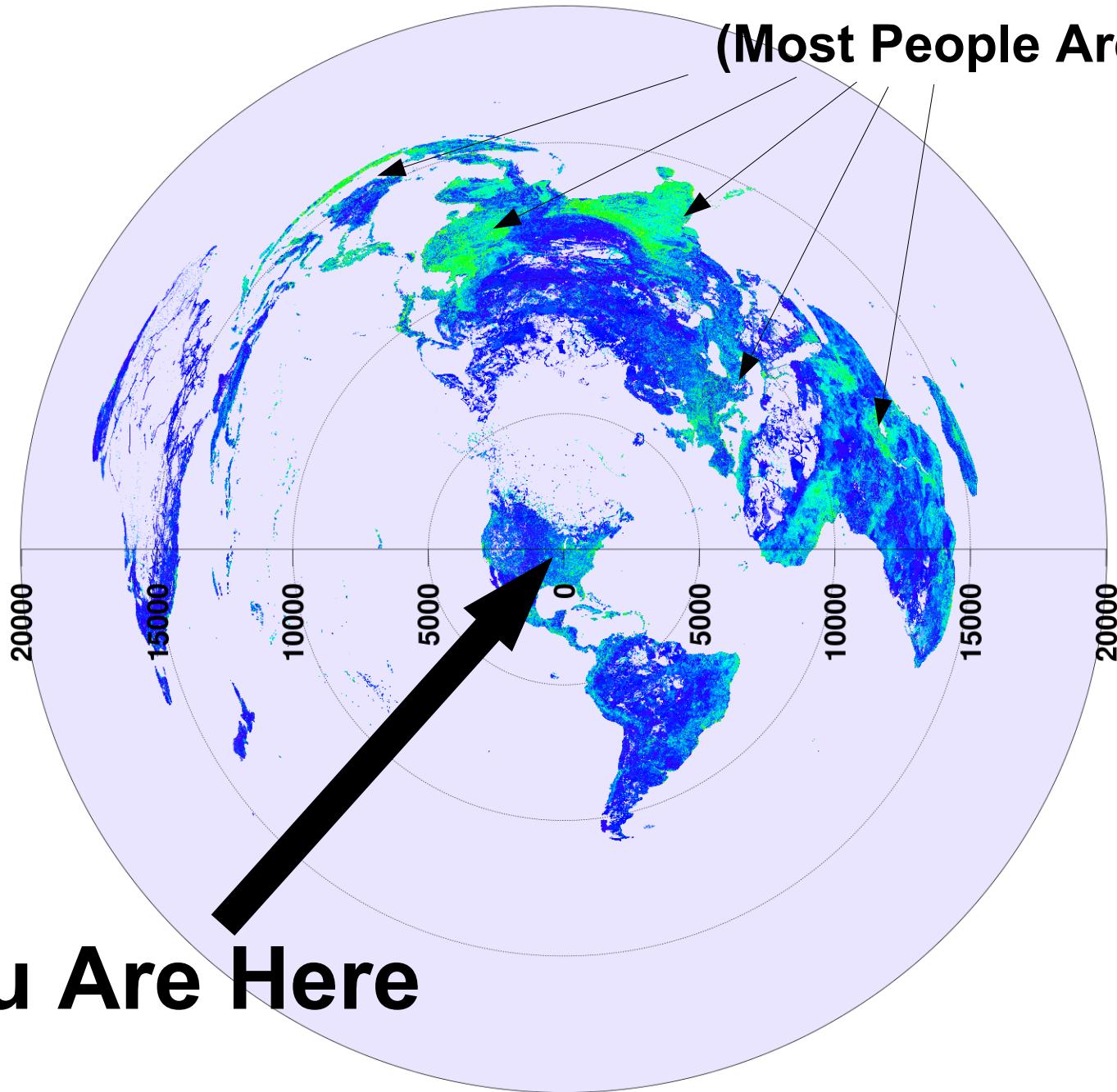


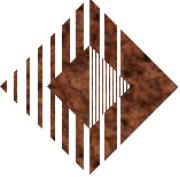
$K'(d)$ for ASFP Egos





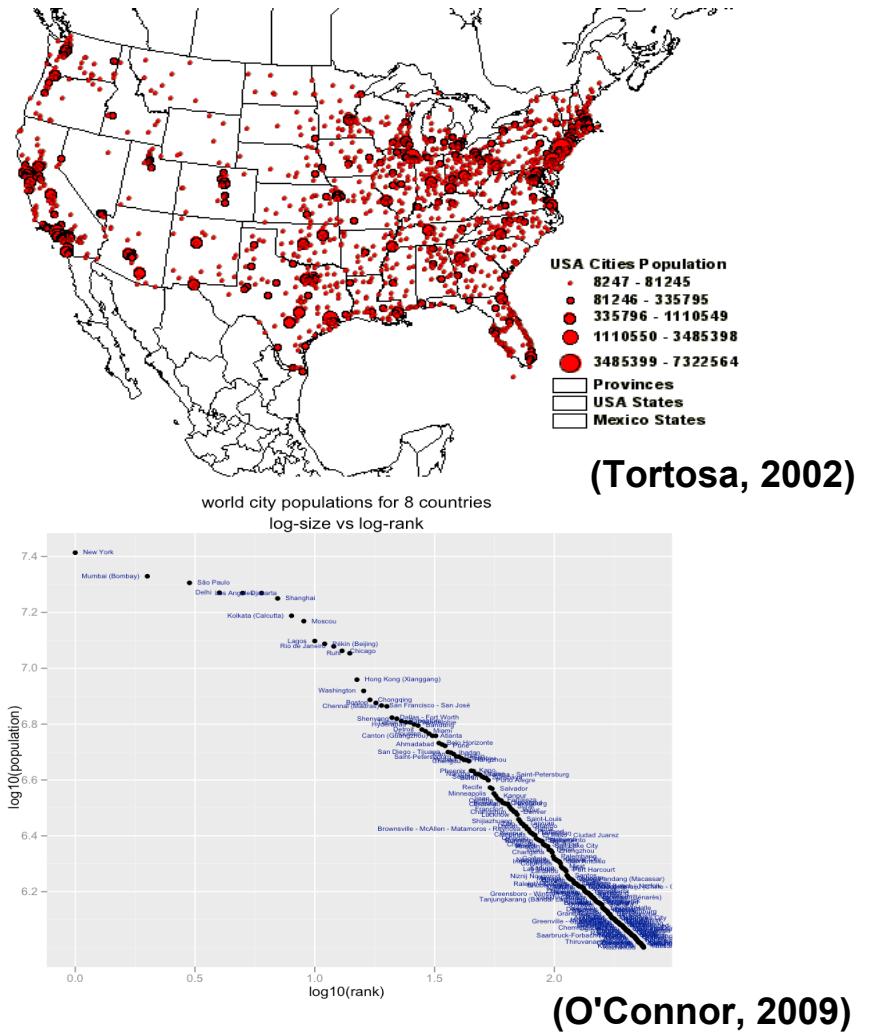
(Most People Are Here)



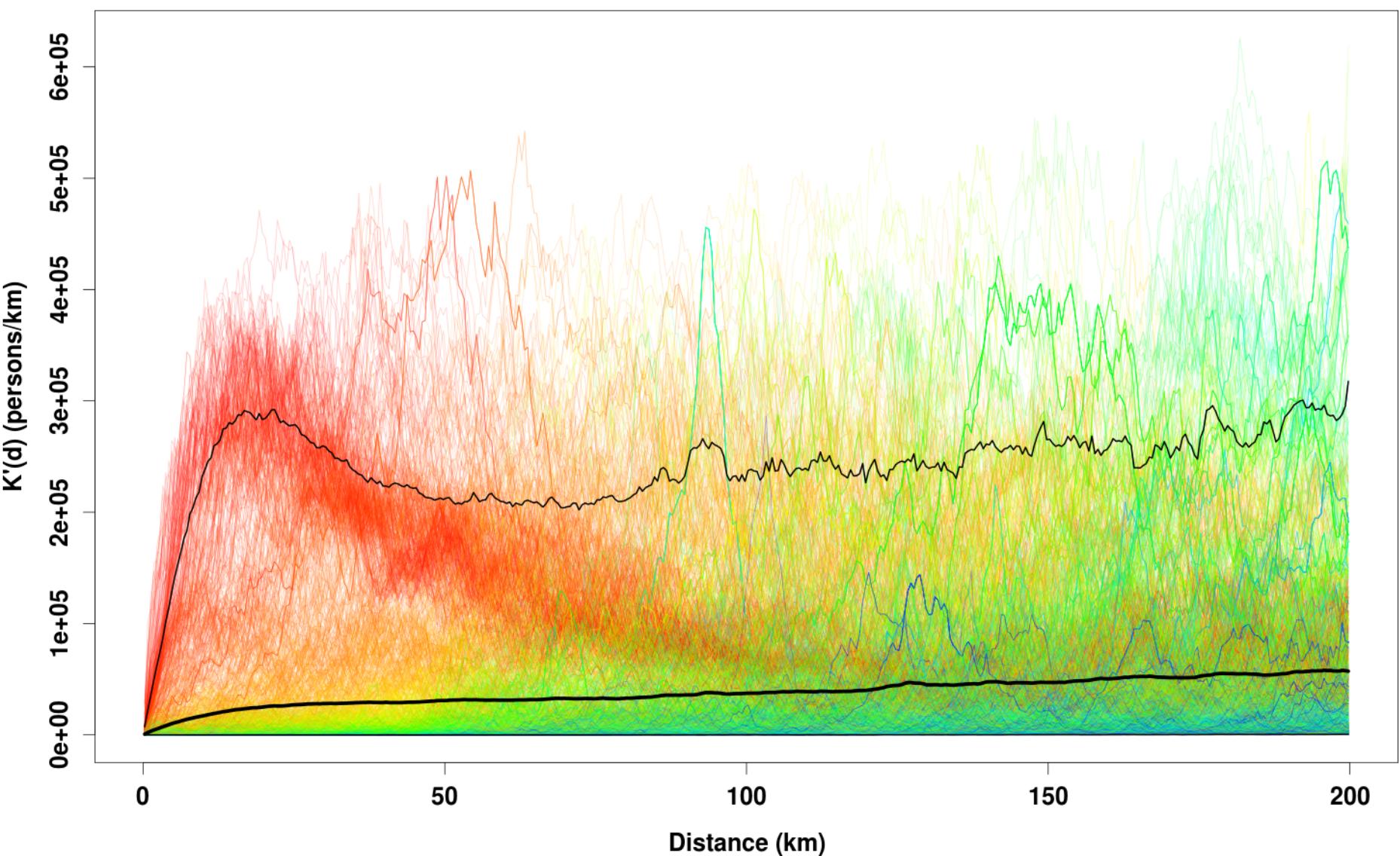


But Why So Rough?

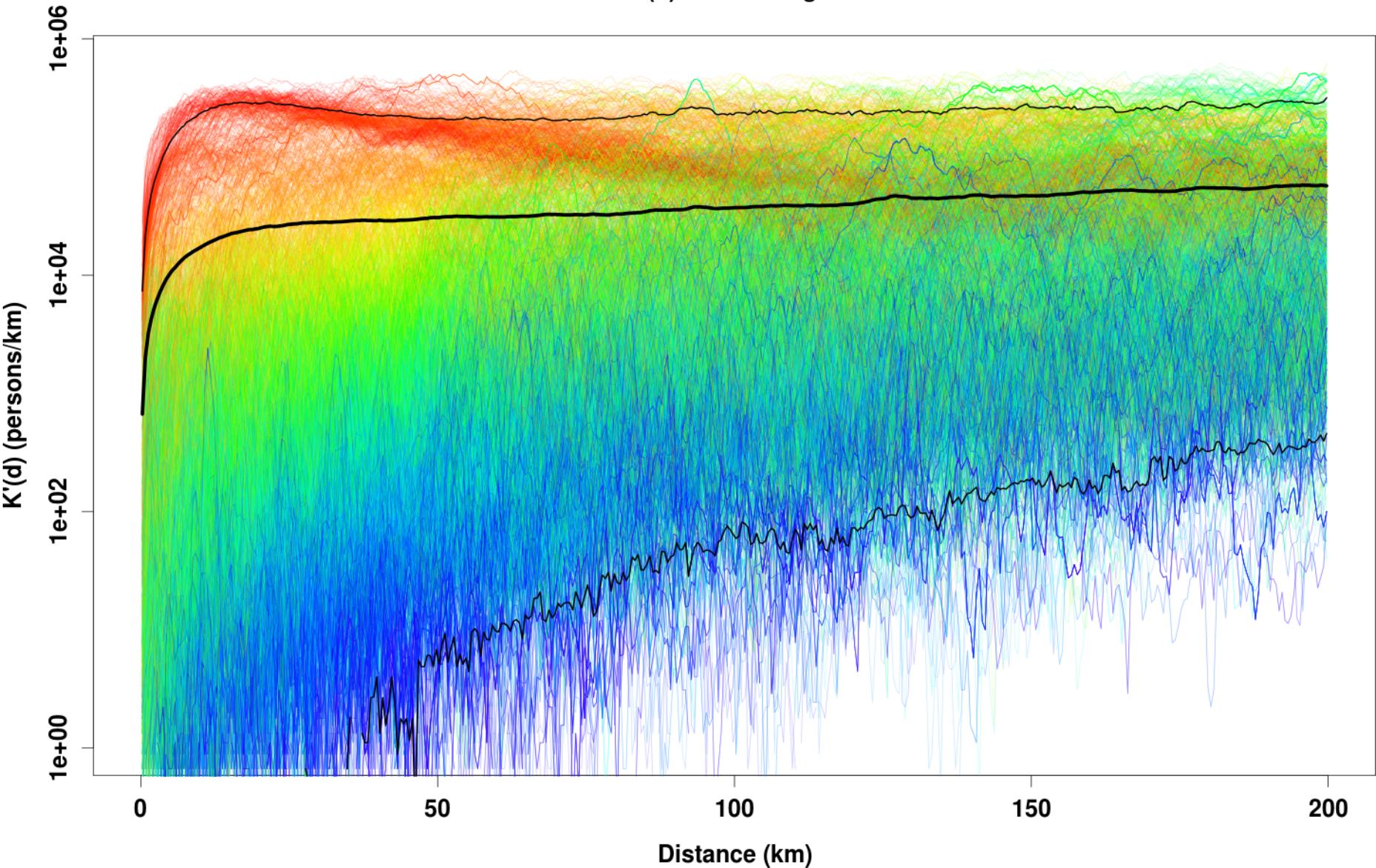
- The “humped” form of K'_i is easy to see, but why is it such a rough function?
- Answer: population distribution is very, very “lumpy”
 - Variation at multiple scales
 - Variation by orders of magnitude; extremely heavy tails
- High-density regions are sparse on the landscape - so a given circle hits few or none (little averaging out)

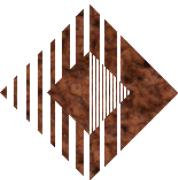


$K'(d)$ for ASFP Egos



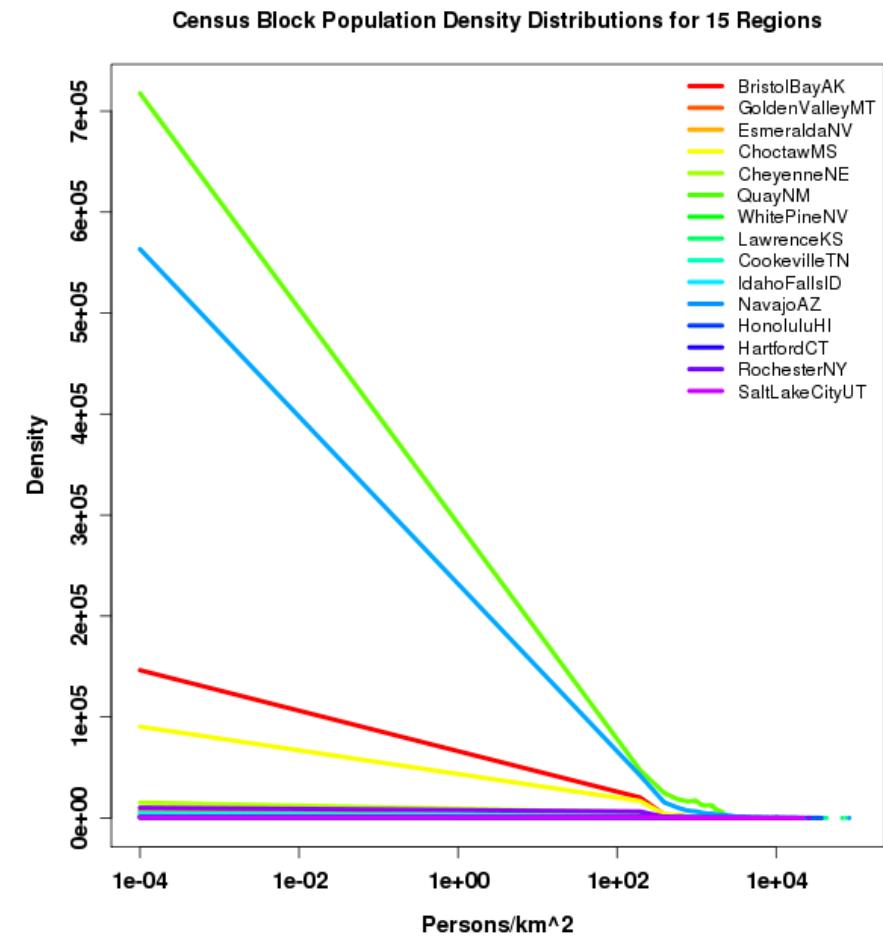
$K'(d)$ for ASFP Egos

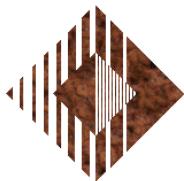




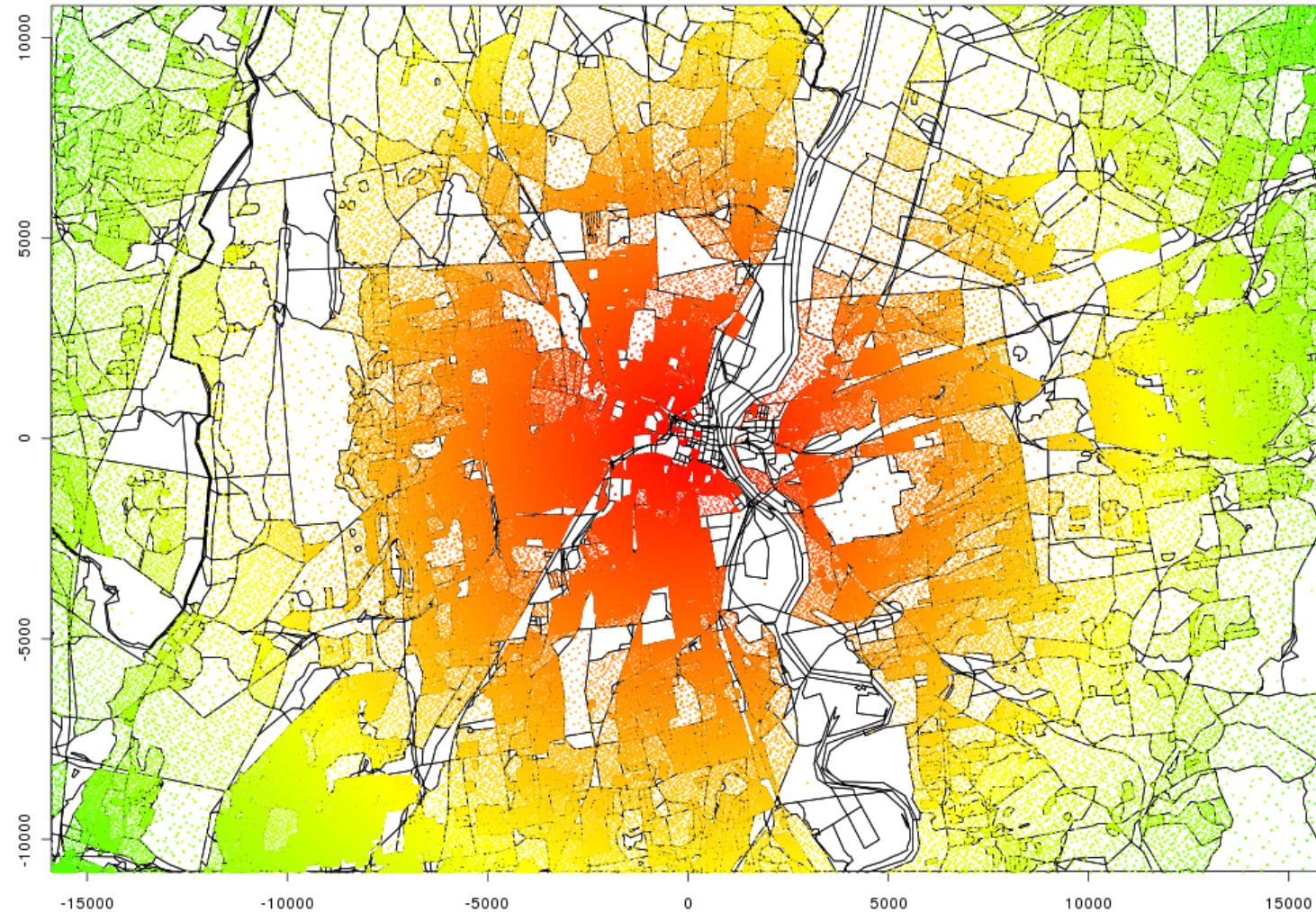
Why K' is Still Rough on Small Scales

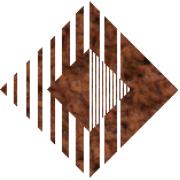
- **Useful illustration: distribution of population densities within US census blocks**
 - Small, relatively compact areas
 - Selected (to an extent) for social "function"
- **Distribution for area/density stratified sample of 15 micro/metropolitan areas (from Butts et al., 2012)**
 - Roughly, $p(x) \approx \alpha \log x$
 - Shouldn't trust this form! But clearly very heavy-tailed
 - **Even within a community, population unevenly dispersed**





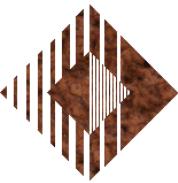
Sample Population Distribution (Hartford, CT)



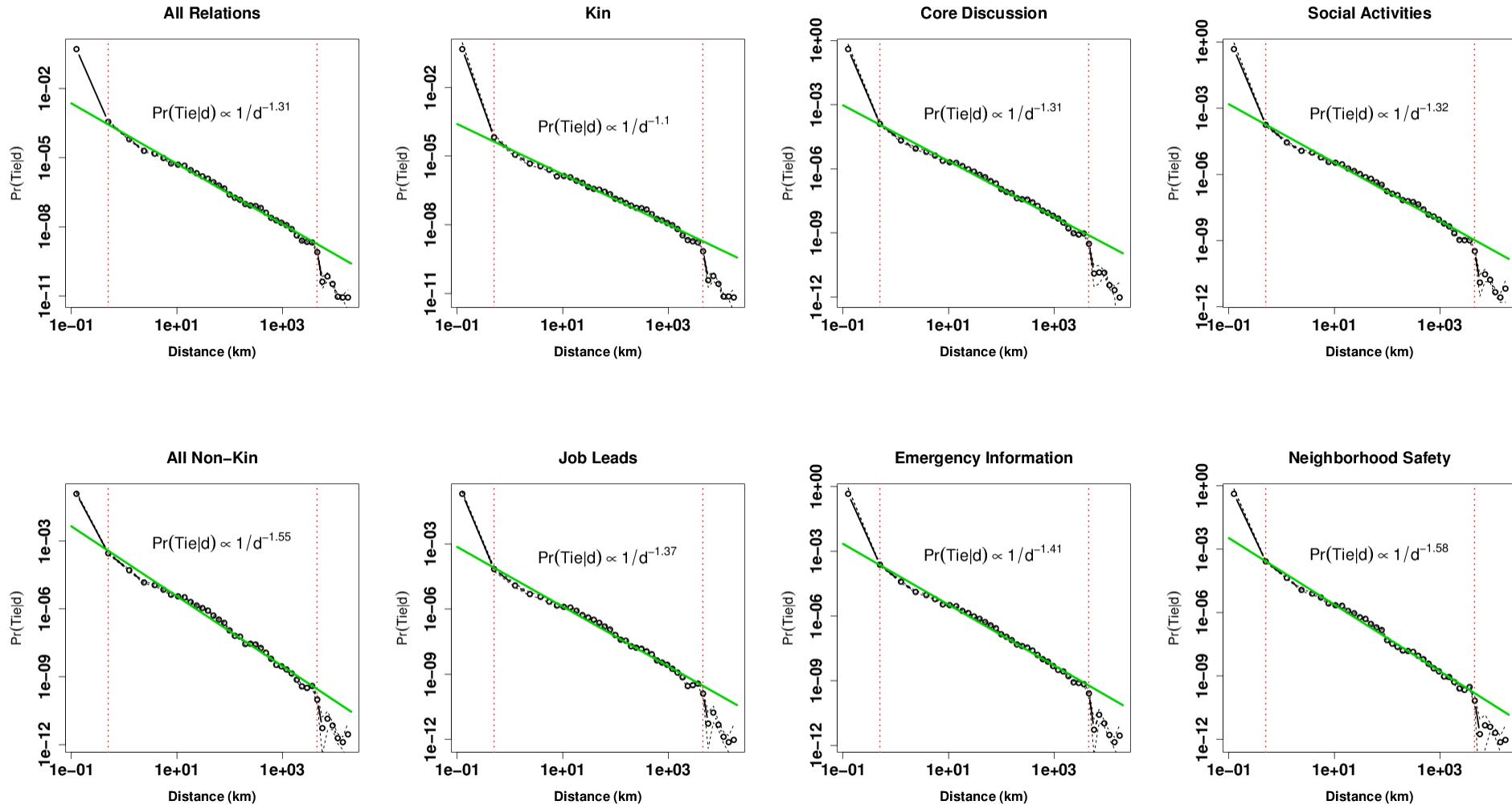


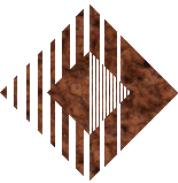
From Opportunity to Realization

- Few opportunities lead to ties, but some are more likely than others
- Given K_i' and realized ties for each ego, can estimate marginal SIF
 - Simple but flexible procedure: logarithmic binning of distances, with counts of realized/potential ties used to estimate mean SIF over interval (here, beta-binomial model w/Jeffreys prior)
 - 500m resolution used for K' calculation (Landscan ~1km raster data used, w/subpixel smoothing)
 - 31,080 geocoded ego/alter ties over 0-16,850km range
- Six ASFP relations:
 - *Kin*: Parents, children, siblings
 - *Core Discussion*: alter with whom ego “discussed matters important” to him/her in past 6 months (Burt, 1984)
 - *Social Activities*: alter ego “engages in social activities with,” e.g. meals, visiting, “going out” (Fischer, 1982)
 - *Job Leads*: alter ego “would contact to obtain job leads or other information” re:employment
 - *Emergency Information*: alter ego “would seek to immediately contact to pass on” warning information
 - *Neighborhood Safety*: alter ego would contact to discuss “crime or other event” raising concern about safety of ego’s neighborhood

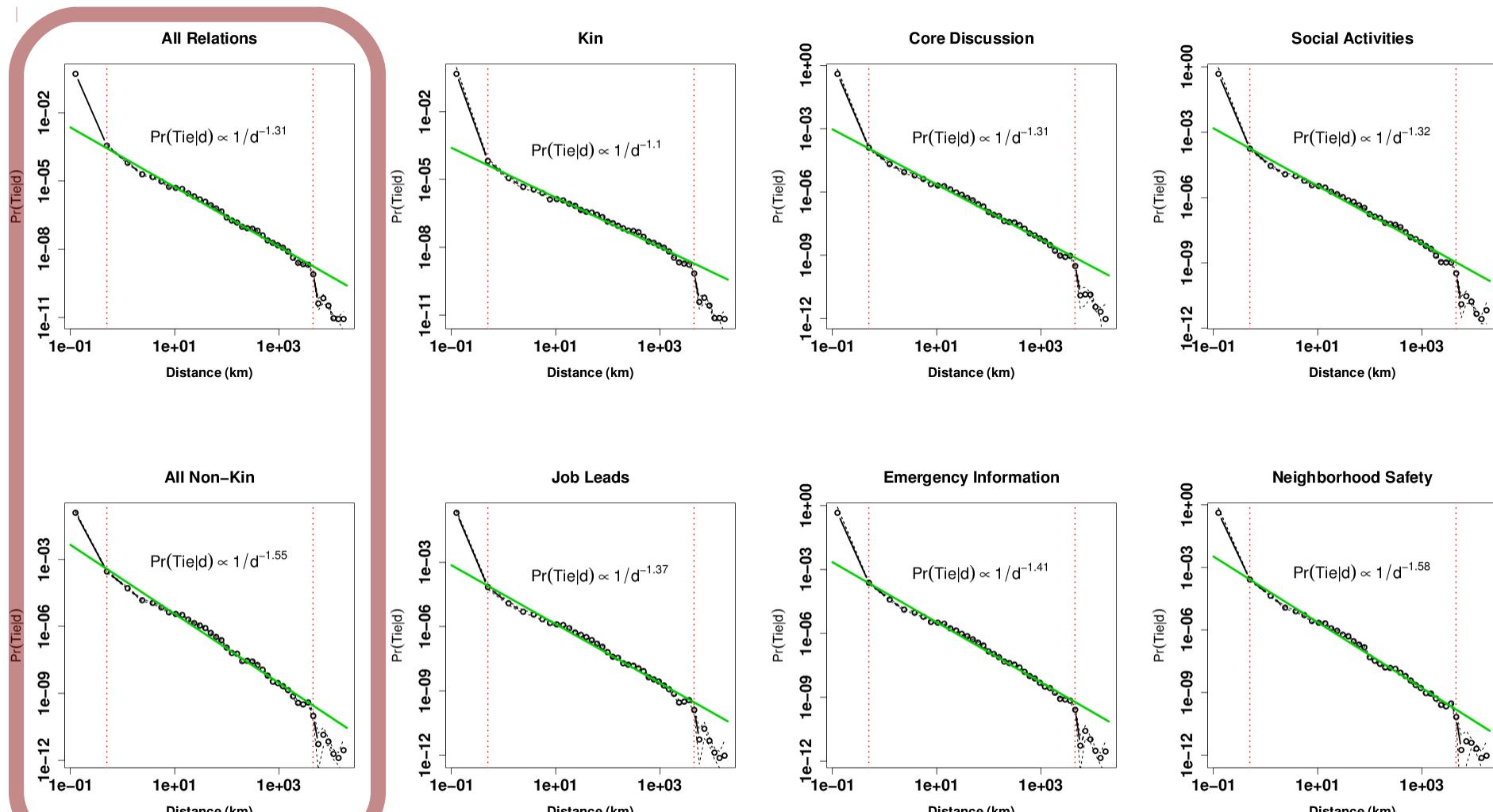


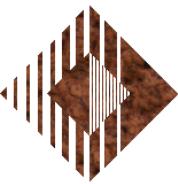
ASFP - Estimated SIFs



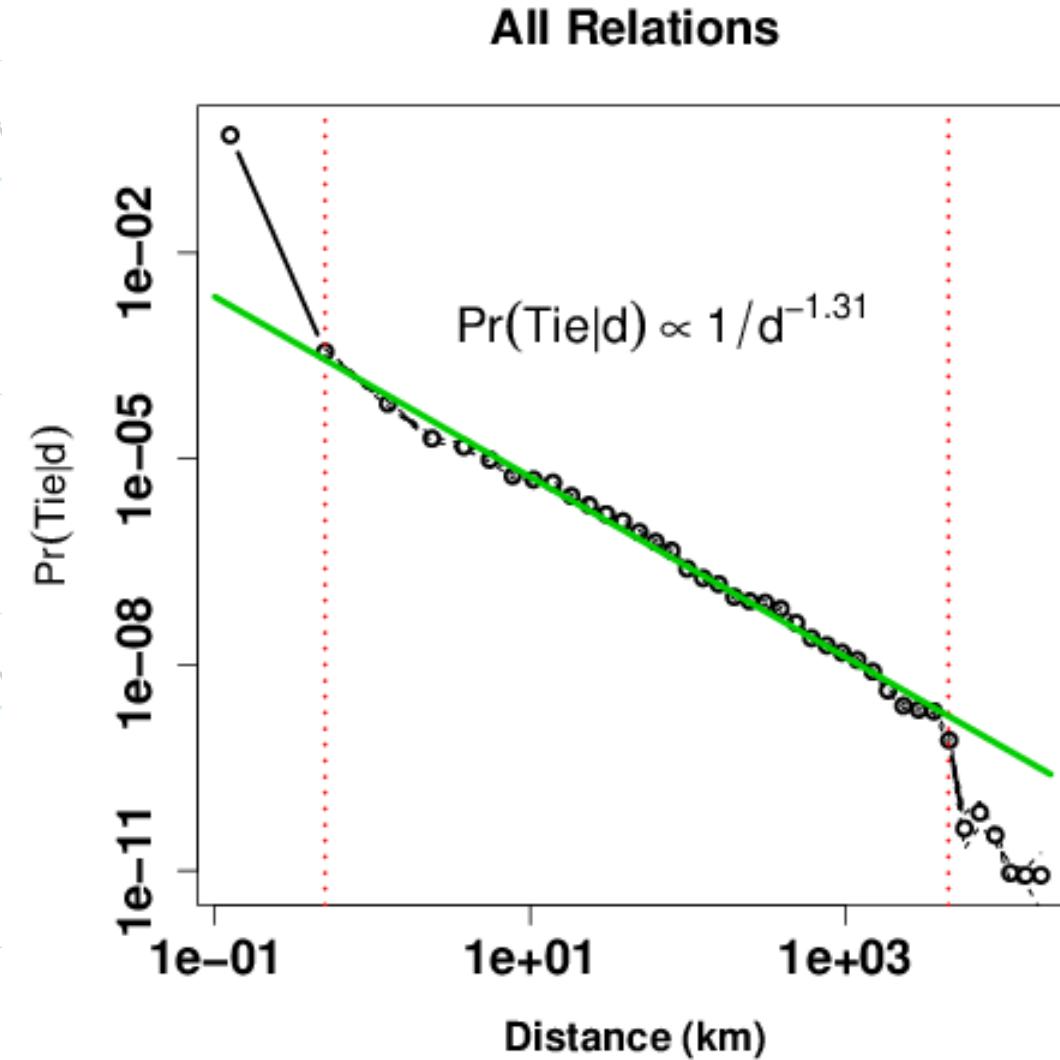
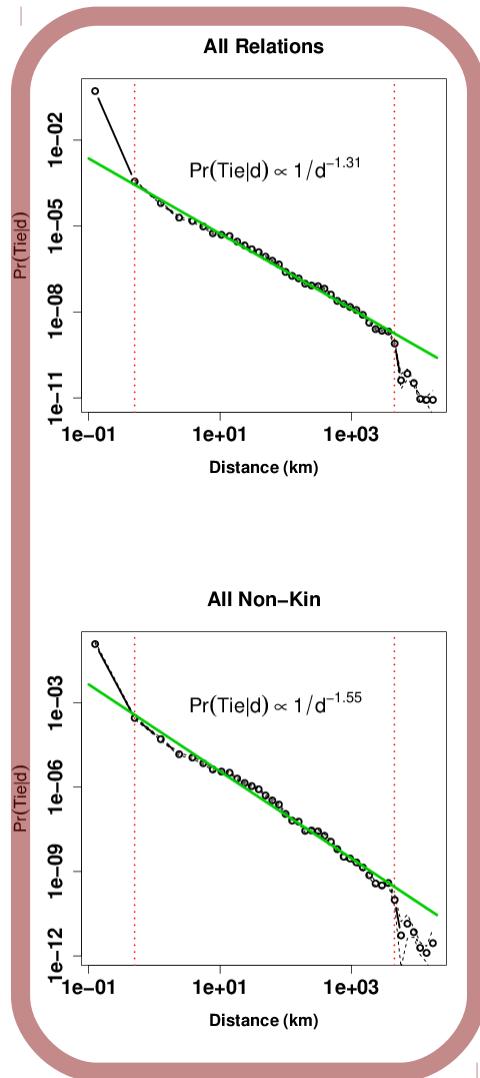


ASFP - Estimated SIFs





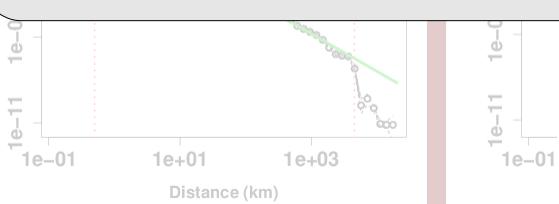
ASFP - Estimated SIFs



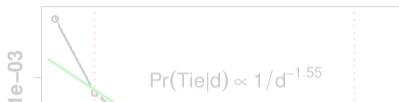


ASFP - Estimated SIFs

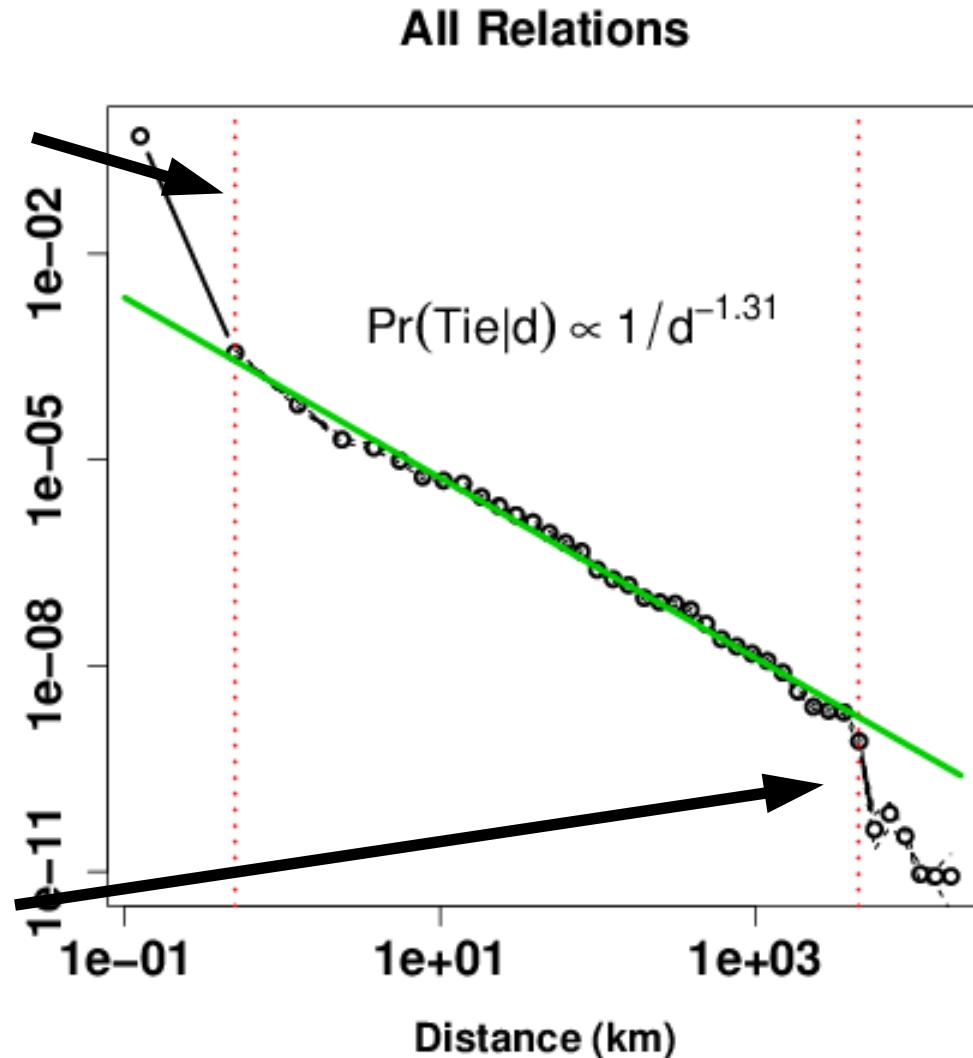
Current resolution limit (~0.5km)



All Non-Kin



Apx diagonal length of continental US (~4500km)



ties

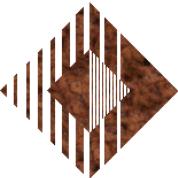
/d^{-1.32}

i)

Safety

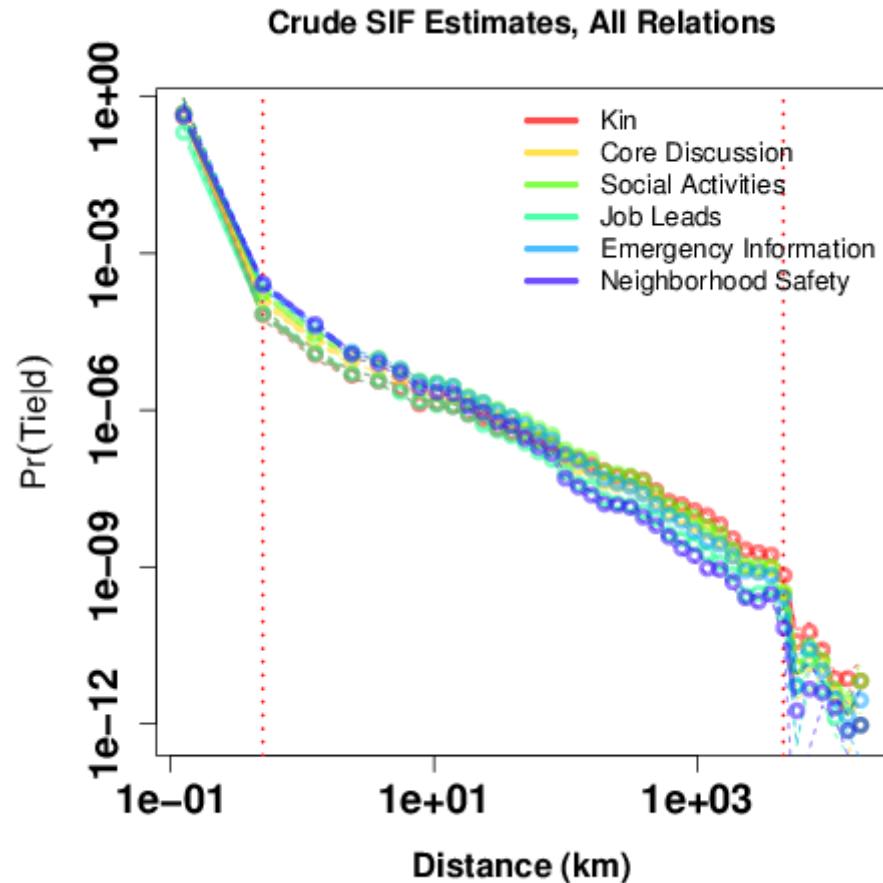
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i)

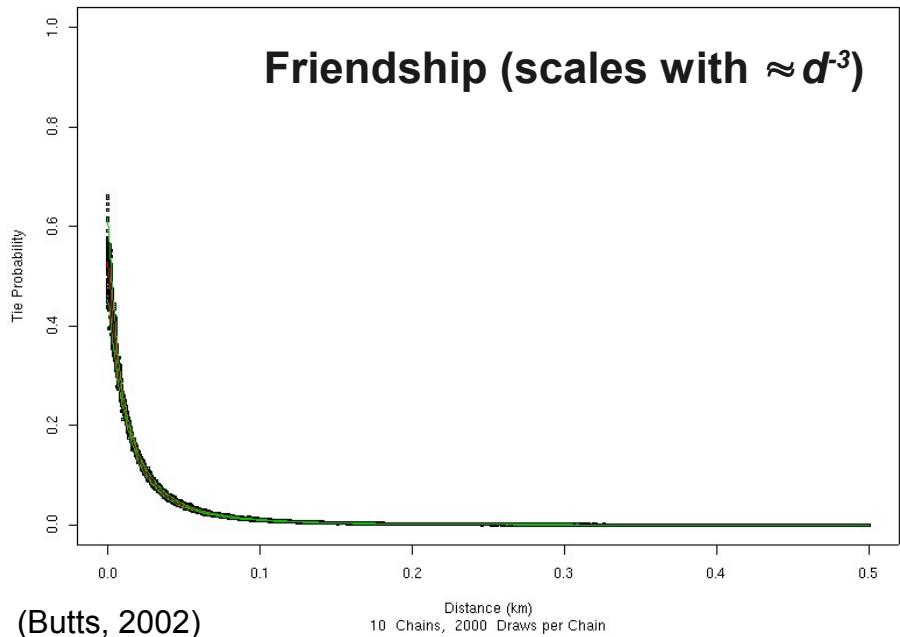


ASFP SIFs: Consistent, Heavy, and Truncated

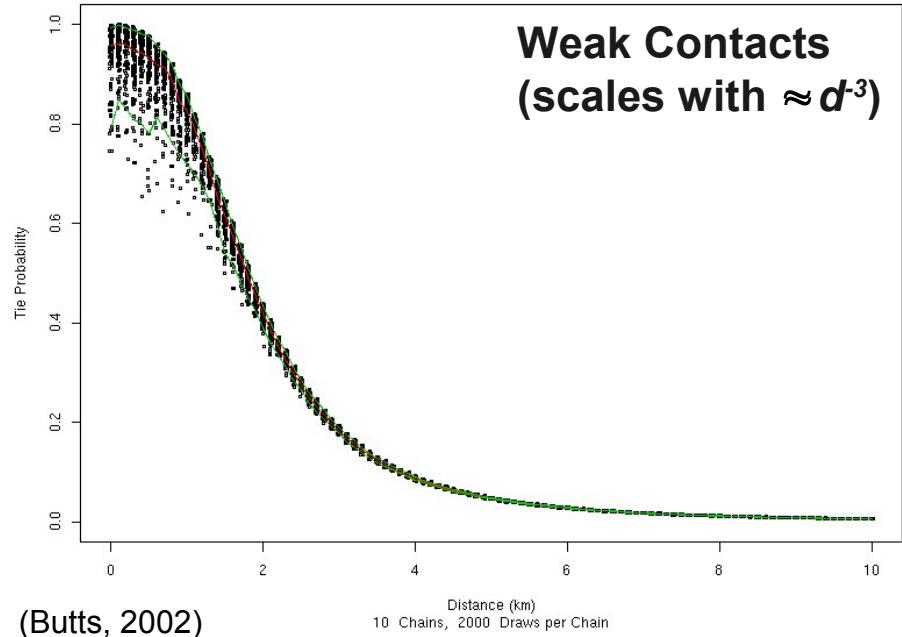
- All relations show similar pattern, with quantitative differences
- Kin, core have heaviest tails, with emergency information and neighborhood safety least
- Form of SIF consistent with earlier work, but much longer tails
- “Crash” at 4500km - qualitative change in interaction barriers



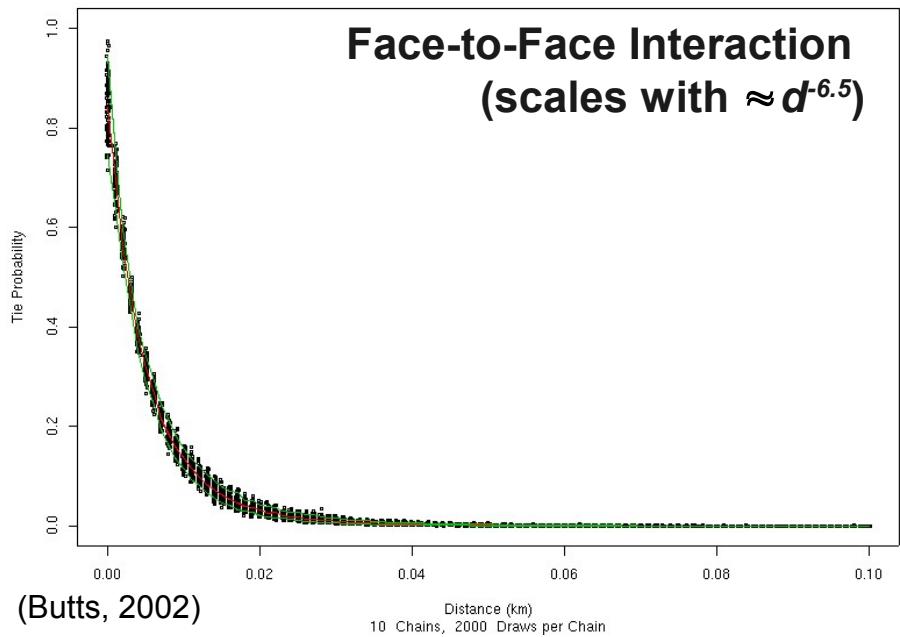
Posterior Predictive – Festinger et al. Data



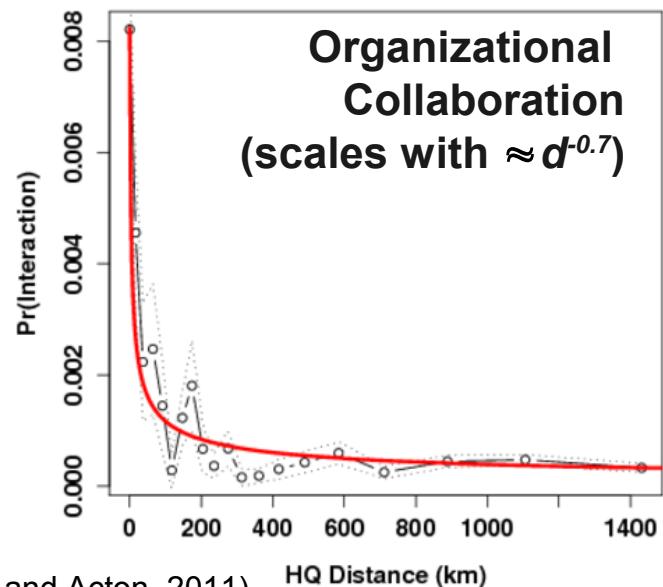
Posterior Predictive – Hagerstrand Data

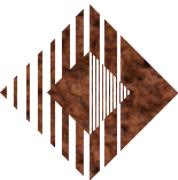


Posterior Predictive – Freeman et al. Data

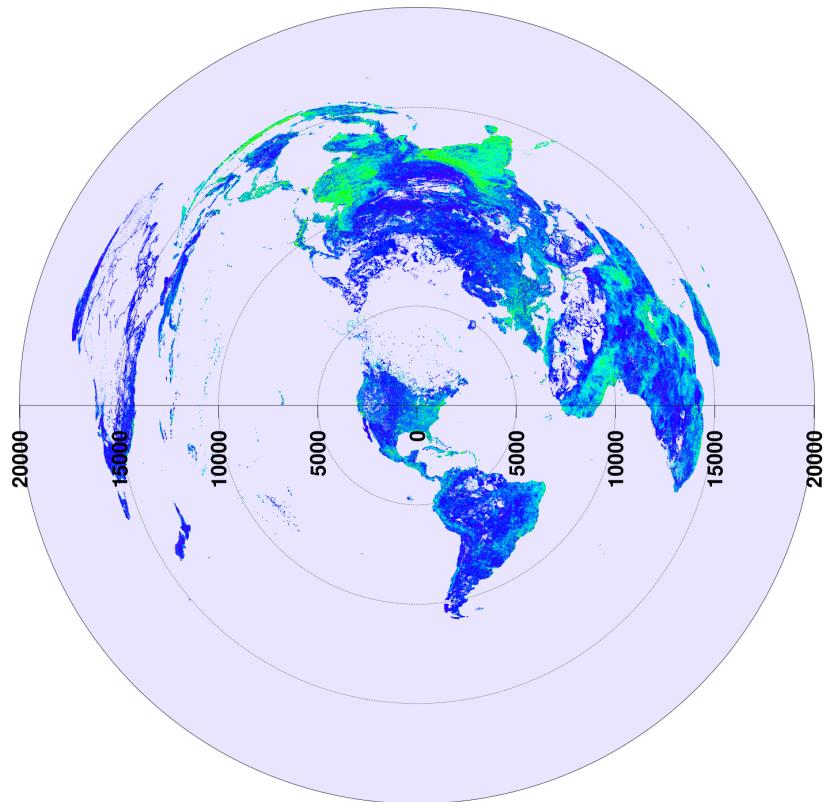


Estimated Interaction Probabilities by HQ Distance

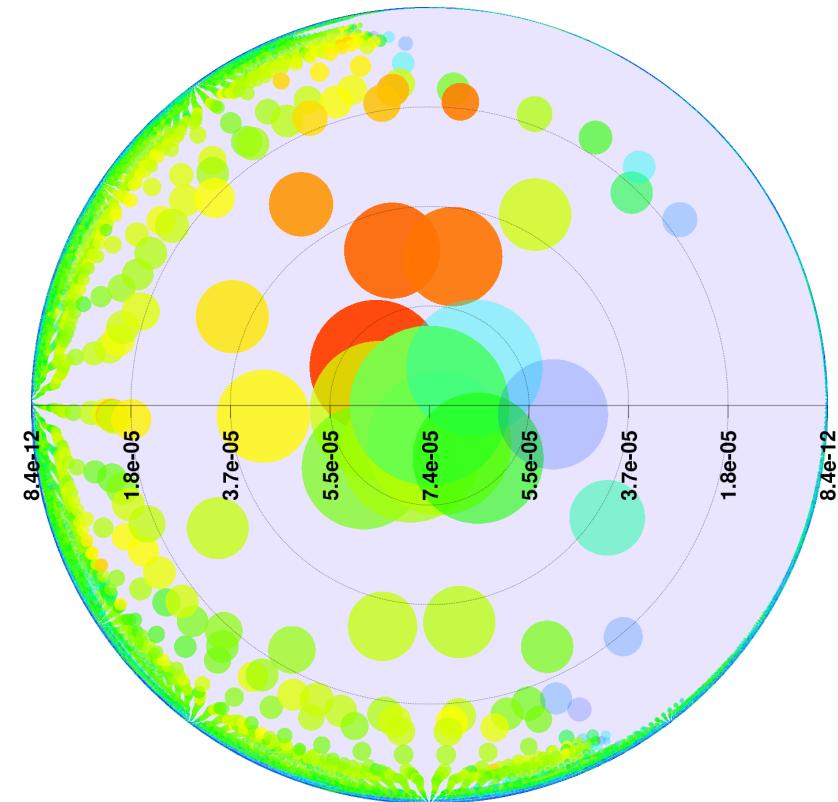




Mixing Ingredients: the SIF as a Spatial Transformation



Physical Space:
Population by Proximity



Social Space: Population
by Propinquity

UPDATED AND EXPANDED



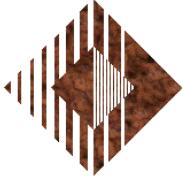
Warped,
↙
(*and Lumpy)

Curved*

The World Is Flat

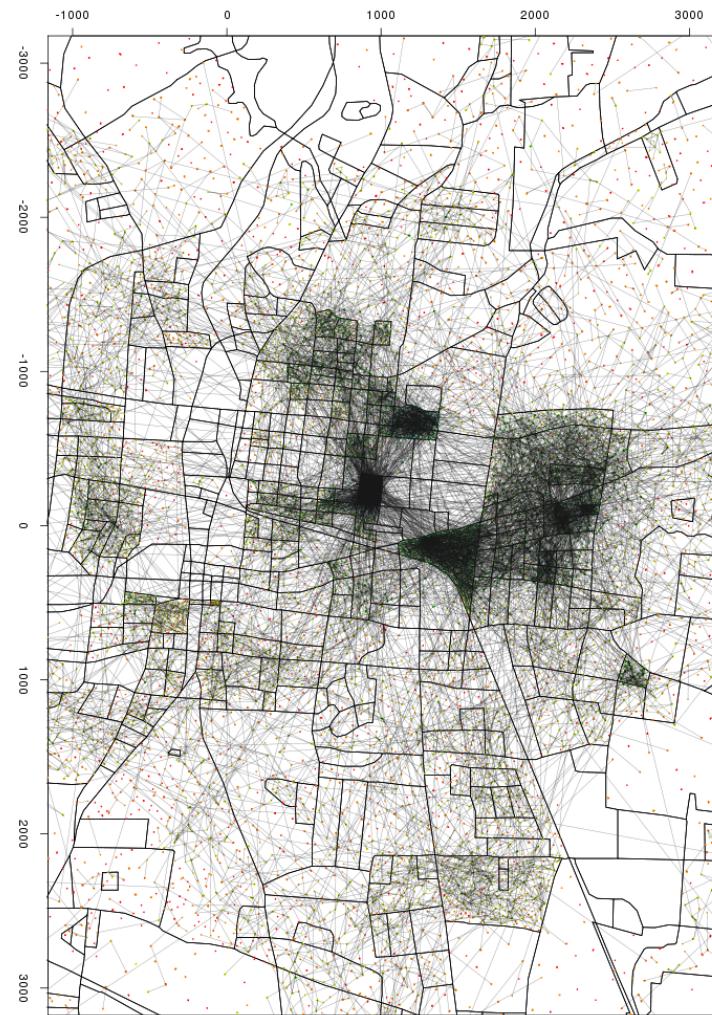
A BRIEF HISTORY OF
THE TWENTY-FIRST CENTURY

Thomas L. Friedman

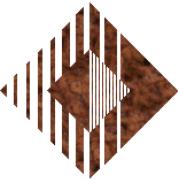


What Does All This Imply?

- **Populations are unevenly distributed in space**
 - Interaction opportunities vary greatly w/distance...
 - ...and with place!
- **Probability of interaction varies systematically with distance**
- **...So, we should see large-scale distortions in social networks associated with population distribution**

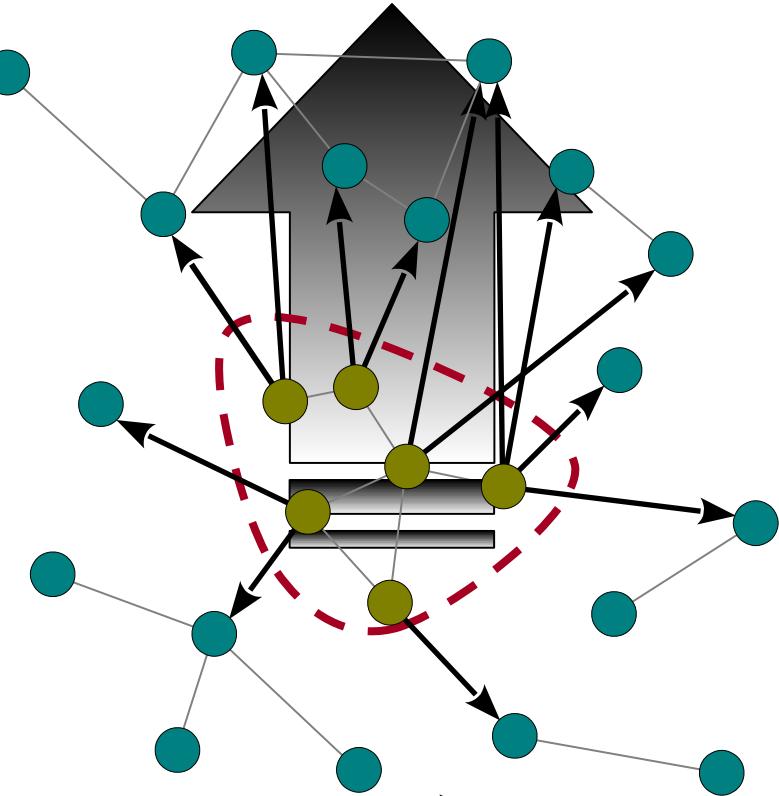


(Butts et al., 2012)



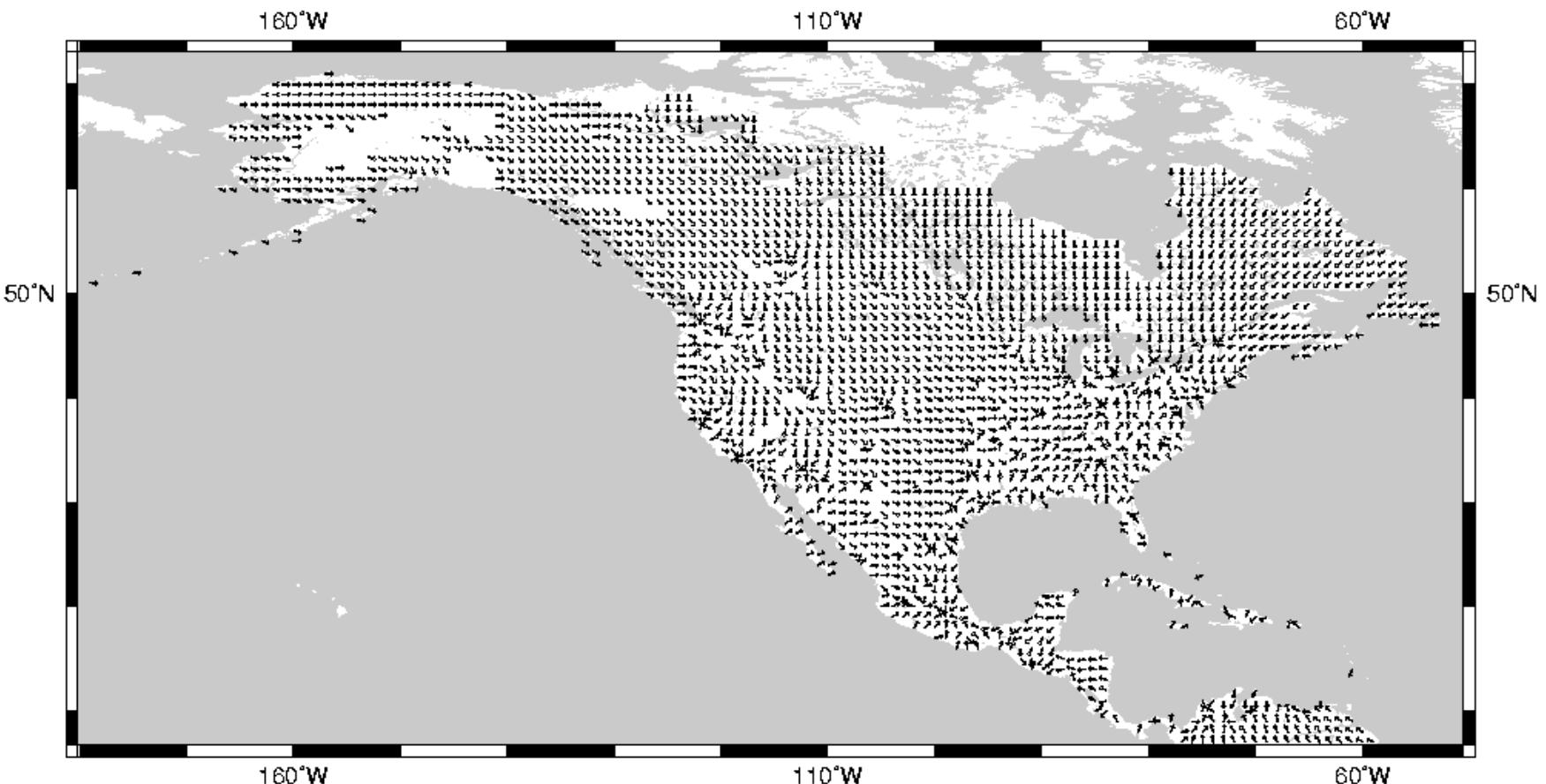
Measuring Distortion in the Social Fabric

- **Anisotropy in the population distribution should alter network structure**
 - Ties “pulled” toward dense regions, creating net average direction
- **Direction of net tie “flow”**
 - For small region A , treat all edges of G w/one endpoint in A as outwardly directed vectors
 - Tie “flow” is in the mean direction of the vector:



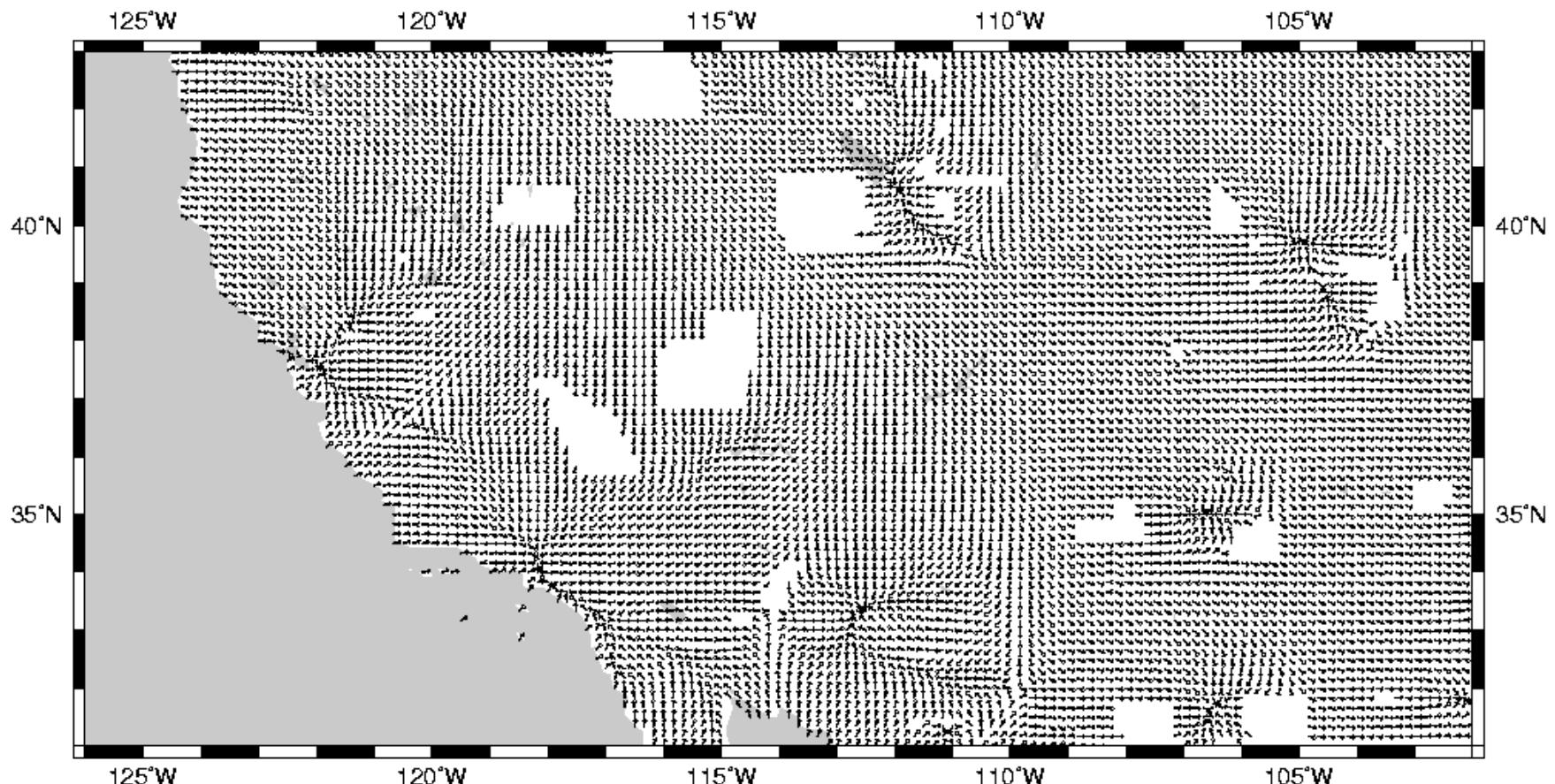
$$\vec{\mathcal{V}}_e(A) = \sum_{e \in G[A, \bar{A}]} \frac{\vec{v}_e}{\|\vec{v}_e\|}$$

Predictions from an Old Model....



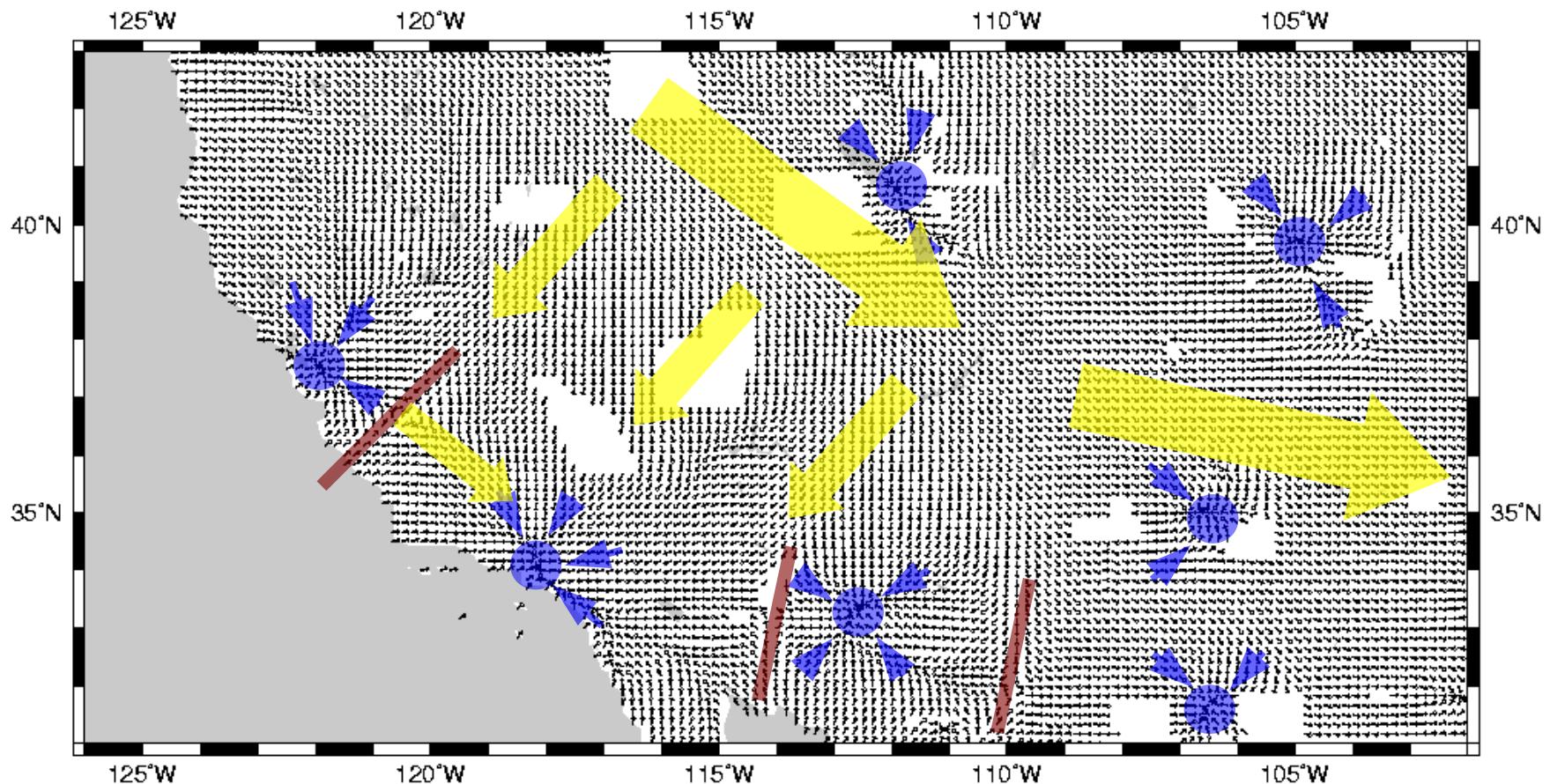
Expected tie flow, social friendship model (Butts, 2002)

Predictions from an Old Model....



Expected tie flow, social friendship model (Butts, 2002)

Predictions from an Old Model....

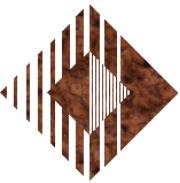


Expected tie flow, social friendship model (Butts, 2002)

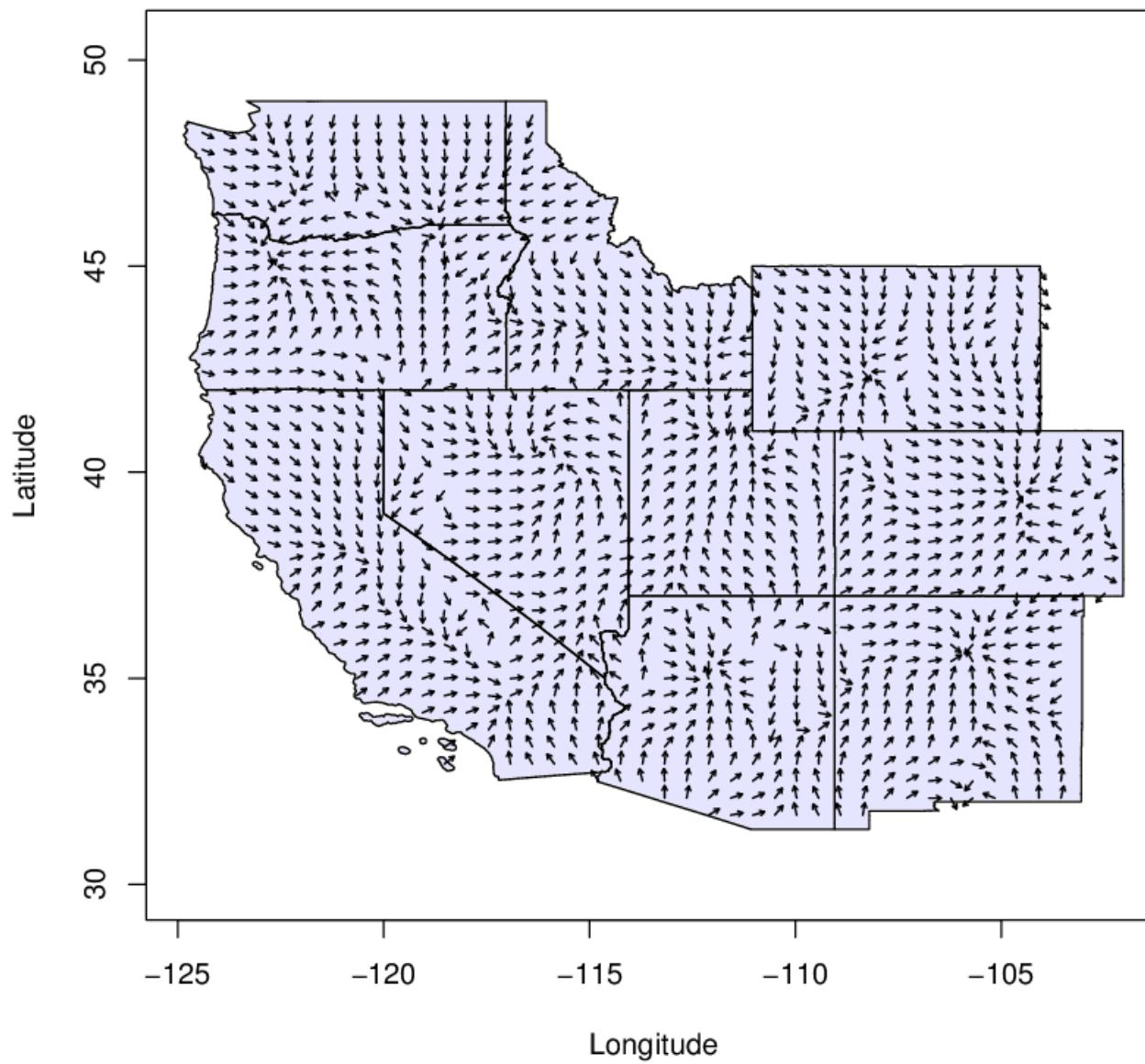


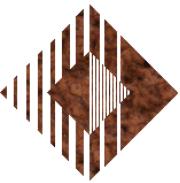
From Prediction to Measurement

- **Initial estimates from the ASFP data**
 - Mean bearings taken for all egos w/at least 1 alter at >5km (bearings taken over all alters >5km distant)
 - $N=2,234$
 - Ego mean bearings converted to complex representation (i.e., $\exp(i*\theta)=\cos(\theta)+i*\sin(\theta)$); observations treated as marked point process with two marks (real, imaginary components)
 - Real, imaginary angular components processed using a Gaussian spatial smoother (`smooth.ppp`, bandwidth selected by cross-validation) to estimate spatial mark means
 - Angles evaluated (i.e., Arg of complex components) on a regular grid over ASFP study area
 - Angular map shows variation in network structure through space

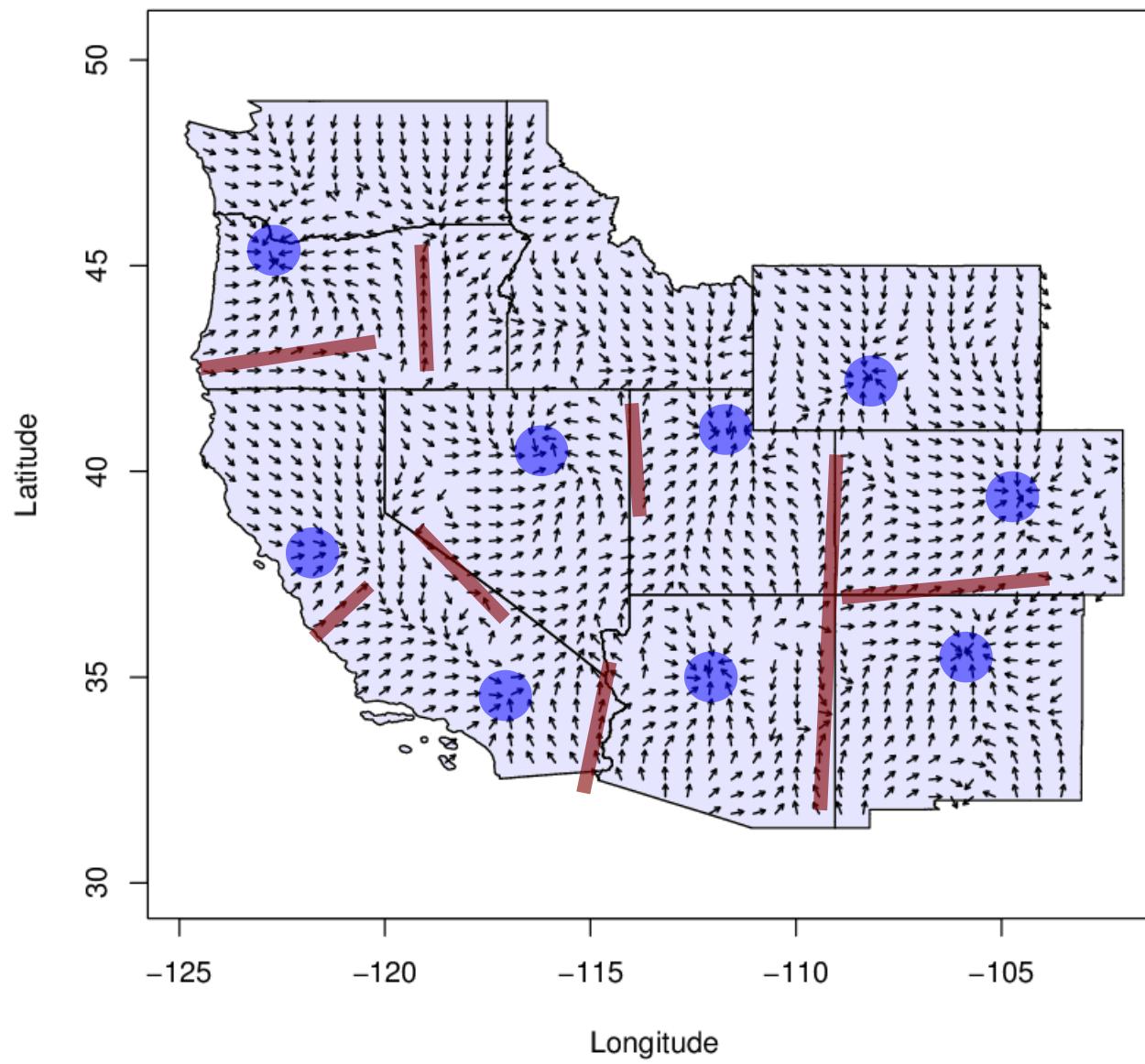


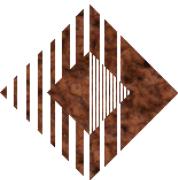
Mean Tie Flow Surface for d>5km, All ASFP Relations





Mean Tie Flow Surface for d>5km, All ASFP Relations





Mean Tie Flow Surface for d>5km, All ASFP Relations

Portland

Elko

Salt Lake City

Riverton

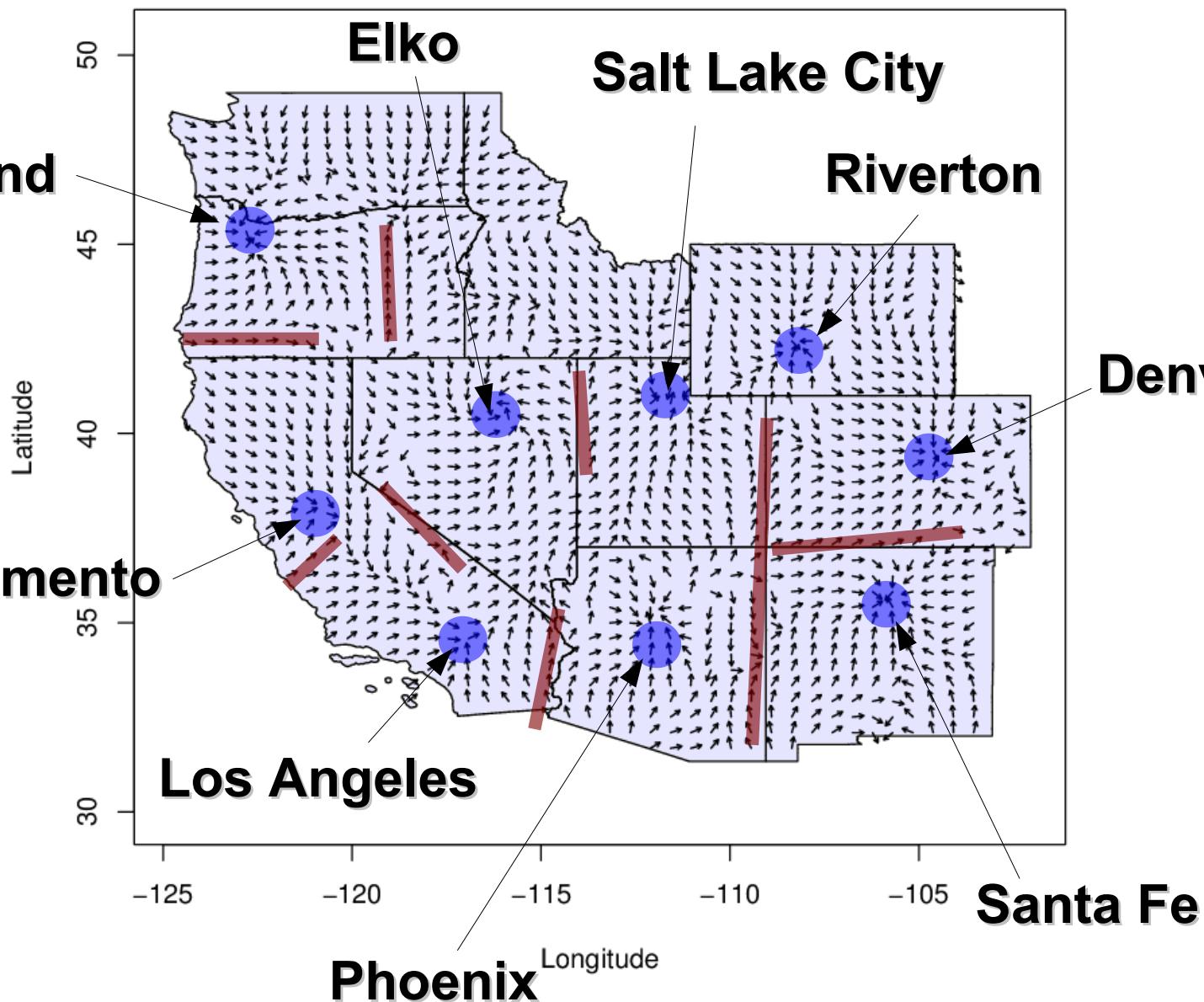
Sacramento

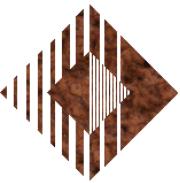
Los Angeles

Denver

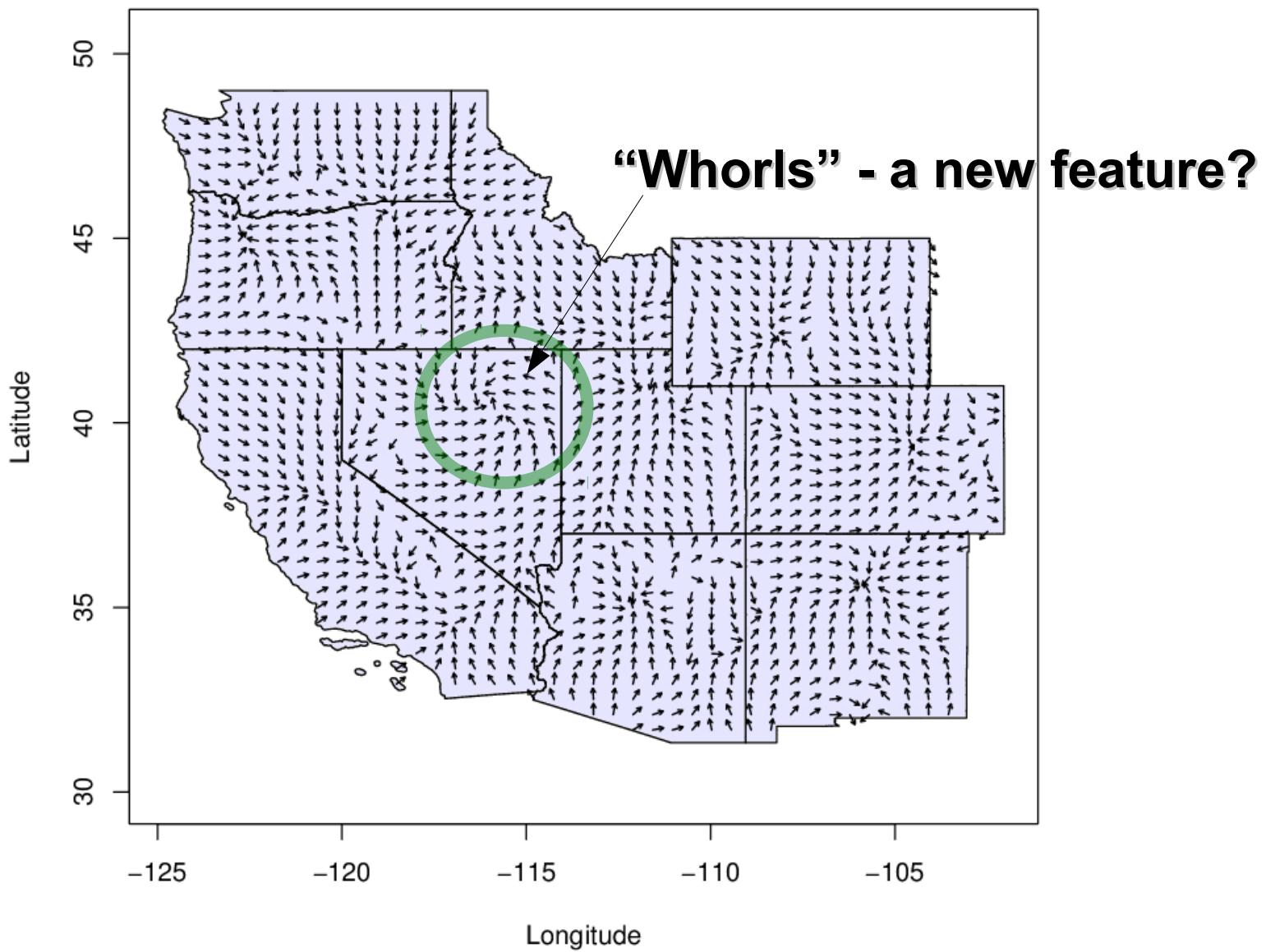
Santa Fe

Phoenix





Mean Tie Flow Surface for d>5km, All ASFP Relations



UPDATED AND EXPANDED



Warped,

↙

(*and Lumpy)

Curved*

↙

The World Is Flat

...and
↗

Twisted!

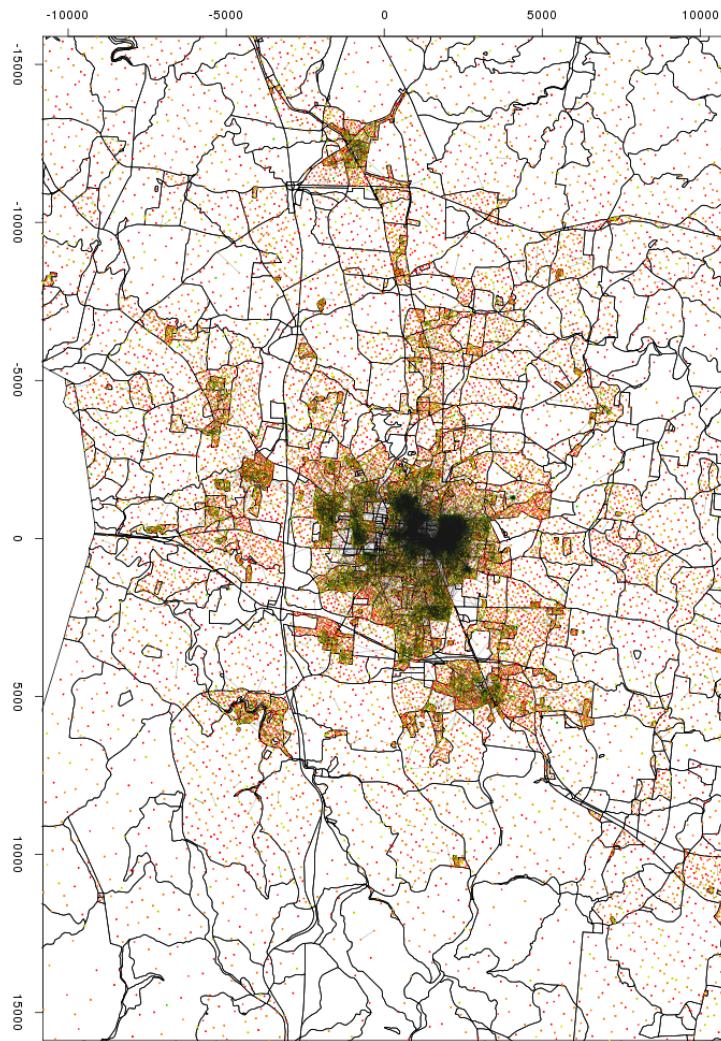
A BRIEF HISTORY OF
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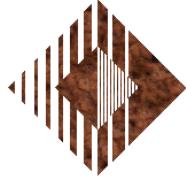
Thomas L. Friedman



Road Map

- **Introduction**
- **A “first-order” look at spatial network structure**
- **Measuring network “distortions” in the western US**
- **Conclusions and such**





Some Initial Take-Aways

- **It's a messy, crumpled world...**
 - Interaction opportunities vary wildly over space, creating complex distortions in network structure
- **...but some order persists**
 - We find stable power law SIFs for the ASFP data on six relations, over a ~4,500km range
 - Upper tail “collapse” suggests additional barriers (language, cost) becomes as important as distance at continental scales
 - Heavy tails indicate that some relations assumed to be local have a long-range component
- **First glimpses of angular distortions in network structure show both confirmation and surprises**
 - Many features predicted in prior models, but some (e.g., “whorls”) not seen; estimates suggest importance of small communities in remote areas



Directions/Work in Progress

- **Bringing other covariates back in**
 - Effects of age, race, income, education, gender on mixing
 - How much attribute-related structure can be explained by population distribution (e.g., segregation)? How much residual heterogeneity is there?
 - Linking network, spatial, demographic, and neighborhood contexts (e.g., vis a vis perceived cohesion, res. tenure)
 - Have had success predicting crime rates, regional identification using crude SIF models; can we do better by leveraging new data?
- **Mapping more characteristics of social space**
 - Areal unit estimates of density, mean degree, social isolation, etc.
 - Studying structure in rural and remote area populations
- **Further data collection**
 - Extending to the entire US; intensive sampling of MSAs



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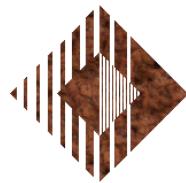
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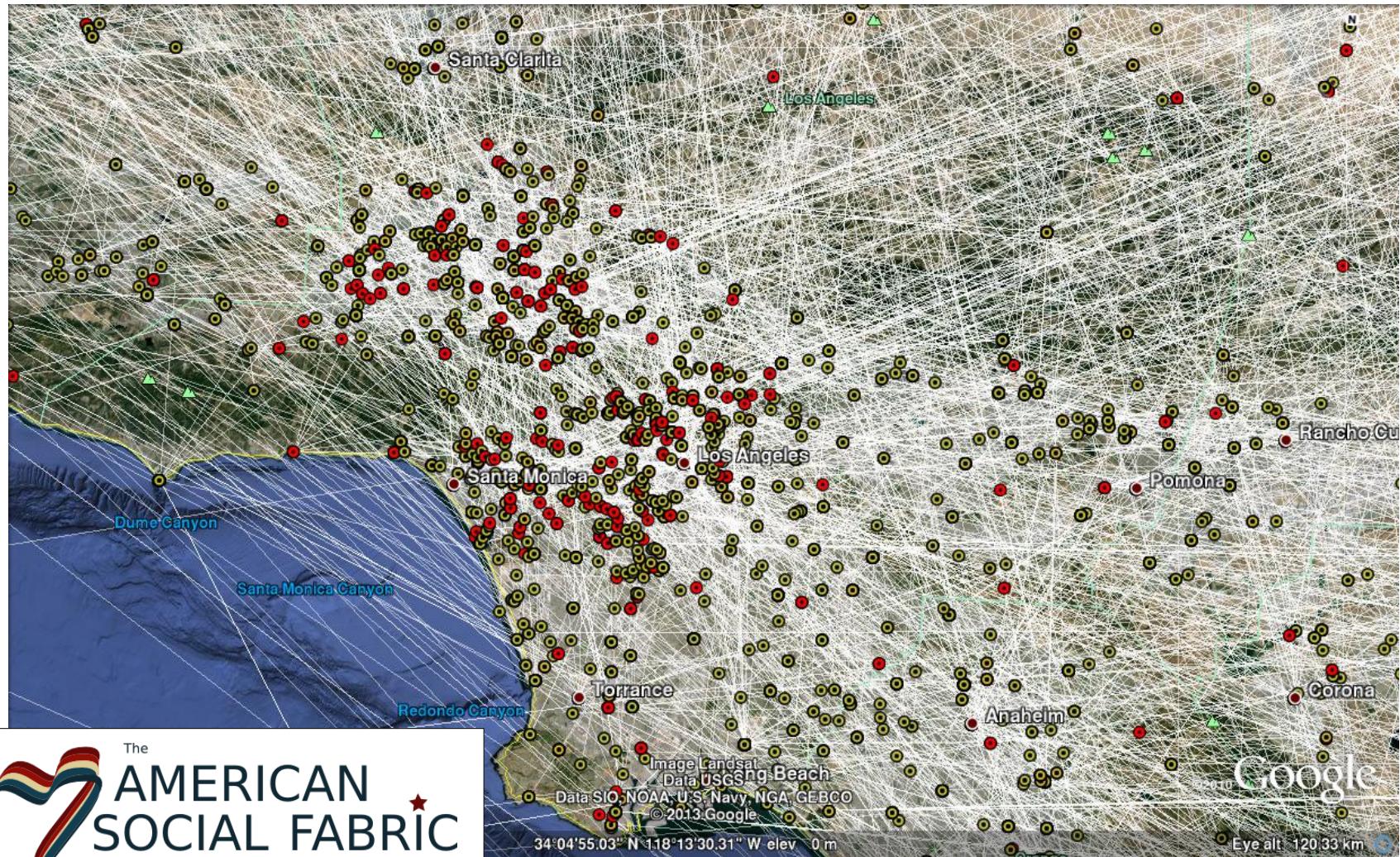
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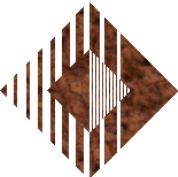


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cannivaltinea@gmail.com

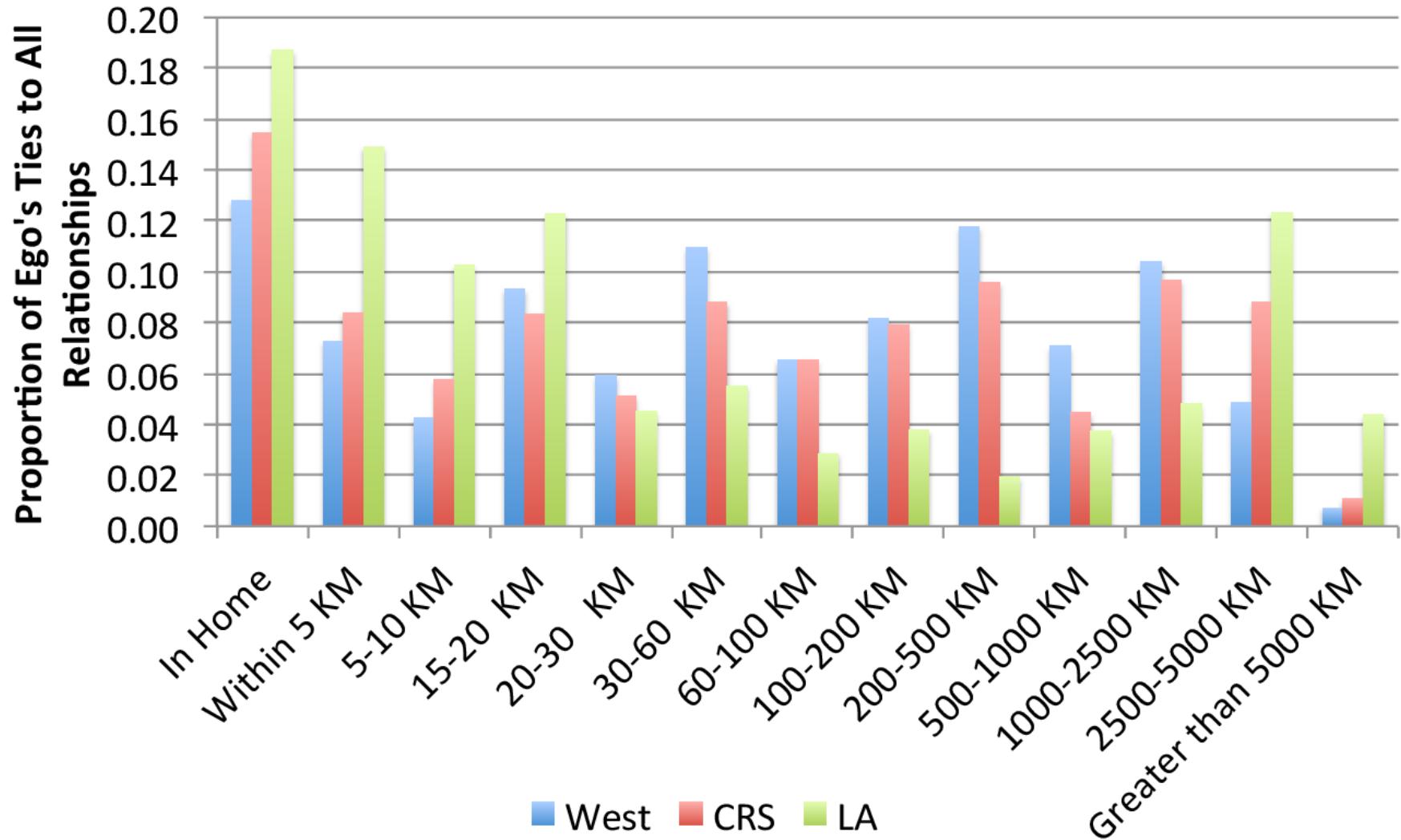


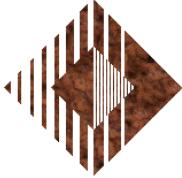
Thank you for your attention!





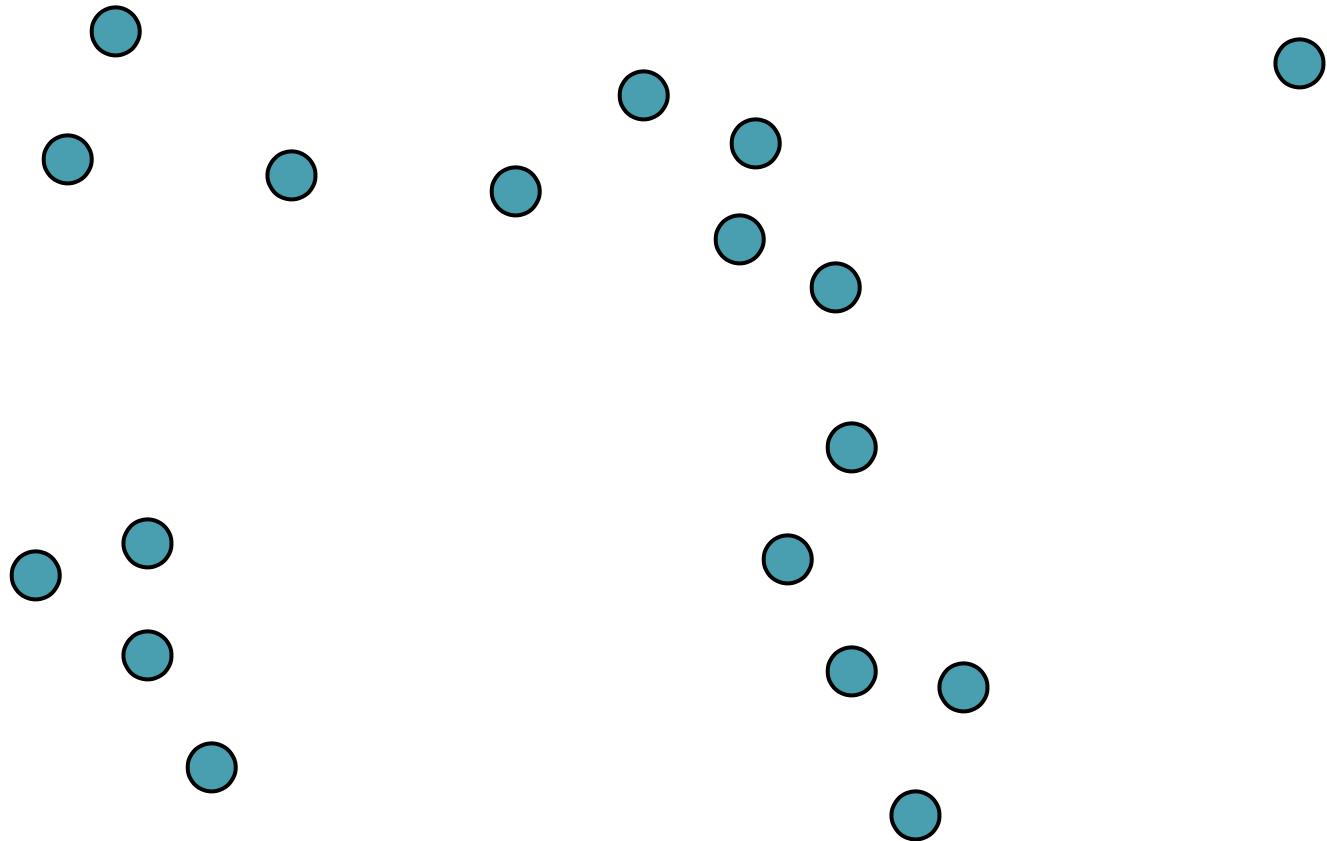
Egocentric Distance Distribution, by Subsample

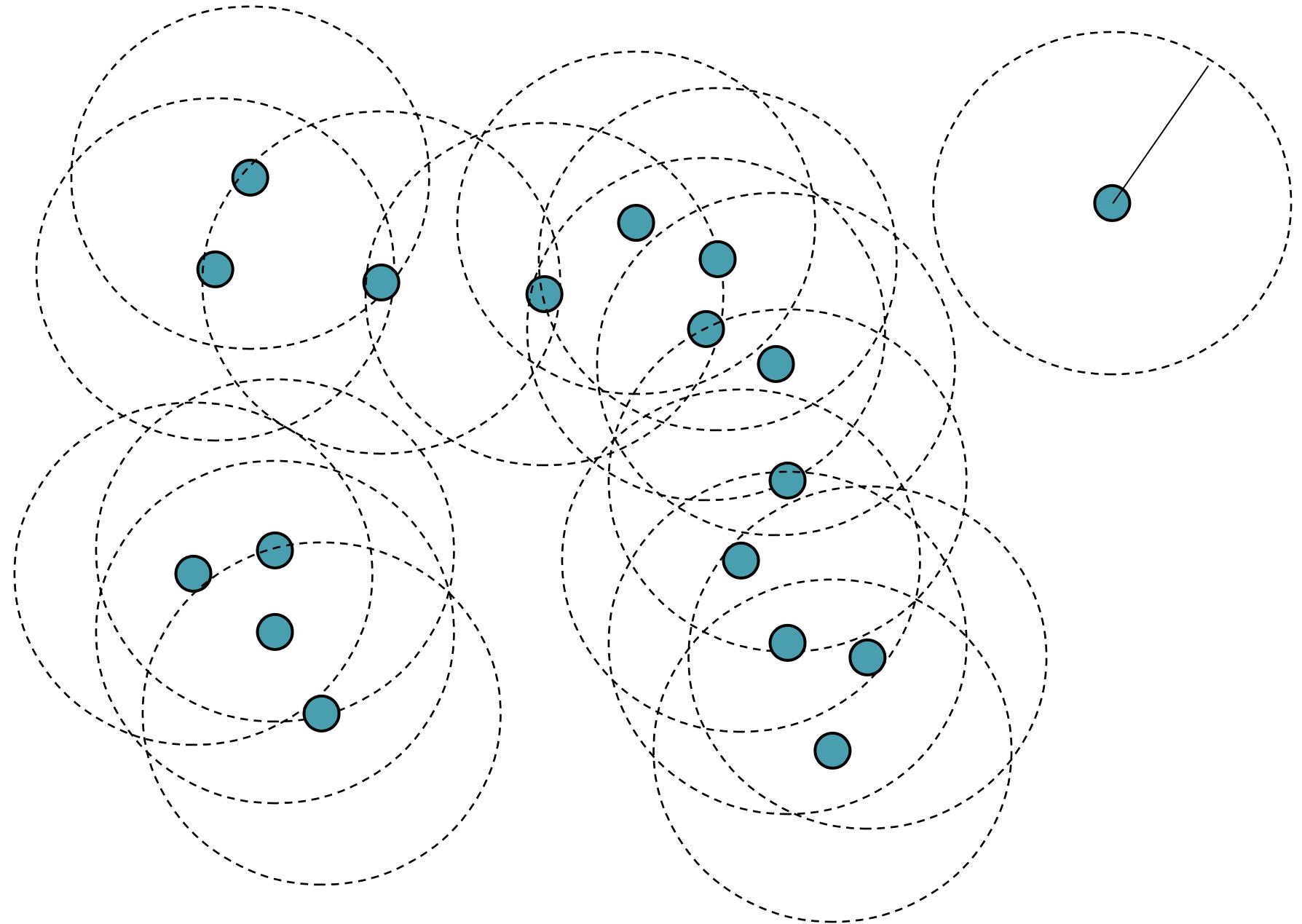


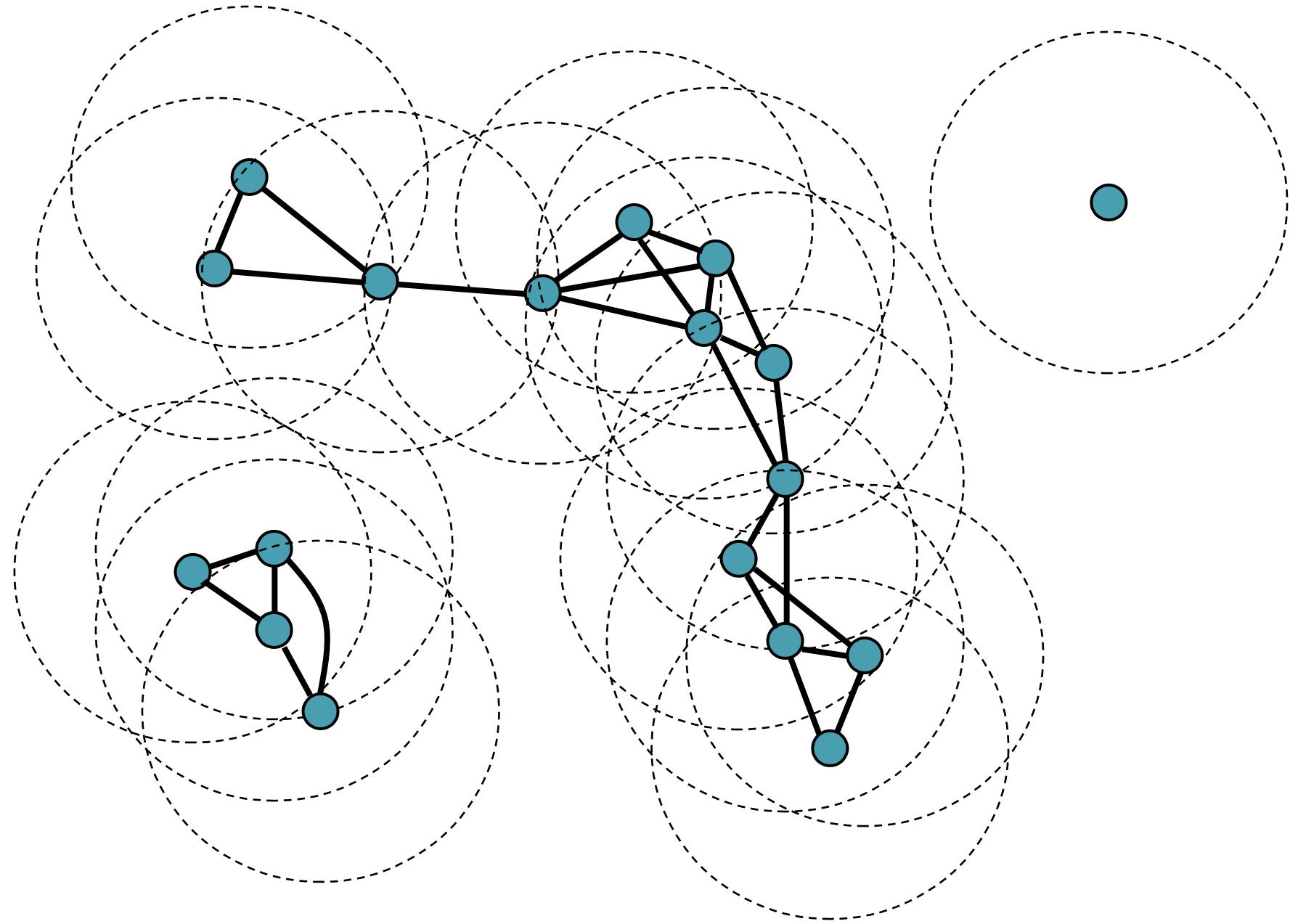


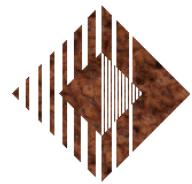
What Does the SIF Give Us?

- Assuming (for the moment) that the estimated SIFs capture edgewise marginals for the distance/tie probability relationship, how much would this tell us about global network structure?
 - Theorem: if edge probability bounded by ε for dyads at distances $>r_c$, lower bound on fraction of entropy explained by d approaches mean $\Pr(d_{ij} > r_c)$ as $\varepsilon \rightarrow 0$
 - In practice, $\varepsilon < 0.001$ is sufficient; scaling is w/ $1 - I(B(\varepsilon))$
 - For uniform population on $\ell \times \ell$ region, can explain over 90% entropy so long as r_c is less than about 20% of ℓ
 - Fitted models easily meet this for 10-100km regions
- Intuition: at large scales, spatial effects dominate
 - Distant dyads almost certainly null
 - Realized graph stochastically approximates subgraph of graph induced by contiguity

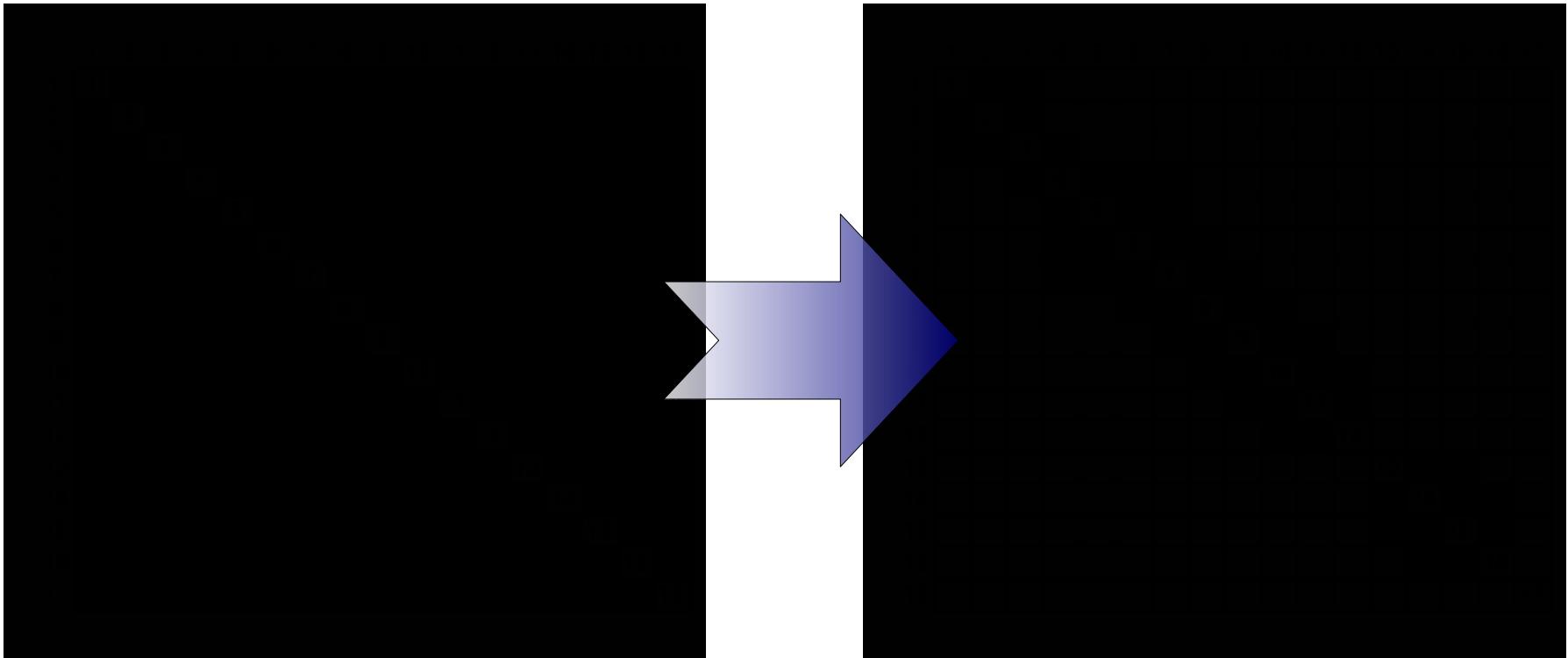






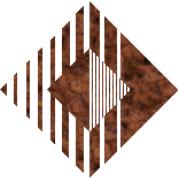


Or, In Matrix Form....



Without Distance

With Distance



An Inhomogeneous Bernoulli Family for Spatially Embedded Networks

- A simple family of models for spatially embedded social networks:

$$\Pr(Y = y | d) = \prod_{\{i, j\}} B(Y_{ij} = y_{ij} | \mathcal{F}(d_{ij}))$$

– where $Y \in \{0, 1\}^{NxN}$, $d \in [0, \infty)^{NxN}$, $\mathcal{F}: [0, \infty) \rightarrow [0, 1]$, B Bernoulli pmf

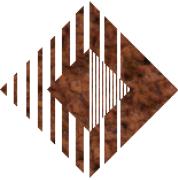
- Special case of the inhomogeneous Bernoulli graph family with parameter matrix $\Phi_{ij} = \mathcal{F}(d_{ij})$

– Assumes that dependence among edges absorbed by distance structure – edges conditionally independent

- Related to the *gravity models*, i.e.

$$\mathbb{E} Y_{ij} = P(i)P(j)F(d_{ij})$$

– where P is an interaction potential, and F is an impedance or spatial interaction function



Generalization to Curved Exponential Random Graph Models

- Increasingly widely used approach – ERG form
- Our likelihood can be rewritten as a curved ERG

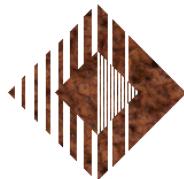
$$\Pr(Y = y | \theta, d) \propto \exp\left(\sum_{\{i, j\}} \eta(\theta, d) y_{ij}\right), \quad \eta(\theta, d) = \text{logit } \mathcal{F}(\theta, d)$$

- Sufficient statistics are the edge indicators of Y ; canonical parameters (η) are logits of marginal edge probabilities
 - $O(N^2)$ canonical parameters – computational savvy advised

- General curved model: space + other effects

$$\Pr(Y = y | \theta_{\mathcal{F}}, \theta, d) \propto \exp\left(\eta(\theta)^T t(Y) + \sum_{\{i, j\}} \eta_{\mathcal{F}}(\theta_{\mathcal{F}}, d) y_{ij}\right)$$

- Allows for integration of complex edge dependence, degree distribution constraints, other covariate effects, etc. (through t)
- Can use to control for social mechanisms when seeking spatial effects, or spatial effects when seeking social mechanisms



Core Discussion Network

http://socialfabric.ss.uci.edu/questionnaire.htm?q=EgoCoreDiscussion — The American Social Fabric Project

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From time to time, most people discuss important matters with other people. Looking back over the last six months, who are the people with whom you discussed matters important to you? Please check all that apply, and list other people (including non-family), one-by-one, in the box below:

- your Spouse/Partner your Mother your Father
 Brad (your sibling) Chris (your sibling) Drew (your child)
 Tom

Please note:
Names will not be submitted with your survey form, and are used here only to help you keep track of people you have listed.

Feel free to use a first name, nickname, initials, or any other name which will allow you to refer to this person later.

Is there anyone else?

If so, type their name below:

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Job Leads Network

http://socialfabric.ss.uci.edu/questionnaire.htm?q=EgoJobLeads — The American Social Fabric Project

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Feel free to use a first name, nickname, initials, or any other name which will allow you to refer to this person later.

When searching for a job, many people turn to friends, family, and/or associates for information regarding potential employment opportunities. Which of the following people would you seek to contact to obtain job leads or other information? Please check all that apply, and list other people (including non-family), one-by-one, in the box below:

your Spouse/Partner your Mother your Father
 Brad (your sibling) Chris (your sibling) Drew (your child)
 Tom

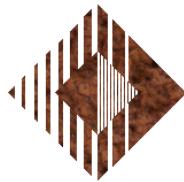
Is there anyone else?
If so, type their name below:

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KNOXVILLE

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Social Activities Network

http://socialfabric.ss.uci.edu/questionnaire.htm?q=EgoSocialActivities — The American Social Fabric Project

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Which of the following people do you engage in social activities with, such as going out for a meal, visiting, going out socially, etc.? Please check all that apply, and list other people (including non-family), one-by-one, in the box below:

Please note:

Names will not be submitted with your survey form, and are used here only to help you keep track of people you have listed.

Feel free to use a first name, nickname, initials, or any other name which will allow you to refer to this person later.

- your Spouse/Partner Drew - Child (your child) your Mother
 your Father Brad - Brother (your sibling) Chris - Brother (your sibling)

Is there anyone else?

If so, type their name below:

 Add

Continue

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Eliciting Spatial Location

http://socialfabric.ss.uci.edu/questionnaire.htm?q=EgoTieLocations — The American Social Fabric Project

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Go Back

For the names you provided, we would like to know how far they live from you. In the boxes below, please provide as much of their address as you know.

You might need to scroll this page down!

Some examples:

Full street address: 423 West Main St., Irvine, CA 92612

Partial street address: 423 West Main St., Irvine, CA

Intersection: West Main St. & Washington Ave., Irvine, CA

Partial street address: W. Main St., Irvine, CA

Partial address: Irvine, CA

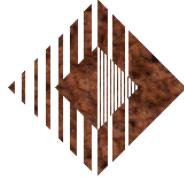
Partial address: Vancouver, BC, Canada

You
your Spouse/Partner
your Mother
your Father
Brad (your sibling)
Chris (your sibling)
Drew (your child)
Tom

Continue

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Confirming Location

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Confirm the address for You

Go Back 1308 Friendship Rd, Jefferson City, MO 65101, US Update Address

This is the location we understood. If it is not correct you can change it and try again.

For the people you've mentioned, we'd like to know how far they live from you and each other. Addresses are helpful for our computers to calculate distances, but addresses **WILL NOT BE SAVED** after you've completed the survey.

Friendship Rd
Steppergate Ct
Steppergate St
Friendship Rd

Continue

POWERED BY Google Map data ©2011 Google - [Terms of Use](#)

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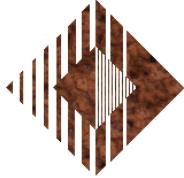




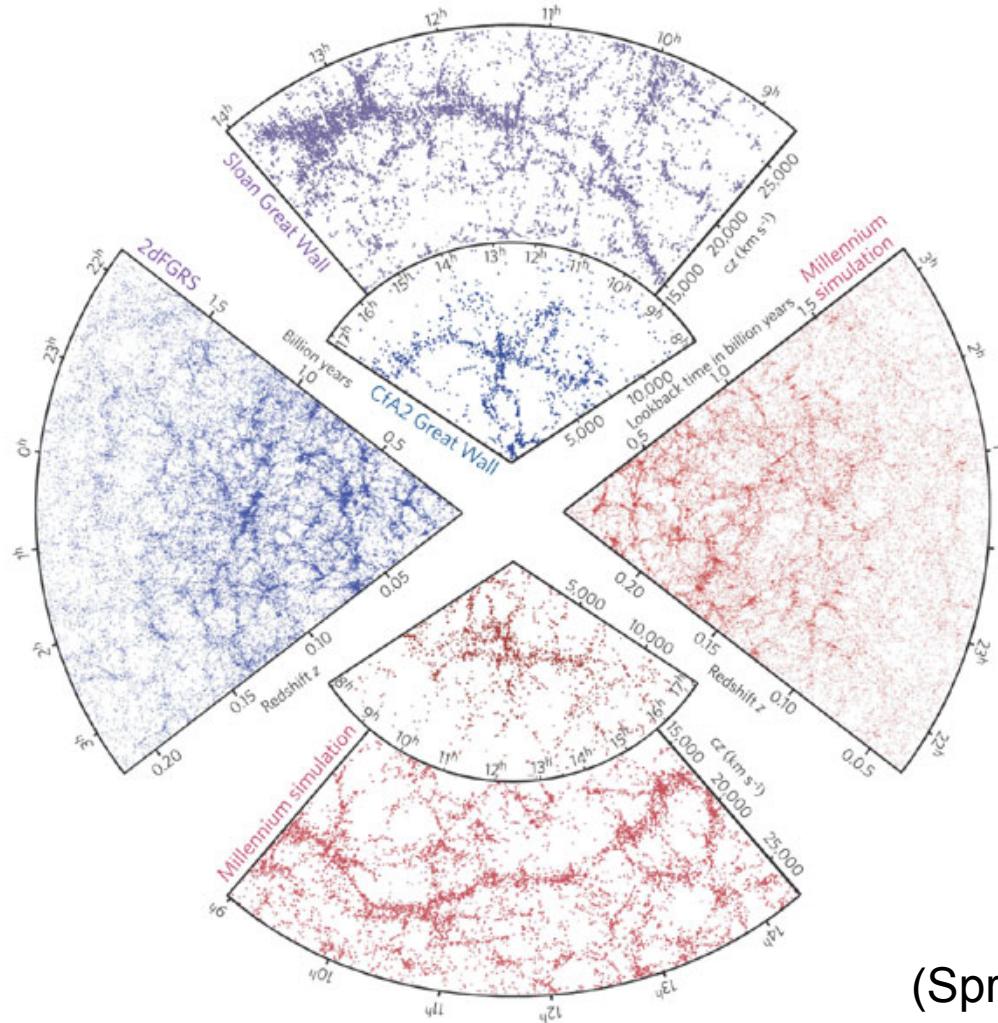
Prelude I: Moreno's Vision

“If we ever get to the point of charting a whole city or a whole nation, we would have an intricate maze of psychological reactions which would present a picture of a vast solar system of intangible structures, powerfully influencing conduct, as gravitation does bodies in space. Such an invisible structure underlies society and has its influence in determining the conduct of society as a whole....”

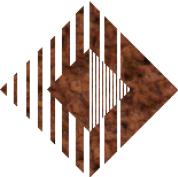
- Jacob Moreno, NYT, April 3 1933



Prelude II: Large-Scale Structure in Nature

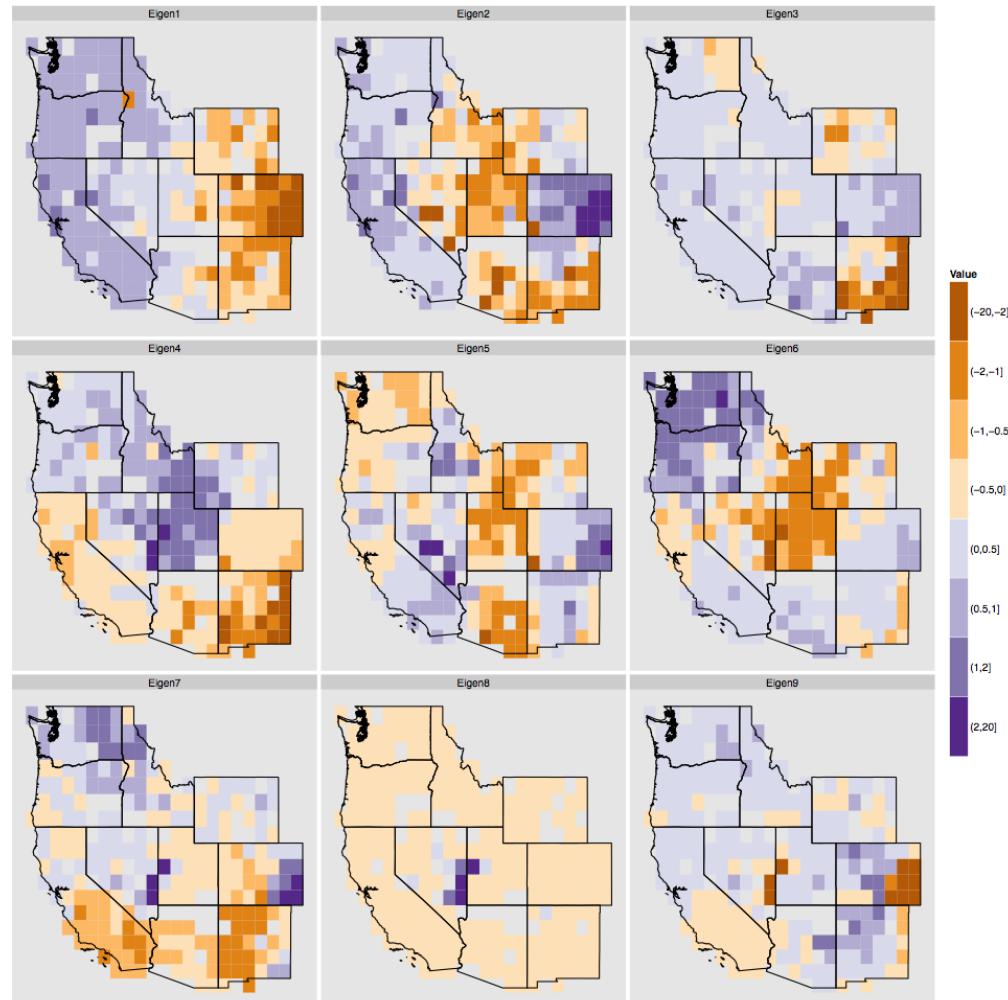


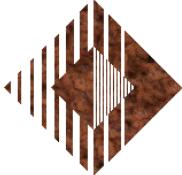
(Springel et al., 2006)



Spectral Decomposition of the Tie Volume Laplacian

- Tie volumes for raster cells estimated using Horvitz-Thompson; self-ties removed
- Row-normalized Laplacian ($L=D-A$) formed (D_{ii} being outdegree)
- Nontrivial eigenvectors with smallest eigenvalues give orthogonal dimensions such that cells with more different values have less total interaction
 - Random walk interpretation: messages would take longer to go between dimensionally distant cells
 - Reveal inhomogeneous connectivity at large scales





Areal Clustering from Laplacian Spectrum

- **6 highest weight Laplacian eigenvectors used to characterize each cell**
 - Values on each normalized to reduce outliers while preserving direction relative to 0
- **Bayesian Gaussian mixture model used to cluster cells based on position in normalized eigenspace**
 - Cells with low Mahalanobis distance tend to be grouped together
 - LA, Denver, Albuquerque, Phoenix, Salt Lake City, Seattle, San Francisco used to “seed”/orient clusters
 - Clusters show urban “reach”

