

COMPREHENSION QUESTIONS

for

NUMERICAL METHODS FOR SCIENTISTS AND ENGINEERS With Pseudocodes

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5.1 Basic Concepts

1. Describe the objective of numerical differentiation.
2. How does numerical differentiation differ from analytical differentiation?
3. How are the basic formulas for numerical differentiation derived?
4. Explain the concept of finite differences and their role in numerical differentiation.
5. How is the accuracy of numerical differentiation measured?
6. Discuss the sources of error in numerical differentiation and how they can be minimized.
7. What is the concept of truncation error, and how does it affect numerical differentiation?
8. What factors should be considered when choosing the step size h for numerical differentiation?
9. How do you balance between step size (h) and computational accuracy in numerical differentiation?
10. What are the challenges associated with numerical differentiation?
11. When might numerical differentiation be preferred over analytical methods, and why?

5.2 First-Order Finite Difference Formulas

1. Describe the forward difference formula and its use in numerical differentiation.
2. Explain the backward difference formula and how it differs from the forward difference.
3. How does the choice of step size affect the accuracy of the forward and backward difference formulas?
4. What is the truncation error associated with first-order finite difference formulas?
5. Discuss the concept of order of accuracy and how it relates to first-order finite difference formulas.
6. How do numerical errors and round-off errors influence the results obtained using first-order finite difference formulas?
7. How are first-order finite difference formulas extended to approximate higher-order derivatives?
8. What is the general process for deriving higher-order finite difference formulas using first-order differences?
9. How does the backward difference formula compare with the forward difference formula in terms of accuracy for higher-order derivatives?

5.3 Second-Order Finite Difference Formulas

1. Describe the general purpose of using second-order finite differences in numerical analysis.
2. How do second-order finite differences differ from first-order finite differences?
3. What is the forward difference formula for a second-order finite difference?
4. How does the forward second-order finite difference formula approximate the second derivative of a function?
5. What is the backward difference formula for a second-order finite difference?
6. How does the backward second-order finite difference formula approximate the second derivative of a function?
7. What is the truncation error associated with second-order finite difference formulas?
8. Discuss the impact of step size h on the accuracy of second-order finite difference approximations.
9. Explain the concept of order of accuracy and how it applies to second-order finite difference formulas.
10. How do numerical errors and round-off errors influence the results of second-order finite difference approximations?
11. Compare the accuracy and efficiency of second-order finite difference formulas with first-order finite difference formulas.
12. Discuss how second-order finite difference methods compare with higher-order finite difference methods in terms of accuracy and computational complexity.

5.4 Central Difference Formulas

1. Describe the general approach for calculating central finite differences for a function.
2. Why is the central difference formula typically preferred over forward or backward differences for approximating the second derivative?
3. What is the formula for the central difference approximation of the first derivative of a function?
4. What is the formula for the central difference approximation of the second derivative of a function?
5. What are the truncation errors associated with the central difference formulas for the first and second derivatives?
6. Discuss the effect of the step size h on the accuracy of central difference approximations.
7. What factors should be considered when choosing the step size h for central finite differences?
8. Compare the accuracy and efficiency of central finite differences with forward and backward finite differences.

5.5 Finite Differences and Direct-Fit Polynomials

1. What is meant by direct-fit polynomial?
2. What role does a direct-fit polynomial play in differentiation?
3. Describe the process of using finite differences to determine the coefficients of a polynomial that fits a set of data points.
4. How does the degree of the polynomial relate to the number of data points used in direct-fit polynomial interpolation?
5. What factors influence the accuracy of a direct-fit polynomial obtained using finite differences?
6. How can the choice of polynomial degree impact the accuracy and stability of the interpolation?
7. Explain how the order of accuracy of derivatives with symmetrical (central) and nonsymmetrical (forward/backward) finite difference forms change with respect to the order of derivative.
8. When might finite difference methods for polynomial fitting be less effective compared to other interpolation methods?
9. What are the advantageous and disadvantageous of using finite differences for direct-fit polynomial interpolation?

5.6 Differentiating Nonuniformly Spaced Discrete Data

1. What is meant by differentiating nonuniformly spaced discrete data, and why is it important?
2. How does nonuniform spacing of data points affect the process of numerical differentiation?
3. What are the challenges associated with differentiating data that is nonuniformly spaced?
4. What is the finite difference method for differentiating nonuniformly spaced data, and how is it adapted for nonuniform spacing?
5. Discuss the accuracy and limitations of using finite differences for nonuniformly spaced data.
6. How does the accuracy of derivative estimates change with varying levels of nonuniformity in data spacing?

5.7 Richardson Extrapolation

1. What is Richardson extrapolation and its purpose in numerical differentiation?
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3. How does Richardson extrapolation enhance the accuracy of derivative estimations?
4. Describe the procedure of applying Richardson extrapolation to improve the accuracy of finite difference estimates.

5. State the steps of applying Richardson extrapolation to numerical differentiation?
6. What role does the step size h play in the application of Richardson extrapolation to numerical differentiation?
7. How does Richardson extrapolation improve the order of accuracy of finite difference methods?