CHAPTER 1 NUMERICAL ALGORITHMS AND ERRORS

COMPREHENSION QUESTIONS

for

NUMERICAL METHODS FOR SCIENTISTS AND ENGINEERS With Pseudocodes

By Zekeriya ALTAÇ

October 2024

1.1 Numerical Algorithms in Pseudocodes

1. What is an algorithm?
2. What is a computer program?
3. Describe the phases of preparing an executable computer program.
4. Name some of the reasons for having many so many computer programming languages.
5. List the elements of a pseudocode.
6. What are the basic constructs in pseudocodes?
7. How do the conditional constructs work?
8. How do the loops and accumulators work?
9. What are the advantages of preparing a pseudocode?
10. What are the disadvantages of pseudocodes?
11. What kind of recommendations would you give for someone to be an efficient programmer?

1.2 Definition of Error

1. What kind of sources of errors do we expect when simulating scientific and engineering problems?
2. How is error in a computed quantity defined?
3. Name and explain the two kinds of errors encountered in experimental measurements.
4. Differentiate between precision and accuracy in the context of numerical algorithms.

1.3 Propagation of Error

1. Explain how uncertainties arise in the experiments and the reasons for applying uncertainty analysis.
2. Define the term propagation of error.
3. How does error propagate through a sequence (summation/subtraction, multiplication, division, exponentiation) of numerical operations?

1.4 Numerical Computations

1. How do you convert a decimal number to a binary number?
2. What digits are used in the binary and octal number system?
3. How would you convert a number from hexadecimal to binary?
4. What is a floating-point number, and how does it differ from an integer?
5. Explain the general structure of a floating-point number in scientific notation?
6. What are the components of a floating-point number according to the IEEE 754 standard?
7. Describe the role of the sign bit in a floating-point number.
8. What is the significance of the exponent in floating-point representation?
9. How is the mantissa used in floating-point representation?
10. How many bits are allocated for the exponent in single- and double-precision floating-point numbers?
11. Explain the concept of precision and how it affects floating-point calculations.
12. What is floating-point rounding, and why is it necessary?
13. Explain why sometimes floating-point arithmetic does not produce the results that you expect?
14. What are some common types of floating-point errors, and how can they be mitigated?
15. Explain the concept of floating-point overflow and floating-point underflow.
16. How does the choice between single-precision and double-precision floating-point numbers affect performance and accuracy?
17. What is the primary difference between rounding and chopping in floating-point arithmetic?
18. What are significant digits in a number, and why are they important?

19. How do you determine the number of significant digits in a given number?
20. How do you perform addition or subtraction with numbers that have different numbers of significant digits?
21. What are the rules for determining the number of significant digits in the result of multiplication or division?
22. What is a decimal place, and how does it relate to the position of digits in a number?
23. Explain the process of rounding a number to a specific number of decimal places.
24. How does the number of decimal places reflect the precision of a measurement instrument?
25. Discuss how the choice of decimal places can impact the accuracy of data presentation in research.
26. Provide an example of a scenario where using too few or too many decimal places could lead to significant errors.
27. What is meant by "loss of significance" in numerical computations?
28. How does loss of significance occur during arithmetic operations?
29. How can understanding loss of significance help in interpreting results from scientific experiments?
30. What strategies can be used to minimize the impact of loss of significance in numerical computations?
31. Describe the role of reformatting or rearranging operations in reducing loss of significance.
32. Explain how using higher precision data types or algorithms can help avoid loss of significance.
33. Why is it important to be aware of loss of significance when designing algorithms for numerical computation?
34. Describe a method to prevent loss of significance when dealing with very large and very small numbers.
35. What is meant by "condition" in the context of numerical problems?
36. What is numerical stability, and why is it important in the context of algorithms?
37. What is the condition number of a problem, and how does it relate to the sensitivity of the solution to changes in input data?
38. What does a high condition number indicate about the sensitivity of a problem?
39. Discuss how an ill-conditioned problem might lead to significant errors in computation even if the algorithm is stable.
40. What are some techniques to improve the stability of numerical algorithms?

1.5 Application of Taylor Series

1. What is a Taylor series, and what is its primary purpose in mathematics?
2. What is a Maclaurin series, and how is it related to the Taylor series?
3. Explain the significance of the center point a in the Taylor series expansion.
4. How do you assess the accuracy of a Taylor series approximation for a function?
5. What does it mean for a Taylor series to converge, and how is convergence determined?
6. Explain the concept of the remainder term (or error term) in the Taylor series expansion.
7. Explain the difference between a Taylor series and a Taylor polynomial.
8. Discuss how Taylor series can be applied in numerical methods for approximating functions.
9. What are the criteria used to obtain an approximate value of a series sum with a certain accuracy, and how are they applied in practice?
10. State the conditions required for the Mean Value Theorem to apply.
11. Provide a geometric interpretation of the First Mean Value Theorem
12. Discuss the implications of the Mean Value Theorem in the context of Taylor's Theorem.
13. What does the Second Mean Value Theorem state, and how does it differ from the First Mean Value Theorem?

1.6 Truncation Error of Series

1. What is truncation error in the context of series approximations?
2. Explain the difference between truncation error and rounding error in numerical computations.
3. How does truncation error arise when using a finite number of terms from an infinite series?
4. When approximating a function with a Taylor series, what is meant by truncating the series?
5. Explain how the truncation error behaves as more terms are added to the series.
6. How does the choice of how many terms to include in a truncated series affect the truncation error?
7. What role does the order of the Taylor series play in the magnitude of truncation error?
8. How can the Lagrange form of the remainder term be used to estimate the truncation error in a Taylor series?
9. What does the term "order of the truncation error" refer to, and how is it determined?
10. Why is it important to consider truncation error when performing numerical approximations in computational methods?
11. Describe how error analysis can guide the decision on how many terms to include in a series approximation.

1.7 Rate and Order of Convergence

1. How does the convergence rate of a series affect the truncation error?
2. What is meant by the rate and order of convergence in the context of iterative methods?
3. Explain how the rate of convergence affects the speed at which an iterative method approaches its solution.
4. What is the significance of a method having linear, quadratic, or cubic convergence?
5. How can you determine the order of convergence by analyzing the asymptotic behavior of the error term?