CHAPTER 6

INTERPOLATION AND EXTRAPOLATION

COMPREHENSION QUESTIONS

for

NUMERICAL METHODS FOR SCIENTISTS AND ENGINEERS With Pseudocodes

By Zekeriya ALTAÇ

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6.1 Lagrange Interpolation

1. How is the Lagrange interpolation polynomial constructed using a set of data points?
2. How do you compute the Lagrange basis polynomials $L_0(x)$, $L_1(x)$, ..., $L_n(x)$ for a set of n+1 data points?
3. What is the general formula for the Lagrange interpolation polynomial?
4. Explain the general concept of polynomial interpolation and how Lagrange interpolation fits into this context.
5. Discuss how the number of data points affects the accuracy of the Lagrange interpolation polynomial.
6. What is Runge's phenomenon, and how does it impact Lagrange interpolation with a high number of data points?
7. How does the spacing of data points affect the performance of Lagrange interpolation?
8. Discuss the advantages and disadvantages of Lagrange interpolation method.
9. How can Lagrange interpolation be extended to handle multidimensional data or functions of several variables?

6.2 Newton's Divided Differences

1. What is Newton's Divided Differences interpolation and its purpose in numerical analysis?
2. How does Newton's Divided Differences interpolation differ from Lagrange interpolation?
3. Explain the concept of divided differences and how they are used in Newton's interpolation method.
4. How do you compute higher-order divided differences?
5. What is the general form of Newton's interpolation polynomial?
6. How do you use the divided differences to determine the coefficients of the Newton interpolation polynomial?
7. What is the concept of interpolation error in the context of Newton's Divided Differences interpolation?
8. What factors can contribute to errors in the Newton interpolation polynomial, and how can they be mitigated?
9. What are the advantages of using Newton's Divided Differences interpolation over Lagrange interpolation?

6.3 Newton's Formualas for Uniformly Spaced Data

1. What are Gregory-Newton formulas and their main purpose in numerical analysis?
2. How do Gregory-Newton formulas relate to numerical differentiation and interpolation?
3. Explain the significance of forward and backward difference formulas in the context of Gregory-Newton formulas.
4. How are forward differences used to construct Gregory-Newton interpolation polynomials?
5. How are backward differences used to construct Gregory-Newton interpolation polynomials?
6. What is the Bessel's interpolation formula and how is it used in interpolation?
7. What is the Stirling's interpolation formula and how is it used in interpolation?
8. How does the choice of forward or backward differences affect the Gregory-Newton interpolation polynomial?
9. What is the concept of interpolation error in the context of Gregory-Newton formulas?
10. Discuss how the choice of step size and data points affects the accuracy of Gregory-Newton interpolation.
11. Compare Gregory-Newton interpolation with Lagrange interpolation in terms of computational efficiency and ease of use.
12. Discuss the advantages and disadvantages of Gregory-Newton interpolation compared to other poly-

nomial interpolation methods.

6.4 Cubic Spline Interpolation

1. What is the primary purpose of cubic spline interpolation in numerical analysis?
2. How does cubic spline interpolation differ from other polynomial interpolation methods?
3. Explain the concept of splines in the context of linear, quadratic and cubic interpolation.
4. Describe the general form of a cubic spline polynomial between two data points.
5. What are the typical conditions or constraints imposed on cubic splines to ensure a smooth interpolation?
6. Explain the concept of continuity and smoothness at the data points in cubic spline interpolation.
7. Describe the natural spline and linear extrapolation end conditions commonly used in cubic spline interpolation.
8. Explain the process of setting up the system of linear equations to solve for the coefficients of the cubic spline.
9. What role do the second derivatives play in determining the cubic spline coefficients?
10. Describe the steps involved in applying cubic spline interpolation to a given set of data points.
11. What is the concept of interpolation error in the context of cubic spline interpolation?
12. Discuss how the accuracy of cubic spline interpolation is affected by the number of data points.
13. What are considered as potential sources of error in cubic spline interpolation, and how can they be minimized?
14. Compare cubic spline interpolation with polynomial interpolation in terms of computational efficiency and accuracy.
15. Discuss the advantages and disadvantages of cubic spline interpolation compared to piecewise linear interpolation and other spline methods.

6.5 Root-finding By Inverse Interpolation

1. What is inverse interpolation, and how does it differ from direct interpolation?
2. Explain the purpose of using inverse interpolation for rootfinding.
3. How does the concept of inverse interpolation relate to finding the roots of a discrete function?
4. Describe the steps involved in applying inverse interpolation to approximate the root of a given discrete function.
5. Explain how Lagrange interpolation can be adapted for inverse interpolation to find roots.
6. Discuss how the choice of interpolation method affects the accuracy of rootfinding by inverse interpolation.
7. What are some common sources of error in inverse interpolation, and how can they be addressed?
8. Discuss the advantages and disadvantages of inverse interpolation in applying to rootfinding.
9. Describe the process of using the inverse interpolation method to find the roots of real function. In which cases can this method be preferred?

6.6 Multivariate Interpolation

1. Explain the difference between univariate and multivariate linear interpolation.
2. In what contexts is multivariate linear interpolation typically used?
3. What is the general formula for linear interpolation in two dimensions?
4. What is bilinear interpolation, and how is it applied to a discrete data points?
5. Explain the process of bilinear interpolation in a rectangular grid, including the steps to compute the interpolated value.

6. What factors affect the accuracy of multivariate linear interpolation?
7. What is the general formula for multivariate Lagrange interpolation in two dimensions?
8. Explain the concept of Lagrange basis polynomials in the context of multivariate interpolation.
9. What factors affect the accuracy of multivariate Lagrange interpolation?
10. What are the advantages and disadvantages of using multivariate Lagrange interpolation?
11. How can multivariate Lagrange interpolation be adapted for nonuniformly spaced data points or non-rectangular grids?

6.7 Extrapolation

1. What is extrapolation, and how does it differ from interpolation?
2. Explain the purpose of extrapolation in the context of predicting values outside the range of known data.
3. Describe the simplest form of extrapolation using linear methods. How is it performed?
4. What is polynomial extrapolation, and how does it extend beyond linear extrapolation?
5. How do you determine the coefficients for polynomial extrapolation, and what are some common forms of polynomial extrapolation?
6. What are the common sources of error in extrapolation?
7. Discuss the concept of extrapolation error and how it can impact the accuracy of predictions.
8. What factors should be considered when choosing between linear and polynomial extrapolation?
9. How does the range of data and the distribution of data points influence the effectiveness of extrapolation?
10. In what situations might extrapolation lead to misleading or inaccurate results?
11. Discuss the advantages and disadvantages of polynomial extrapolation methods.