CHAPTER 11

EIGENVALUES AND EIGENVALUE PROBLEMS

COMPREHENSION QUESTIONS

for

NUMERICAL METHODS FOR SCIENTISTS AND ENGINEERS With Pseudocodes

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11.1 Eigenvalue Problem and Properties

1. What are eigenvalues and eigenvectors?
2. Why is the eigenvalue problem important in various fields?
3. How do you find the eigenvalues of a matrix?
4. What does the characteristic polynomial of a matrix represent?
5. What conditions must be met for a matrix to have eigenvalues and eigenvectors?
6. What is the significance of having multiple eigenvalues (i.e., eigenvalues with multiplicity)?
7. What is the Gershgorin Circle Theorem?
8. How does the Gershgorin Circle Theorem help in finding eigenvalues?

11.2 Power Method

1. What is the primary goal of the power method?
2. How does the power method iteratively converge to the dominant eigenvalue?
3. What is the role of the initial vector in the power method?
4. What are some limitations of the power method?
5. How does the convergence rate of the power method depend on the eigenvalue spectrum?
6. What modification can be made to the power method to find the smallest eigenvalue instead?
7. What is the purpose of scaling the vector in the power method?
8. What is the significance of the Rayleigh Quotient in the power method?
9. What is the Rayleigh Quotient and how is it used to find the largest eigenvalue?
10. What is the difference between the power method and the Rayleigh quotient iteration?
11. How does the inverse power method help in finding the smallest eigenvalue?
12. What is the role of shifting in the shifted inverse power method?
13. What is the typical procedure to apply the inverse power method in practice?
14. What is the significance of choosing an appropriate shift parameter, α?
15. How does the shifted inverse power method help in isolating specific eigenvalues?
16. How does the convergence rate of the shifted inverse power method compare to the standard inverse power method?

11.3 Similarity and Orthogonal Transformations

1. What is a similarity transformation in linear algebra?
2. How do similarity transformations affect the eigenvalues of a matrix?
3. What is the significance of orthogonal transformations in relation to eigenvalues?
4. How does an orthogonal transformation differ from a general similarity transformation?
5. What is the relationship between an orthogonal matrix and its eigenvalues?
6. How can similarity transformations be used to simplify a matrix for eigenvalue computation?
7. Can every matrix be diagonalized using similarity transformations?
8. What is a diagonalizable matrix, and how does it relate to eigenvalues and eigenvectors?

11.4 Jacobi Method

1. What is the Jacobi method used for in numerical linear algebra?
2. How does the Jacobi method iteratively transform a matrix?
3. What is a rotation matrix in the context of the Jacobi method?
4. Why is the Jacobi method particularly suitable for symmetric matrices?
5. What is the goal of each rotation in the Jacobi method?

6. How do you determine the optimal angle for a rotation in the Jacobi method?
7. What is the convergence criterion for the Jacobi method?
8. How does the Jacobi method handle large matrices?
9. What is the main advantage of the Jacobi method over other eigenvalue algorithms?
10. Can the Jacobi method be used for non-symmetric matrices?

11.5 Cholesky Decomposition

1. What is the generalized eigenvalue problem?
2. In the context of the generalized eigenvalue problem, what role does the matrix **B** play?
3. How can the Cholesky decomposition be used to solve the generalized eigenvalue problem?
4. What is the advantage of using the Cholesky decomposition for the generalized eigenvalue problem?
5. What does the Cholesky decomposition of a matrix **B** look like?
6. How does one check if the matrix **B** is suitable for Cholesky decomposition?
7. What is the relationship between the Cholesky decomposition and the standard eigenvalue problem?
8. What are the numerical benefits of using the Cholesky decomposition in eigenvalue problems?

11.6 Householder Method

1. What is the Householder method used for in numerical linear algebra?
2. How does the Householder transformation work?
3. What is the goal of using a Householder reflection in the context of matrix reduction?
4. What is the significance of orthogonality in Householder transformations?
5. What steps are involved in applying the Householder method to find eigenvalues?
6. How does the Householder method handle symmetric matrices differently from general matrices?

11.7 Eigenvalues of Tridiagonal Matrices

1. Why is it advantageous to reduce a matrix to tridiagonal form when computing eigenvalues?
2. How does the tridiagonal form simplify the computation of eigenvalues compared to a general matrix?
3. Which method can be applied to reduce a matrix to tridiagonal form?
4. How does the Sturm sequence help in finding the number of eigenvalues in a given interval?
5. What are the key steps in constructing a Sturm sequence for a polynomial?
6. What are some limitations of using the Sturm sequence method for finding eigenvalues?
7. How does the QR iteration method decompose a matrix?
8. How does the QR iteration work on tridiagonal matrices?
9. What is the significance of the matrix QQQ in the QR iteration method?
10. What are the typical steps involved in the QR iteration process?
11. Why is the QR iteration method considered efficient for large matrices?
12. What are some pros and cons associated with the QR iteration method?
13. What is the purpose of the Gram-Schmidt process in numerical linear algebra?
14. How does the Gram-Schmidt process work to orthogonalize vectors?
15. How does the Gram-Schmidt process relate to eigenvalues and eigenvectors?
16. What is the result of applying the Gram-Schmidt process to a set of linearly independent vectors?
17. What are the key steps in the Gram-Schmidt process for a given set of vectors?
18. Why is orthonormality important when working with eigenvectors?
19. How does the Gram-Schmidt process differ from other orthogonalization methods like the Householder transformation?
20. What type of system is solved to find the eigenvectors for a given eigenvalue?

21. How do you determine if a specified eigenvalue λ is actually an eigenvalue of a matrix A?
22. How can you verify that a vector x is an eigenvector of A for a specified eigenvalue λ?

11.8 Faddeev-Leverrier Method

1. What is the Faddeev-Leverrier method used for?
2. How does the Faddeev-Leverrier method compute the coefficients of the characteristic polynomial?
3. What is the relationship between the characteristic polynomial and the eigenvalues of a matrix?
4. What is the first step in applying the Faddeev-Leverrier method to a matrix A?
5. How are the traces of powers of the matrix used in the Faddeev-Leverrier method?
6. How are the eigenvalues obtained from the characteristic polynomial in the Faddeev-Leverrier method?
7. What are some limitations or challenges of using the Faddeev-Leverrier method?

11.9 Characteristic Value Problems

1. What is a characteristic value problem in the context of ordinary differential equations (ODEs)?
2. How do characteristic values and eigenfunctions relate to boundary value problems in ODEs?
3. How can you find the characteristic values for a second-order linear differential equation with Dirichlet BCs imposed on both sides?
4. What are eigenfunctions in the context of ODEs, and how are they determined?
5. What types of boundary conditions are typically considered in characteristic value problems for ODEs?