CHAPTER 8 NUMERICAL INTEGRATION

# COMPREHENSION QUESTIONS

for

## NUMERICAL METHODS FOR SCIENTISTS AND ENGINEERS With Pseudocodes

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October 2024

## 8.1 Trapezoidal Rule

1. What is the reason for applying numerical integration in practical applications?
2. What is the trapezoidal rule? and How do you apply it to multiple intervals?
3. What factors affect the accuracy of the trapezoidal rule?
4. How does increasing the number of panels affect the error in the trapezoidal rule?
5. Explain if you can apply the trapezoidal rule discontinuous functions?
6. What is the trapezoidal rule with end correction?
7. Why is an end correction necessary in the trapezoidal rule?
8. Under what circumstances can the trapezoidal rule with end correction be applied?
9. What is the main limitation of the trapezoidal rule compared to other numerical integration methods?
10. How do you calculate the end correction term in the trapezoidal rule?
11. What type of functions benefit most from using the trapezoidal rule with end corrections?
12. State the order of accuracy for trapezoidal rule with and without end correction.
13. Can the trapezoidal rule with end corrections be applied to functions with discontinuities or sharp changes?

## 8.2 Simpson's Rule

1. What is Simpson's rule (1/3-rule), and How does Simpson's Rule differ from the trapezoidal rule?
2. What is the error term and order of error in Simpson's rule?
3. When is Simpson's Rule particularly useful?
4. What is Simpson's Rule with end correction?
5. What is the general approach to incorporating end correction into Simpson's Rule?
6. How does the end correction term improve the accuracy of Simpson's Rule?
7. Under which circumstances would Simpson's Rule with end correction be preferred over the basic Simpson's Rule?
8. What are the advantages and disadvantages of using Simpson's Rule with/without end correction?
9. Explain if you could apply Simpson's Rule with or without end correction to functions with discontinuities or singularities?
10. What is Simpson's 3/8 Rule, and How does Simpson's 3/8 Rule differ from Simpson's 1/3 Rule?
11. State the formula for Simpson's 3/8 Rule for a single interval.
12. Derive the composite formula for the Simpson's 3/8 rule.
13. State the error terms and order of errors for Simpson's rule with/without end correction, 3/8-rule.
14. Explain the limitations of Simpson's 1/3- and 3/8- rules.

## 8.3 Romberg's Rule

1. What is Romberg's Rule?
2. How does Romberg's Rule improve upon the basic trapezoidal rule?
3. What is the general approach to applying Romberg's Rule?
4. How is the Romberg table constructed?
5. What is Richardson extrapolation and how is it used in Romberg's Rule?
6. How does Romberg's Rule improve the accuracy of the trapezoidal rule?
7. What is the error term for Romberg's Rule?
8. When would you use Romberg's Rule instead of other numerical integration methods?
9. What are the computational requirements for Romberg's Rule?
10. Can Romberg's Rule be applied to functions with discontinuities or singularities?

## 8.4 Adaptive Integration

1. How does adaptive integration differ from standard fixed panel integration methods?
2. What criteria are typically used to adaptively adjust intervals in integration?
3. How is the local error estimated in adaptive integration?
4. What factors influence the effectiveness of adaptive integration?
5. When is adaptive integration particularly useful?
6. What are some common adaptive integration algorithms or techniques?

## 8.5 Newton-Cotes Rules

1. What are the Closed and Open Newton-Cotes formulas used for?
2. How do Open Newton-Cotes rules differ from Closed Newton-Cotes rules?
3. What are the error terms for the Open Newton-Cotes formulas?
4. Explain how you can implement Open Newton-Cotes formulas to multiple panels (i.e., composite rule).
5. How does increasing n affect the accuracy of the Open Newton-Cotes rules?
6. When would you use Open Newton-Cotes rules over Closed Newton-Cotes rules?
7. What are the potential limitations of Open Newton-Cotes rules?

## 8.6 Integration of Nonuniform Discrete Functions

1. What does nonuniformly spaced discrete data mean?
2. What challenges are associated with integrating nonuniformly spaced discrete functions?
3. How nonuniformly spaced discrete functions are interpolated with quadratic and cubic polynomials?
4. How does the Trapezoidal Rule apply to nonuniformly spaced discrete functions?
5. How does the Simpson's Rule apply to nonuniformly spaced discrete functions?
6. What challenges arise with interpolation and integration of nonuniformly spaced discrete functions?
7. How does the spacing of data points affect the accuracy of numerical integration?
8. How can you estimate the error when integrating nonuniformly spaced discrete functions?
9. What factors contribute to integration error in nonuniformly spaced discrete functions?
10. How do you validate the integration results obtained from nonuniformly spaced discrete functions?

## 8.7 Gauss-Legendre Method

1. What is Gauss-Legendre integration, and How does it differ from other fixed-panel numerical integration methods?
2. What are Legendre polynomials, and how are they related to the Gauss-Legendre method?
3. What are Gauss-Legendre nodes and weights, and How are they determined?
4. How do you apply Gauss-Legendre method to an integration over an arbitrary interval [a,b]?
5. What is the accuracy of Gauss-Legendre integration?
6. What factors influence the error in Gauss-Legendre integration?
7. When is Gauss-Legendre integration particularly useful?
8. What are the advantages and disadvantages of Gauss-Legendre integration?

## 8.8 Computing Improper Integrals

1. What defines an improper integral?
2. What are the two main types of improper integrals?
3. Under what circumstances can you avoid computing a Type I improper integral?

4. Under what circumstances can you avoid computing a Type II improper integral?
5. What is the role of substitutions or transformations in computing improper integrals?
6. How do you handle an improper integral where the integrand has a singularity within the interval?
7. How do you handle an improper integral where the integrand has a singularity on one or both endpoints?
8. What does "subtraction of singularity" mean in the context of integration?
9. How can you apply the subtraction of singularity to an integral with a singularity at x=c within the interval [a,b]?
10. What are potential pitfalls in the subtraction of singularities method?
11. How is an integral with a singularity handled when the singularity is "ignored"?
12. How do you assess the accuracy of an integral when a singularity is ignored?
13. What are potential errors or pitfalls associated with ignoring singularities in integration?
14. Why might one choose to truncate the interval of an integral?
15. How is the process of truncating the interval typically carried out for an integral with an infinite upper limit?
16. How do you assess the accuracy of an integral result after truncating the interval?
17. What potential errors might arise from truncating the interval of integration?

## 8.9 Gauss-Laguerre Method

1. What is the Gauss-Laguerre integration method used for?
2. What are Laguerre polynomials, and how are they related to the Gauss-Laguerre method?
3. How are the nodes and weights for Gauss-Laguerre integration determined?
4. How does the choice of number of nodes affect the accuracy of the Gauss-Laguerre integration?
5. What is the error behavior of the Gauss-Laguerre method?
6. How can you determine if the Gauss-Laguerre method is giving accurate results?
7. In what types of problems is Gauss-Laguerre integration particularly useful?
8. What are the limitations of the Gauss-Laguerre method?

## 8.10 Gauss-Hermite Method

1. What is the purpose of the Gauss-Hermite integration method?
2. What are Hermite polynomials, and how are they used in the Gauss-Hermite method?
3. How are the nodes and weights for Gauss-Hermite integration determined?
4. How does the choice of the number of nodes affect the accuracy of the Gauss-Hermite integration?
5. What is the error behavior of the Gauss-Hermite method?
6. In what types of problems is Gauss-Hermite integration particularly useful?
7. What are the limitations of the Gauss-Hermite method?
8. How can you validate the results of Gauss-Hermite integration?

## 8.11 Gauss-Chebyshev Method

1. What is the purpose of the Gauss-Chebyshev integration method?
2. What are Chebyshev polynomials, and how are they related to the Gauss-Chebyshev method?
3. How are the nodes and weights for Gauss-Chebyshev integration determined?
4. Explain how you can apply the Gauss-Chebyshev integration to an integral with arbitrary interval?
5. What is the error behavior of the Gauss-Chebyshev method?
6. How can you determine if the Gauss-Chebyshev method is giving accurate results?
7. What are the limitations of the Gauss-Chebyshev method?
8. How can you validate the results of Gauss-Chebyshev integration?

## 8.12 Computing Integrals with Variable Limits

1. What is an integral with variable limits?
2. How would you transform an integral with variable limits to a definite integral with constant limits?
3. How do you apply the Trapezoidal or Simpson's rule to an integral with variable limits?
4. What factors affect the accuracy of numerical integration for integrals with variable limits?
5. How can you verify the results of numerical integration for integrals with variable limits?

## 8.13 Double Integration

1. What are some practical applications of double integrals?
2. What are some common numerical methods for evaluating double integrals?
3. How would you apply the midpoint rule to a double integral?
4. How would you extend the trapezoidal rule to a double integral?
5. How would you extend the Simpson's rule to a double integral?
6. What factors influence the accuracy of numerical integration for double integrals?
7. How can you estimate the error in numerical integration of double integrals?
8. How do you handle double integrals with variable limits?