CHAPTER 3 LINEAR SYSTEMS: ITERATIVE METHODS

# COMPREHENSION QUESTIONS

for

## NUMERICAL METHODS FOR SCIENTISTS AND ENGINEERS With Pseudocodes

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## 3.1 Overview

1. What is an iterative method in numerical computing, and what is its primary purpose?
2. Describe the general steps of an iterative method for solving a problem.
3. What is the role of an initial guess in an iterative method?
4. How is the iterative process typically defined or represented mathematically?
5. What criteria are used to determine when to stop the iterations in an iterative method?
6. Explain how convergence tolerance is set and used in iterative methods.
7. How is the error computed and analyzed at each step of an iterative method?
8. What is the significance of convergence in the context of iterative methods?
9. How do iterative methods update solutions from one iteration to the next?
10. Describe how the iterative process is affected by the choice of iterative scheme or algorithm.
11. How does the choice of initial guess affect the performance of an iterative method?
12. Discuss how the convergence rate of an iterative method impacts the number of iterations required to reach an acceptable solution.
13. What are common sources of error in iterative methods, and how can they be mitigated?
14. What are the differences in iterative steps when applying methods to linear vs. nonlinear problems?

## 3.2 Stationary Iterative Methods

1. What are stationary iterative methods, and what types of problems are they typically used to solve?
2. Describe the general steps involved in a stationary iterative method for solving linear systems.
3. What are the main steps of the Jacobi method, and how does it update the solution vector?
4. Outline the steps of the Gauss-Seidel method and explain how it differs from the Jacobi method.
5. Describe the Successive Over-Relaxation (SOR) method and its improvements over the Gauss-Seidel method.
6. Explain the importance of the iterative scheme in stationary iterative methods and how it is derived.

## 3.3 Convergence of Stationary Iterative Methods

1. What is the role of the iterative matrix in a stationary iterative method?
2. What conditions must be met for a stationary iterative method to converge?
3. How is convergence typically analyzed for stationary iterative methods?
4. Discuss the role of the spectral radius of the iteration matrix in determining convergence.
5. What factors contribute to the error propagation in stationary iterative methods?
6. Explain how the choice of relaxation parameters in SOR affects the accuracy and convergence of the method.
7. What are the main advantages of using stationary iterative methods for solving large systems of equations?
8. Discuss how the choice of initial guess affects the performance of stationary iterative methods.
9. What are the common challenges or limitations associated with stationary iterative methods?
10. Discuss the impact of matrix properties (e.g., diagonal dominance) on the effectiveness of stationary iterative methods.
11. Compare the Jacobi, Gauss-Seidel, and SOR methods in terms of convergence speed and computational efficiency.
12. Describe an algorithm that adaptively computes the optimum parameter.

## 3.4 Krylov Space Methods

1. Describe the basic steps of the Conjugate Gradient (CG) method and explain its application to solving linear systems.
2. What is the Conjugate Gradient method used for in numerical linear algebra?
3. Describe the type of problems for which the CG method is most suitable.
4. Explain the basic principle behind the Conjugate Gradient method.
5. How does the CG method generate a new search direction in each iteration?
6. What role does the residual vector play in the Conjugate Gradient method?
7. How is convergence determined in the Conjugate Gradient method?
8. Discuss the significance of the condition number of the matrix in relation to the convergence of the CG method.
9. What factors influence the number of iterations required for convergence in the CG method?
10. What are the advantages of using the Conjugate Gradient method for solving large systems of linear equations?
11. What are the typical challenges or limitations associated with the Conjugate Gradient method?
12. What is the purpose of preconditioning in the Conjugate Gradient method?
13. How does preconditioning improve the convergence rate of the CG method?
14. Describe common types of preconditioners used with the Conjugate Gradient method.
15. What are some variations or extensions of the basic Conjugate Gradient method?
16. Explain the impact of matrix symmetry and positive definiteness on the performance of the CG method.

## 3.5 Improving Accuracy of Ill-Conditioned Systems

1. What is an ill-conditioned linear system, and how does it differ from a well-conditioned system?
2. Why does an ill-conditioned linear system pose challenges in terms of accuracy and numerical stability?
3. How can the condition number of a matrix indicate whether a linear system is ill-conditioned?
4. What is meant by "preconditioning," and how can it improve the accuracy of solutions for ill-conditioned systems?
5. Explain how scaling of the problem or normalization of the matrix can help mitigate issues related to ill-conditioning.
6. How can error propagation be managed in the context of solving ill-conditioned linear systems?
7. Describe the impact of rounding errors on the accuracy of solutions for ill-conditioned systems.
8. What is the role of iterative refinement in improving the accuracy of solutions for ill-conditioned systems?
9. Discuss how modified algorithms or enhanced precision arithmetic can address inaccuracies arising from ill-conditioning.