CEE 4350 Coastal Engineering

Problem Set 4

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Problem 1

Wave data in the form of a time series of surface elevation, η, was provided. Zero-crossings were calculated using a for-loop to find where the sign for η changed. It was found that there were 139 zero crossings, N_z. Then, the maximum of η was found between each peak. The entire dataset was analyzed.

The code above shows the calculation for the wave height and period metrics using the following equations:

$$N = N_z - 1$$

$$H_{\frac{1}{3}} = \frac{3}{N} \sum_{1}^{\frac{N}{3}} H_i$$

$$H_{rms} = \frac{1}{N} \sqrt{(} \sum_{1}^{\frac{N}{3}} H_i^2)$$

$$T_{\frac{1}{3}} = \frac{3}{N} \sum_{1}^{\frac{N}{3}} T_i$$

$$T_c = \frac{T_R}{N_z}$$

Significant wave height and wave period are reported in the following table.

$H_{\frac{1}{3}}$ (cm)	4.88
H_{rms} (cm)	0.244
$T_{\frac{1}{3}}$ (s)	1.08
T_c	0.719

A histogram of the wave heights was made using the wave data provided for analysis.

The supporting R code is shown below.

```
setwd("~/github/Coastal_Engineering")
library(zoo)
```

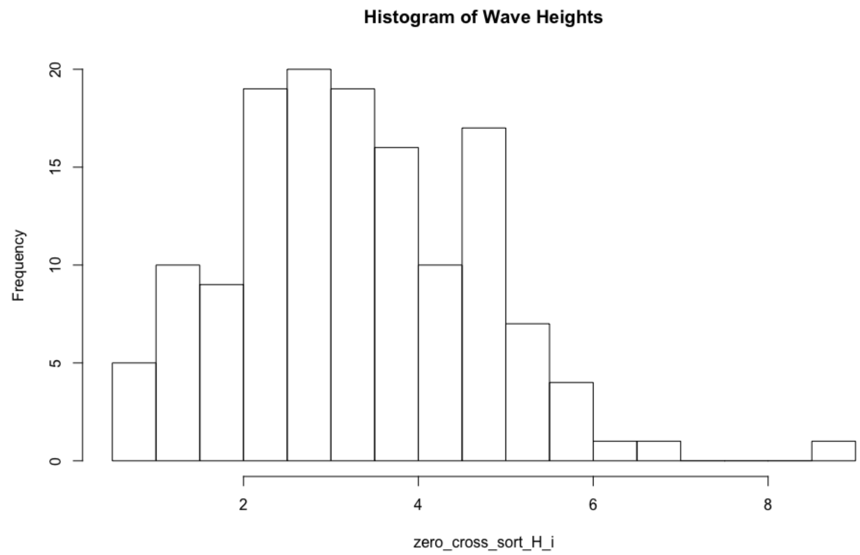


Figure 1:

```
# Load in data from csv file
# Time (s) eta (cm?)
wavedata = read.csv("waverecord_HW4.csv")
wavedata$time_s = wavedata[,1]
wavedata$eta_cm = wavedata[,2]

Nz = 139
N = Nz - 1
N3 = N/3

zero_cross = data.frame(matrix(nrow = Nz, ncol = 0))
num_samples = 1000

# Find the time of zero crossings
# Find the max and min wave height between zero crossings
j = 0
m = 0
for (i in 1:num_samples)
{
  if ((wavedata$eta_cm[i] < 0) & (wavedata$eta_cm[i+1] > 0))
  {
    j = j+1
```

```
    zero_cross$time_s[j] = (wavedata$time_s[i] + wavedata$time_s[i+1])/2
    zero_cross$H_i[j] = max(wavedata$eta_cm[(m:i)]) - min(wavedata$eta_cm[(m:i)])
    zero_cross$T_i[j] = max(wavedata$time_s[(m:i)]) - min(wavedata$time_s[(m:i)]) # wave pe
    m = i
  }
}

# Sort the zero-crossings
zero_cross_sort_H_i = sort(zero_cross$H_i, decreasing = TRUE)
zero_cross_sort_T_i = sort(zero_cross$T_i, decreasing = TRUE)

# Calculate H_1/3
H_onethird = 3/N*sum(zero_cross_sort_H_i[1:N3])
# Calculate H_rms
H_rms = 1/N*sqrt(sum((zero_cross_sort_H_i[1:N3])^2))

# Calculate T_1/3 using zero crossings
T_onethird = 3/N*sum(zero_cross_sort_T_i[1:N3])

# Calculate T_c
# Assume that there are the same number maximum peaks as there are zero crossings
# On average, the period between the peaks will be the same,
# so they can be found by dividing the total time series by the number of zero crossings
T_c = max(wavedata$time_s)/Nz

# Make a histogram of wave heights
hist(zero_cross_sort_H_i, breaks = 20, main = "Histogram of Wave Heights")
```

Problem 2

The Pierson-Moskowitz frequency for wave energy spectrums was given as

$$S_{\eta\eta} = 0.205 H_{\frac{1}{3}}^2 T_{\frac{1}{3}}^{-4} f^{-5} exp[-0.75(T_{\frac{1}{3}} f)^{-4}], (m^2 s)$$

$S_{\eta\eta}$ was calculated using $H_{\frac{1}{3}}$ and $T_{\frac{1}{3}}$ found is Problem 1 as 4.88 cm and 1.08 s, respectively. A vector for f was made from 0.01 to 5 s with a step size of 0.01.

$S_{\eta\eta}$ was plotted against f and is shown in the following spectral plot.

The area under the curve, AUC, for $S_{\eta\eta}$ was calculated to find the following relationships.

$$a_{rms} = \sqrt(2 * AUC)$$

$$H_{rms} = 2 * a_{rms}$$

$$H_{max} = \sqrt{2} * H_{rms}$$

$$H_{\frac{1}{10}} = 20 * \frac{\sqrt{2} * H_{rms} * sin(\frac{\pi}{20})}{\pi}$$

$$H_{\frac{1}{3}} = 6 * \frac{\sqrt{2} * H_{rms} * sin(\frac{\pi}{6})}{\pi}$$

AUC	1.63
a_{rms}	1.804
H_{rms} (cm)	3.609
H_{max} (cm)	5.103
$H_{\frac{1}{10}}$ (cm)	5.082
$H_{\frac{1}{3}}$ (cm)	4.87

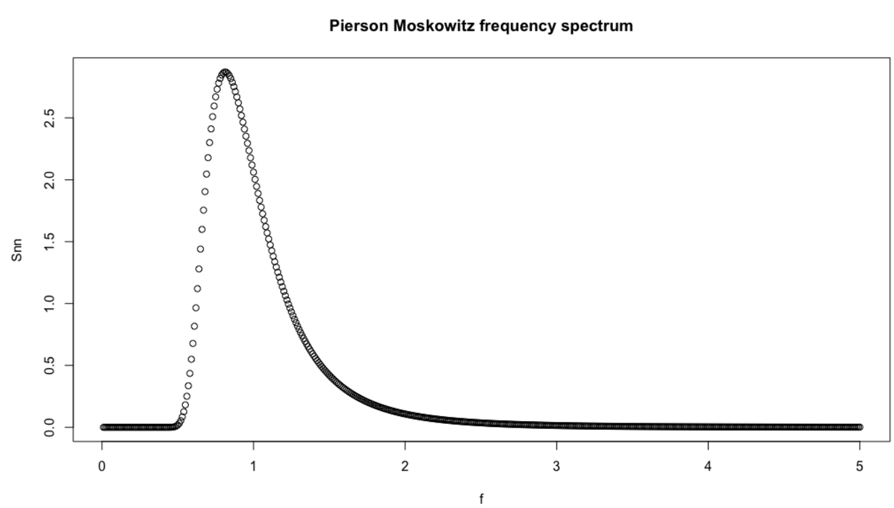
The maximum of $S_{\eta\eta}$ was determined to find the most energetic component of the total energy spectrum. The maximum $S_{\eta\eta} = 2.87$ and was found at $f = 0.81$.

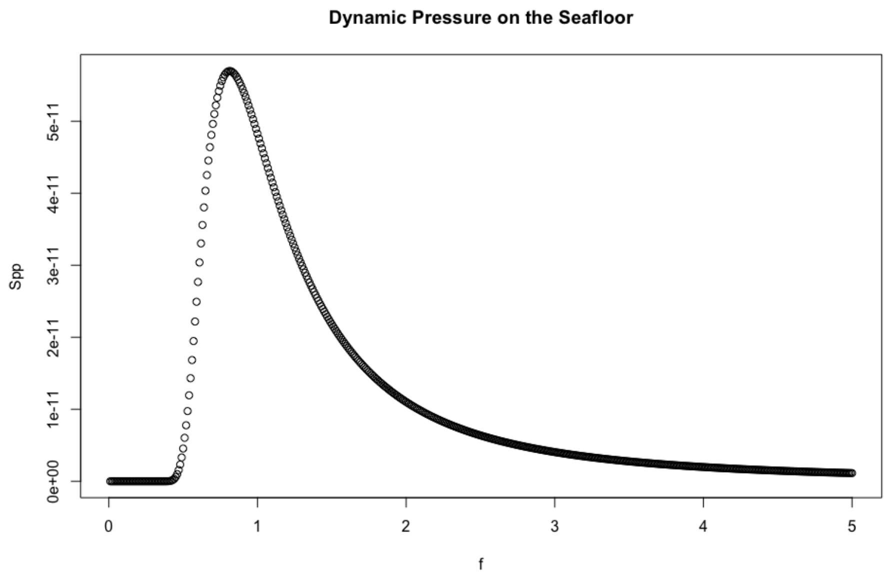
The dynamic pressure, S_{pp}, was calculated by

$$S_{pp} = \frac{\sqrt(S_{\eta\eta}) * g * \rho}{cosh(kh)}$$

using $h = 10\ m$, $k = 3.4\ m^{-1}$, $g = 9.81\ m/s$, and $\rho = 1000\ kg/m^3$.

The following python and R code was used for analysis.






```
from aide_design.play import*
g = pc.gravity

def wavenumber(T, h):
  """Return the wavenumber of wave using period and water height from bed."""
  k = 10  # this is a guess to find what k is
  diff = (((2*np.pi)/T)**2)-(g.magnitude * k * np.tanh(k*h))
  while diff<0:
      LHS = ((2*np.pi)/T)**2
      RHS = g.magnitude * k * np.tanh(k*h)
      diff = LHS - RHS
      k = k - 0.0001
  return k

k = wavenumber(1.083, 10)
```

```
# Pierson Moskowitz frequency spectrum
f = seq(0.01, 5, by = 0.01)
Snn = 0.205 * H_onethird^2 * T_onethird^(-4) * f^(-5) * exp(-0.75 * (T_onethird * f)^(-4))
plot(f, Snn, main = "Pierson Moskowitz frequency spectrum")

# Calculate the area under the curve
id = order(f)
AUC = sum(diff(f[id])*rollmean(Snn[id],2))

# Calculate the variance to check that AUC is reasonable (it is)
```

```
# var = var(wavedata$eta_cm)

a_rms = sqrt(2*AUC)
H_rms2 = 2 * a_rms
H_max2 = sqrt(2)*H_rms2
H_onetenth2 = 20*sqrt(2)*H_rms2*sin(pi/20)/pi
H_onethird2 = 6*sqrt(2)*H_rms2*sin(pi/6)/pi

# Find maximum Snn to find the most energetic component of the total energy spectrum
Snn_max = max(Snn)

# Need to find the index of the max to find the wave frequency that it occurs at
i=0
for (i in 1:length(Snn))
{
  if (Snn[i] == Snn_max)
  {
    f_max = f[i]
  }
}

# Find dynamic pressure on seafloor
g = 9.81 # m/s
rho = 1000 # kg/m^3
h = 10 # m
k = 3.4 # calculated from T_onethird and h using a while loop and function in Python
Spp = sqrt(Snn)*g*rho/cosh(k*h)

plot(f, Spp, main = "Dynamic Pressure on the Seafloor")
```

Problem 3

A sloping bottom of a beach is described by the function

$$h(x) = s(x + \delta x_o exp[-\frac{(x - x_o)^2}{\lambda}])$$

Three different situations were: i) $s = 1/50$, $\delta = 0$ ii) $s = 1/50$, $\delta = 0.5$, $x_o =$ -1000 m, $\lambda = 5000$ m iii) $s = 1/50$, $\delta =$ -0.5, $x_o =$ -1000 m, $\lambda = 5000$ m

To find the waveray and wave amplitude, a for loop was written to evaluate the following equation at every distance from shore:

$$\Delta y = \frac{\kappa}{\sqrt{(k^2 + \kappa^2)}} \Delta x$$

$$\alpha = tan^{-1}(\frac{\Delta y}{\Delta x})$$

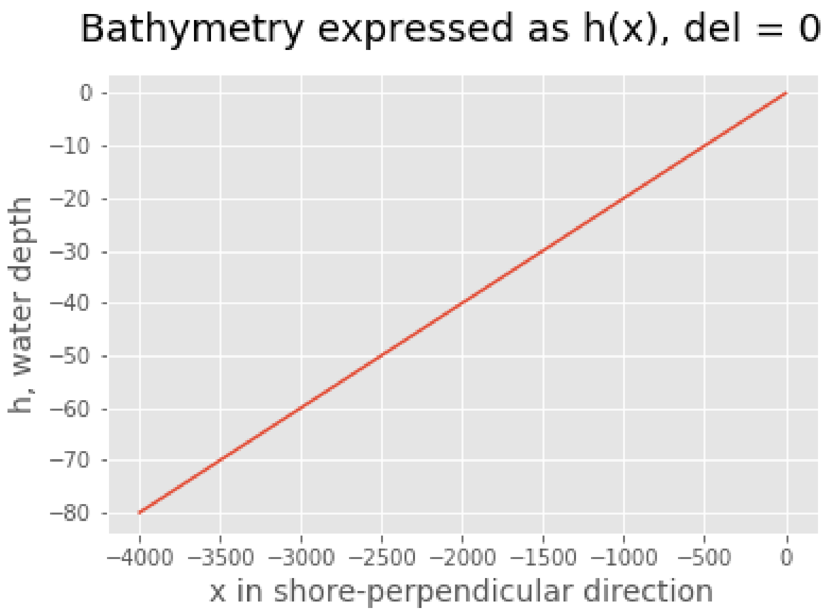
$$C_g = \sqrt(gh)$$

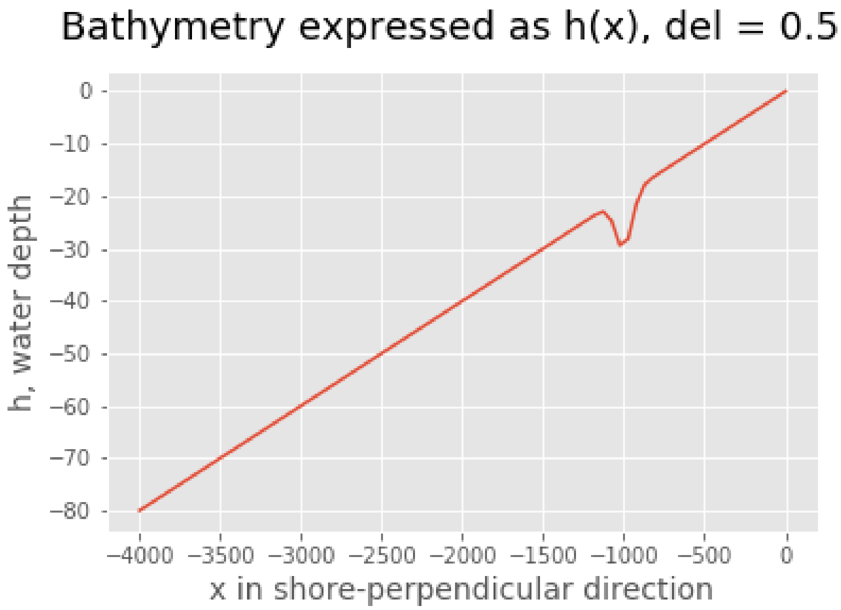
$$b = cos(\alpha)$$

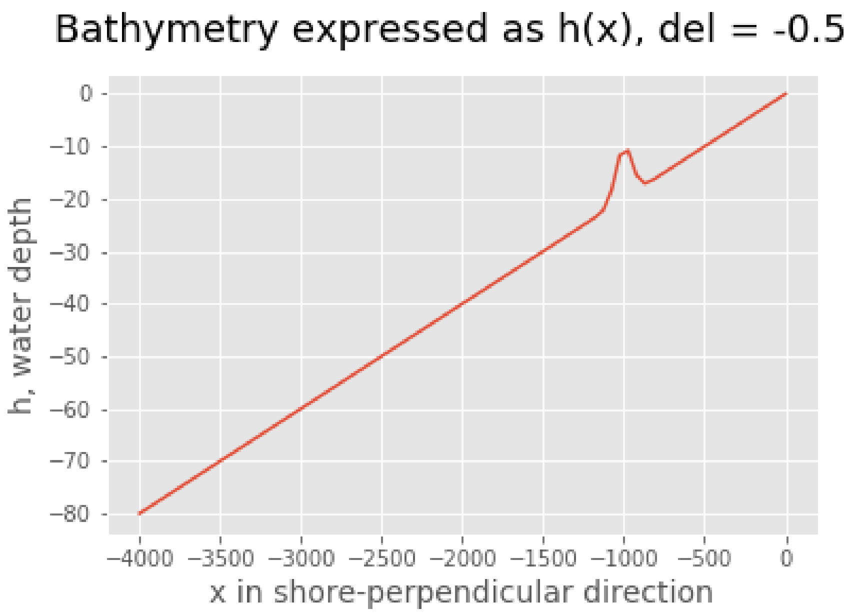
$$a = a_o\sqrt(\frac{C_{go}}{C_g})\sqrt(\frac{b_o}{b})$$

X ranges from -4000 to 0 m and will calculate different values for h and k which are used in calculating y and a. Iteratively looping through all of the x-values produces vectors for y and a which have been plotted for the three conditions.

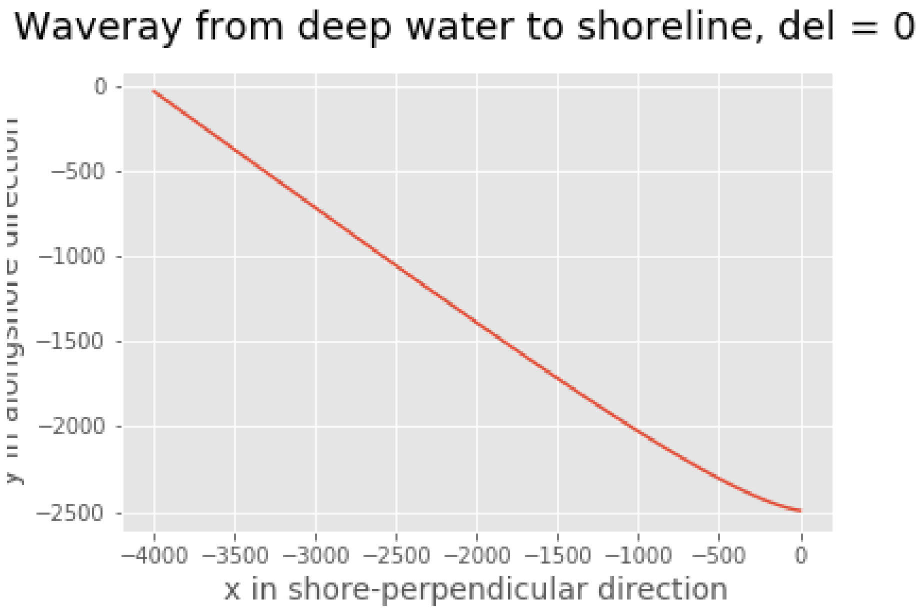
Bathymetry for the three conditions are shown below.

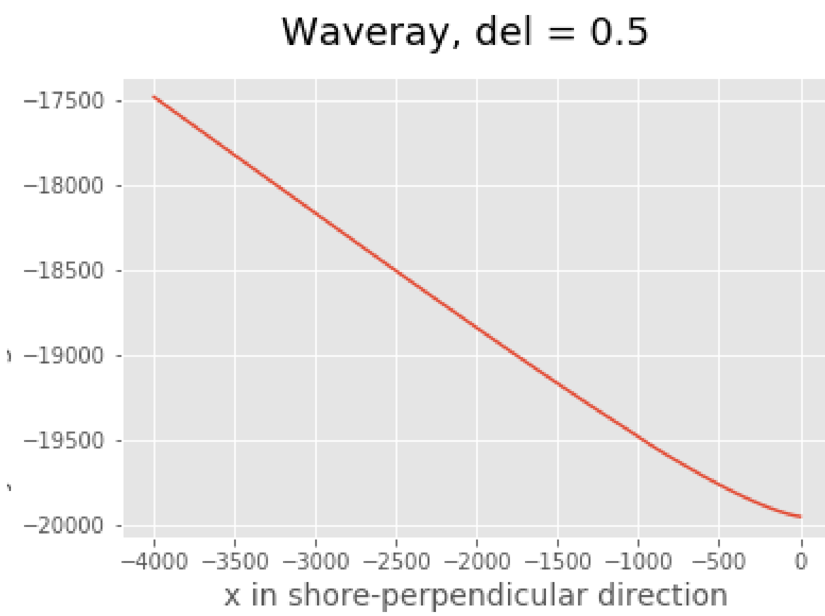


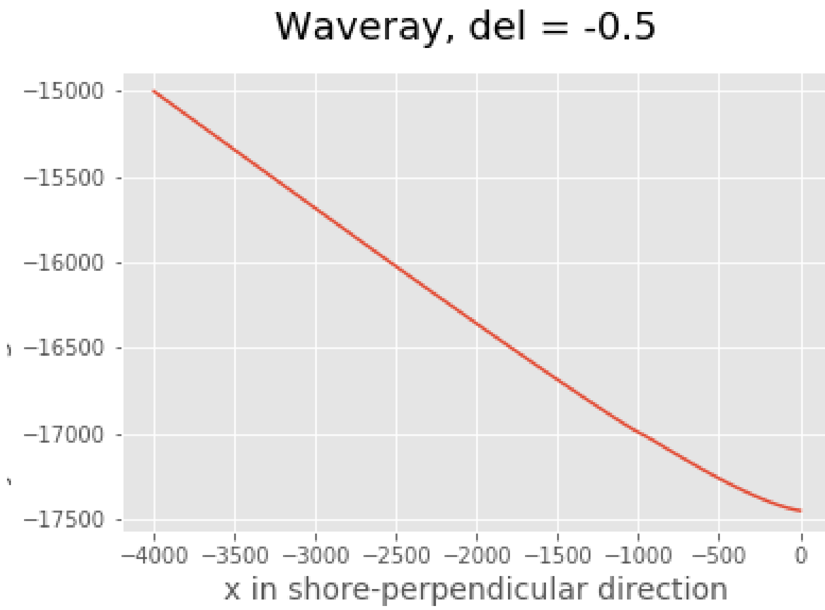




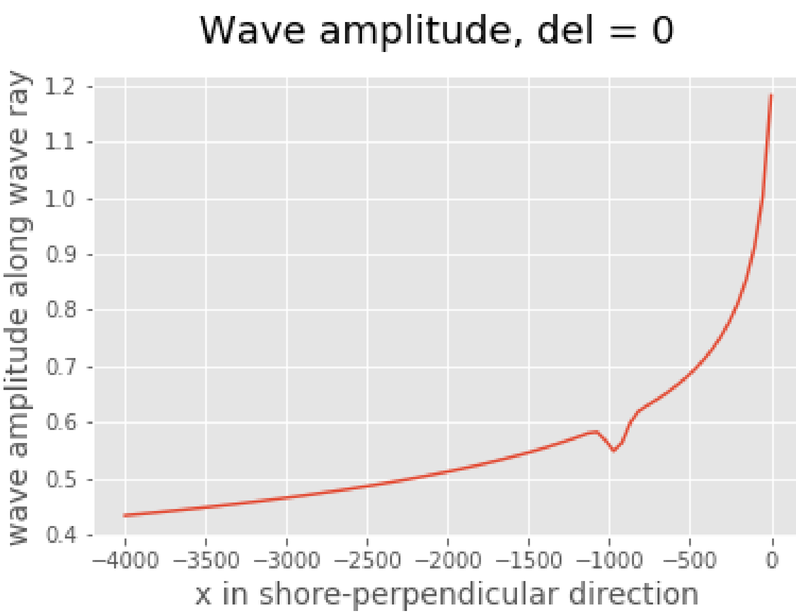
Waverays for the three conditions are shown below.

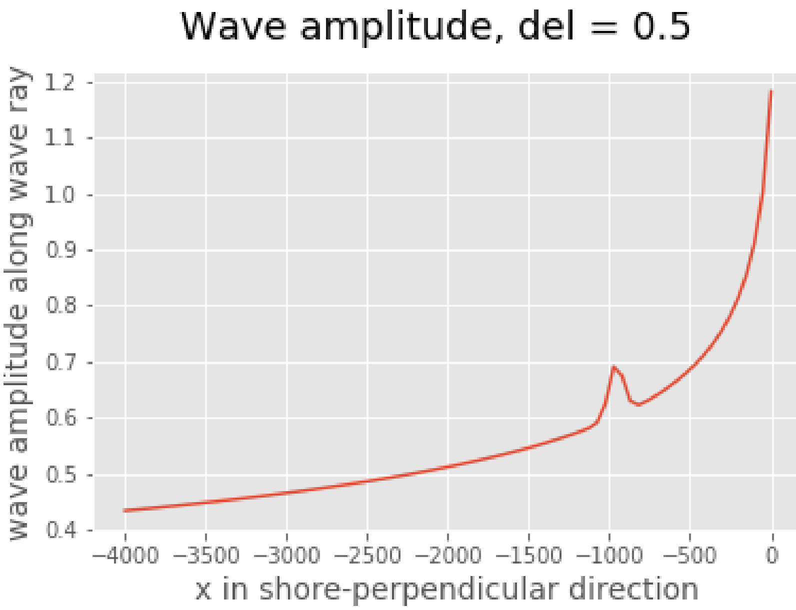

Waveray from deep water to shoreline, del = 0
0
−500
−1000
−1500
−2000
−2500
−4000 −3500 −3000 −2500 −2000 −1500 −1000 −500 0
x in shore-perpendicular direction

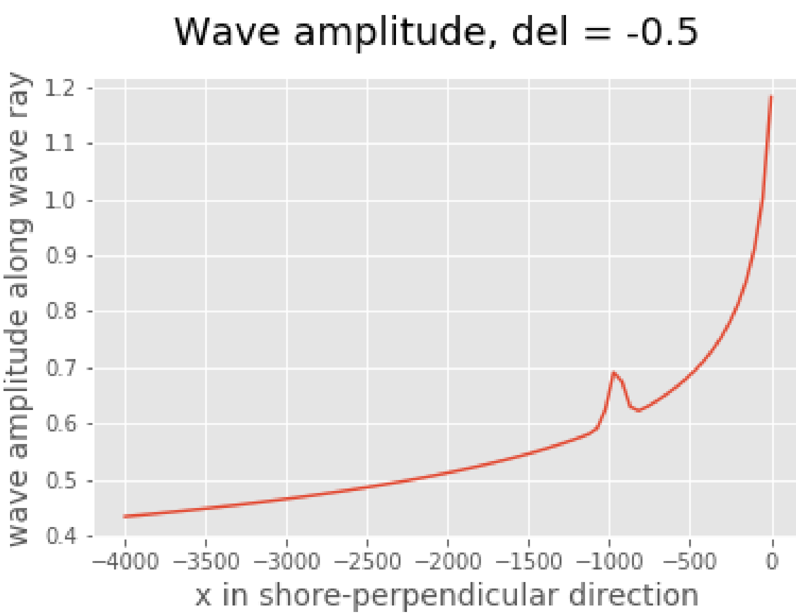

Waveray, del = 0.5
−17500
−18000
−18500
−19000
−19500
−20000
−4000 −3500 −3000 −2500 −2000 −1500 −1000 −500 0
x in shore-perpendicular direction



Wave amplitude for the three conditions are shown below.







The following python code was used for analysis.

```
from aide_design.play import*
import pandas as pd
g = pc.gravity
import timeit
```

```python
def wavenumber(T, h):
  """Return the wavenumber of wave using period and water height from bed."""
  k = 10  # this is a guess to find what k is
  diff = (((2*np.pi)/T)**2)-(g.magnitude * k * np.tanh(k*h))
  while diff<0:
      LHS = ((2*np.pi)/T)**2
      RHS = g.magnitude * k * np.tanh(k*h)
      diff = LHS - RHS
      k = k - 0.0001
  return k

def h_x(x, _del, xo):
  """Return the water height from the bed to determine bathymetry."""
  h_x = slope*(x + _del * xo * np.exp((-(x-xo)**2)/lmbda))
  return h_x

del_x = 50
slope = 1/50
del_zero = 0
del_pos = 0.5
del_neg = -0.5
lmbda = 5000 #m
T = 10 #s
xo = -1000 #m

length = abs(4000/del_x)
print(length)
length_init = int(length - 1)
# Need to initalize all the values
del_y = np.ones(length_init)
del_y[0] = 0

y = np.ones(length_init)
y[0] = 0 # arbitrary coordinate assignment

alpha = np.zeros(length_init)
alpha[0] = 30 # deg

x = np.linspace(-4000, 0, num = 80)
x[0] = -4000 #m

h = np.ones(length_init)
h[0] = abs(h_x(x[0], del_zero, xo))

k = np.ones(length_init)
k[0] = wavenumber(T, abs(h_x(x[0], del_zero, xo)))
```

```python
cg = np.zeros(length_init)
ho = abs(h_x(x[0], del_zero, xo))
cg[0] = np.sqrt(g.magnitude * ho)
cgo = np.sqrt(g.magnitude * ho)

b = np.zeros(length_init)
bo = np.cos(alpha[0])

a = np.zeros(length_init)
a[0] = 1 #m
ao = 1 #m

xplot = np.linspace(-4000, 1, num = 79)
plt.plot(xplot, h_x(xplot, del_zero, xo))
plt.xlabel('x in shore-perpendicular direction', fontsize=14)
plt.ylabel('h, water depth', fontsize=14)
plt.suptitle('Bathymetry expressed as h(x), del = 0', fontsize=18)
plt.savefig('bathydelzero.png')
plt.show()

xplot = np.linspace(-4000, 1, num = 79)
plt.plot(xplot, h_x(xplot, del_pos, xo))
plt.xlabel('x in shore-perpendicular direction', fontsize=14)
plt.ylabel('h, water depth', fontsize=14)
plt.suptitle('Bathymetry expressed as h(x), del = 0.5', fontsize=18)
plt.savefig('bathydelpos.png')
plt.show()

xplot = np.linspace(-4000, 1, num = 79)
plt.plot(xplot, h_x(xplot, del_neg, xo))
plt.xlabel('x in shore-perpendicular direction', fontsize=14)
plt.ylabel('h, water depth', fontsize=14)
plt.suptitle('Bathymetry expressed as h(x), del = -0.5', fontsize=18)
plt.savefig('bathydelneg.png')
plt.show()

# Calculate Kappa using initial k and alpha values
kappa = k[0] * np.sin(alpha[0])
print(kappa)

def bathymetry_wave(_del):
    for i in range(0, length_init):
        #start = time.time()
        #h[i] = abs(h_x(x[i], del_zero, xo))
        k[i] = wavenumber(T, abs(h_x(x[i], _del, xo)))
```

```
        del_y[i] = kappa / (np.sqrt(k[i]**2 + kappa**2)) * del_x
        y[i] = y[i-1] + del_y[i]
        alpha[i] = np.arctan(del_y[i] / del_x)
        cg[i] = np.sqrt(g.magnitude * abs(h_x(x[i], _del, xo)))
        b[i] = np.cos(alpha[i])
        a[i] = ao * np.sqrt(cgo/cg[i]) * np.sqrt(bo/b[i])
        #stop = time.time()
        #print(stop-start)
    return k, del_y, y, alpha, cg, b, a

# Calculate vectors for plotting
xplot = np.linspace(-4000, 1, num = 79)

y_zero = bathymetry_wave(del_zero)[2]
a_zero = bathymetry_wave(del_zero)[6]

y_pos = bathymetry_wave(del_pos)[2]
a_pos = bathymetry_wave(del_pos)[6]

y_neg = bathymetry_wave(del_neg)[2]
a_neg = bathymetry_wave(del_neg)[6]

# del = 0 condition
plt.plot(xplot, y_zero)
plt.xlabel('x in shore-perpendicular direction', fontsize=14)
plt.ylabel('y in alongshore direction', fontsize=14)
plt.suptitle('Waveray, del = 0', fontsize=18)
plt.savefig('waveraydelzero.png')
plt.show()

plt.plot(xplot, a_zero)
plt.xlabel('x in shore-perpendicular direction', fontsize=14)
plt.ylabel('wave amplitude along wave ray', fontsize=14)
plt.suptitle('Wave amplitude, del = 0', fontsize=18)
plt.savefig('waveampdelzero.png')
plt.show()

# del = 0.5 condition
plt.plot(xplot, y_pos)
plt.xlabel('x in shore-perpendicular direction', fontsize=14)
plt.ylabel('y in alongshore direction', fontsize=14)
plt.suptitle('Waveray, del = 0.5', fontsize=18)
plt.savefig('waveraydelpos.png')
plt.show()
```

```
plt.plot(xplot, a_pos)
plt.xlabel('x in shore-perpendicular direction', fontsize=14)
plt.ylabel('wave amplitude along wave ray', fontsize=14)
plt.suptitle('Wave amplitude, del = 0.5', fontsize=18)
plt.savefig('waveampdelpos.png')
plt.show()

# del = -0.5 condition

plt.plot(xplot, y_neg)
plt.xlabel('x in shore-perpendicular direction', fontsize=14)
plt.ylabel('y in alongshore direction', fontsize=14)
plt.suptitle('Waveray, del = -0.5', fontsize=18)
plt.savefig('waveraydelneg.png')
plt.show()


plt.plot(xplot, a_neg)
plt.xlabel('x in shore-perpendicular direction', fontsize=14)
plt.ylabel('wave amplitude along wave ray', fontsize=14)
plt.suptitle('Wave amplitude, del = -0.5', fontsize=18)
plt.savefig('waveampdelneg.png')
plt.show()

# To convert the document from markdown to pdf
pandoc Problem_Set_4.md -o Maisel_Problem_Set_4.pdf
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