CEE 4350 Coastal Engineering

Problem Set 2

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```
from aide_design.play import*
g = pc.gravity
```

Dispersion Equation:

$$\sigma^2 = gk * tanh(kh)$$

Variable definition:

g: gravity

σ: dispersion

a: amplitude

h: water depth

H: distance from wave crest to trough ($2a$)

T: wave period

λ: wavelength

k: wavenumber

c_p: celerity (wave phase speed)

P: pressure

F: force

u, w: x-velocity, z-velocity components

Problem 1

1a) Wave phase speed and wavelength:

$$\lambda = \frac{2\pi}{k}$$

$$c_p = \frac{\lambda}{T}$$

```python
def wavenumber(T, h):
  """Return the wavenumber of wave using period and water height from bed."""
  k = 10  # this is a guess to find what k is
  diff = (((2*np.pi)/T)**2)-(g.magnitude * k * np.tanh(k*h))
  while diff<0:
      LHS = ((2*np.pi)/T)**2
      RHS = g.magnitude * k * np.tanh(k*h)
      diff = LHS - RHS
      k = k - 0.0001
  return k

def wavelength(T, h):
    """Return the wavelength of wave using period and water height from bed."""
    k = wavenumber(T, h)
    wavelength = 2 * np.pi / k
    return wavelength

def celerity(T,h):
    """Return the celerity of wave using period and water height from bed."""
    k = wavenumber(T,h)
    lmbda = 2 * np.pi / k
    celerity = lmbda / T
    return celerity

length_tank = 190 * u.m
width_tank = 4.5 * u.m
depth = 5 * u.m
amp = 1 * u.m
period = 4 * u.s

wavenumber(period.magnitude, depth.magnitude)
wavelength(period.magnitude, depth.magnitude)
celerity(period.magnitude, depth.magnitude)
```

The wavenumber is 0.283, the wavelength is 22.2 meters, and the celerity is 5.55 m/s.

1b) Component water parcel velocities and pressure at 2 meters below the water level and at wave crest:

$$u = a\sigma \frac{cosh(k(h+z))}{sinh(kh)} cos(kx - \sigma t)$$

$$w = a\sigma \frac{sinh(k(h+z))}{sinh(kh)} sin(kx - \sigma t)$$

$$P = -\rho g z + \rho g a \frac{cosh(k(h+z))}{cosh(kh)} cos(kx - \sigma t)$$

```
def u_vel(T, h, a, x, t, z):
    """Return the x-velocity of a wave."""
    sigma = (2*np.pi)/T
    k = wavenumber(T,h)
    u_vel = a * sigma * (np.cosh(k*(h+z))/np.sinh(k*h)) * np.cos(k*x - sigma*t)
    return u_vel

def w_vel(T, h, a, x, t, z):
    """Return the z-velocity of a wave."""
    sigma = (2*np.pi)/T
    k = wavenumber(T,h)
    a = amp
    w_vel = a * sigma * (np.sinh(k*(h+z))/np.sinh(k*h))* np.sin(k*x - sigma*t)
    return w_vel

def pressure(T, h, a, x, t, z):
    """Return the hydrostatic and dynamic pressure from a wave."""
    temp = 25 * u.degC
    sigma = (2*np.pi)/T
    rho = pc.density_water(temp).magnitude
    k = wavenumber(T, h)
    pressure = (-rho * g.magnitude * z) + rho * g.magnitude * a*
    np.cosh(k*(h+z))/np.cosh(k*h) * np.cos(k*x - sigma*t)
    return pressure

z_belowSWL = -2 * u.m
z_crest = 1 * u.m
x = 0 * u.m
t = 0 * u.s

u_vel(period.magnitude, depth.magnitude, amp.magnitude, x.magnitude,
t.magnitude, z_belowSWL.magnitude)
w_vel(period.magnitude, depth.magnitude, amp.magnitude, x.magnitude,
t.magnitude, z_belowSWL.magnitude)
pressure(period.magnitude, depth.magnitude, amp.magnitude, x.magnitude,
t.magnitude, z_belowSWL.magnitude)

u_vel(period.magnitude, depth.magnitude, amp.magnitude, x.magnitude,
t.magnitude, z_crest.magnitude)
w_vel(period.magnitude, depth.magnitude, amp.magnitude, x.magnitude,
t.magnitude, z_crest.magnitude)
pressure(period.magnitude, depth.magnitude, amp.magnitude, x.magnitude,
t.magnitude, z_crest.magnitude)
```

At x = 0 m, t = 0 s and z = -2 m, the u-velocity is 1.12 m/s, the w-velocity is 0.0 m/s, and the pressure is 25.8 kPa. At x = 0 m, t = 0 s and z = 1 m, u-velocity is 2.29 m/s, the w-velocity is 0.0 m/s, and the pressure is 2.89 kPa.

Problem 2

Design a seawall to prevent flooding of a coastal highway. The water depth of in front of the seawall is 3 m. The design wave characteristics are period T = 10 seconds and amplitude a = 0.5 m.

2a) Minimum height of the seawall to prevent flooding:

The height of a seawall (h_s) should be determined from the amplitude of the design wave (a) and the water depth (h).

$$h_s = (2 * a) + h$$

```
a_designwave = 0.5 * u.m
h_designwave = 3 * u.m
h_s = (2 * a_designwave) + h_designwave
print(h_s)
```

The seawall should be designed to be at least 4 meters tall.

2b) Maximum total force on the seawall:

The maximum force on the seawall (F_m) is a function of the hydrostatic and dynamic pressure of the wave. The following equation was modified from "Water Wave Mechanics for Engineers & Scientists" by Dean and Dalrymple.

$$F_m = \rho g \frac{4h^2 + H^2}{2} + \rho g h H \frac{tan(kh)}{kh}$$

```
def max_force(T, h, a, x, t):
    """Return the maximum force that a wave will induce on a seawall."""
    temp = 25 * u.degC
    sigma = (2*np.pi)/T
    rho = pc.density_water(temp).magnitude
    H = 2 * a
    k = wavenumber(T, h)
    max_force = (rho * g.magnitude) * ((4*h**2 + H**2)/2) + (rho * g.magnitude
    * h * H * np.tanh(k*h)/(k*h))
    return max_force

period_designwave = 10 * u.s
print(max_force(period_designwave.magnitude, h_designwave.magnitude,
a_designwave.magnitude, x, t))
```

The maximum force on the seawall is 209 kN.

```
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