CEE 4350 Coastal Engineering

Problem Set 3

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```
from aide_design.play import*
g = pc.gravity
```

Variable definition:

g: gravity

σ: dispersion

a: amplitude

h: water depth

H: distance from wave crest to trough ($2a$)

T: wave period

λ: wavelength

k: wavenumber

c_p: celerity (wave phase speed)

c_g: group velocity

n: ratio of group velocity to phase velocity

P: pressure

F: force

u, w: x-velocity, z-velocity components

$E * c_g$: energy flux

B: width

Governing Equations for Linear Wave Theory Analysis

$$\lambda = \frac{2\pi}{k}$$

$$c_g = nc_p$$

$$E_1 c_{g1} B_1 = E_2 c_{g2} B_2$$

$$n = \frac{1}{2}(1 + \frac{2kh}{sinh(2kh)})$$

$$\frac{a_2}{a_1} = \sqrt{\frac{c_{g1}}{c_{g2}}}$$

$$\sigma = (\frac{2\pi}{T})^2$$

$$c_p = \frac{gT}{2\pi} tanh(kh)$$

$$\sigma^2 = gk * tanh(kh)$$

In deep water,

$$c_p = \frac{gT}{2\pi}$$

$$n = \frac{1}{2}$$

$$kh > \pi$$

In shallow water,

$$c_p = \sqrt{gh}$$

$$n = 1$$

$$kh < \frac{\pi}{10}$$

Problem 1

1) Design a small pier supported by piles. Water depth at front row of piles is h(x) = 2 meters. Beach slope is 1/30. "Deep water" wave buoy measured wave amplitude $a = 0.5$ m and wave period $T = 12$ s.

Givens:

```
slope = 1/30
depth = 5 * u.m
amp1 = 0.5 * u.m
period = 12 * u.s
h2 = 2 * u.m
sigma = 2 * np.pi / period
```

1a) What is the minimum clearance needed to maintain a dry deck?

The waves measured by the buoys are in deep water.

```
c_p1 = g * period / (2*np.pi)
n1 = 0.5
c_g1 = n1 * c_p1
print(c_g1)
```

Assume that the waves at h(x) = 2m are shallow waves. This assumption will be checked in part 1b.

```
c_p2 = np.sqrt(g*h2)
n2 = 1
c_g2 = n2 * c_p2
print(c_g2)

amp2 = amp1 * np.sqrt(c_g1/c_g2)
print(amp2)
```

The minimum clearance needed to maintain a dry deck is 0.73 meters.

1b) Are the waves at pile location "shallow water waves"?

```
def wavenumber(T, h):
  """Return the wavenumber of wave using period and water height from bed."""
  k = 10  # this is a guess to find what k is
  diff = (((2*np.pi)/T)**2)-(g.magnitude * k * np.tanh(k*h))
  while diff<0:
      LHS = ((2*np.pi)/T)**2
      RHS = g.magnitude * k * np.tanh(k*h)
      diff = LHS - RHS
      k = k - 0.0001
  return k
```

```
k = wavenumber(period.magnitude, h2.magnitude) * 1/u.m

shallow_limit = np.pi/10
deep_limit = np.pi
shallow_test = k * h2

if shallow_test < shallow_limit:
  print('The wave is a shallow water wave.')
elif shallow_test > deep_limit:
  print('The wave is a deep water wave.')
else:
  print('The wave is a transitional water wave.')
```

kh	$\pi/10$
0.238	0.314

Using the dispersion relationship and shallow water limits, kh is less than $\pi/10$ and the wave at the piles is a shallow water wave. This confirms the answer found in 1a.

1c) What are the maximum onshore water particle velocity (u & w) at the location of the pile?

The horizontal velocity can be found by

$$u = a\sigma \frac{cosh(k(h+z))}{sinh(kh)} cos(kx - \sigma t)$$

Extreme horizontal velocity values occur at phase positions of kx - σ t = 0, π, . . . , which occur under the crest and trough positions. The cosine term of u goes to 1.

$$u = a\sigma \frac{cosh(k(h+z))}{sinh(kh)}$$

Vertical variation of velocity components should be analyzed at the bottom where

$$k(h+z) = 0$$

and the cosh term of u goes to 1. The next simplification is that in shallow water,

$$sinh(kh) = kh$$

Thus, the equation for maximum horizontal onshore water particle velocity can be given as

$$u = \frac{a\sigma}{kh}$$

```
u_max_shallow = (amp2*sigma)/(k*h2)
print(u_max_shallow)
```

The vertical velocity can be found by

$$w = a\sigma \frac{sinh(k(h+z))}{sinh(kh)} sin(kx - \sigma t)$$

Extreme horizontal velocity values occur at phase positions of kx - σ t = $\frac{\pi}{2}$, $\frac{3\pi}{2}$, ..., which occur at the mean water level (MWL) at $z = 0$.

The sinh terms cancel out and the sine term goes to 1, so the equation for maximum vertical onshore water particle velocity can be given as

$$w = a\sigma$$

```
w_max_shallow = amp2*sigma
print(w_max_shallow)
```

Shallow water onshore particle velocities	m/s
u_{max}	1.597
w_{max}	0.3807

Problem 2

2) A simple harmonic small harmonic amplitude progressive wave train propagates in a rectangular channel with a constant depth, h, with wave frequency σ. The channel width transitions from B_1 with amplitude a_1 to B_2 with amplitude a_2. Find a_2.

Conservation of energy flux can be used to relate progressive energy, group velocity, and width.

$$B_1 E_1 c_{g1} = B_2 E_2 c_{g2}$$

At constant wave depth, $c_{g1} = c_{g2}$.

$$\frac{1}{2}\rho g a_1^2 B_1 c_{g1} = \frac{1}{2}\rho g a_2^2 B_2 c_{g2}$$

$$\frac{a_2^2}{a_1^2} = \frac{B_1}{B_2}$$

$$\frac{a_2}{a_1} = \sqrt{\frac{B_1}{B_2}}$$

$$a_2 = a_1\sqrt{\frac{B_1}{B_2}}$$

```
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