Bending of Beams

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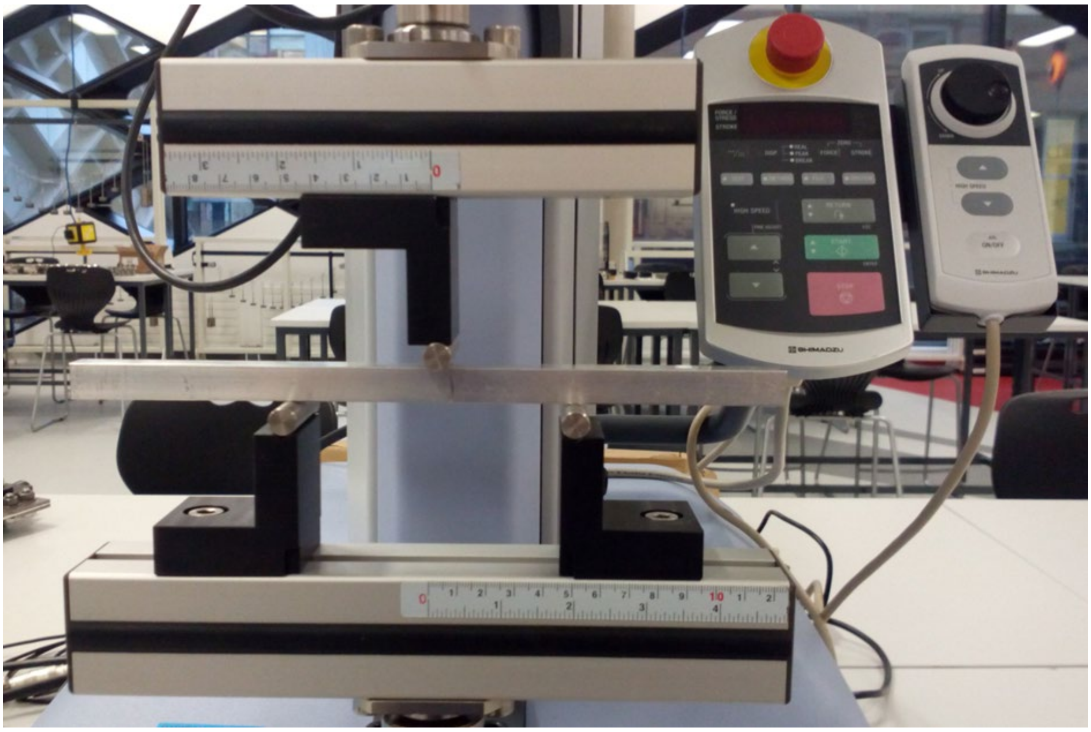

Questions?

You should contact the lead academic if you have any questions about the experiment.

Learning Outcomes

Practical	Conduct mechanical tests to inform your understanding of theory
Design	Understand experimental design to select the most appropriate solution
Analyse	Analyse data and compare it to theoretical models

Pre-experimental Activity

You must complete the pre-experimental activity before you begin an experiment in the laboratory or online. If you do not complete the activity, you may be refused entry into the laboratory or refuse access to the online lab activities.

1. Aims and Objectives

In this lab you will test several different beams under bending loads in order to examine the relationship between beam shape, and its performance. A beams bending performance is its ability to resist a given load while not deflecting much, i.e. its stiffness.

You will see from this lab how changing a beams cross sectional shape can alter its bending stiffness and how shapes such as tubes and I-beams work to reduce weight while maintaining a high stiffness.

1.1. Aims

- To understand how the performance of a beam is affected by its cross sectional shape (its "Second moment of area")
- To learn how this affects selection of materials and structures in practical engineering

1.2. Objectives

- Determine the second moment of area of solid and tubular beams
- Use the second moment of area to calculate the theoretical deflection of beams in bending
- Compare your theoretical predictions to the values measured from a real experiment

2. Background

Beams are a fundamental part of many types of civil engineering structure. Most obviously we see beams used for bridge decks, portal frames in buildings, cranes and many more applications.

Beams are used to resist bending loads in structures so are a key element in the engineers' design toolbox. Understanding beam bending theory is therefore essential for civil engineers.


It is desirable to minimise weight while doing beam design, since weight is heavily linked to material, manufacturing and installation costs. We therefore want to understand how to 'make the most' of our metal.

The shape of a beam is a critical factor in determining its bending stiffness, and thereby how much material we would need to resist a given load. This shape factor is known as "second moment of area".

2.1. Bending Deflection

You may recall the deflection equation for a beam undergoing 3-point bending:

$$\delta = \frac{PL^3}{48EI} \qquad \text{Equation 1}$$

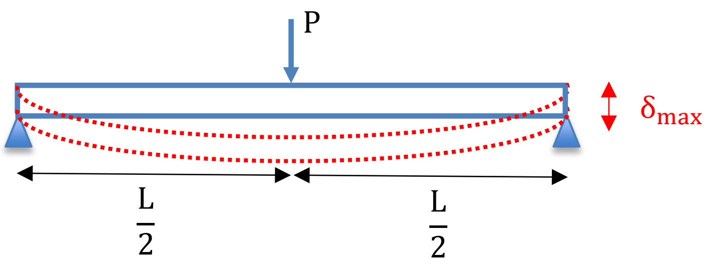

Figure 1 - Three Point Bending

This equation says that the distance the beam deflects (at the mid-point) depends on the size of the load (P), the material (Young's Modulus, E), how long the beam is (L), and also on the Second Moment of Area, I.

So if we know a beams 'I' then we can calculate its deflection when loaded.... but what actually *is* 'I'?

2.2. Second Moment of Area

'I' is usually called the Second Moment of Area in structural equations but sometimes it's referred to as the "Area Moment of Inertia".

'I' is a feature *only* of a beams cross sectional shape and area. Therefore, it doesn't matter how long a beam is, or what it is made from – the only thing that affects the 'I' is the cross section. It is basically a measure of how effective a cross sectional shape is at dealing with bending.

To put this into context, let's take a look at a beam experiencing a bending load:

This beam is loaded in the same way as Figure 1. As the beam deflects the bottom surface is stretched, causing a tensile stress. On the top surface the opposite is true, and we see compression.

In between the top and bottom surfaces of the beam we therefore have a place with neither tension nor compression, and therefore a zero bending stress. This is called the neutral axis of bending.

The greatest bending stresses are found at the top and bottom surfaces, farthest away from the neutral axis.

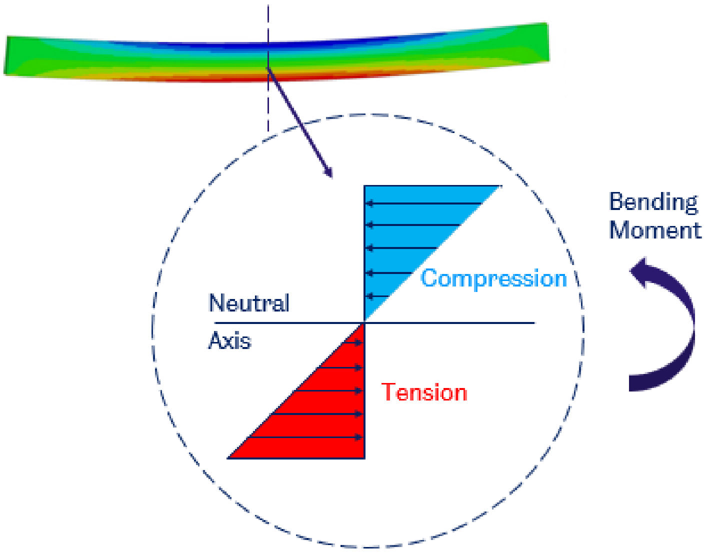

Figure 2 – Bending Stresses in a Beam

So how can we best deal with these bending stresses? That is where the 'I' fits in.

'I' describes how much area there is at a distance from the neutral axis. Clearly the more area that our cross section has away from the neutral axis, then the more easily those high bending loads (tensile/compressive) can be distributed.

This can be appreciated when looking at Equations 2 & 3, along with Figure 3 as an example. Equation 2 is the generalised mathematical definition of 'I' that can be applied to any shape. Equation 3 is this definition applied specifically to a rectangular section.

$$\mathrm{I} = \iint y^2 . dx . dy \quad \text{Equation 2}$$

$$\mathrm{I} = \frac{\mathrm{x.y}^3}{12} \quad \text{Equation 3}$$

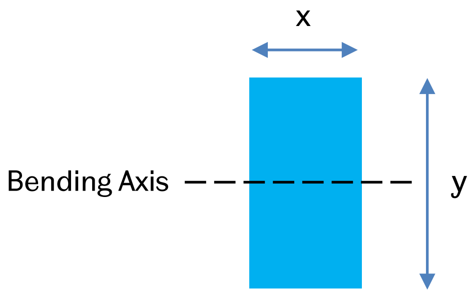

Figure 3 – A Rectangular Cross Section

In either equation you can see that if the area (x.y) is small, or if the shape has a small 'y' (most of the area is positioned close the neutral-axis), then those bending loads would be much more concentrated, making both the stresses and strains (deflection) higher.

To summarise, 'I' is just a measure of how much cross sectional area we have got at distance from the neutral-axis (N-A). The more area there is then the better the bending loads can be distributed; and the further away that area is from the N-A then the more 'leverage' it has to deal with the loads too. Higher 'I' = stiffer beam. The equation for 'I' therefore contains terms of both area, and length providing a value in m^4.

So if the shapes in Figure 4 have the same total area (m^2), which has the greater 'I'?

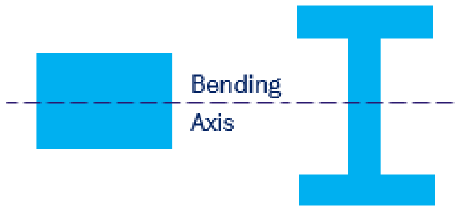

Figure 4 – Example Cross Sections for Comparison

3. Planning and Preparation

Before you attend the lab, you must complete the pre-lab quiz on Blackboard getting all the questions right – you can have as many attempts as you need.

3.1. Equipment List

Below is a list of items that you should bring with you to complete the experiment.

- Lab notebook
- Pen
- Calculator

3.2. Lab Procedure

In this class you will use a Shimadzu mechanical testing machine to perform 3-point flexural tests on a variety of aluminium beams. You will operate the testing machine to apply the load (700N) and record the maximum deflection of the beam. You will then compare this to predicted values you will calculate for the beams using theoretical equations.

3.3. Health and Safety

Please ensure that you read the risk assessment documentation on Blackboard and familiarise yourself with the necessary safety precautions associated with using the testing equipment.

Tasks to complete before the lab	**Completed?**
Read the risk assessment on using the Shimadzu Universal testing Machines.	
Familiarise yourself with the procedure of the lab by reading through this labsheet.	
Successfully complete the pre-lab quiz on Blackboard!	

Date:

Experimental Record

This section is to be completed during the experiment in the laboratory. It must be checked by a member of the teaching staff before leaving the laboratory.

1. Calculate Predictions

Before doing the tests, use the theory (given in Section 2) to predict how far each beam will deflect under 700N. Take the Youngs Modulus of Aluminium to be 72GPa.

- Download and open the file "BendingPredictions.xlsx" from blackboard.
- Do your calculations for Second Moment of Area, and then the Predicted Deflection, recording them into the spreadsheet (a few values have been given to you):

Hint: if you stick to S.I. units [N], [m], [Pa] and [m^4] for your calculation it is much easier to keep track of units and avoid errors.

2. Test Procedure

Now to put those predictions to the test:

- Load the method file "CIV1000-Bending.xmel" onto the Trapezium-X software (within the "CIV1000-21-22" folder on the desktop).
- Ensure that the three-point bending fixtures are securely mounted in the machine, and that the lower supports are spaced approximately 150 mm apart.
- Place the first beam on the supports with equal overhangs at each end.
- Carefully lower the upper support with the Shimadzu panel buttons so that it just touches the beam.
- Start the test, which will run to a load of 700N and then stop.
- Record the deflection in the spreadsheet
- Repeat with the other beams.

3. Discussions and conclusions

- Across all the beams you tested, how closely did your predictions match the experimental results?
- What do you think were the main reasons for any disagreement between theory/experiment?
- What is the weight per/meter of the solid square beam, and the i-beam?
- Which of these two would you use for a primary member when designing a bridge deck and why?