Converting a circle -

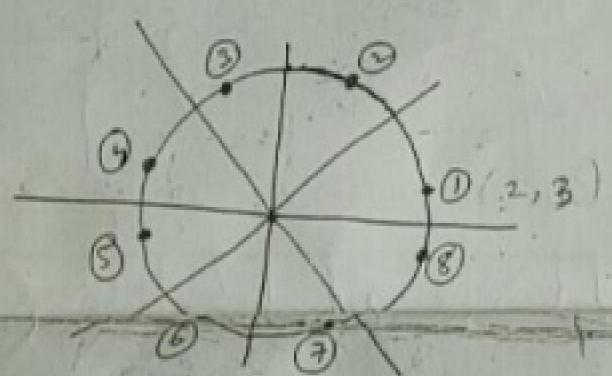
A circle is a symmetrical figure. Eight-way symmetry is used by reflecting each calculated point around each 45° axis.

$\therefore$ $P_1 = (x, y)$ $\quad$ $P_5 = (-x, -y)$

$P_2 = (y, x)$ $\quad$ $P_6 = (-y, -x)$

$P_3 = (-y, x)$ $\quad$ $P_7 = (y, -x)$

$P_4 = (-x, y)$ $\quad$ $P_8 = (x, -y)$



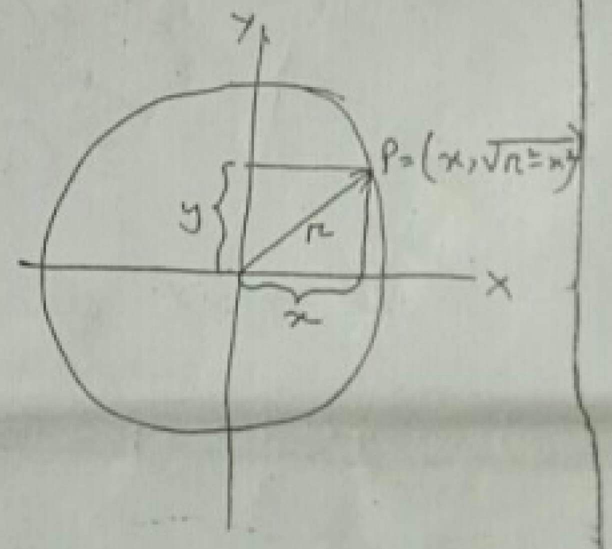
Defining a circle -

A circle of polynomial equation, $y^2 = r^2 - x^2$

Where, x = the x coordinate

y = the y coordinate

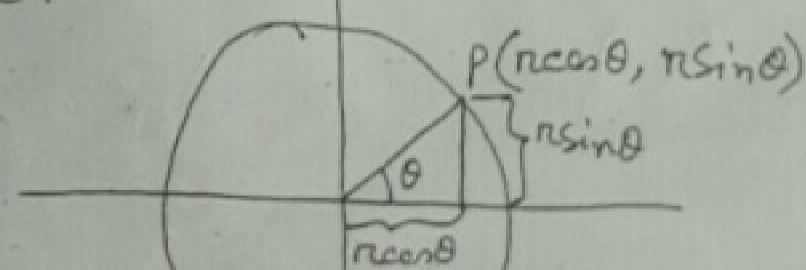
r = the circle radius



A circle of trigonometric function —

$x = r\cos\theta \quad , \quad y = r\sin\theta$

where, θ = current angle.



④ Mid point circle algorithm:-

To apply the midpoint method, we define a circle function

$$f(x, y) = x^2 + y^2 - r^2$$

Any point (x, y) can be determined by checking the sign of the circle function:-

$$f(x,y) = \begin{cases} <0, & \text{if } (x,y) \text{ is inside the circle boundary} \\ =0, & \text{if } (x,y) \text{ is on the circle boundary} \\ >0, & \text{if } (x,y) \text{ is outside the circle boundary.} \end{cases}$$

Now consider the co ordinates of the point halfway between pixel T and pixel S: $(x_k+1, y_k-\frac{1}{2})$. This is called the midpoint and we use it to define a decision parameter:

$$P_k = f(x_k+1, y_k-\tfrac{1}{2})$$

$$= (x_k+1)^2 + (y_k - \tfrac{1}{2})^2 - r^2$$

The next decision parameter by evaluating the circle function at sampling position $x_{k+1}+1 = x_k+2$:

$$P_{k+1} = f(x_{k+1}+1, y_{k+1} - \tfrac{1}{2})$$

$$= [(x_k+1)+1]^2 + (y_{k+1} - \tfrac{1}{2})^2 - r^2$$

$$= (x_k+1)^2 + 2(x_k+1) + 1 + (y_{k+1})^2 - y_{k+1} + (\tfrac{1}{2})^2 - r^2$$

$$= (x_k+1)^2 + (y_k-\tfrac{1}{2})^2 - r^2 + 2(x_k+1) + y_{k+1}^2 - y_{k+1} - y_k^2 + y_k + 1$$

$$= p_k + 2(x_k+1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$$

where y_{k+1} is either y_k or y_{k-1}, depending on the sign of p_k.

Evaluation of the terms $2x_{k+1}$ and $2y_{k+1}$ can also be done incrementally as

$$2x_{k+1} = 2x_k + 2$$

$$2y_{k+1} = 2y_k - 2$$

The initial decision parameter is obtained by evaluating the circle function at the start position $(x_0, y_0) = (0, r)$:

$$p_0 = f(1, r-\tfrac{1}{2})$$

$$= 1 + (r-\tfrac{1}{2})^2 - r^2$$

$$= \frac{5}{4} - r$$

If the radius r is specified as an integer, we can simply round p_0 to

$$p_0 = 1 - r.$$

Algorithm:- (4)

1. Input radius r and circle center (x_c, y_c) and obtain the first point on the circumference of a circle centered on the origin as

$$(x_0, y_0) = (0, r)$$

2. Calculate the initial value of the decision parameter as

$$P_0 = 1 - r$$

3. At each x_k position, starting at $k=0$, perform the following test:

if $P_k < 0$, the next point along the circle centered on ~~at~~ $(0,0)$ is (x_{k+1}, y_k) and

$$P_{k+1} = P_k + 2x_{k+1} + 1$$

Otherwise, the next point along the circle is $(x_{k+1}, y_k - 1)$ and -

$$P_{k+1} = P_k + 2x_{k+1} + 1 - 2y_{k+1}$$

where $2x_{k+1} = 2x_k + 2$ and $2y_{k+1} = 2y_k - 2$.

Determine symmetry points in the other seven octants.

Move each calculated pixel position (x, y) onto the circular path centered on (x_c, y_c) and plot the co-ordinate values:

$$x' = x + x_c, \quad y' = y + y_c.$$

Repeat ~~steps~~ steps 3 through 5 until $x \geq y$.

Example: Given a circle radius $r = 10$.

$\therefore x = 0$ to $x = y$.

$\therefore P_0 = 1 - r = -9$.

initial point $(x_0, y_0) = (0, 10)$

$2x_0 = 0 \quad 2y_0 = 20$

k	P_k	(x_{k+1}, y_{k+1})	$2x_{k+1}$	$2y_{k+1}$
0	-9	(1, 10)	2	20
1	-6	(2, 10)	4	20
2	-1	(3, 10)	6	20
3	6	(4, 9)	8	18
4	-3	(5, 9)	10	18
5	8	(6, 8)	12	16
6	5	(7, 7)	14	14

$= 6 + 6 + 1 - 20$
$= 6 + 8 + 1 - 18$
$= -3$