This problem is to generate control-to-facet controller. Read the paper (in slides) before you proceed with the questions.

Problem 1

Consider the following linear system

$$\dot{x} = \begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix} x + \begin{bmatrix} -2 \\ -2 \end{bmatrix} u + \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Given a polytope with three vertices: $v_1 = (-1, 0)^T$ and $v_2 = (1, 1)^T$ and $v_3 = (1, -1)^T$. Let the facet facing v_i to be F_i with normal vector n_i.

- Solve the normal vectors of three facets.
- For facet F_1, list the set of linear inequalities to be satisfied for u_i, $i = 1, 2, 3$ such that the controller u obtained from $\{u_i\}$ (see the paper) steers any point in the polytope to reach F_1. Based on these linear inequality, does there exist such a controller?
- If the answer is affirmative, then select u_i, $i = 1, 2, 3$. Implement this controller in matlab. Note that you need to find the way that solves the co-efficients of a point in the polytope as the linear combination of the three vertices.
- Solve the question in step 2 for F_2 and F_3. Feel free to implement the controllers for these two cases. But it is not required.