# Dynamic Modeling of an RC car Using System Identification

*RBE 501: Robot Dynamics Course Project Progress Report

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***Abstract*—Dynamical models of the systems are of great use when designing control and planning algorithms of the robots. The probelm with dynamical models is, they are very difficult formulate. We have explored graybox modeling technique of system identification to estimate the parameters of dynamics of RC Car. Three identification experiments are conducted that estimates different parameters.**

## I. Introduction

In broad view, a mathematical model of a dynamical system gives insights that relate physical input and output quantities which can also be observed experimentally. Therefore, it helps in estimating the dynamic behavior of the system in response to a given set of external inputs [1]. The process of devising a precise and clearly defined mathematical model is not straight forward. Though, in many cases, a set of well established models already exist, a model needs to be generated for specific cases as well. [2]

Figure 1 shows the block diagram of a generic dynamic system model with the Dynamic system $\Sigma$, characterized by a set of state variables *x(t)*, with input variables *u(t)* and the output variables *y(t)* represent the response of the system. [2]

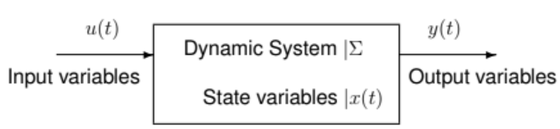


Fig. 1. General Dynamic System Model

System identification is the methodology to estimate the parameters of the mathematical model from the data obtained from wide range of experiments. Due to the emergence of high performance graphic cores and processors, the parameter fitting from the input output graphs obtained form experimental data using machine learning algorithms can be done in minuscule time periods.

## II. Problem Description

The dynamic modelling can be done using purely the first principles. This method involves tiresome process of coming up with the parameters and the inaccuracies might peek into when the system involved is complex. On the other hand, data driven models seek to match input-output predictions to data but cannot explain the internal dynamics. Internal dynamics help in giving a deep understanding of the system where high performance is critical. With these in mind, a hybrid approach of modelling using first principles and data-driven modelling is being used in this project.

A Kinematic [3] and Dynamic [4] bicycle model were derived using the first principles and data-driven approach is used to fit the parameters from the experimental results.

### A. Objectives

Going forward in this direction, the objective can be met with these steps in order:

1) Come up with a few candidate base models of an RC Car.
2) Design and Perform experiments on the RC Car to generate response data using VICON Motion Capture System.
3) Use System Identification techniques to identify the parameters for each model from the collected data.
4) Validation: Compare the results of the proposed model with Vicon data for new sets of trajectories. Choose the model with highest resemblance to the RC car.

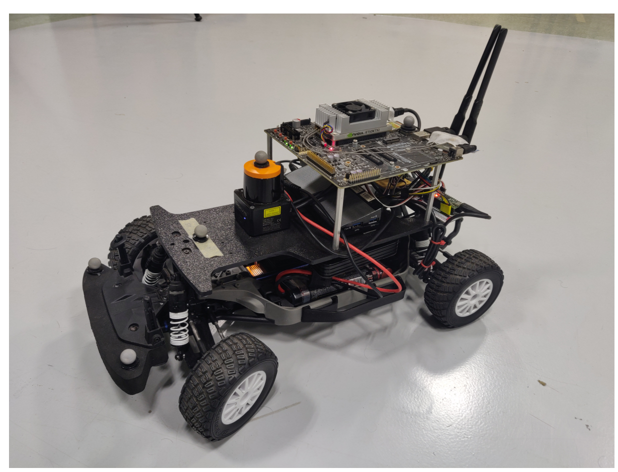

Fig. 2. F1/10 Race Car

This work was done at Control and Intelligent Robotics Laboratory (CIRL) at Worcester Polytechnic Institute, MA, USA

## III. Related Work

In this section, we first look at the various methods followed in literature to identify the dynamic model of the RC car.

A linear bicycle model in continuous-time using the Newton's equations of motion has been used to estimate a set of model parameters in [5]. The identification of the physical parameter set is done using the prediction error method that utilizes the Kalman predictor in a grey-box structure. The estimation problem is then converted to an optimization problem where a cost function is to be minimized. Steering angle is the only input whereas lateral acceleration and yaw rate form the output equations. Input and output data are measured on the vehicle for identification. The predicted data has a comparable performance to that of the experimental data. The authors suggest using data different from the identification to validate the model.

Further [6] shows the equations governing the dynamics of the vehicle using the Fiala tire model. The lateral velocity $v_y$ and the yaw rate $\psi$ are shown related to the tire normal forces. This relation could be used to estimate the parameters of vertical dynamics (suspension) model [7]. Further, it is shown that the linear velocities $v_x$, $v_y$, parameters $l_f$, $l_r$ and steering input $\delta$ are sufficient to determine the slip angles $\alpha_f$ and $\alpha_r$ for the front and rear tires respectively. The slip angles could then be used to obtain the lateral tire forces as shown in equations (10)(11).

## IV. Dynamic Models

In this section, Kinematic and Dynamic bicycle models are described. Bicycle model (Single Track Model) is one of the simplest models, in which left and right wheels are joined together, both on the front and rear axis. These models form the basis of many control and motion planning algorithms. Our aim is to find the parameters of these models from the experimental data.

### A. Kinematic Bicycle Model

The Non-Linear continuous time equations that describe a kinematic bicycle model [3] in the inertial frame are given by:

$$\dot{x} = v\cos(\psi + \beta) \tag{1}$$

$$\dot{y} = v\sin(\psi + \beta) \tag{2}$$

$$\dot{\psi} = \frac{v}{l_r}\sin(\beta) \tag{3}$$

$$\dot{v} = a \tag{4}$$

$$\beta = \arctan\left(\frac{l_r}{l_f + l_r}\tan(\delta_f)\right) \tag{5}$$

Here, $x$ and $y$ are the coordinates of the Center of Mass in inertial frame $(X, Y)$.

System identification on kinematic model is easier as compared to higher fidelity vehicle models as there are only two parameters to identify: $l_f$ and $l_r$.

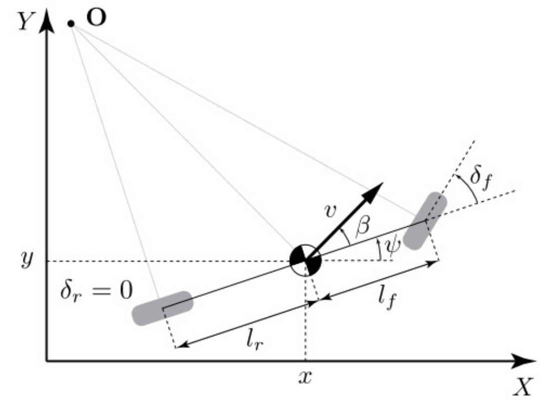


Fig. 3. Kinematic Bicycle Model

| Parameter | Description |
|---|---|
| $\psi$ | Yaw Angle |
| $\beta$ | Velocity Angle |
| $l_f, l_r$ | Distance of front and rear axle from CoG |
| $\delta$ | Steering Angle |
| $a$ | Acceleration |

### B. Lateral Dynamics of Bicycle Model

A 3-DOF dynamic model as given in [4] is considered here as it represents all possible movements of the vehicle on the plane; translations along $X$ and $Y$ axis and rotation about $Z$ axis. When deriving Lateral Dynamics of the Single Track Model, following assumptions are made:

- Longitudinal and lateral load transfer is neglected.
- Body is not subjected to rolling and pitching motion.
- Aerodynamic effects are neglected.
- Suspension system does not affect the calculation.
- Steering angle is small.

These assumptions imply that the model is good under the conditions of cornering, which will be utilized in the experiments to find the tire model of the vehicle.

Equations of lateral dynamics of bicycle model are given by:

$$\dot{r} = \frac{aF_y^f\cos(\delta) - bF_y^r}{I_z} \tag{6}$$

$$\dot{\beta} = \frac{F_y^f\cos(\delta) - F_y^r}{mU_z} - r; \tag{7}$$

$$\dot{U}_x = \frac{F_x^r - F_y^f\sin(\delta)}{m} + rU_x\beta \tag{8}$$

$$\beta = \arctan\left(\frac{U_y}{U_x}\right) \tag{9}$$

This model utilizes lateral ($F_y$) and longitudinal ($F_x$) forces at front and rear wheel as shown below.

During a cornering manoeuvre, the lateral forces of a tire are greater than its friction resistance. This gives rise to sideways

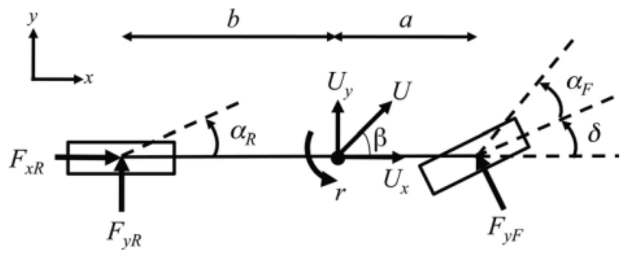


Fig. 4. Lateral Dynamics of Bicycle Model

motion of the tire which can be characterized by Tire-Slip Angle:

$$\alpha = \arctan\left(\frac{v_y^w}{v_x^w}\right)$$

where $v_y^w$ and $v_x^w$ are heading and lateral velocities of the wheel.

| Parameters | Description |
|---|---|
| $\beta$ | Velocity Angle |
| $I_z$ | Yaw Moment of Inertia |
| $m$ | Vehicle Mass |
| $U_x, U_y$ | Velocity in x and y direction |
| $r$ | Yaw Angle |
| $a, b$ | Distance of front and rear axle from CoG |
| $\delta$ | Steering Angle |
| $\alpha_f.\alpha_r$ | Front and Rear Tire Side Slip Angles |
| $F_y^r, F_y^f$ | Front and Rear Lateral Forces |
| $F_x^r, F_x^f$ | Front and Rear Lateral Forces |

### *C. Tire Dynamics*

Tire is responsible for the generation of the traction forces that propel the car. Therefore, it is necessary to understand the relationship between tires, their operating conditions and the resulting forces and moments developed at the contact patch. Tire serves three basic functions:

- Supports vertical load.
- Develops longitudinal forces for acceleration and braking.
- Develops lateral forces for cornering.

Highly Non-Linear Tire models exists such as Fiala Tire Model and Pacejka Magic Formula, but a simplified model of tire is used here. As the RC car is not subjected to very high velocities, simplified tire models can be used.

$$F_{c,r} = -C_{\alpha_r}\alpha_r \tag{10}$$
$$F_{c,f} = -C_{\alpha_f}\alpha_f \tag{11}$$

Lateral dynamics equations require tire forces which can be calculated from the tire models. As we are interested in finding lateral tire forces, we will use (10) and (11) which relates the tire slip angles to the lateral tire forces. Tire cornering stiffness coefficients ($C_a$) can be identified by conducting experiments.

## V. System Identification Experiments

An RC car that is a one-tenth model of an actual car with realistic dynamics is used to perform experiments. The pose of the car, undergoing various trajectories, is tracked from VICON Motion Capture system. Pose data can be used to find Yaw Angle and Yaw Rate ($\psi$, $\dot{\psi}$).

We get the values of $x$, $y$, $z$, and $\psi$ from VICON. These values can be differentiated to calculate the Velocities in x, y and z direction. The problem with these velocities is that they are very noisy.

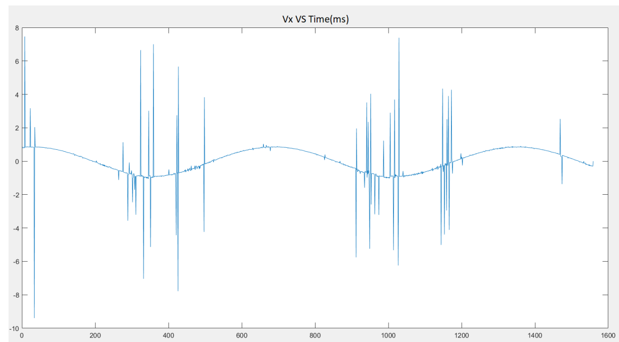


Fig. 5. Differentiating the Position obtained from VICON, we get very noisy velocity.

Hample filter is used to remove the outliers. After passing the data from Hample filter, some stray points still persists so the data is passed through Hample filter twice. This ensures that no outliers remain. Now the velocity doesn't have outliers but contains local noise. This is pretty easy to reove and Lowpass filter is used to rmove the noise.

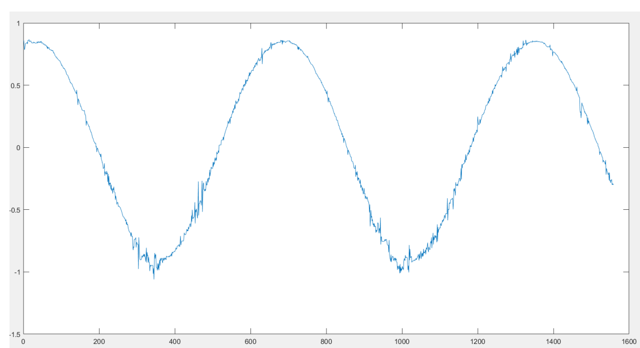

Fig. 6. Passing the data through Hampel filter twice ensures that no outliers remain.

Now that the data is filtered, it can be used for parameter estimation. Three System Identification experiments are performed to estimate different values. These experiments are as follows:

### *A. Estimation of distance of axles from COG*

Kinematic Bicycle model can be used to estimate the distance of axles from COG. $l_f andl_r canbeestimatedfromeq(3)andeq(5)asthosearetheonlyunkown$

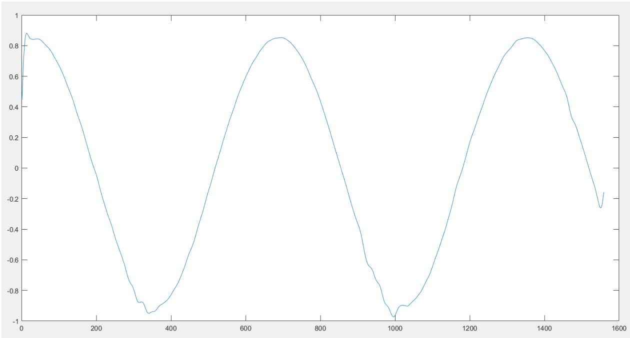


Fig. 7. Local noise can be removed easily by using a lowpass filter

| Parameters | Measured | Estimated |
|---|---|---|
| $l_f$ | 19.5cm | 18.84cm |
| $l_r$ | 14.5cm | 13.92cm |

*B. Steering Model Identification*

Steering on the RC Car is performed by servo. To drive the car with different steering angles, servo counts are given through ROS. But it is necessary to identify the relationship between servo counts and steering angle in degrees or radians.

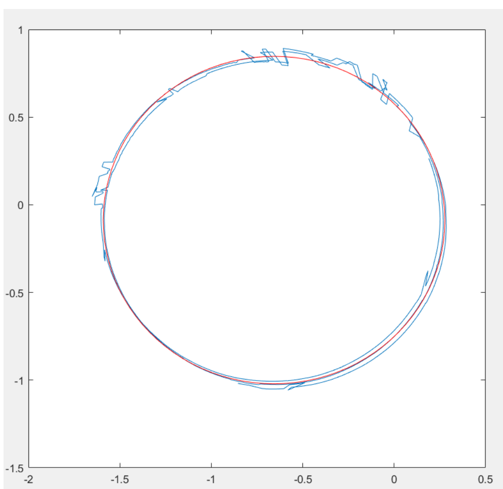

Fig. 8. A circle is fit into the path traced by the car using Taubin's method. The radius of the circle is then used to estimate steering angle.

Circular Motion Test is used to establish this relation. The car is driven on a circle with different combinations of velocities and servo counts. The VICON data is plotted and a circle is fit into that data using Taubin's Method. Folloing relation is used to find steering angle $\delta$:

$$\delta = \frac{wheelbase}{R}$$

. Here R is the radius of th circle obtained from the circular fit. This process is repeated for all the trails. In total, 40 trials were condcted and the results are plotted in the figure.

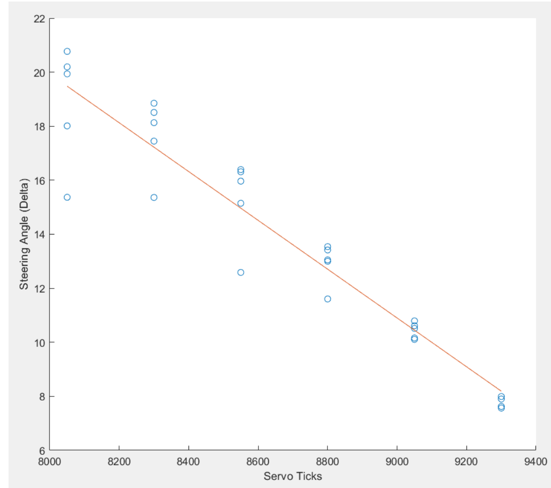


Fig. 9. Steering model can be estimated by best fit line

The estimated steering model obtained is as follows:

$$\delta = -0.009ticks + 92.196$$

*C. Tire Model Identification*

Identification of tire model is very difficult as it depends on the physical properties and quality of the tire. A tire made of poor quality would not be able to produce tire slip angle which is exactly the case here. The tires in the RC car are not of good quality and this led to suboptimal results.

Tire model relates the lateral force produced at the front and rear tires to the tire slip angles. These two are related non-linearly by a coefficient called tire cornering stiffness $C_\alpha$. The cornering stiffness value for both front and rear tires are different and needs to be identified separately.

Another important relation that we need to find is between $\alpha$ and $\delta$. This tells us how much tire slip angle is generated with steering angle. Anf the tire slip angle is in turn used to estimate the lateral force that would develop at rear and front tires. These forces are used as parameters in the dynamic model.

Two important relations needs to be established to relate $\alpha$ to $F$. The first relation is between Lateral force and lateral acceleration (in steady state). This is gives as follows:

$$F_y^f = \frac{mb}{(a+b)\cos\delta} a_y^{ss}$$

$$F_r^y = \frac{ma}{(a+b)} a_y^{ss}$$

Another relation is between tire slip angle $\alpha$ and steering angle $\delta$. It is given as follows:

$$\alpha_f = \arctan \frac{v_y + ar}{v_x} - \delta$$

$$\alpha_r = \arctan\frac{v_y + br}{v_x} - \delta$$

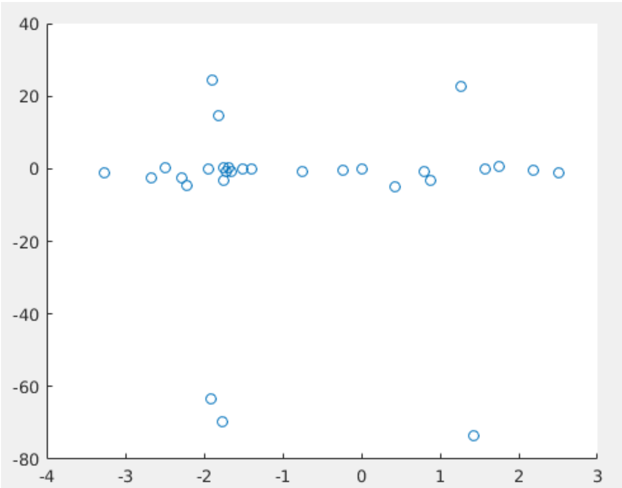


Fig. 10. This is the plot between lateral force and tire slip angle alpha.

Using above equations, we can plot the $F_y$ vs $\alpha$. A 2nd degree polynomial is fit into the data points and this gives us the tire cornering stiffness function. Thus, tire model identification is complete.

## VI. Conclusion and Future Work

All the necessary parameters that are used to describe kinematic and dynamic bicycle model are estimated using system identification experiments.

This work can be extended by using Full Track model of the vehicle, which used four wheels instead of bicycle model. This model will be more realistic and would give accurate results. Another extention of this work is finding parameters for verticle dynamics of the car.

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