Dynamic Modeling of an RC Car using System Identification

RBE 501: Robot Dynamics Course Project

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Problem Statement and Motivation

- Multiple approaches to a dynamic vehicle model
    - From first principles
    - Data driven models
- First principles approach can be tiresome and inaccurate
- Typical data driven approaches use input-output relationship to describe the model but fail to explain internal dynamics
- Most control algorithms and planners end up utilizing simplified kinematic model

Approach/ Methodology

- Remedy the problem by developing a hybrid approach
- Combine both first principles and data driven approach
- We start from the available models - kinematic bicycle, lateral vehicle dynamics and tire models
- Operate the car in various scenarios to collect data
- Obtain the coefficients by fitting the data to the mathematical model equations

Dynamic Models

- Kinematic Bicycle Model
- Lateral Dynamics of Bicycle Model
- Tire Dynamics

Kinematic Bicycle Model

$$\dot{x} = v\cos(\psi + \beta) \tag{1}$$

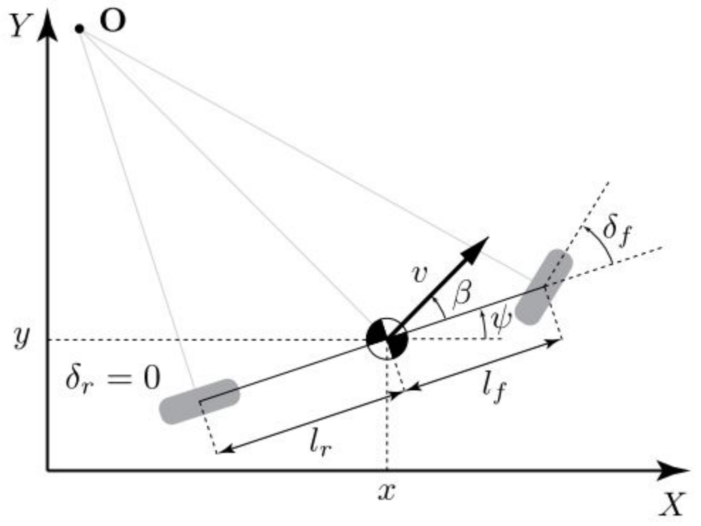
$$\dot{y} = v\sin(\psi + \beta) \tag{2}$$

$$\dot{\psi} = \frac{v}{l_r}\sin(\beta) \tag{3}$$

$$\dot{v} = a \tag{4}$$

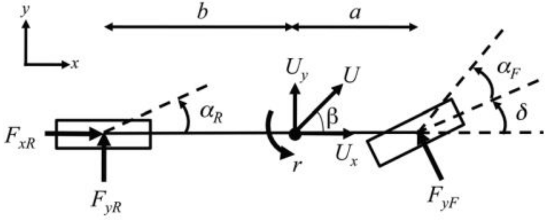
$$\beta = \arctan\left(\frac{l_r}{l_f + l_r}\tan(\delta_f)\right) \tag{5}$$

Parameter	Description
ψ	Yaw Angle
β	Velocity Angle
l_f, l_r	Distance of front and rear axle from CoG
δ	Steering Angle
a	Acceleration



Lateral Dynamics of Bicycle Model

parameters of lateral dynamics model



$$\dot{r} = \frac{aF_y^f \cos(\delta) - bF_y^r}{I_z} \tag{6}$$

$$\dot{\beta} = \frac{F_y^f \cos(\delta) - F_y^r}{mU_z} - r; \tag{7}$$

$$\dot{U}_x = \frac{F_x^r - F_y^f \sin(\delta)}{m} + rU_x\beta \tag{8}$$

$$\beta = \arctan\left(\frac{U_y}{U_x}\right) \tag{9}$$

$$\alpha = \arctan\left(\frac{v_y^w}{v_x^w}\right)$$

Parameters	Description
β	Velocity Angle
I_z	Yaw Moment of Inertia
m	Vehicle Mass
U_x, U_y	Velocity in x and y direction
r	Yaw Angle
a, b	Distance of front and rear axle from CoG
δ	Steering Angle
$\alpha_f.\alpha_r$	Front and Rear Tire Side Slip Angles
F_y^r, F_y^f	Front and Rear Lateral Forces
F_x^r, F_x^f	Front and Rear Lateral Forces

Tire Dynamics

Why Tire Dynamics?

Fiala Tire Model

$$F_y(\alpha) = \begin{cases} -C_\alpha \tan\alpha + \dfrac{C_\alpha^2 (2 - \mu_s/\mu_p)}{3\mu_p F_z} |\tan\alpha| \tan\alpha \\ \qquad - \dfrac{C_\alpha^3 (1 - 2\mu_s/3\mu_p)}{9\mu_p^2 F_z^2} \tan^3\alpha & , \quad |\alpha| < \alpha_{sl} \\ -\mu_s F_z \mathrm{sgn}\alpha, & \quad |\alpha| \geq \alpha_{sl} \end{cases}$$

$$\alpha_{sl} = \arctan \frac{3\mu_p F_z}{C_\alpha}$$

Where:

C_α = tire cornering stiffness
F_z = normal load applied to the tire
μ_p = peak friction coefficient between the tire and the ground
μ_s = sliding coefficient of friction between the tire and the ground
α = tire slip angle

Tire Dynamics

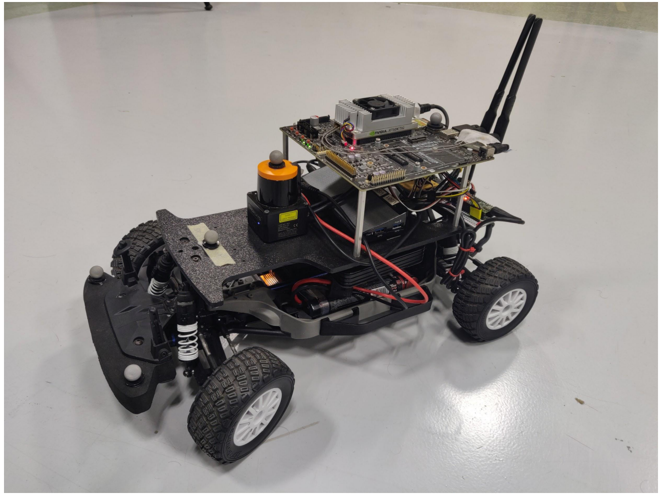
Simplified Tire Model:

How to estimate $C_{a,\ r}$

$$F_{c,r} = -C_{\alpha_r}\alpha_r$$
$$F_{c,f} = -C_{\alpha_f}\alpha_f$$

System Identification Experiments

- Used F1/10 RC Car for experiments.
- Used VICON Motion Capture system to get pose of the car.
- Conducted at CIRL, WPI



Car Details:

- Distance of axles from COM:
  - Lf = 19.5cm, Lr = 14.5cm
- Wheelbase: 42.5 cm
- Track-width: 25.4 cm

System Identification Experiments

Experiments Conducted:

- Estimating a and b (distance of front and rear axles from COM) using Linear Kinematic Model.
- Estimated Steering Model.
- Tire Model Identification.

What Values Do We Have?

In Kinematic and Dynamic Bicycle Model, there are common set of values that we need. These are:

- Position Px, Py, Pz
- Velocity Vx, Vy, Vz
- Acceleration Ax, Ay, Az
- Yaw angle, Yaw rate.
- Steering Angle

We get these values from VICON.

- But it is very noisy.

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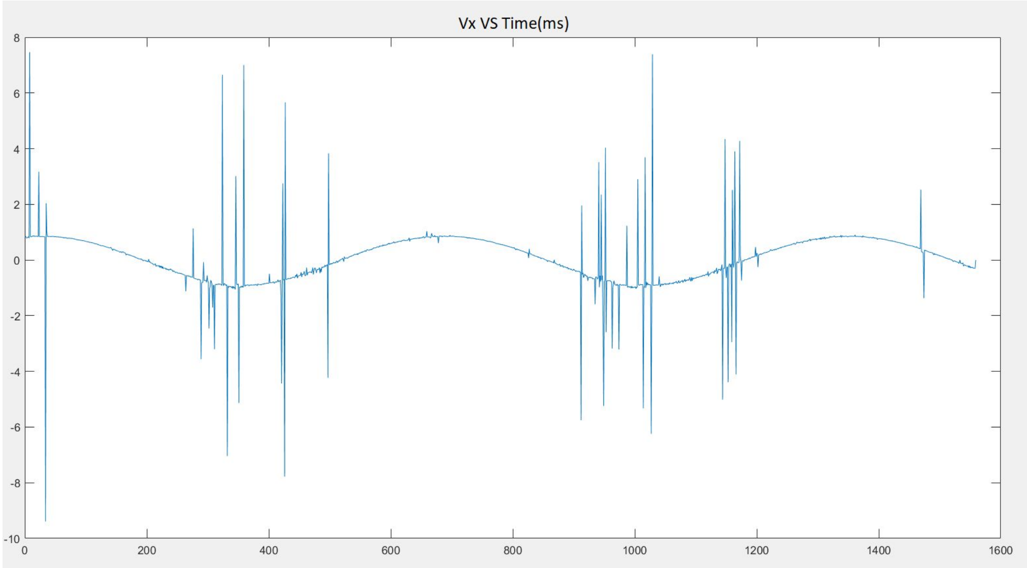
Filtering VICON Data

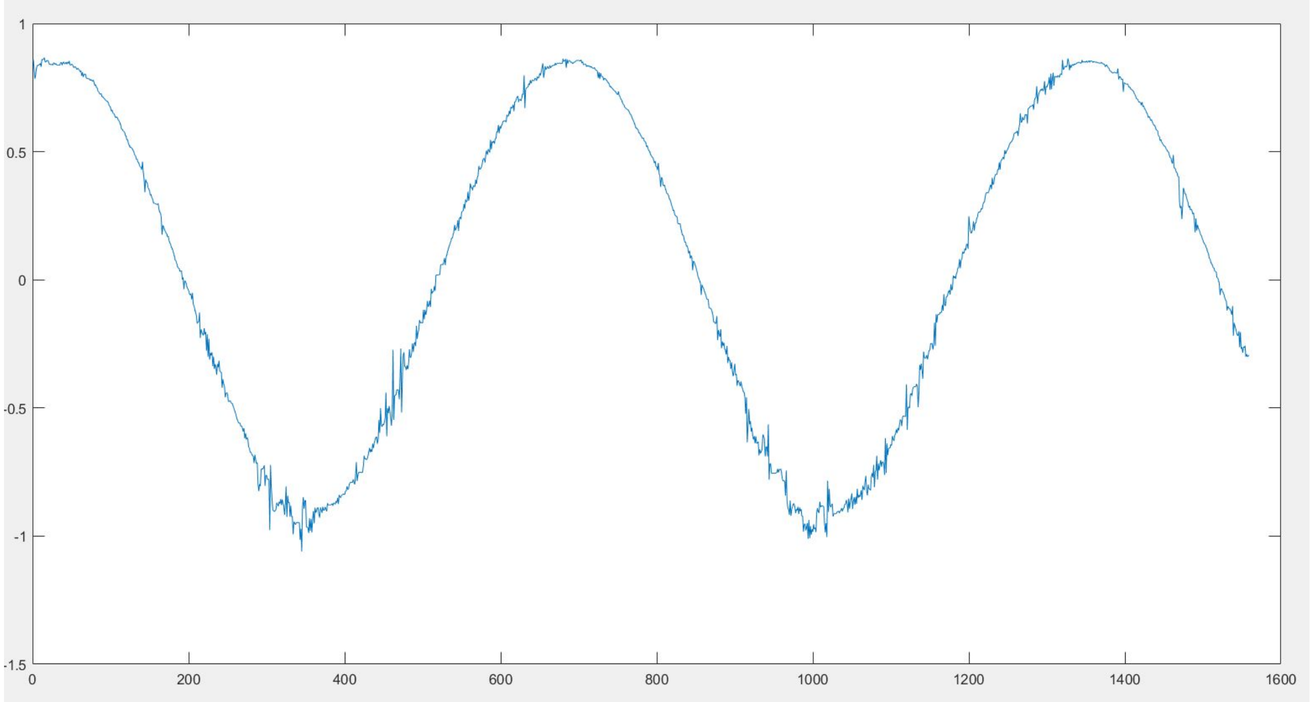
We get Px, Py, Pz, Roll, Pitch and Yaw angles from VICON

Differentiating it, we get Velocity.



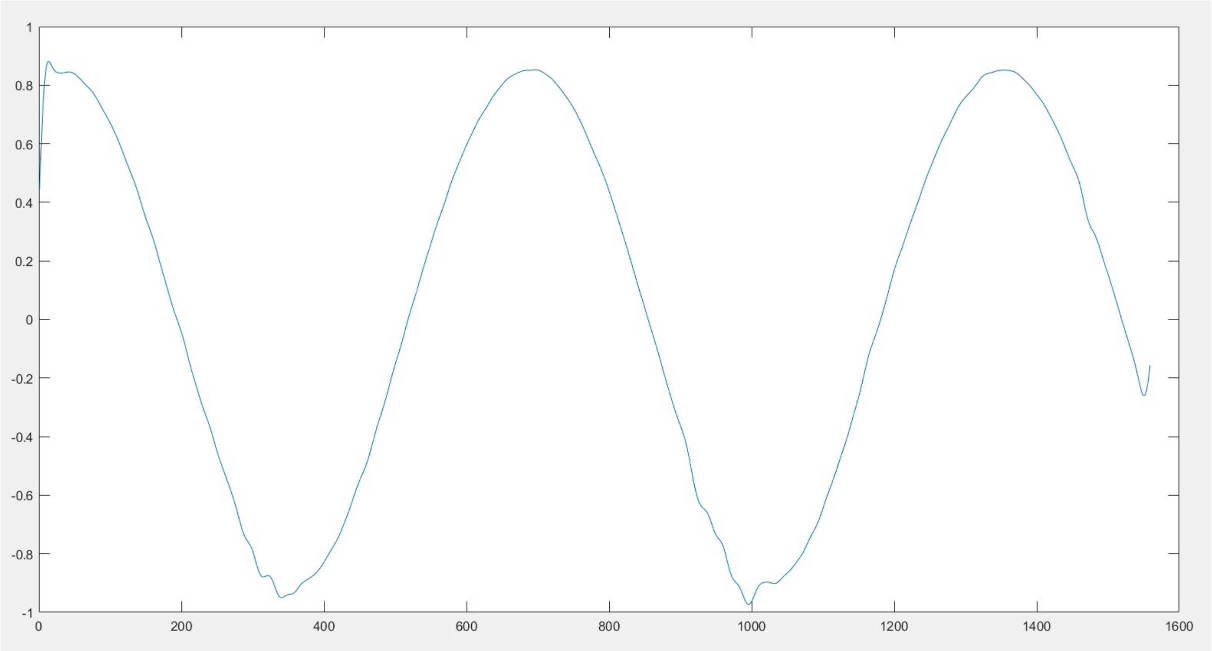
Filtering VICON Data

Use Hampel Filter, twice to remove the outliers.



Filtering VICON Data

Use Low Pass Filter to remove noise.



Estimation of distance of axles from COG

Measured data:

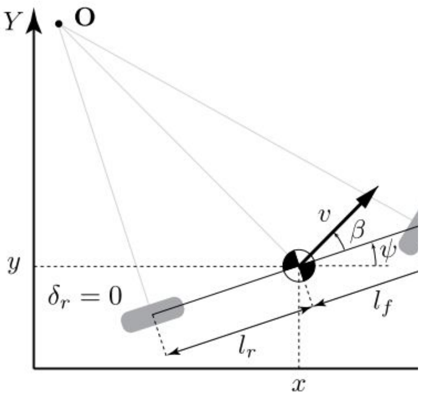
Lf = 19.5 cm

Lr = 14.5 cm

Estimated data:

Lf = 18.84 cm

Lr = 13.92 cm



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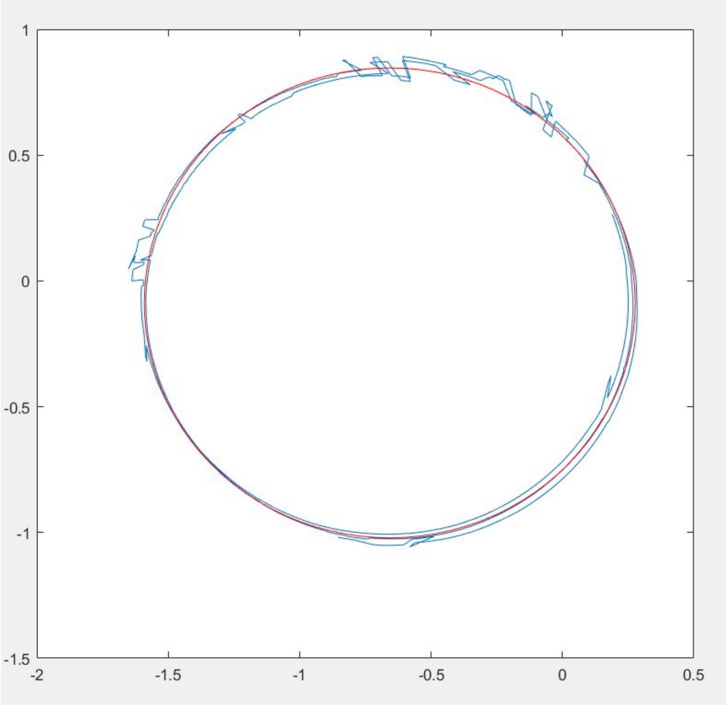
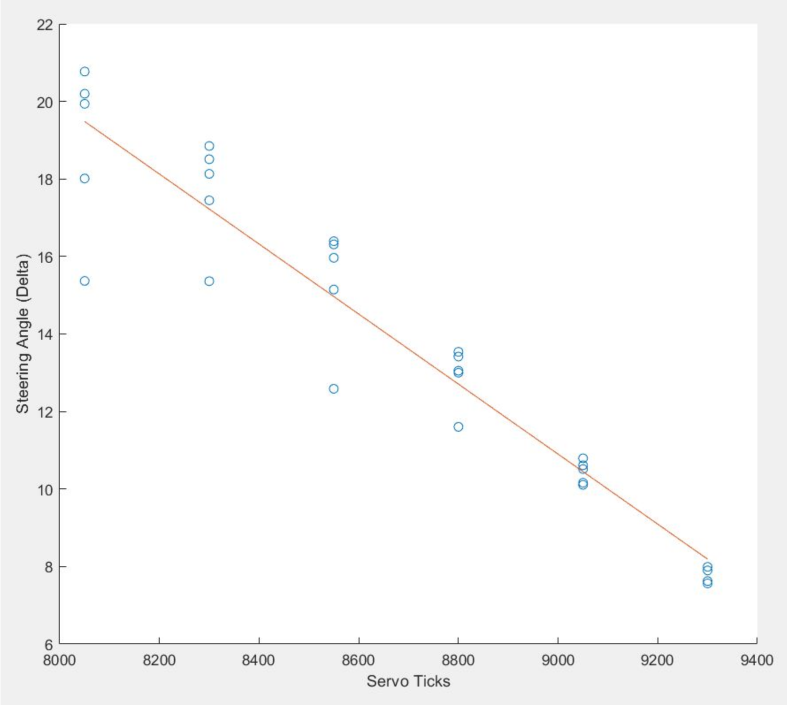
Estimation of Steering Model

- Steering performed by Servo.
- Need to identify the relation between Servo counts and Steering angle Delta.
- Perform Circular Motion Test:
  - Measure the circle traced by car for different velocities and steering angle.
  - Total of 40 trials were conducted.
  - Fit the circle (using Taubin's Method) on the traced circular path and find its Radius.
  - Use the below relation to find steering angle.
    - delta = (wheelbase/R)
  - Repeat it for all the trials and fit a line.

Estimation of Steering Model





Steering Angle (δ) = x*-0.0090 + 92.1960

Estimation of Tire Model

- Tire Model related Force produced at the wheels to the tire slip angle α
- These are related by Cornering Stiffness C_{α}
- Cornering Stiffness is a non-linear function of 2nd or 3rd order.
- We also need to find a relation between α and steering angle δ.

$$F_{c,r} = -C_{\alpha_r}\alpha_r$$

$$F_{c,f} = -C_{\alpha_f}\alpha_f$$

Estimation of Tire Model

Relation between Steady State Lateral Acceleration and Lateral Force

$$F_y^f = \frac{mb}{(a+b)\cos\delta} V_x r = \frac{mb}{(a+b)\cos\delta} a_y^{ss}$$

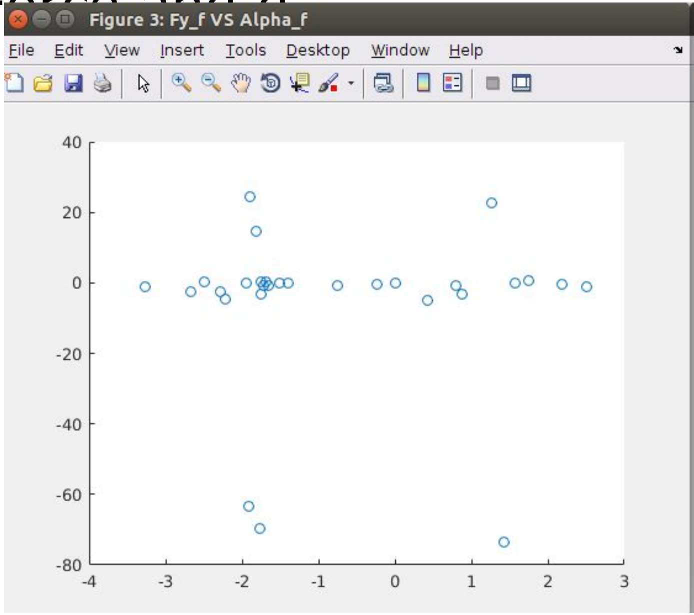
$$F_y^r = \frac{ma}{a+b} V_x r = \frac{ma}{a+b} a_y^{ss}$$

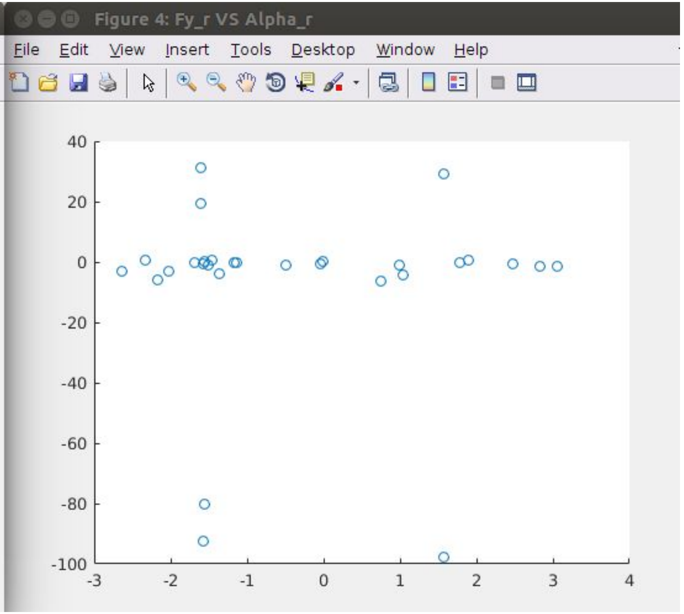
Relation between α and δ is linear:

$$\alpha_f = \arctan\frac{V_y + ar}{V_x} - \delta \qquad \alpha_r = \arctan\frac{V_y - br}{V_x}$$

Estimation of Tire Model

Using previous two equations, we can find relation between Lateral Force and α





Conclusion & Future Work

- All the necessary parameters that characterize the Kinematic and Dynamic Bicycle Model of the RC are estimated using System Identification experiments.

Future Work

- This work can be extended to model Full-Track model of the Car.
- Vertical dynamics of the Car that model the suspension can be estimated using data driven techniques as well.