Nominal MPC on Race Car

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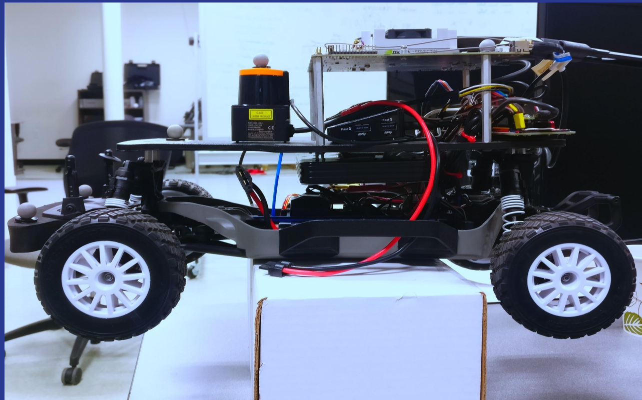
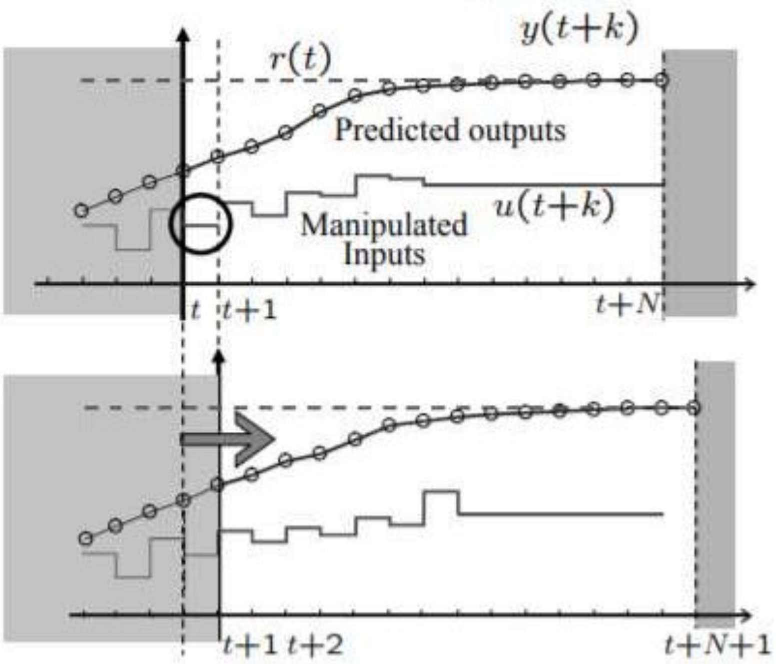
Team 88

Conducted at CIRL Lab,
Worcester Polytechnic Institute

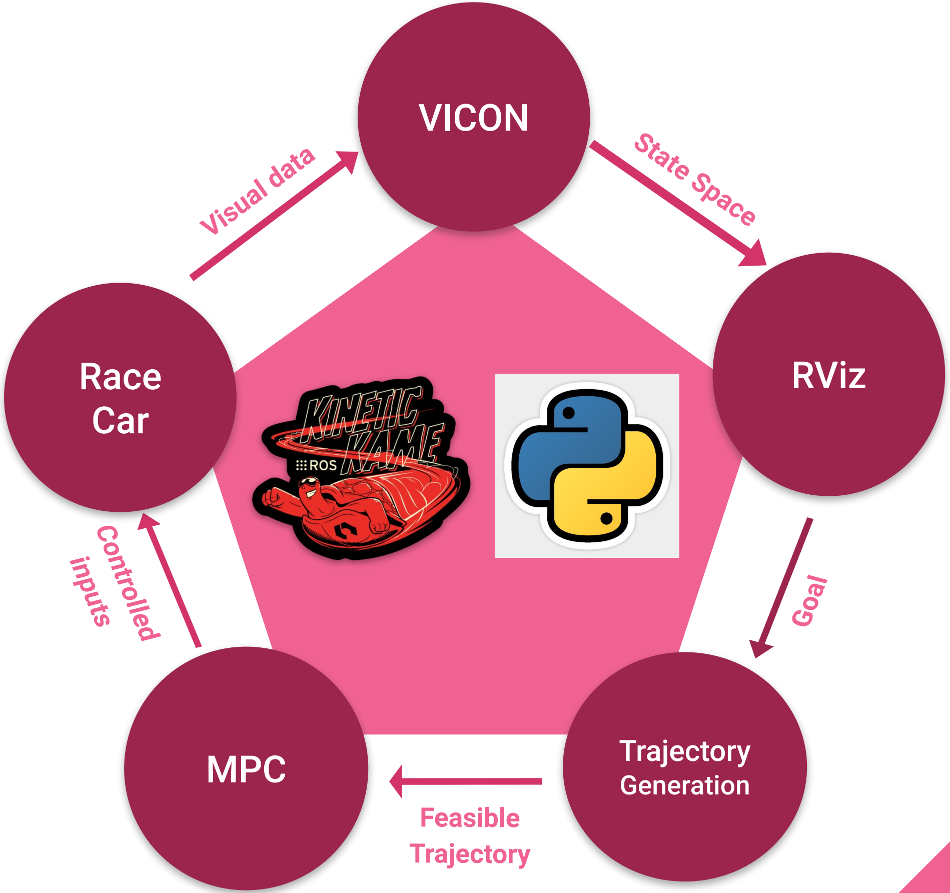


Problem Statement

- Implement Nominal MPC on the Race Car to track given trajectory
- Set up ROS-Kinetic on the robot and base station
- Set up VICON state estimation for race car



Approach


VICON
State Space
RViz
Goal
Trajectory Generation
Feasible Trajectory
MPC
Controlled inputs
Race Car
Visual data
KINETIC KAME
ROS

State Space Representation

Kinematic Model

$$\dot{x} = v\cos(\psi + \beta)$$

$$\dot{y} = v\sin(\psi + \beta)$$

$$\dot{\psi} = \frac{v}{l_r} sin(\beta)$$

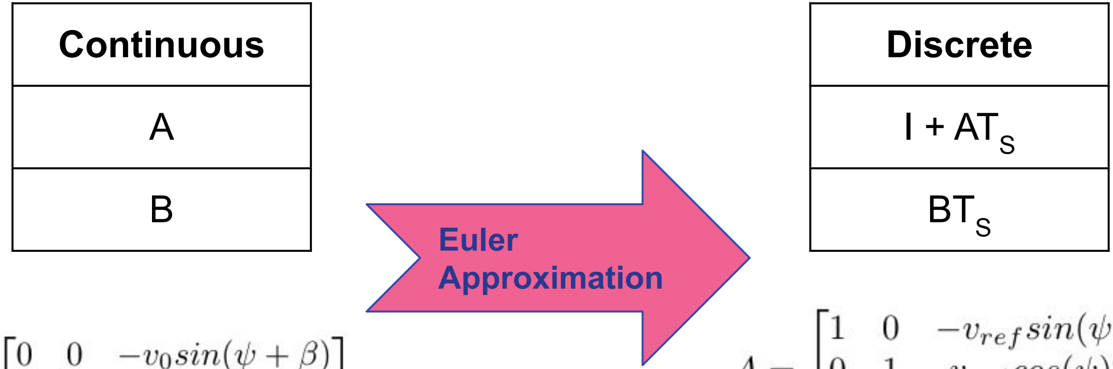
$$\beta = \tan^{-1}(\frac{l_r}{l_f + l_r} \tan(\delta_f))$$

State Space

$$z = \begin{bmatrix} x \\ y \\ \psi \end{bmatrix} \qquad u = \begin{bmatrix} \delta \\ v \end{bmatrix}$$

$$\dot{z} = Az + Bu$$

Discretization of State Space



Continuous
A
B

Euler Approximation

Discrete
I + AT$_S$
BT$_S$

$$A = \begin{bmatrix} 0 & 0 & -v_0 sin(\psi + \beta) \\ 0 & 0 & v_0 cos(\psi + \beta) \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & -v_{ref} sin(\psi) T_s \\ 0 & 1 & v_{ref} cos(\psi) T_s \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & cos(\psi + \beta) \\ 0 & sin(\psi + \beta) \\ \frac{v_0(1+tan^2(\delta))}{L_f + L_r} & \frac{tan(\delta)}{L_f + L_r} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & cos(\psi) T_s \\ 0 & sin(\psi) T_s \\ \frac{v_{ref}(1+tan^2(\delta))}{L_f + L_r} T_s & \frac{tan(\delta)}{L_f + L_r} T_s \end{bmatrix}$$

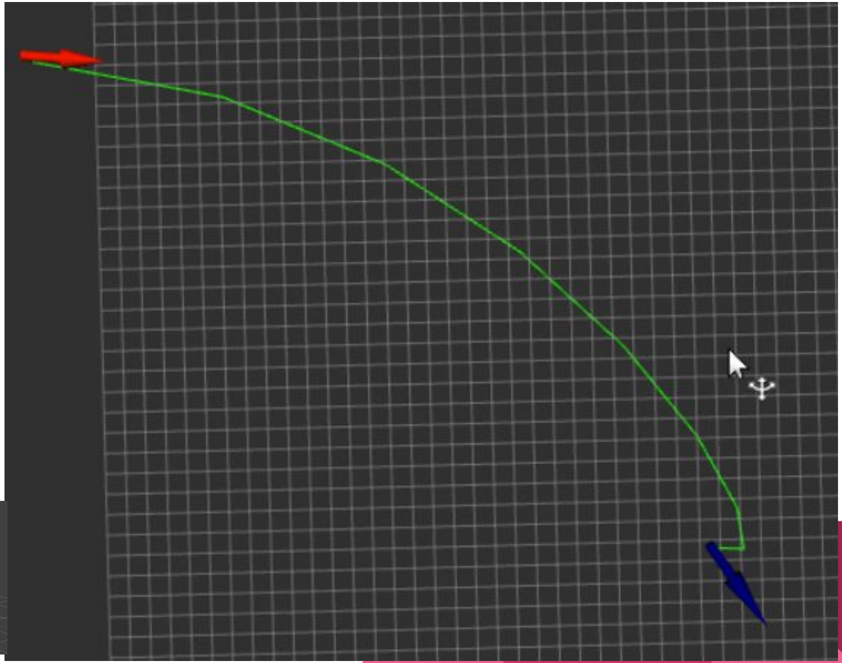
Trajectory Generation

$$a_0 + a_1 t^1 + a_2 t^2 + a_3 t^3 = f_x(t)$$

$$b_0 + b_1 t^1 + b_2 t^2 + b_3 t^3 = f_y(t)$$

$$\dot{x}\sin(\theta) - \dot{y}\cos(\theta) = 0$$

$$\dot{x}\cos(\theta) + \dot{y}\sin(\theta) = v$$



MPC Implementation

- Using CVXPY APIs with ECOS solver in Python 2.7
- Average run time of optimiser in python is 0.7 seconds

$$\min_{u} \sum_{i=0}^{N-1} (z_i - z_{ref,i})^2 Q + \sum_{i=0}^{N-1} [(u_i - u_{ref})^2 R + (z_N - z_{ref,N})^2 Q_f]$$

$$\begin{aligned} \text{s.t.} \quad & z_0 = z_t, u_{-1} = u_{(}t - t_s), \\ & z_{i+1} = f(z_i, u_i), i = 0, ..., N-1, \\ & u_{min,i} \leq u_i \leq u_{max,i} \forall i, \\ & y_{min,i} \leq y_i \leq y_{max,i} \forall i \end{aligned}$$

Result of tuning
Horizon, N = 2 to 4
State cost, Q = **5***[1 0 0;0 1 0;0 0 2]
Input cost, R = 2*[3 0; 0 1]
Terminal cost, Q_f = 2*I_{3x3}
Time Step, T_S = 1s

Experiments and Results

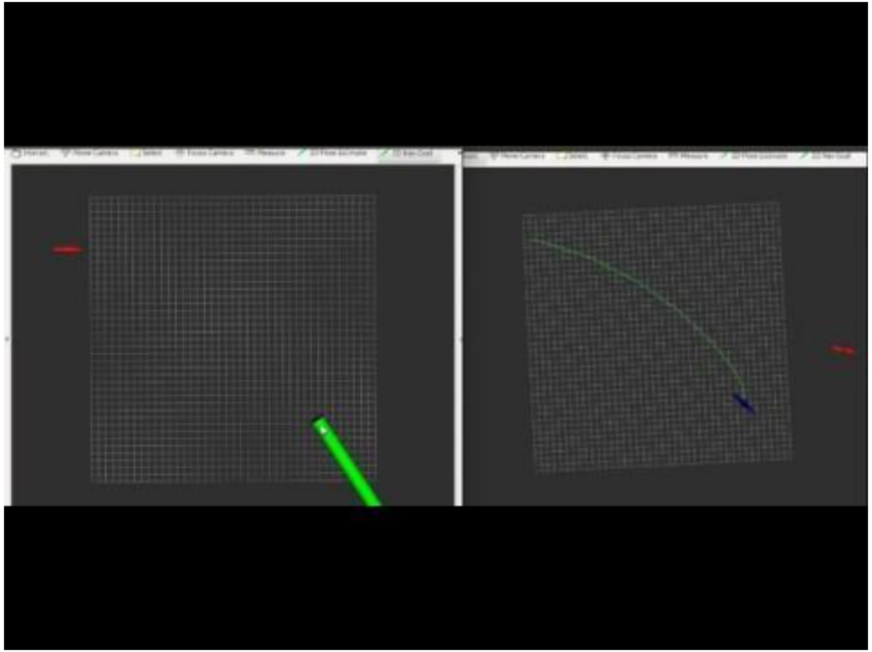
Straight Line Tracking

https://drive.google.com/open?id=1JZ82dnKNvFir5AtCJnfWLO4l2wO-cAml
https://drive.google.com/open?id=1qZM7rGrK4eMngJhqQiW2mtoTccorw1aF

Curved Path Tracking

https://drive.google.com/open?id=10NlNcgxJBXRgyID9-NWyHwcAAXzhkcgV

Challenges: High Velocity Behavior



Access at:
https://drive.google.com/open?id=156UWnCwBG0SrJAuVlH1vuh-yXeToJQAN

Challenges: Time Step Match

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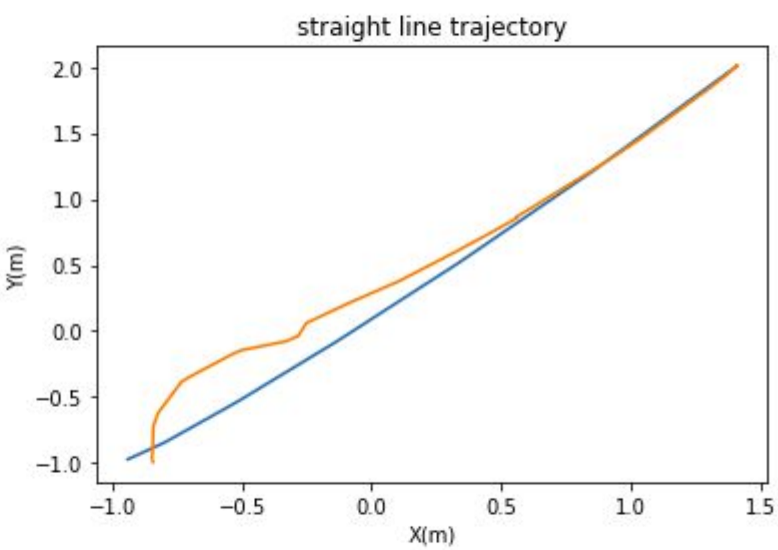
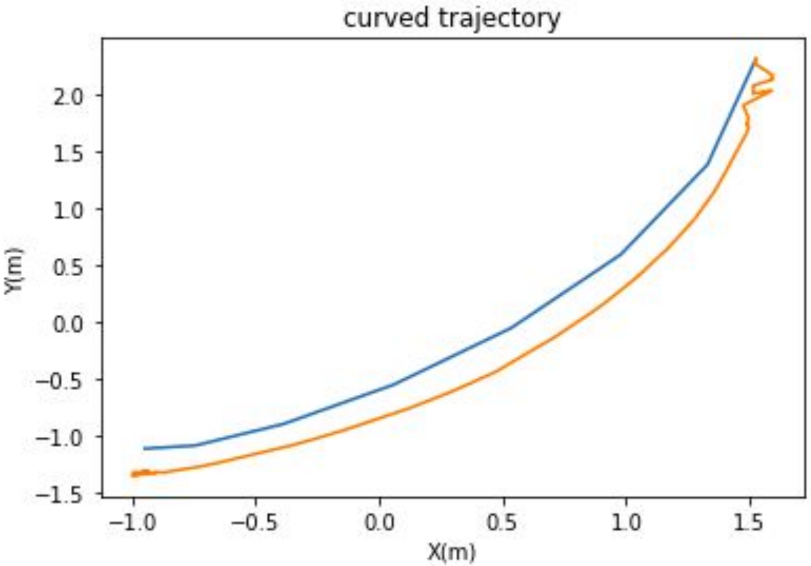
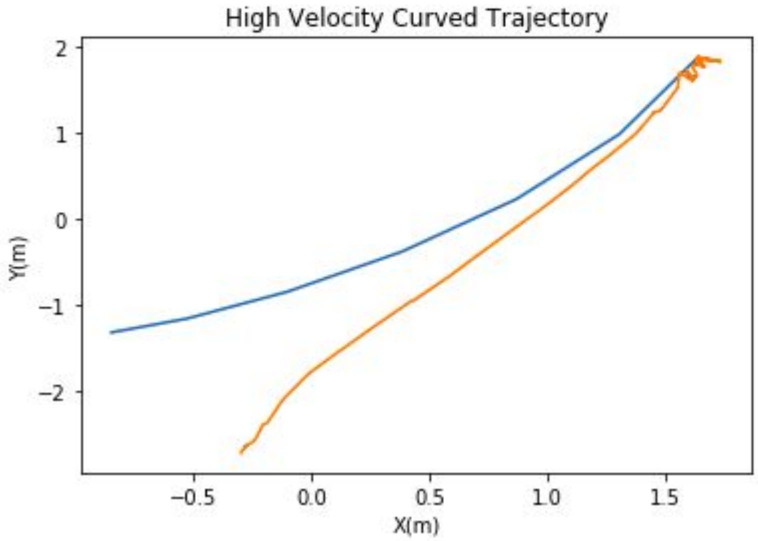
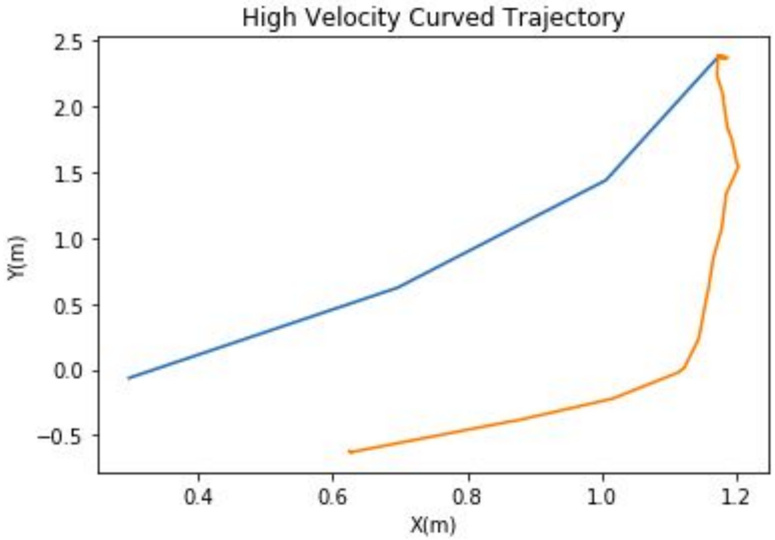
Performance





Performance





Conclusion

- MPC provides large range of flexibility to improvise cost function and gain matrices to meet desired requirements
- For lower speeds nominal MPC works perfectly fine but for higher speeds we will need a more robust MPC
- Time step determines the length of the horizon and pose sampling rate

Future Scope

- Implementation of Robust MPC
- Achieve velocity control with timestep considerations
- Tackle time step errors by considering nearest state in the desired trajectory
- Update state and input matrices (A, B) with respect to the predicted states

What did we learn?

- Practical implementation of MPC control system
- ROS
- Python
- Debugging software issues on hardware
- Team and time management

References

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- Model Predictive Control of Hybrid Systems of Hybrid Systems by Alberto Bemporad; http://cse.lab.imtlucca.it/~bemporad/hybrid/school07/pdf/08.Bemporad.pdf
- "MPC Course Material." *MPC Lab @ UC-Berkeley,* www.mpc.berkeley.edu/mpc-course-material.
- **arXiv:1808.10703 [cs.RO]**
- **arXiv:1603.00943 [math.OC]**

Questions?

Thank You!