Assignment I

MEMAD

August 16, 2025

Abstract

The goal is to revise some relevant theory and key results from vector calculus. Please upload your solutions in one compressed file to Classroom before August 26th. Use a PDF or Jupyter Notebook file for each problem.

Problem 1

Find the angles (both of them) between the vectors $p = (-1, 3, 1)$ and $q = (-2, -3, -7)$.

Problem 2

Find two non parallel vectors which are orthogonal to $p = (1, 1, 1)$.

Problem 3

Let n be a natural number and A be as follows:

$$A = \begin{vmatrix} n & n+1 & n+2 \\ n+3 & n+4 & n+5 \\ n+6 & n+7 & n+8 \end{vmatrix}.$$

Show that $det(A)$ remains constant with respect to n.

Problem 4

Solve the following system of linear equations using determinants:

$$\begin{array}{ccccccccc} x & + & y & + & z & & & = & 1 \\ & & 2y & - & 2z & - & w & = & -7 \\ x & + & y & - & z & & & = & -3 \\ x & & & + & z & + & w & = & 2 \end{array}$$

Problem 5

Show whether or not the following points are concyclic:

$$p_1 = (-1, 6),\ p_2 = (-1, 2),\ p_3 = (1, 4),\ p_4 = (0, 4 - \sqrt{3}).$$

Problem 6

Compute the following limits:

a)

$$\lim_{x \to 0} \frac{e^x - 1}{x}.$$

b)

$$\lim_{(x,y) \to (0,0)} \frac{e^{xy} - 1}{y}.$$

Problem 7

Compute $\nabla f(1,1,1)$ and $\nabla^2 f(1,1,1)$ where $f(x,y,z) = (x+z)e^{x-y}$.

Problem 8

Given a function $f : \mathbb{R}^n \to \mathbb{R}$ and an initial point $\mathbf{x}_0 \in \mathbb{R}^n$, Newton's method is defined as follows:

$$\mathbf{x}_{k+1} = \mathbf{x}_k - [\nabla^2 f(\mathbf{x}_k)]^{-1} \nabla f(\mathbf{x}_k), \quad k = 0, 1, 2, \cdots, K,$$

where $\nabla f(\mathbf{x}_k)$ and $\nabla^2 f(\mathbf{x}_k)$ are respectively the gradient and Hessian of f at $\mathbf{x}_k$. Write a Python script that, granted a natural number K, computes the value of $\mathbf{x}_k$ for the function $f(\mathbf{x}) = x_1^2 + 3x_2^2$. Consider $\mathbf{x}_0 = [1, 1]$. Make a plot with your results.

Problem 9

Let $\mathbf{x}$ and $\mathbf{1} := (1, 1, \cdots, 1)$ lie both in $\mathbb{R}^n$. Consider the Rosenbrock's function:

$$f(\mathbf{x}) = \sum_{i=1}^{n-1} \Big[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \Big]. \tag{1}$$

Compute $\nabla f(\mathbf{1})$.

Problem 10

Write a script in Python that, granted a natural number n and a point $\mathbf{x} \in \mathbb{R}^n$, approximates $\nabla f(\mathbf{x})$ of (1) using finite differences. Use this program to verify the results you got in problem 9. Make a table or a plot showing the value of the approximation $\|\nabla f(\mathbf{1})\|$ for some decreasing values of h.