Raciocínio probabilístico – redes Bayesianas

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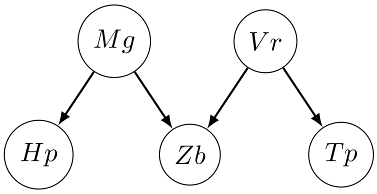
$Mg = \text{S}$	$Mg = \text{N}$
0.1	0.9

$Vr = \text{S}$	$Vr = \text{N}$
0.2	0.8



	$Hp = \text{S}$	$Hp = \text{N}$
$Mg = \text{S}$	0.8	0.2
$Mg = \text{N}$	0.7	0.3

	$Tp = \text{S}$	$Tp = \text{N}$
$Vr = \text{S}$	0.3	0.7
$Vr = \text{N}$	0.1	0.9

		$Zb = \text{S}$	$Zb = \text{N}$
$Mg = \text{S}$	$Vr = \text{S}$	0.6	0.4
$Mg = \text{S}$	$Vr = \text{N}$	0.5	0.5
$Mg = \text{N}$	$Vr = \text{S}$	0.4	0.6
$Mg = \text{N}$	$Vr = \text{N}$	0.01	0.99

1) Observando a tabela para o nodo Zb:

$$P(Zb = N \mid Mg = N \wedge Vr = N) = \mathbf{0.99}$$

2) Multiplica-se a probabilidade de todos os eventos serem verdadeiros entre si:

$$\begin{aligned}
&P(Mg = S \wedge Vr = S \wedge Hp = S \wedge Zb = S \wedge Tp = S)\\
&= P(Mg = S) \times P(Vr = S) \times P(Hp = S \mid Mg = S) \times P(Zb = S \mid Mg = S \wedge Vr = S) \times P(Tp = S \mid Vr = S)\\
&= 0.1 \times 0.2 \times 0.8 \times 0.6 \times 0.3 = \mathbf{0.00288}
\end{aligned}$$

3) Multiplica-se a probabilidade dos nodos predecessores de Zb, quando este tem valor verdadeiro, e soma-se todos estes valores:

$$\begin{aligned}
p_1 &= P(Mg = S \wedge Vr = S \wedge Zb = S) = 0.1 \times 0.2 \times 0.6 = 0.012\\
p_2 &= P(Mg = S \wedge Vr = N \wedge Zb = S) = 0.1 \times 0.8 \times 0.5 = 0.4\\
p_3 &= P(Mg = N \wedge Vr = S \wedge Zb = S) = 0.9 \times 0.2 \times 0.4 = 0.072\\
p_4 &= P(Mg = N \wedge Vr = N \wedge Zb = S) = 0.9 \times 0.8 \times 0.01 = 0.0072
\end{aligned}$$

$$P(Zb = S) = \sum_{i=1}^{4} p_i = 0.012 + 0.04 + 0.072 + 0.0072 = \mathbf{0.1312}$$

4) Multiplica-se a probabilidade de Vr e Zb quando estes têm valor verdadeiro, aplicando também a probabilidade de Mg nos dois casos, e soma-se estes valores:

$$\begin{aligned}
p_1 &= P(Mg = S \wedge Vr = S \wedge Zb = S) \times P(Mg = S) = 0.6 \times 0.1 = 0.06\\
p_2 &= P(Mg = N \wedge Vr = S \wedge Zb = S) \times P(Mg = N) = 0.4 \times 0.9 = 0.36
\end{aligned}$$

$$P(Zb = S \mid Vr = S) = \sum_{i=1}^{2} p_i = 0.06 + 0.36 = \mathbf{0.42}$$

5) Primeiramente, é necessário descobrir a probabilidade de Zb dado que Mg pode ou não acontecer:

$$\begin{aligned}
p_1 &= P(Mg = S \wedge Vr = S \wedge Zb = S) \times P(Vr = S) = 0.6 \times 0.2 = 0.12\\
p_2 &= P(Mg = S \wedge Vr = N \wedge Zb = S) \times P(Vr = N) = 0.5 \times 0.8 = 0.4
\end{aligned}$$

$$P(Zb = S \mid Mg = S) = \sum_{i=1}^{2} p_i = 0.12 + 0.4 = 0.52$$

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$$p_1 = P(Mg = N \wedge Vr = S \wedge Zb = S) \times P(Vr = S) = 0.4 \times 0.2 = 0.08$$
$$p_2 = P(Mg = N \wedge Vr = N \wedge Zb = S) \times P(Vr = N) = 0.01 \times 0.8 = 0.008$$
$$P(Zb = S \mid Mg = N) = \sum_{i=1}^{2} p_i = 0.08 + 0.008 = 0.088$$

Então, utilizando probabilidade condicional e resultados anteriores:

$$\begin{aligned}
P(Hp = S \mid Zb = S) &= \frac{P(Hp = S \wedge Zb = S \wedge Mg = S) + P(Hp = S \wedge Zb = S \wedge Mg = N)}{P(Zb = S)} \\
&= \frac{P(Mg = S) \times P(Hp = S \mid Mg = S) \times P(Zb = S \mid Mg = S)}{P(Zb = S)} \\
&+ \frac{P(Mg = N) \times P(Hp = S \mid Mg = N) \times P(Zb = S \mid Mg = N)}{P(Zb = S)} \\
&= \frac{0.1 \times 0.8 \times 0.52 + 0.9 \times 0.7 \times 0.088}{0.1312} = \mathbf{0.739\overline{63414}}
\end{aligned}$$

6) A probabilidade de Hp ser verdadeiro precisa ser calculada:

$$\begin{aligned}
P(Hp = S) &= P(Hp = S \mid Mg = S) \times P(Mg = S) + P(Hp = S \mid Mg = N) \times P(Mg = N) \\
&= 0.8 \times 0.1 + 0.7 \times 0.9 = 0.71
\end{aligned}$$

Então, pelo Teorema de Bayes:

$$P(Zb = S \mid Hp = S) = \frac{P(Hp = S \mid Zb = S) \times P(Zb = S)}{P(Hp = S)} \approx \frac{0.74 \times 0.1312}{0.71} \approx \mathbf{0.1367}$$

7) A probabilidade de Tp ser verdadeiro precisa ser calculada:

$$\begin{aligned}
P(Tp = S) &= P(Tp = S \mid Vr = S) \times P(Vr = S) + P(Tp = S \mid Vr = N) \times P(Vr = N) \\
&= 0.3 \times 0.2 + 0.1 \times 0.8 = 0.14
\end{aligned}$$

Então, utilizando probabilidade condicional e resultados anteriores:

$$\begin{aligned}
p_1 &= \frac{P(Zb = S \wedge Tp = S \wedge Hp = S \wedge Mg = S \wedge Vr = S)}{P(Tp = S \wedge Hp = S)} \\
&= \frac{P(Mg = S) \times P(Hp = S \mid Mg = S) \times P(Zb = S \mid Mg = S \wedge Vr = S) \times P(Vr = S) \times P(Tp = S \mid Vr = S)}{P(Hp = S) \times P(Tp = S)} \\
&= \frac{0.1 \times 0.8 \times 0.6 \times 0.2 \times 0.3}{0.71 \times 0.14} \approx 0.02897
\end{aligned}$$

$$\begin{aligned}
p_2 &= \frac{P(Zb = S \wedge Tp = S \wedge Hp = S \wedge Mg = S \wedge Vr = N)}{P(Tp = S \wedge Hp = S)} \\
&= \frac{P(Mg = S) \times P(Hp = S \mid Mg = S) \times P(Zb = S \mid Mg = S \wedge Vr = N) \times P(Vr = N) \times P(Tp = S \mid Vr = N)}{P(Hp = S) \times P(Tp = S)} \\
&= \frac{0.1 \times 0.8 \times 0.5 \times 0.8 \times 0.1}{0.71 \times 0.14} \approx 0.03219
\end{aligned}$$

$$\begin{aligned}
p_3 &= \frac{P(Zb = S \wedge Tp = S \wedge Hp = S \wedge Mg = N \wedge Vr = S)}{P(Tp = S \wedge Hp = S)} \\
&= \frac{P(Mg = N) \times P(Hp = S \mid Mg = N) \times P(Zb = S \mid Mg = N \wedge Vr = S) \times P(Vr = S) \times P(Tp = S \mid Vr = S)}{P(Hp = S) \times P(Tp = S)} \\
&= \frac{0.9 \times 0.7 \times 0.4 \times 0.2 \times 0.3}{0.71 \times 0.14} \approx 0.1521
\end{aligned}$$

$$\begin{aligned}
p_4 &= \frac{P(Zb = S \wedge Tp = S \wedge Hp = S \wedge Mg = N \wedge Vr = N)}{P(Tp = S \wedge Hp = S)} \\
&= \frac{P(Mg = N) \times P(Hp = S \mid Mg = N) \times P(Zb = S \mid Mg = N \wedge Vr = N) \times P(Vr = N) \times P(Tp = S \mid Vr = N)}{P(Hp = S) \times P(Tp = S)} \\
&= \frac{0.9 \times 0.7 \times 0.01 \times 0.8 \times 0.1}{0.71 \times 0.14} \approx 0.00507
\end{aligned}$$

$$P(Zb = S \mid Tp = S \wedge Hp = S) = \sum_{i=1}^{4} p_i \approx 0.02897 + 0.03219 + 0.1521 + 0.00507 \approx \mathbf{0.21833}$$