

Reduction of Key Sizes on Rainbow-like Multivariate Signature Schemes

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Context

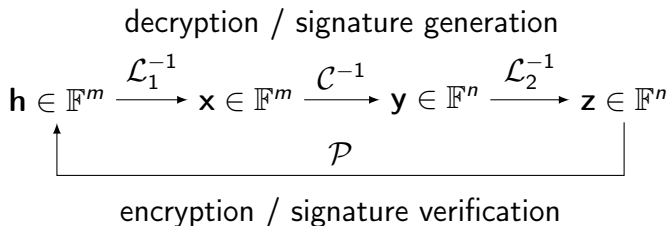
- ▶ Guarantee protection and privacy of messages sent digitally
- ▶ Security of digital signature schemes is based on problems from number theory
 - ▶ Integer factorization, discrete logarithm
- ▶ There exist quantum algorithms [Sho97] that solve such problems efficiently
- ▶ Post-quantum cryptography aims to create cryptosystems based on problems immune to quantum speed-ups

Motivation

- ▶ Foreshadowing of quantum computers
- ▶ Several active branches of post-quantum cryptography based on distinct mathematical structures
- ▶ Standardization calls by institutions such as NIST, IRTF and ETSI
- ▶ We focus on cryptosystems that are built upon the difficulty of solving systems of equations

Multivariate cryptography

- Cryptography based on systems of multivariate quadratic equations over finite fields
- Bipolar construction:

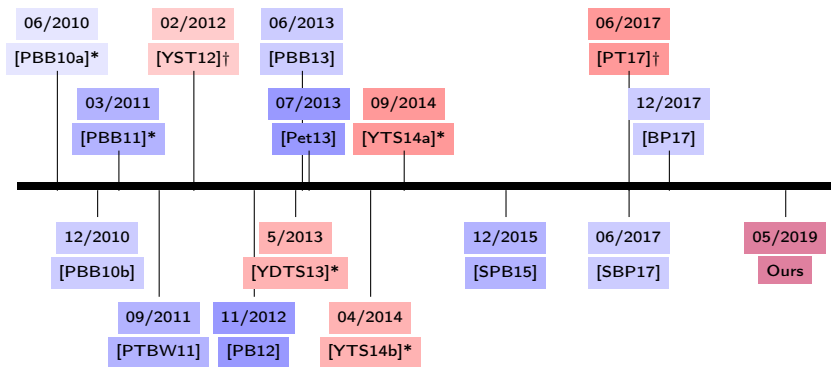


- Fast operations, small signature sizes and large keys, compared to conventional schemes

Research object

- ▶ Focus on the Rainbow signature scheme [DS05], currently on Round 2 of the NIST standardization process
- ▶ Easy description, good balance between signature and key sizes
- ▶ Keys are one to two orders of magnitude greater than conventional ones
- ▶ Generalized version of Unbalanced Oil and Vinegar [KPG99]

Related works



Works in blue optimise public keys, while red ones reduce private keys. Asterisks denote reparametrized works and crosses denote broken schemes.

Hypothesis

- ▶ To the best of our knowledge, works have reduced either private or public keys
- ▶ Introduction of structures in the keys may lower security
- ▶ **Can both reductions be achieved simultaneously?**

Rainbow signature scheme

Preliminaries

- ▶ Parameters are a finite field \mathbb{F}_q , integers u, n such that $u \leq n$ and $0 < v_1 < \dots < v_u < v_{u+1} = n$
- ▶ For $1 \leq \ell \leq u$, set vinegar variables $V_\ell = \{1, \dots, v_\ell\}$ and oil variables $O_\ell = \{v_\ell + 1, \dots, v_{\ell+1}\}$
- ▶ Define vector spaces spanned by quadratic Oil-Vinegar polynomials

$$P_\ell = \sum_{i,j \in V_\ell} \alpha_{ij} x_i x_j + \sum_{i \in V_\ell, j \in O_\ell} \beta_{ij} x_i x_j + \sum_{i \in V_\ell \cup O_\ell} \gamma_i x_i + \delta,$$
$$\alpha_{ij}, \beta_{ij}, \gamma_i, \delta \in \mathbb{F}_q$$

Rainbow signature scheme





Key generation

- ▶ Let $m = n - v_1$ and $o_\ell = v_{\ell+1} - v_\ell$
- ▶ Randomly pick two affine transformations $\mathcal{L}_1 : \mathbb{F}_q^m \rightarrow \mathbb{F}_q^m$ and $\mathcal{L}_2 : \mathbb{F}_q^n \rightarrow \mathbb{F}_q^n$
- ▶ Central map is a function $\mathcal{C} : \mathbb{F}_q^n \rightarrow \mathbb{F}_q^m$
 - ▶ A total of o_ℓ polynomials and respective coefficients are randomly chosen from each P_ℓ
- ▶ Private key is the 3-uple $(\mathcal{L}_1, \mathcal{C}, \mathcal{L}_2)$, public key is the composition $\mathcal{P} = \mathcal{L}_1 \circ \mathcal{C} \circ \mathcal{L}_2$

Rainbow signature scheme

Inversion of the central map

- ▶ Vinegar variables of a layer are exactly the oil and vinegar variables from the previous layer
- ▶ This enables the inversion of each Oil-Vinegar layer recursively
- ▶ With $u = 2$, the initial configuration of \mathcal{C} is

	$V_1 \times V_1$	$V_1 \times O_1$	$O_1 \times O_1$	$V_1 \times O_2$	$O_1 \times O_2$	$O_2 \times O_2$	V_1	O_1	O_2	δ
$\ell = 1$		0	0	0	0		0	*		
$\ell = 2$		0		*						

Rainbow signature scheme

Inversion of the central map

- Randomly choose variables in V_1 and substitute them

	$V_1 \times V_1$	$V_1 \times O_1$	$O_1 \times O_1$	$V_1 \times O_2$	$O_1 \times O_2$	$O_2 \times O_2$	V_1	O_1	O_2	δ
$\ell = 1$	*		0	0	0	0	*		0	*
$\ell = 2$	*					0	*			*

- Solve linear o_1 equations in the first layer to obtain V_2 , and then solve the remaining o_2 equations

	$V_1 \times V_1$	$V_1 \times O_1$	$O_1 \times O_1$	$V_1 \times O_2$	$O_1 \times O_2$	$O_2 \times O_2$	V_1	O_1	O_2	δ
$\ell = 2$	*	*	*		*	0	*	*		*

Rainbow signature scheme

Signature generation

- ▶ Consider a cryptographic hash function $\mathcal{H} : \{0, 1\}^* \rightarrow \mathbb{F}_q^m$ and a message M , and compute the digest $\mathbf{d} = \mathcal{H}(M)$
- ▶ Obtain the value $\mathbf{x} = \mathcal{L}_1^{-1}(\mathbf{d})$
- ▶ Generate the pre-image of \mathbf{x} under the central map, $\mathbf{y} = \mathcal{C}^{-1}(\mathbf{x})$, as per the operations above
- ▶ Compute the final signature $\mathbf{z} = \mathcal{L}_2^{-1}(\mathbf{y})$

Rainbow signature scheme

Signature verification

- ▶ Obtain \mathbf{d} from the message M
- ▶ Compute $\mathbf{d}' = \mathcal{P}(\mathbf{z})$
- ▶ The signature is valid if $\mathbf{d} = \mathbf{d}'$, and invalid otherwise

Our proposal

- ▶ Introduction of structures in the private key is dangerous and may lead to security holes
- ▶ Observe that, to invert the central map, vinegar variables are chosen randomly every time a preimage is computed
- ▶ If these variables are changed less often, or fixed, they simplify the central map matrix representation
- ▶ Indeed, the central map may be stored in a linearized fashion, and regenerated only occasionally

Our proposal

- ▶ We propose to fix the V_1 variables throughout the central map
- ▶ A possible implementation of this method is to use a PRNG to regenerate \mathcal{C} every time it is needed
- ▶ The linear relations described in CyclicRainbow [PBB10b] are another way to obtain the original central map
- ▶ The EUF-CMA variant described in [DCP⁺17] provides a salt that can be modified instead of vinegar variables

Our proposal

- ▶ Only variables in V_1 are fixed, since V_2 and beyond need the digest to be calculated
- ▶ Strategy is not hindered by current cryptanalytic methods, since the choice of parameters does not change
- ▶ General framework for every Rainbow-like scheme
- ▶ Can be applied on top of variants that reduce the public key, confirming our hypothesis

Open problems

- ▶ Given multiple signatures created with the same set of vinegar variables, is it possible to unveil information about the private key?
- ▶ Does every signature needs its own set of vinegar variables or is the cost of regenerating the central map amortized?
- ▶ Is it possible to create a constant-time implementation with this strategy?
- ▶ Do there exist parameter sets which optimize the private key size?

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