# Homework 4

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## 1 Multilayer Perceptron

There are 2 units on input layer, 6 units on hidden layer and 2 units on output layer. The $W, b$ for each layer edges are as following:

$$W^{(1)} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} b^{(1)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$W^{(2)} = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} b^{(2)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

How does it work?

$$\begin{aligned} h1 &= ReLU(x_1 - x_2) \\ h2 &= ReLU(x_2 - x_1) \\ h3 &= ReLU(x_1) \\ h4 &= ReLU(x_2) \\ h5 &= ReLU(-x_1) \\ h6 &= ReLU(-x_2) \end{aligned}$$

We can get $y_1$, $y_2$

$$\begin{aligned} y1 &= -\frac{1}{2}h1 + -\frac{1}{2}h2 + \frac{1}{2}h3 + \frac{1}{2}h4 + -\frac{1}{2}h5 + -\frac{1}{2}h6 \\ &= -\frac{1}{2}h1 + -\frac{1}{2}h2 + \frac{1}{2}(h3 - h5) + \frac{1}{2}(h4 - h6) \\ &= -\frac{1}{2}h1 + -\frac{1}{2}h2 + \frac{1}{2}(max(0, x_1) - max(0, -x_1)) + \frac{1}{2}(max(0, x_2) - max(0, -x_2)) \end{aligned}$$

If $x_1 \geq 0$, $max(0, x_1) - max(0, -x_1) = x_1 - 0 = x_1$;

If $x_1 < 0$, $max(0, x_1) - max(0, -x_1) = 0 - (-x_1) = x_1$; So is $x_2$. Then,

$$\begin{aligned} y1 &= -\frac{1}{2}h1 + -\frac{1}{2}h2 + \frac{1}{2}x_1 + \frac{1}{2}x_2 \\ y2 &= \frac{1}{2}h1 + \frac{1}{2}h2 + +\frac{1}{2}x_1 + \frac{1}{2}x_2 \end{aligned}$$

If $x_1 \geq x_2$,

$$h1 = x_1 - x_2$$
$$h2 = 0$$

Then,

$$\begin{aligned} y1 &= -\frac{1}{2}(x_1 - x_2) + -\frac{1}{2} * 0 + \frac{1}{2}x_1 + \frac{1}{2}x_2 \\ &= -\frac{1}{2}x_1 + -\frac{1}{2}(-x_2) + \frac{1}{2}x_1 + \frac{1}{2}x_2 \\ &= x_2 \\ y2 &= \frac{1}{2}(x_1 - x_2) + \frac{1}{2} * 0 + \frac{1}{2}x_1 + \frac{1}{2}x_2 \\ &= x_1 \end{aligned}$$

If $x_1 < x_2$,

$$h1 = 0$$
$$h2 = x_2 - x_1$$

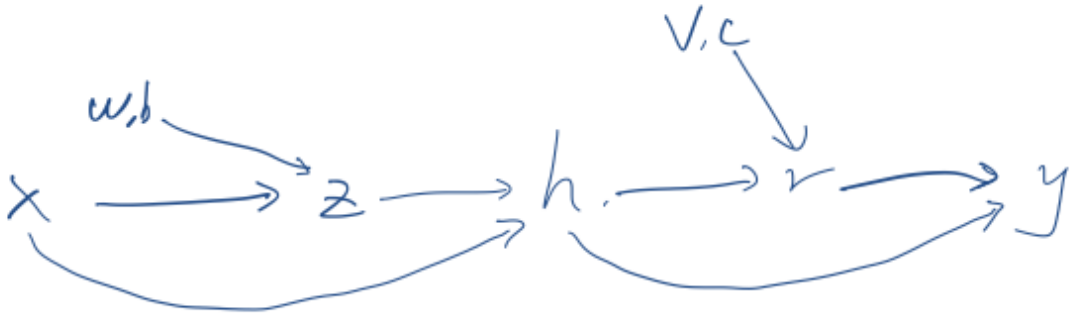
Then,

$$\begin{aligned} y1 &= -\frac{1}{2}(x_2 - x_1) + \frac{1}{2}x_1 + \frac{1}{2}x_2 \\ &= x_1 \\ y2 &= \frac{1}{2}(x_2 - x_1) + \frac{1}{2}x_1 + \frac{1}{2}x_2 \\ &= x_2 \end{aligned}$$

They are sorted in order.

## 2 Backprop

### (a)




### (b)

Assume we know $\overline{y} = \frac{\mathrm{d}L}{\mathrm{d}y}$