Algebraic Geometry

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THIS IS A DRAFT. Expect inaccuracte and incomplete information.

Introduction

Algebraic Geometry is a field of mathematics with roots as far back as at least the 19^{th} century, and with major advancement and applications in recent years. At its core, Algebraic Geometry is concerned with studying the sets of zeroes of a system of polynomials, in particular their algebraic structure. Though real polynomials are surely the first that come to mind, much of the power in Algebraic Geometry comes from the generalization to polynomials with coefficients in arbitrary fields.

The undergraduate mathematics student would hopefully find this paper accessible, but experience with algebra and geometry would help. A "Basics" section is included, but will not be exhaustive. As unhelpful as it seems, it's recommended to do independent research as necessary using Wikipedia, YouTube (videotaped lectures) and the referenced textbooks.

1 Basics

Here's where I'll put all the theorems and definitions that are simply necessary. I'll definitely talk about projective space here, and homogenous equations.

2 Conic Sections

Use conic sections to illustrate some basics here and provide necessary theorems/tools for elliptic curve section. Will draw heavily from the book

3 Bezout's Theorem

Bezout's theorem will be important later, but we probably won't be able to fully prove it here. However, the book does prove the limited cases that we'll most rely on.

4 Elliptic Curves

One of the more interesting sections. Definitely need to cover proving the given construction on the non-singuler elliptic curve creates a group. Should also cover how in practice for crypto the identity is chosen as the point at infinity, allowing for much cheaper computation.

5 Regular Polyhedra??

If we even get here, this should be a neat topic to talk about for a bit to offset the earlier material.