APPM 4600 Lab 3
Playing with the bisection and fixed point

1 Overview

In this lab, you will play with some root finding algorithms from class. You will explore the limitations and capabilities of the different methods.

2 Before lab

There is not much for you to do before this lab. You should rename your repo files so that they are named APPM 4600 and you have both a homework and lab folder. These should have subfolders for each of the labs and homeworks. Then you should have the bisection and fixed point example codes from lecture downloaded so you are ready to make the class at the beginning of the lab session.

3 Lab day: Exploring the capabilities of the root finding algorithm

To start lab you will begin by refreshing your memory on the different algorithms.

For the bisection method

- What does the method do?
- What is required for the method to work?

For the fixed point iteration

- What problem is the fixed point iteration trying to solve?
- How do you recast that problem as a root finding problem?
- What properties are required for the method to work?

Depending on how far we get in class you may or may not be able to answer the last 2 questions.

4 Exercises

1. Consider the function $f(x) = x^2(x-1)$ and use bisection with the following starting intervals.

   (a) $(a, b) = (0.5, 2)$
   (b) $(a, b) = (-1, 0.5)$
   (c) $(a, b) = (-1, 2)$

   What happens for each choice of interval? If the method is not successful, explain why. Is it possible for bisection to find the root $x = 0$?

2. The following four methods are proposed for computing $7^{1/5}$. Rank them in order, based on their apparent speed of convergence, assuming $p_0 = 1$.

(a) $p_n = p_{n-1}\left(1 + \frac{7-p_{n-1}^5}{p_{n-1}^2}\right)^3$

(b) $p_n = p_{n-1} - \frac{p_{n-1}^5-7}{p_{n-1}^2}$

(c) $p_n = p_{n-1} - \frac{p_{n-1}^5-7}{5p_{n-1}^4}$

(d) $p_n = p_{n-1} - \frac{p_{n-1}^5-7}{12}$

How do you determine the speed of convergence?

3. Use your fixed point iteration code to see if your solutions to the previous problem are correct. Does the stopping tolerance impact if your answers from the previous problem are correct? For example, if you set the tolerance to 10^{-2} and 10^{-13}, does this impact how correct your answers the previous question were? If so, try to explain why?

4. Use a fixed point iteration method to determine a solution accurate to within 10^{-2} for

$$x^4 - 3x^2 - 3 = 0$$

on $[1, 2]$. Use $p_0 = 1$ as your initial guess.

5 Deliverables

To receive full credit for this lab, you need to commit and push your lab codes to git.