Rank Revealing QR Factorizations

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December 19, 2023

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Definition: Numerically Rank Deficient

A matrix is numerically rank deficient if there are $k < n$ singular values of A that are larger than $\epsilon > 0$, or equivalently $n - k$ singular values that are smaller than $\epsilon > 0$.

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- Several common RRFs used in practice
  1. SVD
  2. QR Factorization
- The RRF of a matrix $A \in \mathbb{R}^{m \times n}$ can be defined as

Definition: RRF

$A = XDY^T, X \in \mathbb{R}^{m \times p}, D \in \mathbb{R}^{p \times p}, Y \in \mathbb{R}^{n \times p}$ where

1. $p < min\{m, n\}$
2. D is diagonal and nonsingular
3. X and Y are well conditioned

QR Factorization as an RRF

Definition: RRF

$A = XDY^T, X \in \mathbb{R}^{m \times p}, D \in \mathbb{R}^{p \times p}, Y \in \mathbb{R}^{n \times p}$

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- This basic definition of the RRF with the QR factorization can lead to potential problems

Numerical Example

- Consider the following matrix and its QR factorization

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- What about Y?
  - D is a non-invertible matrix, so we cannot find Y

QR Factorization with Column Pivoting

The QR factorization with column pivoting for a matrix $A \in \mathbb{R}^{m \times n}$ is

Definition: QR with Pivoting

$AP = QR$ where

1. $P \in \mathbb{R}^{m \times m}$ is a permutation matrix
2. $Q \in \mathbb{R}^{m \times m}$ has orthonormal columns
3. $R \in \mathbb{R}^{m \times n}$ is upper triangular where $|r_{11}| \geq |r_{22}| \geq \cdots \geq |r_{nn}|$

Numerical Rank with Column Pivoting

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$$AP = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{bmatrix} = Q_1 \begin{bmatrix} R_{11} & R_{12} \end{bmatrix} + \begin{bmatrix} 0 & Q_2 R_{22} \end{bmatrix}$$

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2. $Q_1 \in \mathbb{R}^{m \times k}$ and $Q_2 \in \mathbb{R}^{m \times (n-k)}$

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- R_{22} is numerically negligible if the following inequality holds, where F is the Frobenius norm

$$||R_{22}||_2 \leq ||R_{22}||_F \leq 2^{-\frac{1}{2}}(n-k+1)\epsilon \tag{3}$$

Development of the QR Algorithm

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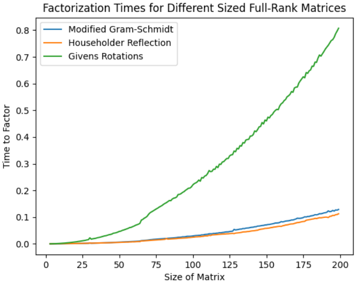
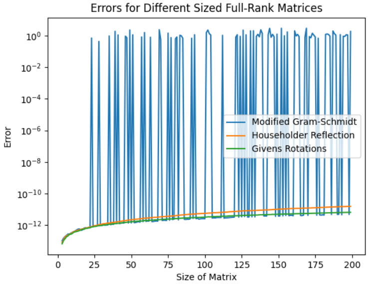
Comparing QR Implementations





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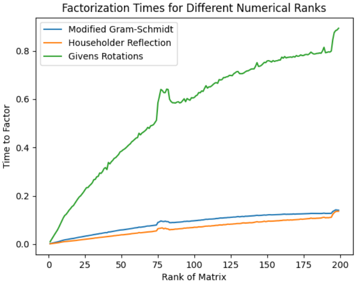

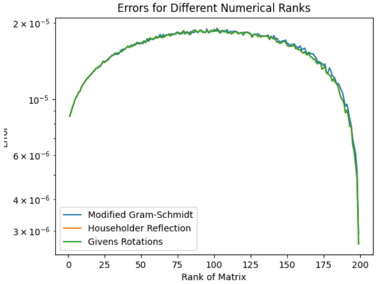

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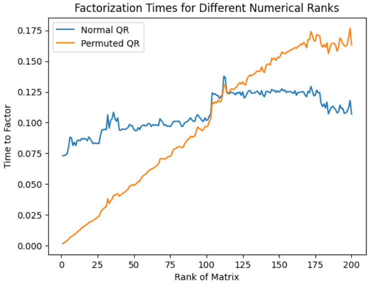

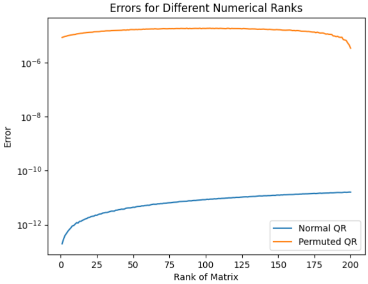

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  - Inspired by recent machine learning techniques to produce the song *Now and Then* by the Beatles

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  - Consider a signal $s \in \mathbb{R}^N$ where $s = \begin{bmatrix} s_1 & s_2 & \dots & s_N \end{bmatrix}^T$
  - The corresponding Hankel matrix is

$$\mathbb{H}(s) = \begin{bmatrix} s_1 & s_2 & s_3 & \dots & s_n \\ s_2 & s_3 & s_4 & \dots & s_{n+1} \\ s_3 & s_4 & s_5 & \dots & s_{n+2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ s_m & s_{m+1} & s_{m+2} & \dots & s_N \end{bmatrix}$$

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- What is a covariance matrix?
  - A square matrix C where the diagonal entries are the variance and the remaining entries are the covariance.

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  5. Force rank k on Hankel matrix, which gives us an estimate of the pure signal
  6. Extract pure signal from rank k Hankel approximation through summing the anti-diagonal elements

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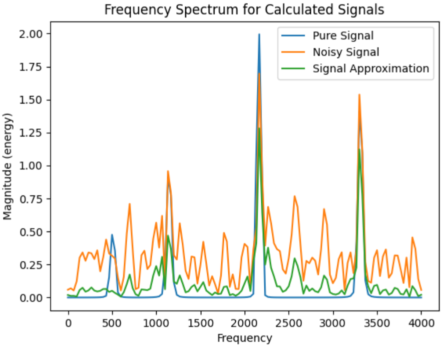

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- Then for each interval, we complete steps 2 - 6 previously mentioned
- Recombine estimated signal segments

Speech Signal Results

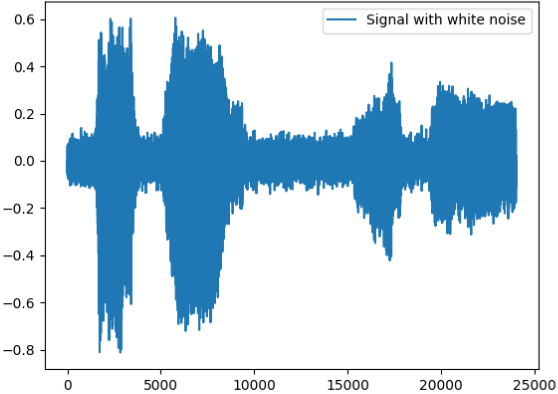

Figure: Noisy Speech Signal

Speech Signal Results

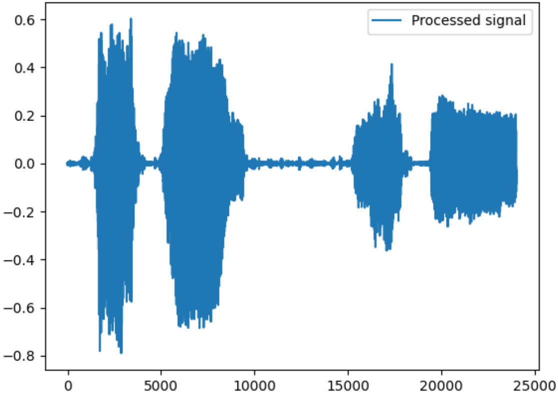

Figure: Processed Signal

Speech Signal Results

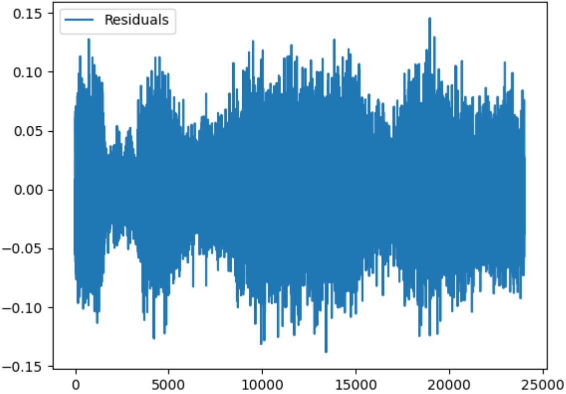

Figure: Residuals

Speech Signal Results

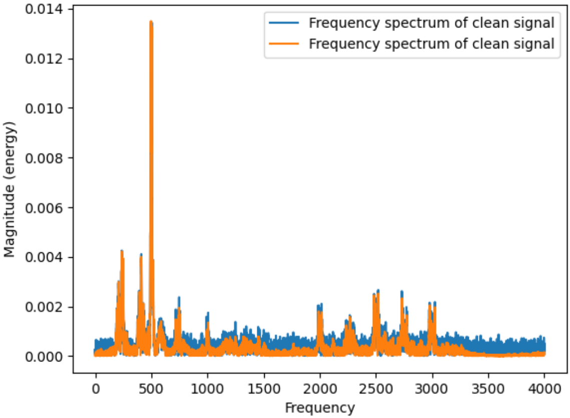

Figure: Spectra Error

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- Householder reflection matrices provided the best balance between speed of factorization and stability when compared against both matrix size and matrix rank
- The RRQRF can extract pure and noisy signals from pure sine waves and speech signals
- Can form good estimates without having any knowledge about the noise in the signal itself.

Future Work

- Continuing to increase the efficiency of the QR algorithm

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- Further explore non-white noise methods mentioned in the paper
  - General noise
  - Rank-deficient noise

References

[1] Per Christian Hansen and Søren Holdt Jensen. "Subspace-Based Noise Reduction for Speech Signals via Diagonal and Triangular Matrix Decompositions: Survey and Analysis". In: *EURASIP Journal on Advances in Signal Processing* 2007.1 (2007), p. 092953. DOI: 10.1155/2007/92953. URL: https://doi.org/10.1155/2007/92953.