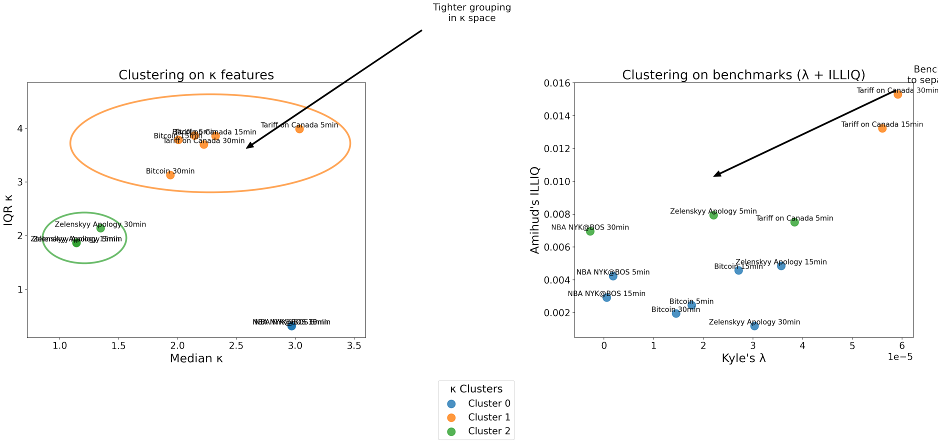
Clustering Results



Clustering: κ vs. classical benchmarks

market	freq	κ_{med}	κ_{IQR}	Kyle λ	Amihud ILLIQ
Bitcoin	5min	2.14	3.86	1.8×10^{-5}	0.0025
Bitcoin	15min	2.01	3.79	2.7×10^{-5}	0.0046
Bitcoin	30min	1.94	3.13	1.4×10^{-5}	0.0020
NBA NYK@BOS	5min	2.97	0.32	1.8×10^{-6}	0.0042
NBA NYK@BOS	15min	2.97	0.32	5.4×10^{-7}	0.0029
NBA NYK@BOS	30min	2.97	0.32	-2.8×10^{-6}	0.0070
Zelenskyy	5min	1.14	1.86	2.2×10^{-5}	0.0079
Zelenskyy	15min	1.14	1.87	3.6×10^{-5}	0.0049
Zelenskyy	30min	1.35	2.14	3.0×10^{-5}	0.0012
Tariff	5min	3.04	3.99	3.8×10^{-5}	0.0075
Tariff	15min	2.32	3.86	5.6×10^{-5}	0.0132
Tariff	30min	2.22	3.70	5.9×10^{-5}	0.0153

Silhouette (κ features): **0.82** Silhouette (λ + ILLIQ): **0.65**

- **Kyle's λ:** In equities it is "\$ per share." Here it becomes log-odds per USDC flow, so values scale with market size (Bitcoin vs celebrity) rather than true liquidity.
- **Amihud's ILLIQ:** In stocks, returns and volume are standardised. In bounded $[0, 1]$ prices, returns compress near $0/1$ and volume depends on arbitrary contract units, giving design-driven distortions.
- **Fragility:** Both shift by orders of magnitude if you rescale flow or change window size. Silhouette ≈ 0.65 reflects scale artefacts, not real structure.
- **Contrast:** $\kappa = b/\tau$ is dimensionless and stable. A $\kappa = 2$ has the same meaning in Bitcoin, politics, or sports: resistance of price to belief gaps.

LMSR mechanics: cost, price, log-odds, curvature

Cost (binary LMSR).

$$C(q_{\text{yes}}, q_{\text{no}}) = b \, \ln\left(e^{q_{\text{yes}}/b} + e^{q_{\text{no}}/b}\right), \qquad b > 0$$

Gradient (prices).

$$\frac{\partial C}{\partial q_{\text{yes}}} = \frac{e^{q_{\text{yes}}/b}}{e^{q_{\text{yes}}/b} + e^{q_{\text{no}}/b}} = p, \qquad \frac{\partial C}{\partial q_{\text{no}}} = 1 - p$$

Log-odds.

$$\ell = \text{logit}(p) = \frac{q_{\text{yes}} - q_{\text{no}}}{b} \quad \Rightarrow \quad \Delta\ell = \frac{1}{b}\,\Delta q.$$

CARA certainty equivalent and optimal trade

Wealth increment:

$$\Delta W = \Delta q Y - \Delta C = \quad Y \sim \text{Bernoulli}(\pi).$$

Under CARA utility and normally distributed wealth increments, expected utility is equivalent to maximising a certainty equivalent:

$$CE = \mathbb{E}[\Delta W] - \tfrac{1}{2\tau}\mathrm{Var}(\Delta W).$$

$$CE = \Delta q(\pi - p) - \tfrac{1}{2}\left[\tfrac{p(1-p)}{b} + \tfrac{1}{\tau}\pi(1-\pi)\right]\Delta q^2.$$

FOC:

$$\frac{dCE}{d\Delta q} = (\pi - p) - \left[\tfrac{p(1-p)}{b} + \tfrac{1}{\tau}\pi(1-\pi)\right]\Delta q = 0$$

Solution:

$$\Delta q \approx \tau\left[\text{logit}(\pi) - \text{logit}(p)\right]$$

From demand to κ estimator

LMSR mapping:

$$\Delta\ell = \frac{1}{b}\Delta q \approx \ \frac{\tau}{b}(\text{logit}(\pi) - \ell).$$

Define risk-adjusted liquidity:

$$\kappa = \frac{b}{\tau}.$$

Estimator (window w):

$$\widehat{\kappa}_w = \frac{\text{logit}(\pi_w) - \ell_{w,\text{end}}}{\Delta\ell_w}, \qquad \Delta\ell_w = \ell_{w,\text{end}} - \ell_{w,\text{start}}.$$