Updates and Outstanding Questions

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Easy Changes (Completed)

- Added explicit statement: a trader who buys at price p must believe the event probability exceeds p.
- Corrected spelling and removed unnecessary `\times` symbols.
- Replaced *ILLIQ* notation with scalar A.
- Reduced repetition between Sections 1, 2.2, and 3.
- Standardised notation: *parametrise*, and adopted a clean binary LMSR form.
- Binary LMSR cost function:

$$C(q_{\mathrm{Yes}}, q_{\mathrm{No}}) = b \log\left(e^{q_{\mathrm{Yes}}/b} + e^{q_{\mathrm{No}}/b}\right).$$

- Price defined via explicit partial derivative:

$$p = \frac{\partial C}{\partial q_{\mathrm{Yes}}} = \frac{e^{q_{\mathrm{Yes}}/b}}{e^{q_{\mathrm{No}}/b} + e^{q_{\mathrm{Yes}}/b}}.$$

- Mapping to log-odds:

$$Q = q_{\mathrm{Yes}} - q_{\mathrm{No}}, \quad \ell = \log \frac{p}{1-p} = \frac{Q}{b}, \quad \Delta \ell \approx \frac{1}{b} \Delta Q.$$

Medium Changes (Mathematical Clean-Up)

- A key property of CARA utility is that uncertain payoffs can be summarised by a *certainty equivalent*:

$$\mathrm{CE} \approx \mathbb{E}[\Delta W] - \frac{1}{2\tau}\,\mathrm{Var}(\Delta W).$$

- **Why the penalty is $\frac{1}{2\tau}$:** For small trades, a second-order Taylor expansion of expected utility shows that curvature of exponential utility is constant and equal to $1/\tau$. Thus the first-order term gives the mean, and the second-order term gives a variance penalty weighted by that curvature:

$$\text{risk penalty} = \frac{1}{2} \cdot \frac{1}{\tau} \cdot \mathrm{Var}(\Delta W).$$

- Interpretation:
  - mean term: expected profit of the trade,
  - variance term: discounted by risk tolerance,
  - larger τ = smaller penalty = more willingness to take risk.
- small trade: $\Delta W \approx \Delta Q(Y - p)$ with $Y \sim \mathrm{Bernoulli}(\pi)$, giving

$$\mathbb{E}[\Delta W] = \Delta Q(\pi - p), \quad \mathrm{Var}(\Delta W) = \Delta Q^2 \pi(1-\pi).$$

- Final expression used throughout the model:

$$\mathrm{CE}(\Delta Q) = \Delta Q(\pi - p) - \frac{1}{2\tau}\Delta Q^2 \pi(1-\pi).$$

Belief Rationality Constraint (Why $\pi > p$ for Buys)

Why the model requires $\pi > p$ for buys and $\pi < p$ for sells

- Under CARA utility, the trader chooses ΔQ to maximise:

$$\mathrm{CE}(\Delta Q) = \Delta Q(\pi - p) - \frac{1}{2\tau}\Delta Q^2 \pi(1-\pi).$$

- The **sign of the optimal trade** comes entirely from the expected-gain term:

$$\mathrm{sign}(\Delta Q^*) = \mathrm{sign}(\pi - p).$$

- Intuition (non-economist friendly):
  - If the trader thinks the event is *more likely* than the market price implies ($\pi > p$), buying increases wealth in expectation.
  - If the trader thinks the event is *less likely* than the price implies ($\pi < p$), selling increases wealth in expectation.
- This matches Chakraborty–Das (2015): trades always push the market price *toward the trader's belief*, never away from it.
- Therefore, allowing a "buy" with $\pi < p$ (or a "sell" with $\pi > p$) is a structural error: the model would imply the trader knowingly accepts a negative expected return.

Consequences for Estimation and the Chakraborty–Das Fixed Point

Consequences of enforcing $\pi > p$ for buys and $\pi < p$ for sells

- All $\kappa = b/\tau$ estimates must be recomputed using only direction-consistent π.
- Fixed grids such as $\pi = 0.6$ are invalid when many observed trades occur at $p > 0.6$.
- The previous specification breaks the first-order condition and produces invalid trade magnitudes.

Chakraborty–Das (2015) fixed-point issue

- Optimal demand ΔQ^* depends on (p, π, τ, b).
- But LMSR price p depends on Q, so beliefs $\rightarrow$ trades $\rightarrow$ price feedback creates a fixed-point problem.
- **Key methodological decision needed:**
  - Whether to solve the *full Chakraborty–Das fixed point* (belief $\rightarrow$ demand $\rightarrow$ price $\rightarrow$ belief) at the trade level.
  - Or to maintain the *local linearised adjustment rule* (using $\Delta\ell \approx (\tau/b)(logit(\pi) - \ell)$), which is what κ estimation currently relies on.