Laying the Theoretical Foundation for a Game Theory Application to Pitch Sequencing and the Pitcher-Batter Matchup

University of San Francisco Baseball

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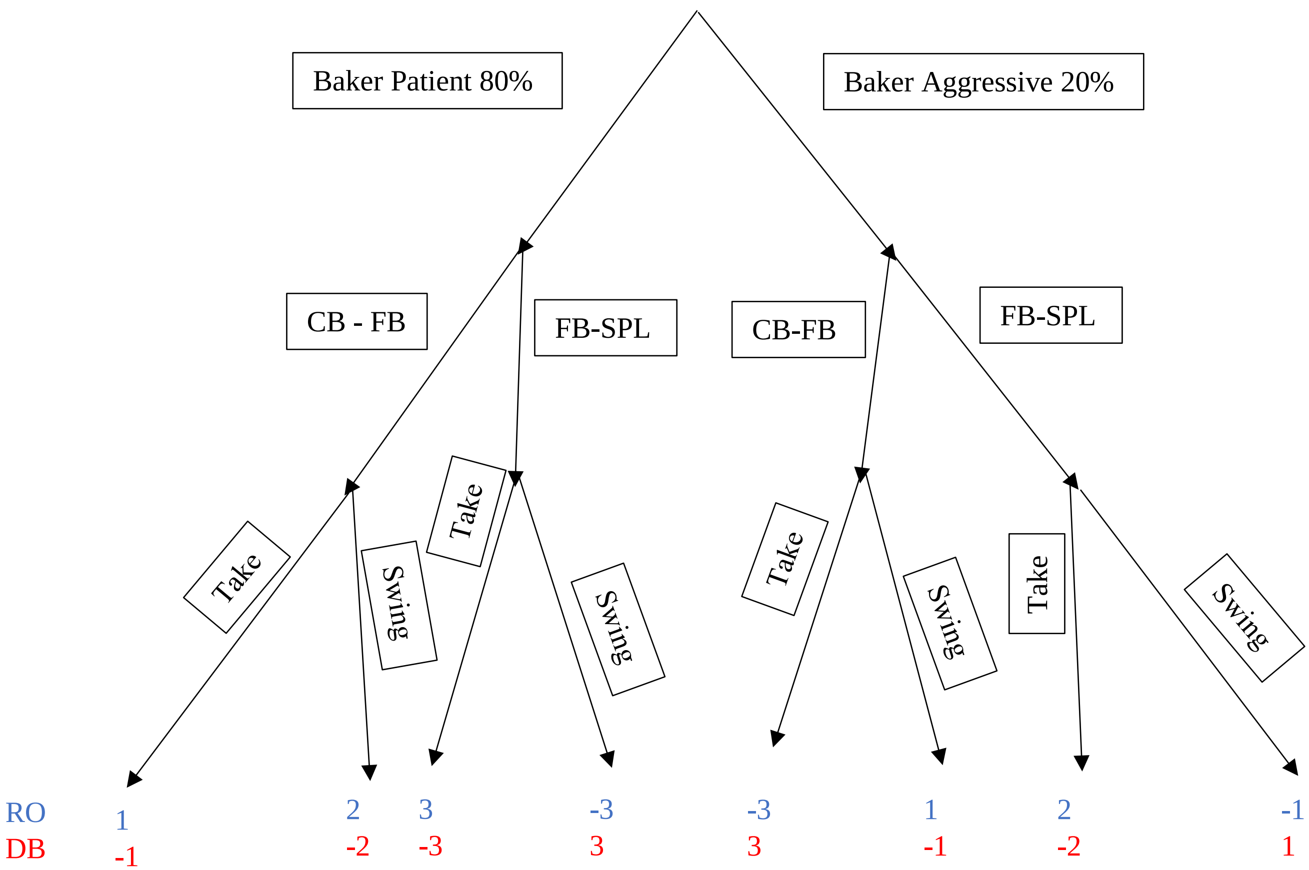


Introduction

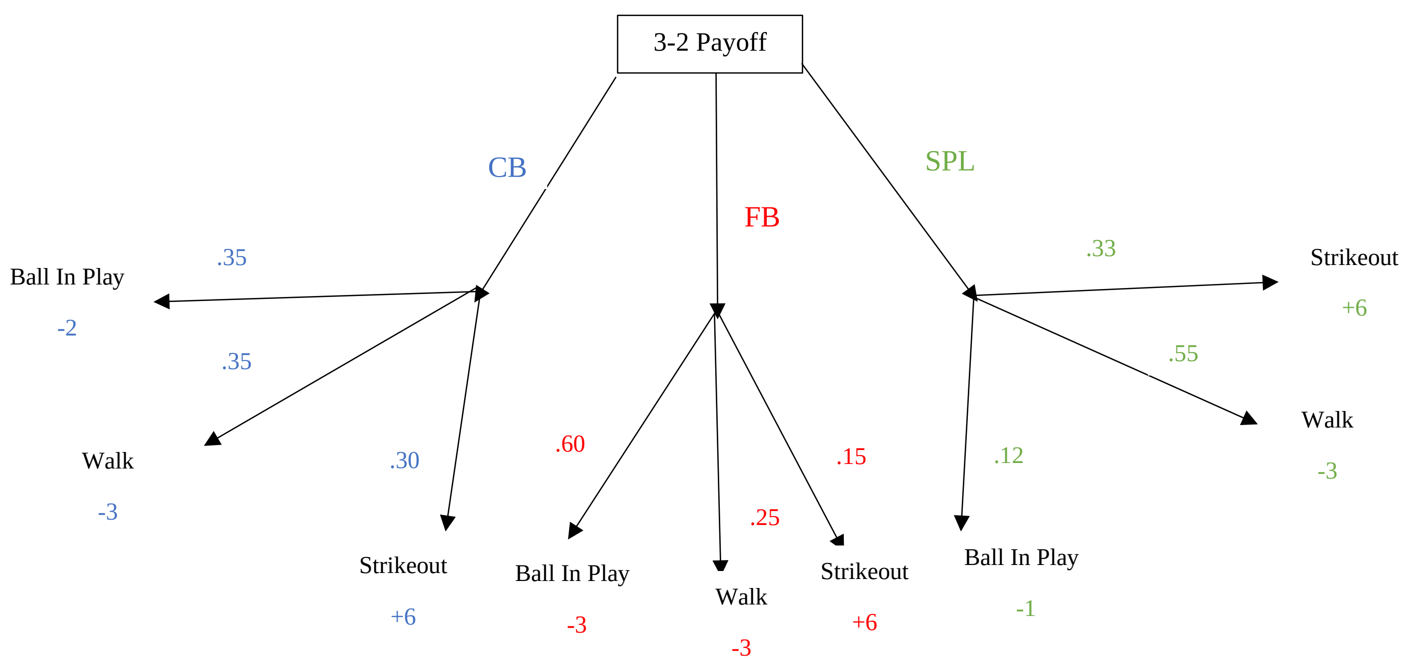
This writing is a theoretical exploration of different Game Theory frameworks and how they could eventually be applied and scaled to thinking about a plate appearance for a pitcher. Game Theory is the study of the interaction between both rational and irrational agents in a variety of contexts. The definition is broad and a lot fits under the Game Theory umbrella. This writing serves the purpose of examining different items theoretically across the Game Theory spectrum and laying a foundation for what could turn into actionable, scaled Game Theory models for use in Advanced Scouting / Game planning contexts…

The writing will include notation, examples, and hopefully easy to understand commentary. The role of Nature in Payoffs and Probability, Normal and Extensive Form games, Mixed Strategies, Nash Equilibrium, and Bayesian Games will be elaborated upon. *Numbers used in examples and/or explanation are arbitrary in nature, unless otherwise stated*.

Nature, Payoffs, and Probability

On March 7, 2020, USF's Riley Ornido faced Darren Baker. The sequence of pitches went like this: FB Ball (1-0), SPL Ball (2-0), SPL Ball (3-0), FB middle-middle, take strike (3-1), FB middle-up take strike (3-2), SPL down-down, swing and miss (K).

This plate appearance will be referenced throughout this writing, but the intuitive connections must start with nature, payoffs, and probability. Understanding how nature, payoffs, and probability interact is the goal. Was the 3-2 Splitter the right pitch called? Nature ultimately made its choice and Baker swung over the splitter in the dirt for strike 3. Nature is the force that pushes a dice in every which direction when it is rolled. Decisions should be made controlling for as much outside of Nature. It is too easy to look back now and say that Splitter was the right pitch if there is no examination of the probabilities and payoffs outside of Nature. The situation is modeled below:



Pitch Selection and Outcome Probabilities

	K	Walk	Ball In Play
FB	.15	.25	.60
CB	.30	.40	.30
SPL	.33	.55	.12

Pitch Selection and Outcome Payouts

	K	Walk	Ball In Play
FB	+6	-3	-3
CB	+6	-3	-2
SPL	+6	-3	-1

In this example we know a few things: pitches that can be called, the probabilities of observing specific outcomes for the different pitch types, and the different outcome payoffs for the different pitch types. What is the best pitch selection given different probabilities and payoffs for balls in play for different pitch types? Intuitively, we want to place greater weight on higher probability events. We weight the payoffs by the probabilities to create a value of each pitch selection:

$$Pitch\,Value = (K\ Payoff * K\ Probability) + (Walk\ Payoff * Walk\ Probability) + (BIP\ Payoff * BIP\ Probability)$$

$$FB = (.15 * 6) + (.25 * -3) + (.60 * -3) = -1.65$$

$$CB = (.30 * 6) + (.35 * -3) + (.35 * -2) = .05$$

$$SPL = (.33 * 6) + (.55 * -3) + (.12 * -1) = .21$$

Given the information of payoffs and probabilities, Splitter is on average the best pitch selection. That is the best we can do. Nature is going to reach into the jar of Splitters comprised of 33 Strikeouts, 55 Walks, and 12 Balls in Play and randomly select one. At this point, we let nature run its course, but we have confidence and objective backing in our reasoning that we did not have without the above analysis.

Normal Form Games and Dominated Strategies

1 / 2	Rock	Paper	Scissors
Rock	0 , 0	-1, +1	+1 , -1
Paper	+1 , -1	0 , 0	-1, +1
Scissors	-1, +1	+1, -1	0 , 0

The above game is rock-paper-scissors. In this game, a finite set of players have a set of pure strategies (rock, paper, or scissors) to choose from and are subject to payoff functions depending on what the other player does. This is a Normal Form Game.

In the Normal Form Game, a *Pure Strategy* is when a player makes their move deterministically before engagement. Choices are deterministic if they are made without any component of randomness, that the player CHOOSES the certain strategy. Before player 1 engages, he knows he will play paper. Before player 2 engages, he knows he will play rock. (Reference above table)

Rock-Paper-Scissors-SHOOT! Player 1 chooses Rock, player 2 chooses Scissors. Player 1's payoff is +1, player 2's payoff is -1.

Again…

Rock-Paper-Scissors-SHOOT! Player 1 chooses Scissors, player 2 chooses Rock. Player 1's payoff is -1, player 2's payoff is +1

Player 1 is the row player. The row player's payoff is always the left value in a cell. Player 2 is the column player. The column player's payoff is always the right value in a cell.

Back to Baseball

Ornido / Baker	Sit Fastball	Expand Zone	See it Late
FB	-5 , +5	-2, 0	+3 , -2
SPL	+4 , -3	+3, +1	-2, +2
CB	+2, -4	+2, -1	0 , +2

The table displays different payoffs for different combinations of Darren Baker approaches and Riley Ornido pitch selections. Before the pitch is made, Ornido and Baker select a strategy. The specific payoff cell in the table is a result of both strategies.

In our example, why would Ornido ever throw the Curveball? In a rational state, he would not. Why? Under no Darren Baker strategy, does the Curveball have the highest payoff. If Baker sits Fastball, Ornido's highest payoff pitch is the Splitter. If Baker expands the zone, Ornido's highest payoff pitch is the Splitter. And if Baker sees it late, Ornido's highest payoff pitch is Fastball.

Ornido Dominant Strategy if Baker Sits Fastball

Ornido / Baker	Sit Fastball
FB	-5 , +5
SPL	+4 , -3
CB	+2, -4

Ornido Dominant Strategy if Baker Expanded the Zone

Ornido / Baker	Expand Zone
FB	-2, 0
SPL	+3, +1
CB	+2, -1

Ornido Dominant Strategy if Baker Sees it Late

Ornido / Baker	See it Late
FB	+3 , -2
SPL	-2, +2
CB	0 , +2

At this point, Curveball is a *strictly dominated* strategy. Under no circumstance, in this game, should a Curveball be thrown on the payoff pitch. We can now make Ornido's selections a bit more concise by eliminating the row of Curveballs.

Ornido / Baker	Sit Fastball	Expand Zone	See it Late
FB	-5 , +5	-2, +0	+3 , -2
SPL	+4 , -3	+3, +1	-2, +2

We can examine Baker's payoffs and apply the same exact reasoning in the same way in the table above. For Baker, should he ever choose to employ an expand the zone strategy? He should not. This strategy is also strictly dominated. If Ornido were to throw a Fastball, the highest payoff strategy would be to Sit Fastball. If Ornido threw a Splitter, Baker's best strategy would be to See it Late. Ornido should never throw a Curveball, so we do not need to examine that pitch.

Ornido / Baker	Sit Fastball	Expand Zone	See it Late
FB	-5 , +5	-2, +0	+3 , -2

Ornido / Baker	Sit Fastball	Expand Zone	See it Late
SPL	+4 , -3	+3, +1	-2, +2

At this point, in theory, the assumption is that neither player would ever play a strategy that was strictly dominated by the other strategies. For Ornido, this would mean never throwing Curveballs. And for Baker, this would mean never Expanding the Zone.

Final Game if Players acted Rationally

Ornido / Baker	Sit Fastball	See it Late
FB	-5 , +5	+3 , -2
SPL	+4 , -3	-2, +2

Welcoming Back Probabilities and Introducing Nash Equilibrium

How should Riley Ornido approach the game above? With a *Mixed* Strategy, players randomize from a unique probability distribution of strategies / moves. With what probability distribution of Fastballs and Splitters should Riley Ornido randomly draw from in payoff pitches to Darren Baker?

With some algebra (in the back), we can find the strategy distribution of both players that would make them both indifferent. In Game Theory, this is a *Nash Equilibrium*. In a Nash Equilibrium of a mixed strategy game, both players in a rational state would not deviate from that unique strategy probability distribution, as it is a best response to what the other player is doing.

Nash Equilibrium serves as the compass for evaluating strategies and expected payoffs in nearly all strategic games. For either player, we can use the Nash Equlibrium as a point of reference and compass for what to do if the opponent is acting outside of it.

Using Algebra, we back out the unique Nash Equilibrium. The derivation is not necessary for intuition so I will not include it here:

Riley Ornido: Randomize between (41.6% Fastballs, 58.3% Splitters)

Darren Baker: Randomize between (35.7% Sit Fastball, 64.2% See it Late)

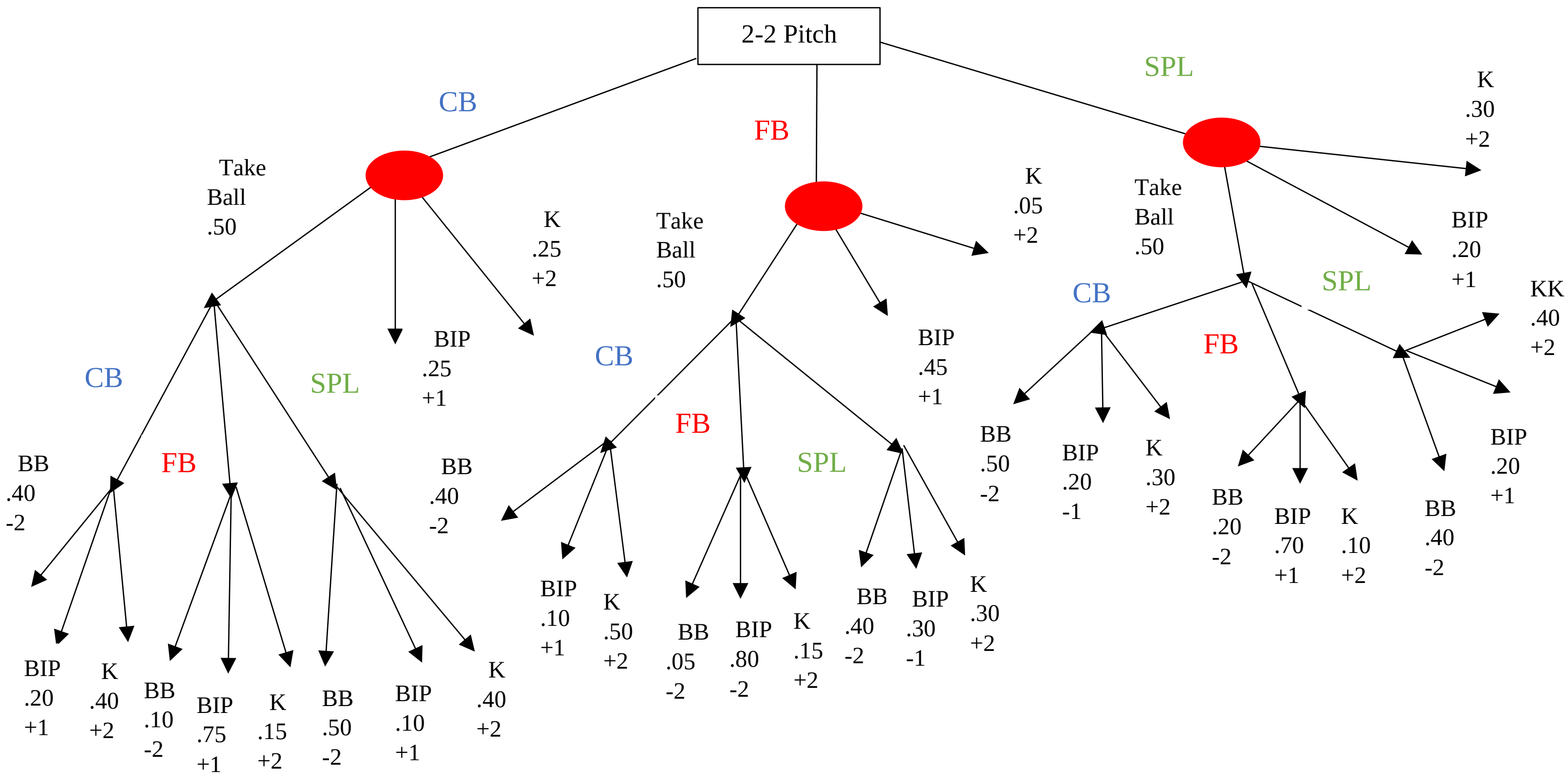
Again, Nash Equilibrium serves as a reference point and compass. A deviation outside of a player's Nash Equilibrium probability distribution would mean the player is not playing a Best Response given the opponent's strategy is still fixed within the Nash Equilibrium.

While not in Nash Equilibrium, Best Responses for both players are perpetually changing. Players attempt to form new Best Responses given the defection of one player. Only in Nash Equilibrium will both players be in a state of mutual Best Response.

The distribution Baker chooses to randomize from, if different than the Nash Equilibrium, is how Ornido decides to best respond. In short, if Baker Sits Fastball 35.7% of the time, Ornido is indifferent. If Baker Sits Fastball less than 35.7% (5/14) of the time, Ornido should best respond with Fastballs. In the scouting report, if Baker sits Fastball more than 35.7% (9/14) of the time, Ornido should best respond with Splitters.

This idea of best response, given difference to Nash Equilibrium, in something like a scouting report will be a crucial paradigm moving forward. Knowing the state where both players are indifferent provides the compass for what to do if one player deviates.

Examining the Extensive Form / Multi-Stage Game and Lead Up to Bayesian Games



This is a multi-stage or "extensive form" game. This is a 2-pitch sequence starting with the 2-2 pitch. For this game, assume there are no foul balls. Payoffs at the end of every arrow are quoted for Riley Ornido. They are the opposite for Darren Baker. For the 2-2, Darren Baker is presented the opportunity to swing or not. A Baker decision to swing or not swing on the 2-2 is represented by each red oval.

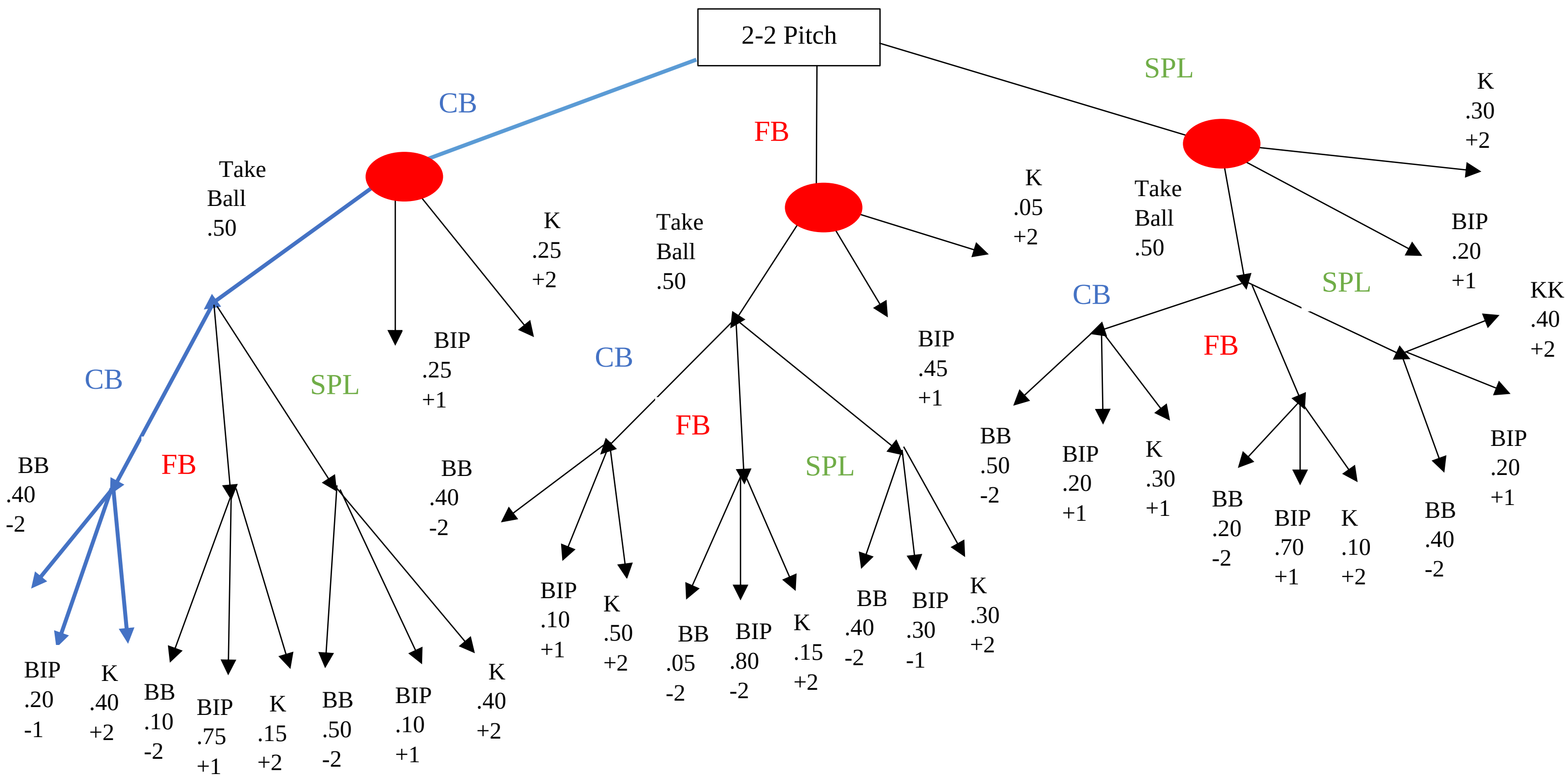
The second pitch guarantees an outcome, so while intuitively Baker faces a second decision, just for this example there is not a second round of decision making for Baker. The decision-making logistics for this game is as follows:

- Riley Ornido: What pitch to throw on 2-2?
- Darren Baker: Given pitch selected, take or swing on 2-2? If swing, payoff is subject to nature taking the form of strikeout or ball in play.
- Riley Ornido: What pitch to throw on 3-2? Payoff is subject to nature taking the form of walk, strikeout, or ball in play.

Payoffs following the principles in the *Payoffs, Nature, and Probability* section are represented below given Baker's decision to either swing or take the 2-2:

RO/DB	Take	Swing
FB-FB	-1.40, 1.40	.10, -.10
SPL-SPL	.20, -.20	.60, -.60
CB-CB	-.20, .20	.50, -.50
FB-CB	.30, -.30	.10, -.10
FB-SPL	-.50, -.50	.10, -.10
SPL-FB	.50, -.50	.60, -.60
SPL-CB	-.60, .60	.60, -.60
CB-FB	.85, -.85	.50, -.50
CB-SPL	-.10, -.10	.50, -.50

For example, the expected payoff from (CB-CB, Take): (Path highlighted in blue)



$$Ornido\ Expected\ Payoff\ from\ (CB - CB, Take) = -.20$$

If Riley Ornido knew that Baker was taking, what should Ornido's strategy be? In our example, the best response given the computation of expected payoffs in the table above, would be for Riley Ornido to sequence CB-FB. In addition, if Ornido knew that Baker was looking to be aggressive/swing on the 2-2, then Ornido would be indifferent between SPL-SPL and SPL-CB, SPL-FB sequences.

This is incomplete though. In the game as is, *the expected payoff does not capture the reality that Ornido will sometimes end up facing an aggressive Baker or a patient Baker on the 2-2*. Riley

Ornido's payoff is partially subject to the strategy Baker chooses. Enter Bayesian Games, and we can evaluate this game with more clarity…

Bayesian Games

Original Game

RO/DB	Take	Swing
FB-FB	-1.40, 1.40	.55, -.55
SPL-SPL	.20, -.20	.80, -.80
CB-CB	-.20, .20	.75, -.75
FB-CB	.30, -.30	.55, -.55
FB-SPL	-.50, -.50	.55, -.55
SPL-FB	.50, -.50	.80, -.80
SPL-CB	-.60, .60	.80, -.80
CB-FB	.85, -.85	.75, -.75
CB-SPL	-.10, -.10	.75, -.75

In our original game, where the event tree was the single determinant of payoff, there was no account for Baker's approach/strategy. The payoffs and actions in the table above correspond to payoffs and actions found in the tree. In reality, Ornido does not know Baker's approach before the 2-2 pitch is thrown.

If the scouting report stated that Baker is far more likely (assume 80%) to adopt a patient approach on 2-2 and less likely to adopt an aggressive approach on 2-2, we would need to map the realities of Ornido confronting two different Darren Baker strategies. For computational and example simplicity, our 2-2 game is going to be reduced with respect to Ornido pitch sequence strategies.

Riley Ornido could receive a payoff where Baker uses an aggressive approach, or he could receive a payoff where Baker uses a patient approach:

Riley Ornido could end up in either table/game

Patient (prob. 80%)				Aggressive (prob. 20%)		
RO/DB	Take	Swing		RO/DB	Take	Swing
CB-FB	-3, 3	1, -1		CB-FB	1, -1	2, -2
FB-SPL	2, -2	-1, 1		FB-SPL	3, -3	-3, 3

With respect to expected payoffs, Riley Ornido could end up in entirely different worlds given Darren Baker approach. If Ornido speculated Baker was taking, in both worlds he would best respond with a FB-SPL. The problem is that if Baker plays an aggressive strategy and swings, Ornido gets burned -3.

Patient (prob. 80%)				Aggressive (prob. 20%)		
RO/DB	Take	Swing		RO/DB	Take	Swing
CB-FB	-3, 3	1, -1		CB-FB	1, -1	2, -2
FB-SPL	2, -2	-1, 1		FB-SPL	3, -3	-3, 3

The objective best response would be calculated with Nash Equilibrium in mind. Nash Equilibrium again serves as the compass. If Nash Equilibrium is the state of play where both players are playing a best response, if one player does not respond in Nash Equilibrium, Nash Equilibrium will provide the compass for Riley Ornido's best response to what the scouting report says.

For this game, Nash Equilibrium calls for Riley Ornido to mix between 48% Curveball-Fastball and 52% Fastball-Splitter. If we changed Baker's strategy profile to a 50/50 split, Ornido would optimize his mix by using a 62% Curveball-Fastball and 38% Fastball-Splitter mix.

Conclusion

In my two years doing this work, I have found the best models we in QST have come up with and developed are the ones that scale. Scale is now always in the front of my mind when pursuing any model development for the program. Scale can be defined loosely as how efficiently we can run a model and more importantly how efficiently feed the inputs it needs. Scale measures how much we can get done in one 'click'.

I do not know if these ideas / theories can be scaled yet in a model. If there are any changes with Synergy data, given other MLB clubs are not happy that the Dodgers ownership group purchased it and can scale that data, we would stand to greatly benefit while developing Game Theory models using Synergy data as the inputs.

In addition, I think I would need to do additional exploration of the theoretical frameworks because there is just so much to bite off. I am always working and am always looking for new applications / ideas to use.

Calculations

Nash Equilibrium in *Welcoming Back Probabilities and Introducing Nash Equilibrium*:

For RO to be Indifferent in Nash Equilibrium:

$$5Q + 3(1\text{-}Q) = 4Q – 2(1\text{-}Q)$$

$$-5Q + 3 – 3Q = 4Q – 2 + 2Q$$

$$-8Q + 3 = 6Q -2$$

$$-14Q = -5$$

$$Q = 5/14 \sim .357$$

For DB to be Indifferent in Nash Equilibrium:

$$5P – 3(-P) = -2P + 2(1 - P)$$

$$5P – 3 + 3P = -2P + 2 – 2P$$

$$8P – 3 = -4P + 2$$

$$12P = 5$$

$$P = 5/12 \sim .416$$

Nash Equilibrium in *Bayesian Games*:

$$u(Take): .8[2P - 2(1 - P)] + .2[-P – 3(1\text{-}P)]$$

$$.8[2P – 2 + 2P] + .2[-P – 3 + 3P]$$

$$.8[4P – 2] + .2[2P – 3]$$

$$3.2P – 1.6 + .4P - .60$$

$$3.6P – 2.2$$

$$u(Swing): .8[-3P + 2(1\text{-}P)] + .2[-2P + 3(1\text{-}P)]$$

$$.8[-3P + 2 – 2P] + .2[-2P + 3 – 3P]$$

$$.8[-5P + 2] + .2[-5P + 3]$$

$$-4P + 1.6 – P + .60$$

$$-5P + 2.2$$

For DB to be indifferent:

$$3.6P – 2.2 = -5P + 2.2$$

$$P = .48$$

$$u(CB\text{-}FB): .8[-3Q + 2(1-Q)] + .2[Q + 2(1-Q)]$$

$$.8[-3Q + 2 – 2Q] + .2[Q + 2 – 2Q]$$

$$.8[-5Q + 2] + 2[-Q + 2]$$

$$-4Q + 1.6 - .2Q + .4$$

$$-4.2Q + 2$$

$$u(FB\text{-}SPL): .8[3Q – 1(1-Q)] + .2[3Q – 3(1-Q)]$$

$$.8[3Q – 1 + Q] + .2[3Q – 3 + 3Q]$$

$$.8[4Q – 1] + .2[6Q – 3]$$

$$2Q – 2$$

For RO to be indifferent:

$$-4.2Q + 2 = 2Q - .2$$

$$2.2 = 6.2Q$$

$$Q = .354$$