Power and Bit Allocation for Multicarrier Modulation in Multi-User Environments

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Barcelona, 2005

This work is dedicated to
my wife Verónica and our son Julián.

Abstract

In this thesis, we study how to distribute power and bits among the subcarriers in an adaptive multicarrier communications (MCM) system. We start introducing the basic aspects of a MCM system and then we discuss the benefits of the adaptive modulation to improve the link performance. In order to relate the actual performance (spectral efficiency) of an adaptive QAM system to the channel capacity, two approaches are followed and compared. The first one that uses the so called "gap approximation", and the second one that uses exact signal-to-noise ratios (SNR) obtained from a BER analysis.

The first part of the project dealt with the resource-allocation problem for the single-user scenario (all the subcarriers used by a single transmitter / receiver pair) as an introduction to the multi-user case. Three well known power-minimization algorithms were described and evaluated here, as well as two capacity-maximization ones. Simulation results showed that Chow's algorithm is the most cost-effective one among the first group (it makes the transmitter use only 0.53dB more power than Hughes-Hartogs' but it works 5 times faster). In the second group the suboptimal method devised by Yu and based on a constant power-allocation strategy achieves nearly the same spectral efficiency than the optimal Hughes-Hartogs' algorithm while it runs 25 times faster.

The main section of this work analyzes how to distribute subcarriers, and power and bits onto them, among multiple users sharing a MCM system. Here, we have both examined existing solutions and proposed new ones for different applications. Among the well-known power-minimization algorithms that were evaluated, the algorithm proposed by Pfletschinger et al. showed the best cost-effective performance (only 1.06dB worse than the optimal method but up to 10 times faster in a 6-users scenario).

A novel spectrally efficient method was proposed to save as much bandwidth as possible at the expense of a small transmit-power increase. From simulations, we observed that our method saved up to 9% of bandwidth when using just 1dB more power than the optimal method in a high load context. The low complexity of the proposed method is also shown.

In the last part, two well known optimal capacity maximization algorithms are described, evaluated and compared against a newly proposed suboptimal resource-allocation algorithm. When observing the obtained spectral efficiencies per cell, our numerical results showed that the proposed method is only 0.19 bps/Hz away from the optimal one. Besides, our algorithm is 10 times faster and it does not use iterative routines (it produces results after a fixed number of operations).

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1. Introduction

Although in use since many years ago, Multi-Carrier Modulation (MCM) has recently become an attractive technique to many digital communication systems. The main idea of a MCM system is to split a high-rate data stream into several slow-rate data sequences and to use them to modulate a set of parallel subchannels that make full use of the available bandwidth. When the channel introduces Inter-Symbol Interference (ISI), a system using MCM does not need a complex equalizer at the receiver end. This is a nice property when dealing with very time-dispersive channels and high data rates.

Orthogonal Frequency Division Multiplexing (OFDM), one of the most popular types of MCM schemes, is an enhanced extension of Frequency Division Multiplexing (FDM), where the parallel subchannels have overlapped spectra. However, due to their orthogonality, they can be recovered at the receiver without major problems. As a direct consequence, these systems have high spectral efficiency. And this is another precious feature that modern digital transceivers demand.

Multitone systems have been used in military systems since the 1960's. At this early stage of MCM, the set of parallel subcarriers was implemented using banks of sinusoidal generators and demodulators. The complexity was huge. The interested reader may refer to [Kel00] and the references therein to have a good historical perspective of this technology.

It was not until 1971 that Weinstein and Ebert introduced the Discrete Fourier Transform (DFT) to generate harmonically related frequencies to be used as OFDM carriers. Their classical paper [Wei71] proposed a completely digital OFDM modem using the Fast Fourier Transform (FFT). However, only the recent advances in Very-Large-Scale-Integration (VLSI) technology have made OFDM commercially possible by means of the FFT,. It took some time to the silicon industry to produce a dedicated chip that could solve a large-size FFT in the order of microseconds. Once this issue was solved a number of commercial applications using OFDM appeared in the broadband-data-communications arena.

Standards as the European Digital Audio Broadcasting (DAB), terrestrial Digital Video Broadcasting (DVB-T), and IEEE 802.11 and Hiperlan/2 wireless local-area networks (WLANs) are good examples of OFDM wireless applications. In [Rap02], a broader description of different wireless technologies is given. This paper suggests that OFDM is a serious candidate to become the modulation of the fourth-generation (4G) broadband wireless systems. For fixed-wired applications OFDM served as the core technology for the Asynchronous Digital Subscriber Line (ADSL) and High-Bit-Rate Digital Subscriber Line (HDSL) systems. In these standards OFDM is better known as Discrete Multitone Modulation (DMT). A great research effort has also been placed to study the benefits of MCM for Power-Line Communications (PLC) and for broadband return channels in Cable TV networks.

OFDM is usually used jointly with adaptive modulation techniques to improve the performance of the MCM system. This strategy requires knowledge of the channel conditions at the transmitter, which is a difficult task but it rewards since the transmitter can efficiently set its parameters (modulation type and coding scheme) according to the instantaneous quality of the channel.

The work presented in this report is related to the study and development of methods to produce optimal transmitter adaptations in a multi-user OFDM system. For single-user OFDM systems (e.g. ADSL, where all the available subchannels are used by a single transmitter-receiver pair) the optimal transmitter adaptation, also known as resource allocation problem, has been deeply studied for many years during the research and development of the DSL technology as it will be shown later on. But for the much more complex case of multi-user OFDM (e.g. a wireless network where the available subchannels are distributed among the existing users following a multiple access scheme), the growing interest in finding robust solutions to the resource allocation problem has started only a short time ago, at the same time as the wireless communications demanded improvements to the current air interfaces in terms of higher data rates and lower power consumptions.

The outline of this report is as follows. Section 2 comprises a review about the main aspects of MCM and OFDM that will be needed in subsequent sections. Special attention will be placed on adaptive modulation techniques and the analysis of the channel capacity related to the optimal bitloading (transmitter adaptation) strategy. Also a description of the WLAN channel models used for the simulations is provided. Section 3 deals with the power and bit allocation across the OFDM subcarriers in a single-user communication system, a problem that is also present inside the multi-user scenario. Both the description and evaluation of several relevant algorithms are presented. More references to the literature in this field are provided here too. Section 4 focuses on the resource allocation problem when a set of users share an OFDM Multiple Access (OFDMA) system. After the system description is presented, a subsection discussing the multiple optimization criteria and their applications provides some insight into the variety of related problems that the multi-user scenario has. Then, two main subsections follow. The first one studies and evaluates several algorithms that have a power-minimization objective. The second subsection groups algorithms performing capacity maximization (or an aggregate-data-rate maximization). In both of these sections novel methods are proposed and their performance evaluated and compared against existing well known algorithms. Finally, section 5 brings the conclusions and the suggested guidelines for future work.

2. Multicarrier Modulation for Digital Communications

2.1. Orthogonal Frequency Division Multiplexing (OFDM)

2.1.1 OFDM Basics

In a digital communications system it is well known that the effects of ISI on the received bit error rate (BER) can be neglected if the time dispersion of the channel (measured by e.g. its delay spread) is much shorter than the length of one transmitted symbol. When we attempt to transmit at a symbol rate beyond this limit, channel equalization must be introduced. Such an equalization method involves some complexity that may be high depending on the specific communications medium and required data rates.

The innovative approach that a MCM system provides is that instead of attempting to cancel the multipath structure of the channel, it splits the data stream into parallel subchannels. Then every subchannel conveys a lower-rate data stream whose symbol period is much longer than the original data sequence. This fact makes ISI a minor problem compared to a single-carrier system with the same data throughput.

Moreover, the previous idea can be improved to get extra benefits if we use a FDM scheme for the different subchannels and if we are able to overlap their spectra without loosing the orthogonality between them. Such a configuration would additionally give us a good spectral efficiency. And that is what OFDM does. A communication system that it is resistant to ISI and impulsive noise due to its long symbol period and that it is spectrally efficient due to the way the subchannels are generated.

Of course, nothing is for free and OFDM imposes some drawbacks that need to be considered when designing a MCM data modem. We will come back to this point later on.

Since a FDM scheme was chosen to demultiplex the original high-rate data stream, it can be shown that a minimum inter-carrier spacing of $\mathbf{f_b = 1/T}$, with $\mathbf{T}$ equal to the symbol period, keeps the subcarriers orthogonal between them. This statement can be proofed by computing the dot product between any two of the participating waveforms and showing that (2.1) holds.

$$
\begin{aligned}
&\int_0^T \sin\left(\frac{2p\ i\,t}{T}+q\right)\sin\left(\frac{2p\ j\,t}{T}+j\right)dt = 0\\
&\forall i,\, j \in \mathbb{Z}\\
&i \neq j
\end{aligned}
\qquad (2.1)
$$

Figure 2.1 illustrates the spectra of a 16-carriers OFDM system (without guard interval) with rectangular pulse-shaping showing the overlapping between the subchannels.

When the channel is time dispersive, multiple delayed versions of the transmitted signal reach the receiver and this causes that the received signal be a distorted version of the transmitted one. Let's see how an OFDM system with $\mathbf{N}$ subcarriers usually deals with this. If we know that the time span of the channel is $\mathbf{T_G}$ samples long, we can decide the length of the symbol period just by choosing an appropriate $\mathbf{N}$. Then, the effect of the ISI is the distortion of the only first $\mathbf{T_G}$ samples of the symbol. A first attempt to remove completely the ISI would be to add $\mathbf{T_G}$ dummy (zero) samples at the beginning of the symbol and discard them at the receiver (since we know that they are distorted). This is the so-called zero-padded (ZP) guard interval which is shown in figure 2.2. It works, though it worsens the spectral efficiency of the system (we waste time during the $\mathbf{T_G}$ period) and at the same time it generates a new problem: Inter-Carrier Interference (ICI) because of the loss of orthogonality that happens among the effectively received waveforms.

We have previously highlighted that the subchannels could be overlapped without loosing orthogonality, but now, with the addition of the ZP guard interval, the receiver takes $\mathbf{T_D}$ out of $\mathbf{T_D} + \mathbf{T_G}$ samples from each received symbol to reconstruct the original signal. Since the channel introduces echoes of the subchannels' waveforms at the receiver and these echoes (that also have the ZP guard interval) are misaligned in several samples respect to the direct path, they produce ICI to the direct-path waveforms of the subcarriers. The solution to this added problem is to replace the simple ZP guard interval by the cyclic prefix (CP) guard interval which is a replica of the last $\mathbf{T_G}$ samples of the OFDM symbol (Fig 2.2).

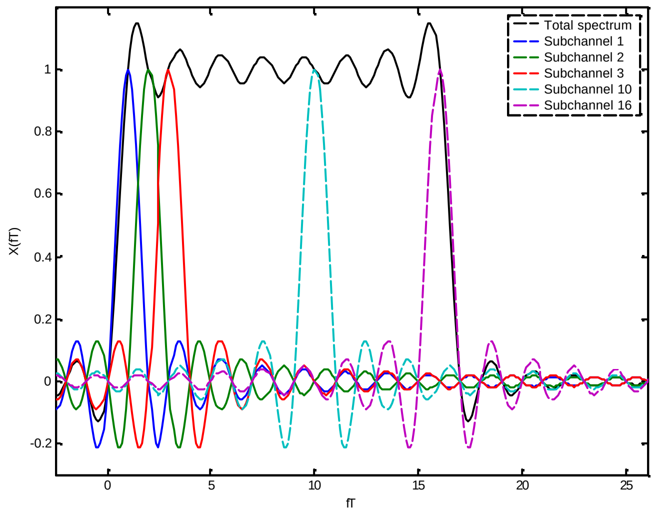
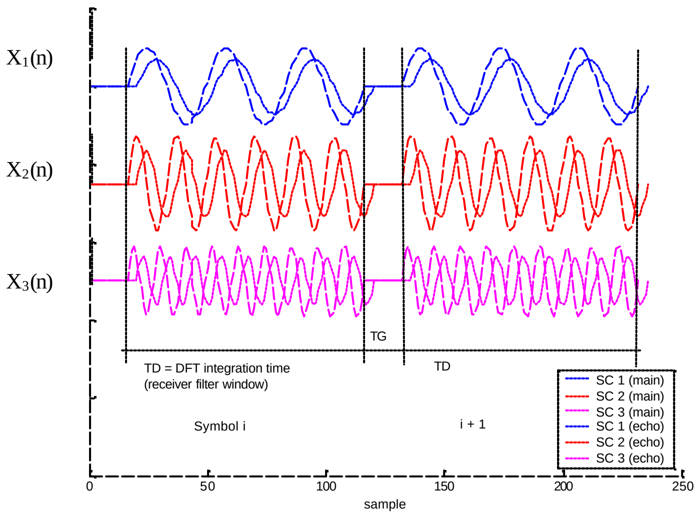


Fig.2.1. OFDM Subchannels and Total Spectra (16 carriers) before the addition of the guard interval



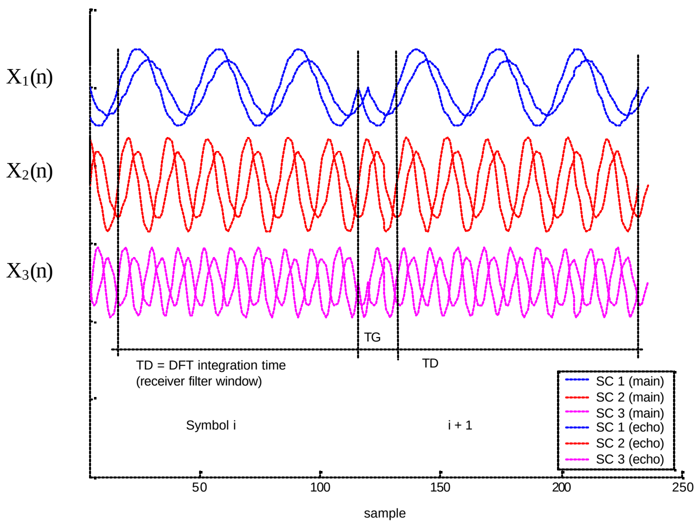


Fig.2.2.OFDM operation with ZP guard interval (upper) and CP guard interval (lower) showing 3 received subcarriers after a two-path channel.

It can be observed in the example proposed in figure 2.2. (3 subcarriers across a 2-path channel) that if the ZP guard interval is used, the received waveforms are no longer orthogonal along the DFT integration time (e.g. the dot product between sc 1 and the echo of sc 2 is clearly NOT equal to zero). But when the CP guard interval is used in the same example, the orthogonality is kept between the subchannels within $\mathbf{T_D}$ (any dot product between the received waveforms equals to zero). A very good introduction to the basic aspects of MCM is presented at [Sch02] for the interested reader.

2.1.2 Implementation of an OFDM System

The principles of MCM and OFDM described in the previous paragraphs can be efficiently implemented by means of digital signal processing using the FFT algorithm to perform the N-point IDFT and DFT at the transmitter and at the receiver respectively. Figures 2.3 and 2.4 display a basic scheme of an OFDM modem (advanced aspects as pulse shaping, synchronization, and bit interleaving are not included).

At the transmitter, a high data-rate sequence **b[n]** is sent to a serial to parallel buffer from which **N** groups of bits $\{\mathbf{m_1},\mathbf{m_2},\ldots,\mathbf{m_N}\}$ are mapped into **N** complex points $\mathbf{X_i}$ corresponding to each one of the subcarriers that the system handles. This mapping process is done according to the power and bit allocation that is decided for *each* subchannel (constellation size and dimension). The process can be further simplified if the mapping is performed only once before the serial to parallel conversion (and consequently all the subchannels are equally modulated), but we will keep the option drawn in figure 2.3 that provides more flexibility to perform the adaptation to the channel conditions when this information is known at the transmitter.

Then, the $\mathbf{X_i}$ points are converted into a time-domain sequence $\mathbf{x_i}$ via an FFT operation and a parallel to serial conversion. As it was discussed before, the addition of the Cyclic Prefix is necessary at this stage to combat the time dispersion of the channel and to avoid the ICI at the receiver. This can be easily done in a digital hardware implementation with a few memory storage and retrieval operations before the digital to analog conversion and frequency upconversion are performed. The interface towards the wireless channel is constructed with a High Power Amplifier (HPA) matched to the transmitter antenna.

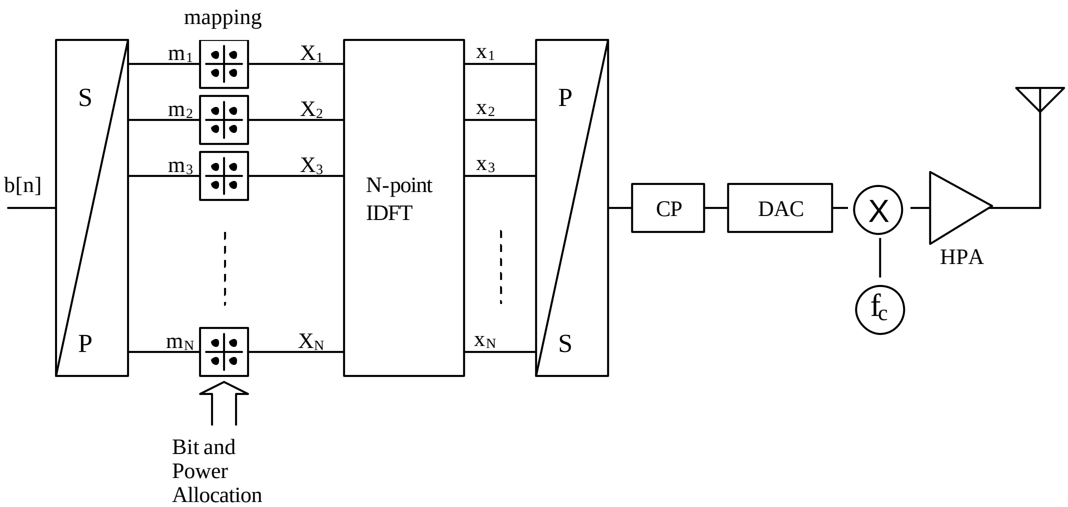


Fig. 2.3 Basic OFDM transmitter

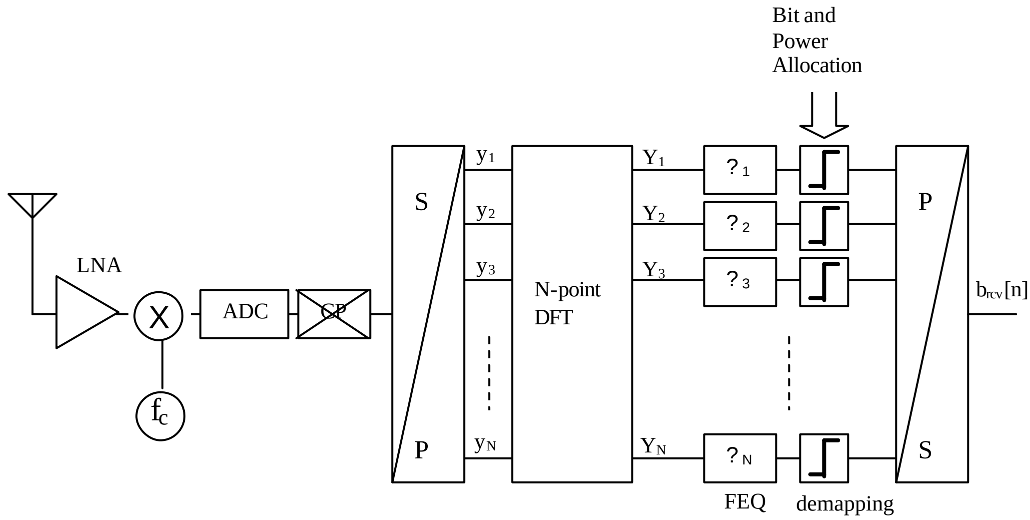


Fig. 2.4 Basic OFDM receiver

The basic OFDM receiver depicted in figure 2.4 takes the RF signal from the antenna and from its Low Noise Amplifier (LNA). Then, the signal is downconverted and transformed into a digital sequence after it passes an Analog-to-Digital Converter (ADC). If the system is perfectly synchronized, the CP is easily removed by cutting off the first $\mathbf{T_G}$ samples of each OFDM symbol. The following step is to pass the remaining $\mathbf{T_D}$ samples through a parallel-to-serial converter and to compute a N-point DFT. The resulting $\mathbf{Y_i}$ complex points are the complex baseband representation of the **N** modulated subcarriers.

As the broadband channel has been decomposed into **N** parallel subchannels, each subchannel needs an equalizer (usually a 1-tap equalizer) in order to compensate the gain and phase introduced by the channel at the subchannel's frequency. These blocks are the so-called Frequency Domain Equalizers (FEQ).

The demapping from the complex symbols to bits according to the original bit and power allocation tables follows. In this way, the groups of bits that had been placed on the subcarriers at the transmitter are recovered at the receiver as well as the high data-rate sequence.

The total OFDM spectrum displayed in figure 2.1 is not usually observed in practical implementations. In real systems, some subcarriers at the beginning and at the end of the broadband channel are left unmodulated (not used). In a wireless application, the central OFDM subcarrier is not used either. This is necessary to physically allocate the system carrier frequency after the upconversion process at the transmitter. Since this process is reverted at the receiver, no data can be transmitted through this subchannel. Finally, some subcarriers are used as pilots to produce a good reference at the receiver for synchronization and channel estimation purposes. In summary, we loose spectral efficiency (due to the carriers that do not carry data payload) as well as power efficiency (during the CP transmission) when we go from theory to practice. Two examples of a real-life OFDM subcarrier-usage are the well known IEEE 802.11 and Hiperlan/2 WLAN standards. Figure 2.5 shows how their 64 subcarriers are employed.

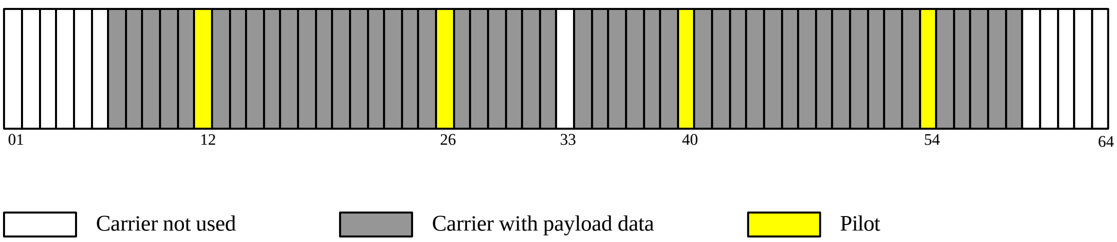


Fig. 2.5. Subcarrier usage for IEEE 802.11 and Hiperlan/2 standards.

[Bin90] is a good starting reference for practical MCM. In [Lit01], a list of several implementation issues and non-ideal effects in an OFDM system is presented in a very clear manner. Such problems that arise and need to be taken into account when designing and building an OFDM modem include:

- Local oscillators frequency offsets
- Local oscillators phase offsets
- FFT window location offsets
- Sampling frequency offsets
- Peak-to-Average Power Ratio (PAPR)

2.2. Adaptive Modulation Techniques

2.2.1 How to Improve the Link Performance

Adaptive modulation is an interesting strategy to increase the data rate (or decrease the transmitted power) in a digital communications link with fixed resources. The idea is opposed to the classical approach of designing the system for the worst-case scenario. Here the modulation parameters are set according to the channel conditions or channel state information (CSI). Hence, we do not waste resources (power, or complex channel coding) when the channel is known to be bad. And analogously we can benefit from a good channel by using, for instance, higher order constellations to increase the data rate while keeping the average transmitted power nearly constant.

Channel estimation is then a main issue, and we will discuss it more in detail later on. For the time being, we need the transmitter have a perfect knowledge of the CSI in order to settle its constellation size and transmitted power. Moreover this transmitter configuration must be available at the receiver with perfect synchronization such that the demodulation of the transmitted symbols can be carried out without problems. For a single carrier system we will refer to the system model depicted in figure 2.6 that has been presented in [Chu01] jointly with an in-depth analysis on the tradeoffs in adapting all combinations of different modulation parameters. A more advanced and realistic model is presented in [Gol97] with a proposal of adaptive modulation for fading channels.

For a MCM system, we will consider a group of parallel blocks like the ones in figure 2.6 provided that the subchannels are independent and they are processed in parallel.

To clarify the used notation, it must be noticed that **r[i]** represents the data sequence to be transmitted, **x[i]** is the waveform sent to the channel, and **g[i]** and **n[i]** are the squared value of the instantaneous channel gain and the noise sequence respectively. **S[i]** denotes the transmit power. At the receiver a channel estimator produces an estimate of the channel state that is used both at the transmitter and the receiver for both the adaptation and demodulation processes. It is assumed that the return path for the CSI is perfect (instantaneous and error free).

The proposed model in figure 2.6 implicitly proposes a time adaptation of the system, i.e. the time-varying channel makes the transmitter change its parameters accordingly. However, this is not the only kind of adaptation that can be done in a digital communications system. A frequency adaptation and a time-frequency adaptation are also possible in a MCM system.

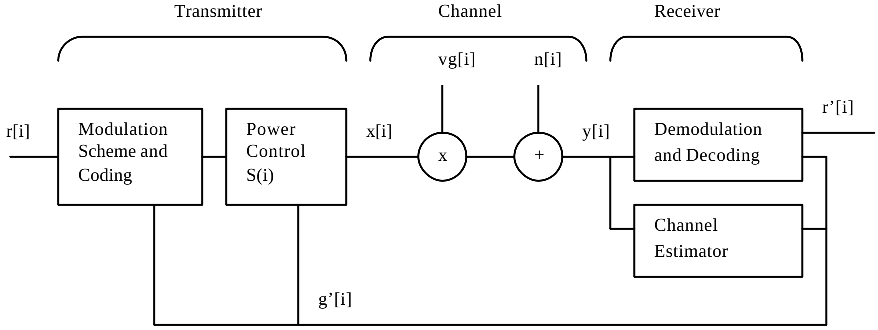


Fig. 2.6. System Model for a Single-Carrier Adaptive-Modulation Scheme

As soon as the broadband channel presents a multipath structure and the transmitter or the receiver are not static at their location, a situation that is typical in many wireless fading channels, each $\mathbf{g_j[i]}$ with $\mathbf{j}$:={$\mathbf{1}$,..,$\mathbf{N}$} has a different value. This fact makes that each subchannel be adapted individually to obtain the desired performance enhancement. Of course, the price to pay is a larger overall system complexity placing high demands on the channel estimators, the feedback-channel requirements and the resource allocation algorithms to optimally decide the transmitter parameters.

2.1.2 About Channel Estimation, Signalling and Transmitter Configuration

We cannot think an adaptive modulation system without having the CSI at the transmitter end. And to do so, a channel estimator is necessary. The features of this estimator strongly depend on the type of channels we are dealing with. Wired media like a Digital Suscriber Line (DSL), a Hybrid Fiber-Coaxial (HFC) network, or the Power-Line network have time-invariant characteristics (if no spurious or interference is considered). This means that the channel estimator needs not a periodic update, which is good in terms of lower complexity and less information signalled between the receiver and the transmitter. But media like the wireless channels between moving terminals usually have a complex time-frequency varying structure [Skl97] that require complex and fast estimators for an accurate representation of the CSI to be available at the transmitter. Besides, a larger bandwidth is required at the feedback channel to signal these data.

Some complexity reduction can be accomplished by using an *open-loop adaptation* regime [Kel00]. When two stations have a bidirectional link with reciprocal channels, e.g. two stations using the same channel with time division multiplexing (TDM), then each station can estimate the channel quality on the basis of the received symbols and adapt the local transmitter to this estimation. Hence, no feedback is necessary between both parts.

But if the channels are not reciprocal (e.g. when FDM is used), a *closed-loop adaptation* regime is required. This is the case represented in figure 2.6 where the receiver estimates the signal quality and signals this information to the transmitter through the feedback channel.

Another important issue is the delay between the channel estimation and the actual transmission of data in relation to the maximal Doppler frequency of the channel: if the transmitter adaptation is done according to an obsolete CSI, a bad system performance will be achieved. For this reason, closed-loop schemes are feasible only when operating over slow-fading environments.

OFDM modems usually estimate the channel transfer function using pilot tones interleaved among the rest of the subcarriers and along the whole broadband channel.

At every moment, the receiver must know which transmission parameters were used by the transmitter in order to demodulate the current symbol properly.

In a MCM open-loop adaptation scheme there are two alternatives to coordinate the receiver and the transmitter together. One is to explicitly signal or send the modulation parameters using dedicated subcarriers at the expense of a reduction in the effective data throughput. The second one is to perform blind detection by SNR estimation. This can be done by comparing the received data symbol to the ones that are available in all the possible modulation modes and finding the closest point that gives a minimal energy difference.

When a closed-loop adaptation scheme is used, the receiver explicitly indicates to the transmitter which configuration to use. As a separated special channel is used, it is expected that the spectral and power efficiency of this link be as good as possible and that the introduced delay be affordable.

Once the transmitter has the CSI available and the receiver is, at the same time, able to demodulate what the transmitter sends, a resource allocation algorithm (or bit loading algorithm) determines which modulation to use on each subchannel (bit allocation) and how much power will be placed on it (power allocation). These algorithms' operation is based not only on the channel conditions but also on the overall system constraints. These constraints will be described more in depth in the following sections. Basically they are related with minimum required data rates, maximum available transmit powers, maximum constellation sizes or maximum allowed power per subchannel, among others. The combination of constraints usually depends on the specific features of the system.

Nevertheless, the bit loading algorithms have in common that they always follow an optimization criterion that can be any of the following three:

- Aggregate-bit-rate maximization constrained to a certain maximal available power and to a maximal allowed error rate.
- Transmit-power minimization constrained to a minimal required data rate and to a maximal allowed error rate.
- Error-rate minimization constrained to a minimal required data rate and to a maximal available power.

For most applications, the first two criteria are relevant, while the last one is of less practical interest. Along this report we will focus on these resource allocation algorithms, their applications and their performance in terms of how well they optimize the system and at what computational complexity or speed.

2.3. Channel Capacity and Bit Loading

2.3.1 The Water-Filling Theorem

According to the analysis presented in [Sch02] and [Gal68], the channel capacity or the maximum achievable error-free bit rate over a band-limited AWGN channel is

$$C_{\max} = \frac{w_B}{2p} \log_2 \left(1 + \frac{s_y^2}{s_r^2} \right) \tag{2.2}$$

where **? $_{\mathbf{B}}$ = N.??** is the total channel bandwidth, $\mathbf{s^2_y}$ is the received signal power and $\mathbf{s^2_r}$ is the noise power. The capacity of subchannel **µ** with very small bandwidth **? ?** is:

$$C_m = \frac{\Delta w}{2p} \log_2 \left(1 + \frac{\Delta w \, \Phi_{xx}\left(w_m\right) \left|H\left(w_m\right)\right|^2}{\Delta w \, \Phi_{rr}\left(w_m\right)} \right) \tag{2.3}$$

Here $\mathbf{F_{xx}}$**(?)** denotes the PSD of the transmitted signal **x[n]**, **H(? $_{\mathbf{µ}}$)** is the channel transfer function for the particular subchannel and $\mathbf{F_{xx}}$**(?)** is the PSD of the noise **r[n]** at the receiver. The total capacity of all subchannels in the limit for **?? ? 0** with **? $_{\mathbf{B}}$ = N.??** gives:

$$C = \frac{1}{2p} \int_0^{w_B} \log_2 \left(1 + \frac{\Phi_{xx}\left(w\right) \left|H\left(w\right)\right|^2}{\Phi_{rr}\left(w\right)} \right) dw \tag{2.4}$$

When we try to maximize the channel capacity **C**, subject to a limited transmitter power,

$$\frac{1}{2p} \int_0^{w_B} \Phi_{xx}\left(w\right) dw = P_t \tag{2.5}$$

we search for a function $\mathbf{F_{xx}}$**(?)** that maximizes (2.4) subject to (2.5). This can be done by means of classical Lagrangean methods (Lagrange function and multipliers [Rad03]). The solution to this problem is

$$\Phi_{xx}\left(w\right) = \begin{cases} \Phi_0 - \dfrac{\Phi_{rr}}{\left|H(w)\right|^2} & for \left|w\right| < w_B \\ 0 & elsewhere \end{cases} \tag{2.6}$$

which is known as the water-filling solution. Its curious name comes from its graphical interpretation: if we define the channel to noise ratio **T** as

$$T(w) = \frac{|H(w)|^2}{\Phi_{rr}(w)} \tag{2.7}$$

then $\mathbf{T^{-1}}$ can be drawn as the bottom of a bowl in which we fill an amount of water corresponding to $\mathbf{P_t}$. The water will distribute in a way that the depth represents the wanted function $\mathbf{F_{xx}(?)}$. This is depicted in figure 2.7.

In this way, to maximize the channel capacity, the PSD of the transmitter output has to approximate as closely as possible the water-filling solution. By discretising the frequency axis of the water-filling diagram, we can adapt the solution to a MCM system:

$$E_v = \left[c_0 - \frac{s_v^2}{|H_v|^2} \right]^+ \quad where \quad [x]^+ = \begin{cases} x & for\ x > 0 \\ 0 & else \end{cases} \qquad v = 1,...,N \tag{2.8}$$

The "water level" $\mathbf{c_0}$ is chosen such that

$$E_{tot} = \sum_{v=1}^{N} E_v \tag{2.9}$$

$\mathbf{E_v}$ represents the average symbol energy per subchannel (or the average transmitter power on subchannel **v** if the symbol period **T** is considered). The total transmit energy is $\mathbf{E_{tot}}$. Additionally, $\mathbf{s^2_v}$ is the noise variance on subchannel **v** (which is not restricted to have the same value for all the subchannels) and $\mathbf{H_v}$ represents the aforementioned discretisation of **H(?)**.

MCM with relatively narrowband subchannels can approximate the desired optimum PSD by applying appropriate constellation sizes and gain factors for the bit-to-symbol mapping on each subchannel (bitloading process).

Thus, the subchannel capacity in an MCM system can be written (see[Sha48],[Pfl03]) as:

$$C_v = \log_2 \left(1 + \frac{E_v |H_v|^2}{s_v^2} \right) \tag{2.10}$$

measured in bits per symbol or bits per channel use.

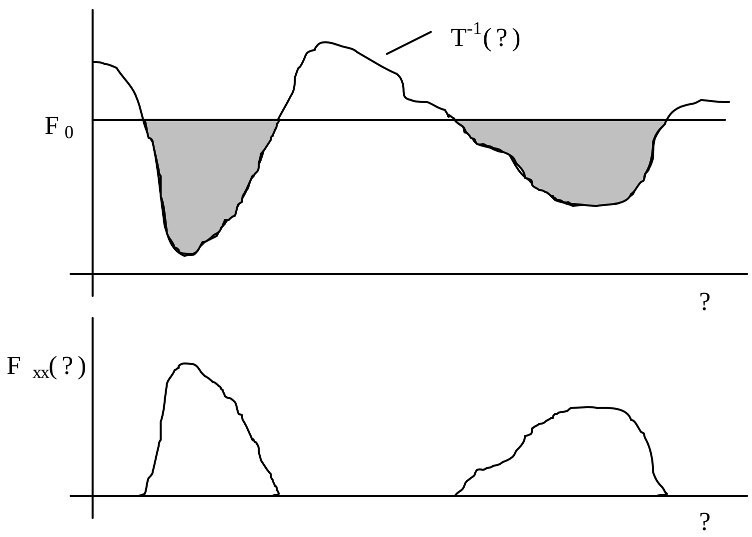


Fig. 2.7 Water-filling diagram (upper)and PSD of the transmitted signal that maximizes the channel capacity(lower)

2.3.2 The SNR Gap vs. Exact SNR Tables

In a MCM system, to achieve the channel capacity described in (2.10), we can use the water-filling theorem to allocate powers on the subcarriers in an optimal way. But this is a concept from an information-theory point of view. In a real system we need to work with modulation schemes (e.g. QAM –Quadrature Amplitude Modulation-) that do not reach the channel capacity at all. In these cases we are constrained to send **b** bits per symbol at a certain error rate with the given available symbol energy. Hence, when designing the adaptation of the transmitter to the channel conditions, we can follow any two of the following ways to settle the parameters of the MCM (OFDM) system:

- By using the SNR Gap analysis we relate the performance of the QAM scheme to the theoretical channel capacity and keep using the capacity formulae to compute the achievable data rates.
- By using an exact SNR table we forget about the channel capacities and concentrate on the exact achievable data rate that a QAM system can provide at a certain bit error rate (BER).

Along the following paragraphs, we will discuss about both approaches that are widely used in bitloading algorithms to determine the number of bits and average power per OFDM subcarrier.

SNR Gap Analysis

Presented with great detail in [Cio91], the SNR Gap **G** is a convenient quantity defined as:

$$\Gamma = \frac{1}{3}\left[Q^{-1}\left(\frac{P_s}{4}\right)\right]^2 \quad ,\Gamma \geq 1 \tag{2.11}$$

where $\mathbf{P_s}$ is the required symbol error probability and **Q(.)** the well-known Q-function. With it, we can write the following approximation for the necessary symbol energy to transmit **b** bits using QAM at a given $\mathbf{P_s}$ [Pfl03]:

$$E_v = \frac{\Gamma s_v^2}{|H_v|^2}\left(2^b - 1\right) \tag{2.12}$$

If we derive **b** from (2.12), we get

$$b = \log_2\left(1 + \frac{E_v |H_v|^2}{\Gamma s_v^2}\right) \tag{2.13}$$

and if we compare the bit rate achieved by QAM (2.13) with the channel capacity formula in (2.10), we see that the only difference is **G**. We can interpret **G** (and therefore its name) as the gap or the additional amount of SNR that QAM requires to transmit a bit rate equal to the channel capacity.

Moreover, a possible coding gain $\mathbf{?_c}$ and a system margin $\mathbf{?_m}$ (both in dB) can be incorporated into the SNR gap if necessary (i.e. when coded QAM is used or when we reserve a margin for unforeseen channel impairments):

$$\Gamma = \frac{1}{3}\left[Q^{-1}\left(\frac{P_s}{4}\right)\right]^2 .10^{\frac{(g_c - g_m)}{10}} \tag{2.14}$$

As it was shown before, the gap approximation is a clever way to keep the analysis simple and elegant. That's why it has been widely used in MCM bitloading algorithms for many years, as we will see later on. However there are two drawbacks that must be indicated or taken into account when using it: first the SNR gap is computed using the symbol error probability instead of the BER and the second one is that the QAM constellations were originally assumed square, so some extra inaccuracies are introduced when non-square constellations are used or PSK is used at low SNR (see [Czy96]).

In spite of this, the gap approximation is useful because the water-filling theorem can be implemented directly to configure the OFDM subcarriers -via equations (2.8) and (2.13)-.

Exact SNR Tables

In order to avoid the aforementioned approximations and inaccuracies and to perform an actual optimal transmitter adaptation to the channel conditions we can make use of the knowledge about how a digital modulation scheme (PSK, QPSK, R-QAM, etc) performs over an AWGN channel. That is, if all the subchannels are required to be demodulated with a BER of 10^{-5}, we want to know the minimum SNR at the receiver which allows a subcarrier to transmit 1, 2, 3,… or **b** bits. If these SNR values are determined, the subchannel transmitter powers can be obtained provided the channel transfer function is well known.

These SNR values are frequently known as switching levels since when a subchannel operates above one of these values it can switch its operating mode to a 1-step higher constellation and vice versa when the received SNR decays to a subsequent lower value in the table.

To compute these SNR switching levels we need the BER plots from each modulation scheme over an AWGN channel. Such information is analytically well defined for PSK and QPSK since quite long ago. However it is only recently that a closed-form BER analysis of M-QAM (square) and R-QAM (rectangular) constellations has been developed (refer to [Cho01] and the references therein). Previously the BER of R-QAM was obtained via bounds or approximations and/or simulations.

Following [Cho01] we plotted the exact BER curves of different R-QAM schemes. They are displayed in figure 2.8. Then, to obtain the SNR switching values for a BER of 10^{-5}, we take from the plots the bit SNR (**E_b/N0**) at the reference BER and compute the symbol SNR (**E_s/N0**) with **E_s=b*E_b**. The obtained symbol SNR values are the required switching levels of the adaptive system that, in this example, has the possibility to transmit **b** $\in$ {0,1,2,…,10} bits per symbol.

To physically implement this table look-up in a bitloading algorithm two alternatives can be selected. One is to allocate the table in memory and go to it when required and the second option is to produce a linear approximation (in a Least Squares sense) of the computed SNR points that yields to a linear equation that is easy to handle by a bitloading algorithm. Table 2.1 shows both the exact and linearized SNR switching levels. Figure 2.9 depicts the values from Table 2.1.

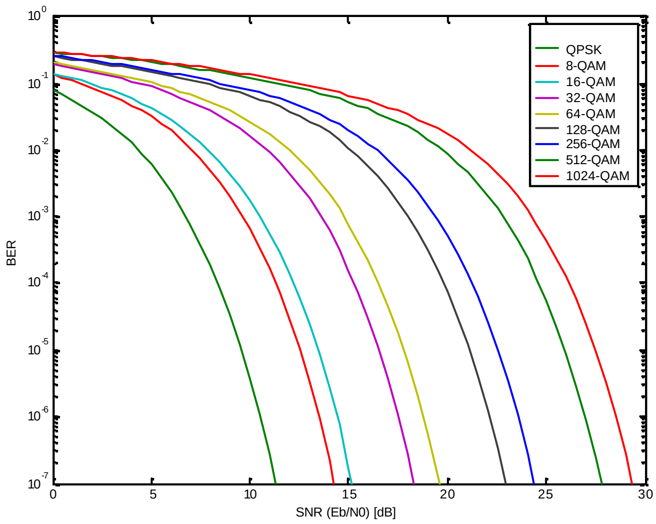


Fig. 2.8. BER plots of different R-QAM schemes

Table 2.1. Exact and linearized SNR switching levels

Bits/sym	1	2	3	4	5	6	7	8	9	10
SNR_{exact} [dB]	9.59	12.60	17.29	19.46	23.54	25.57	29.54	31.54	35.48	37.48
SNR_{linear} [dB]	10.17	13.29	16.41	19.53	22.65	25.77	28.89	32.01	35.13	38.25

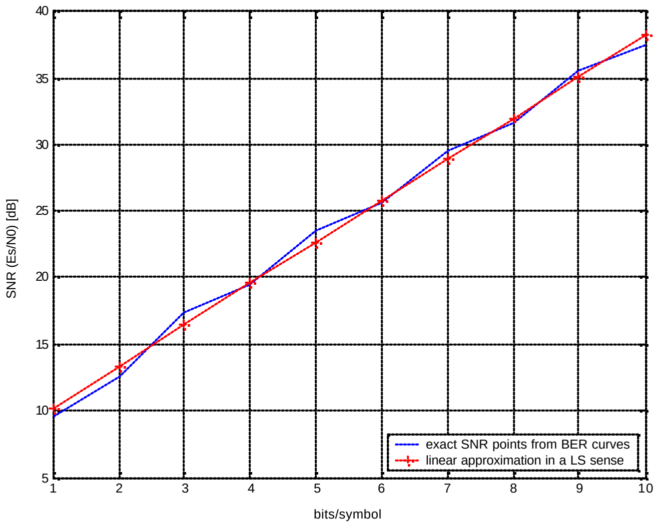


*Fig. 2.9. Required SNR to receive **b** bits per symbol at a BER = 10^{-5}*

The equation that describes the linearized version of the switching levels is

$$\frac{E_s}{N_0}[dB] = 3.1194 * b + 7.0529 \tag{2.15}$$

2.4. Channel Models for OFDM Wireless LANs

As it was stated before, any adaptive modulation system requires the knowledge of the CSI available both at the transmitter and the receiver. As the work developed along the current project concerned with resource allocation algorithms for MCM (OFDM) systems we considered that a Wireless Local Area Network (WLAN) environment would be appropriate for testing the evaluated bitloading methods. The main reasons for such a choice were that the channel models for standard Hiperlan/2 WLANs have been well studied and described [ETS98], and that their implementation can be done without major troubles.

In general terms, as future adaptive OFDM systems are being thought for improving current air interfaces, it is natural to test the bitloading-algorithms performance within the hard context of the wireless rayleigh fading channels. However, the resource-allocation methods are, in general, channel-type independent if some conditions are fulfilled as we will see.

[Skl97] and [And03] are recommended references about radio fading channels. From them we know that the inherent structure of a broadband radio channel, used for digital communication of two terminals in both indoor and outdoor environments, is typically characterized by its time dispersion or frequency selectivity and its time variance or Doppler-frequency shift. Different types of fading (flat, frequency-selective, slow, fast) are usually defined according to these parameters.

Two basic mechanisms define the features of a mobile radio channel. One is the so-called Large-Scale Fading due to the relative motion of the terminals over large areas covered with obstacles and scatterers. Then, the mean signal attenuation vs. distance is considered as well as its variations about the mean. The path-loss models according to the specific urban or indoor environments are considered here. The signal attenuation is often modelled from measurements with a log-normal distribution

$$L_p(d)_{dB} = L_s(d_0)_{dB} + 10n\log_{10}\left(\frac{d}{d_0}\right) + X_{s\ \ dB} \tag{2.16}$$

with

$L_p(d)$: the signal path-loss at distance ***d***.

$L_s(d_0)$: the path-loss to the reference distance ***d_0*** from the transmitter (taken from measurements in the far field of the TX antenna or from free-space models)

n: the path-loss exponent depending on the frequency, antenna heights and propagation environment.

X_s: a zero-mean Gaussian random variable (in dB) with standard deviation **s** (also in dB) that usually comes from field measurements.

The second mechanism that is present in a fading channel is the Small-Scale Fading. Here, two physical phenomena arise in this fading manifestation:

1- When the received signal is formed of multiple reflective rays plus a line-of-sight (LOS), its envelope amplitude has a Rician pdf. For the frequent case that there is no line-of-sight (NLOS), the Rician pdf becomes a Rayleigh pdf [Skl97]. The fact that the received signal is the superposition of multiple echoes of the transmitted signal (each one with a different relative amplitude and delay), makes that the channel be described as time dispersive (i.e. frequency selective). The receiver suffers the induced ISI effects that the channel has placed on the transmitted signal.

2- The previous point assumed static terminals and scatterers[1]. If we consider now that the terminals move from each other at a relative speed ***V***, the channel becomes also time-variant. The multipath structure of the channel changes over time and we can observe either a fast or slow fading process.

To measure or model the first point, communication engineers use a power delay profile, which is a plot that displays how the received average relative power varies as a function of time delay for a transmitted impulse. For a typical wireless radio channel, the received power delay profile consists of several multipath components (echoes) also known as *fingers*. Figure 2.10 shows the power delay profile of a Hiperlan/2 model-A channel (Typical Office Environment with NLOS Conditions) [ETS98].

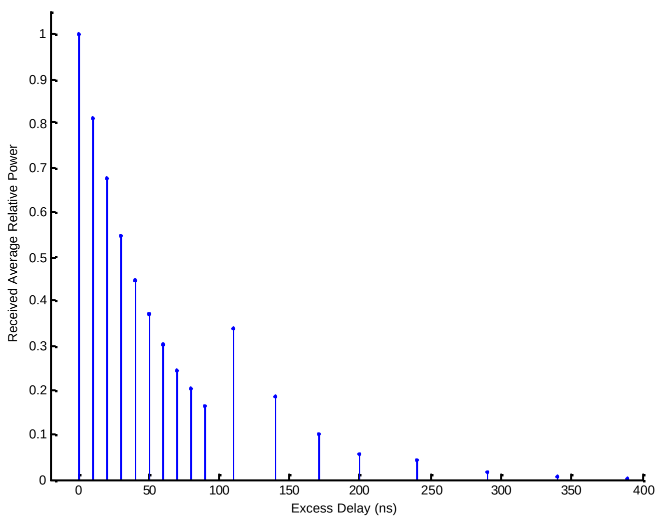


Fig.2.10. Power delay profile of the Hiperlan/2 Model-A channel.

[1] Along this report we will assume static scatterers (i.e. the Doppler-shift is introduced only by the relative motion of the terminals)

Given the power delay profile of the channel, a useful quantity that describes the effective delay spread of the channel is the root-mean-squared (rms) delay spread $\mathbf{s_t}$. With it, a channel coherence bandwidth is defined as [Skl97]

$$B_c = \frac{1}{5s_t} \tag{2.17}$$

This definition refers to the bandwidth or frequency interval over which the channel's complex frequency transfer function has a correlation of at least 0.5. Hence if we relate our transmitted signal's bandwidth (***W***) to this coherence bandwidth we have:

- A flat-fading channel if

$$B_c > W \tag{2.18}$$

- A frequency-selective-fading channel if

$$B_c < W \tag{2.19}$$

To measure or model the time variance of the channel, we need first to determine the maximum Doppler frequency as

$$f_d = \frac{V}{l} \tag{2.20}$$

where ***V*** is the relative velocity between the terminals and ***?*** the signal wavelength. This maximum carrier frequency shift can be positive or negative depending on the relative motion direction of the terminals (towards or away from each other respectively). A coherence time is analogously defined here to determine the time duration over which the channel's transfer function is highly correlated (i.e. the channel's response to a sinusoid has a correlation > 0.5)[2]

$$T_c \approx \frac{9}{16p\, f_d} \tag{2.21}$$

When comparing the coherence time with the transmitted symbol period (***T***) we can observe:

- A slow-fading channel if

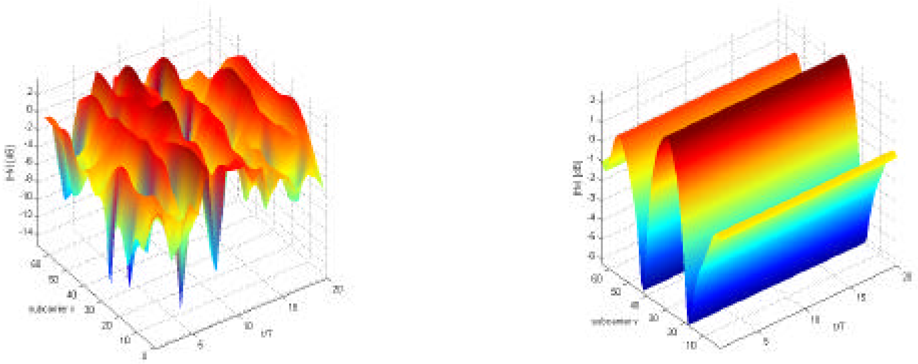
$$T_c > T \tag{2.22}$$

- A fast-fading channel if

$$T_c < T \tag{2.23}$$

In summary, a broadband wireless fading channel has a complex time-frequency structure that depends on many propagation parameters of the environment as well as the relative location and motion of the terminals. For illustration purposes, we present in figure 2.11(a) an Hiperlan/2 model-A channel realization (64 complex baseband subchannels) along 20 symbol periods for two terminals moving at a very high speed (frequency-selective fast fading structure). In figure 2.11(b) another realization of the same channel model in a completely static environment (frequency-selective slow (time-invariant) fading) is depicted.

[2] Other definitions are also considered in the literature, refer to [Skl97]



(a) (b)

Fig.2.11.(a)Frequency-selective fast-fading channel, (b)Frequency-selective slow(time-invariant) fading channel. Both are Hiperlan/2 model-A channel realizations along 20T

For the specific case of our project, we used the Hiperlan/2 channel models defined in [ETS98]. We implemented a complex baseband representation of the broadband channel assuming the following channelization parameters:

$f_c = 5.10^9$ Hz (carrier frequency)
$N = 64$ (number of subchannels)
$W = 2.10^7$ Hz (total occupied bandwidth)

We also considered channels with slow fading when using OFDM symbol periods in the order of T=3.2 10^{-6} s and terminals that were static or moving at walking speed (V = 6 km/hr). This assumption yields to channels with a coherence time in the order of T_c=6.2 10^{-3} s -via equations (2.20) and (2.21)- which is much longer than the considered T.

In practice we will use individual channel realizations (represented with the complex channel transfer functions) for our adaptive schemes and will regard them as time-invariant during the coherence time. After that, new channel estimation should be produced and the transmitter parameters should be updated accordingly.

Another simplification used along the project, in order to illustrate power and bit distributions better, was that all of the 64 OFDM subchannels were used for transmitting payload data (disregarding the sub-channel usage shown in Fig. 2.5). It is the same for the bitloading methods, for illustration purposes, to work with all the subchannels than with a set of them as we will show. Figure 2.12 shows a single realization of a Hiperlan/2 model-A channel (as an example of a CSI that we will use to perform the adaptation at the transmitter).

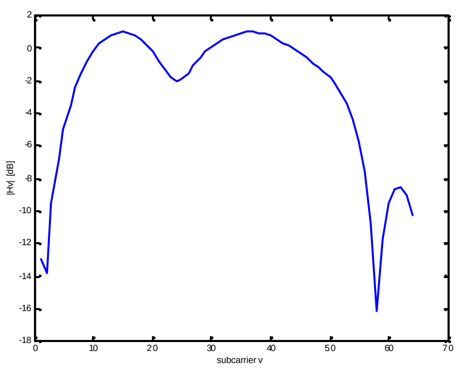


Fig.2.12 Hiperlan/2 model-A channel realization (without large-scale fading effects)

3. Resource Allocation for the Single-User Scenario

3.1. System Description

The single-user scheme is the one where all the available subcarriers in an OFDM system are dedicated to only one transmitter-receiver pair which is assigned to one user or service. Following the description of the basic OFDM system shown in the previous section, we assume that the cyclic prefix is longer than the channel impulse response. Then, each OFDM subcarrier faces an independent, flat-fading, narrowband channel. With this, we can use [Pfl03] the equivalent channel model in complex baseband representation illustrated in figure 3.1. With it, the overall OFDM system drawn in figures 2.3 and 2.4 can be modelled with a simpler block diagram as in figure 3.2.

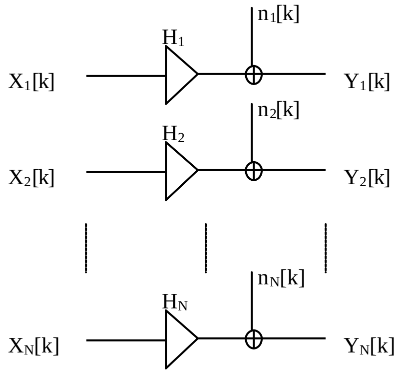


Fig. 3.1. Channel model for single-user OFDM with cyclic prefix

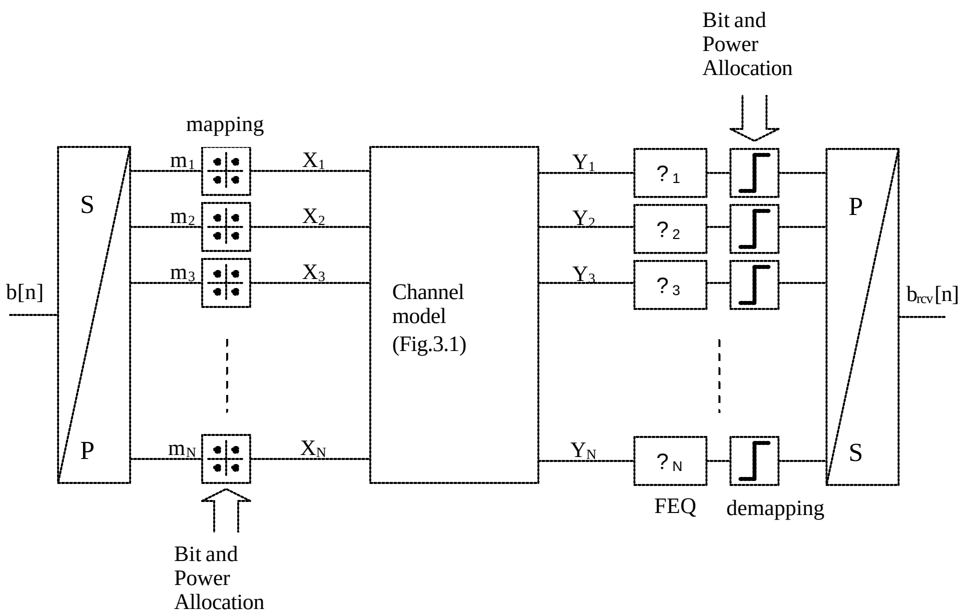


Fig.3.2 Single-User OFDM system (including the channel model in Fig3.1)

X_n represents the complex symbol that has been allocated to the sub-carrier n (mapped according to the bit and power allocation algorithm). H_n are the channel coefficients (i.i.d. Rayleigh distributed and generated according to section 2.4), n_n is the noise sequence affecting the sub-carrier n (with variance $s_n^2 = E[|n_n[k]|^2]$) and Y_n is the received symbol from the sub-channel n.

3.2. Optimization Criteria and Applications

Several bitloading optimization criteria are considered when designing adaptive algorithms for multicarrier modulation systems:

1. Minimu m transmit power:

$$\begin{aligned} &\min \sum_n p_n \\ &s.t. \\ &\sum_n b_n \geq B_{\min} \\ &P_e \leq P_{e,\max} \end{aligned} \tag{3.1}$$

2. Maximum bit rate:

$$\begin{aligned} &\max \sum_n b_n \\ &s.t. \\ &\sum_n p_n \leq P_{\max} \\ &P_e \leq P_{e,\max} \end{aligned} \tag{3.2}$$

3. Minimum error probability:

$$\begin{aligned} &\min P_e \\ &s.t. \\ &\sum_n b_n \geq B_{\min} \\ &\sum_n p_n \leq P_{\max} \end{aligned} \tag{3.3}$$

where $\boldsymbol{p_n}$ and $\boldsymbol{b_n}$ are power and bit allocation for sub-channel $\boldsymbol{n}$ respectively. $\boldsymbol{B_{min}}$ is the minimum bitrate that the user (or service) requires. $\boldsymbol{P_{max}}$ is the available transmitter power to distribute among all the existing subcarriers.

Most of the applications (mainly in wireless communication) use either criteria 1 or 2 while criterion 3 has less practical interest. These three optimization criteria are the basic ones. More complex requirements may also impose restrictions on the maximum transmitter power per subchannel (PSD transmitter mask) or on the minimum required operating noise margin (amount of additional noise that can be tolerated without degrading the designed performance).

A certain limit on the error rate is usually defined for a digital communication link. If this limit is exceeded the system is considered faulty. In criteria 1 and 2 this constraint is regarded and it is not common that we see an application requiring an error rate minimization (criterion 3).

When the physical layer of a communications system transports bits of real time services (voice, video, etc) a minimum required bit rate has to be fulfilled (delays are not allowed). Then criterion 1 is usually used. Therefore, it is a good choice to use the lowest total transmitter power when assigning power and bits to the subchannels.

For the case that the transmitter is power-constrained (e.g. mobile terminals operating with batteries) or when we need the maximum data throughput (e.g. non-real-time data transfer), criterion 2 is often selected to perform the transmitter adaptation.

Along the following sub-sections several classic algorithms using the concepts defined here are presented and evaluated within a Hiperlan/2 WLAN context.

3.3. Hughes-Hartogs' algorithm

This multicarrier loading algorithm described in [Hug87] and outlined in [Pfl03], implements the water-filling theorem adapted to QAM by using the "gap approximation" (SNR gap in section 2.3.2) to relate capacity to the achievable bit rate. The algorithm can operate following optimization criterion 1 or 2, as it will be explained. Fig. 3.3 shows the flow diagram of the procedure.

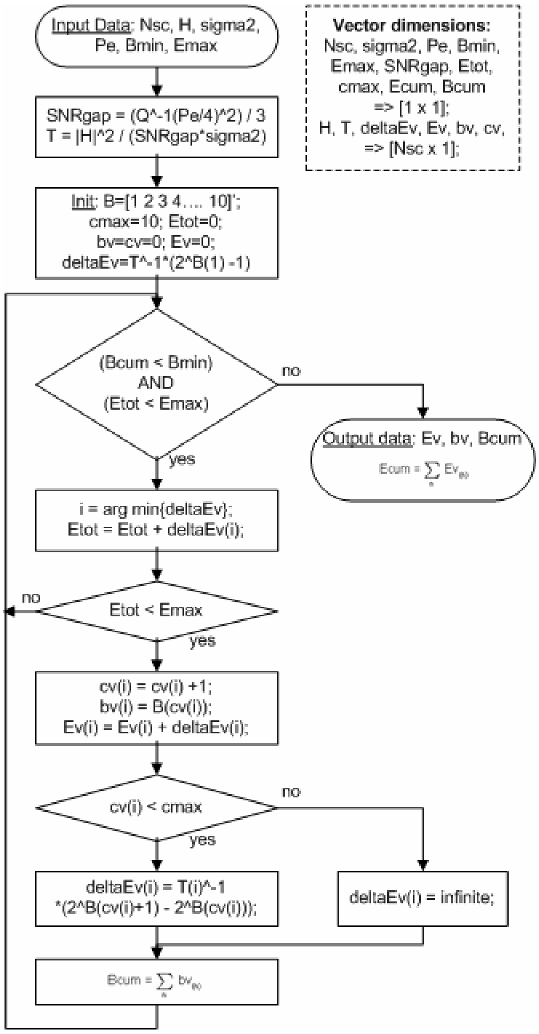


Fig. 3.3.Hughes-Hartogs' bitloading algorithm

As in every algorithm of this kind, a perfect knowledge of the channel is needed at the transmitter, something that in practice is not so easy to obtain, mainly if the CSI needs not to be delayed or corrupted to achieve the optimal solution of the bitloading process. **H** represents the vector with the complex channel coefficients affecting each active data subcarrier; **sigma2 (s^2)** is the variance of the AGN on each carrier at the receiver (it can be the same value for all the sub-channels or not), and $\mathbf{N_{sc}}$ is the number of active subcarriers in the OFDM system.

$\mathbf{B_{min}}$ and $\mathbf{E_{max}}$ are the minimum bit rate that each OFDM symbol has to carry and the maximum available power to distribute among the $\mathbf{N_{sc}}$ carriers, respectively. $\mathbf{P_e}$ is the symbol error probability.

When receiving this input data, the algorithm computes the **SNR gap** and the Channel Gain to Noise Ratio (**T**) for each sub-channel.

Next, and upon an available set of constellations defined in **B**, the program assigns iteratively and starting from $\mathbf{b_v} = 0$ ($\mathbf{b_v}$ is the vector with the bit allocations), bits to the subcarriers that require the lowest energy increments (**deltaEv** is evaluated each time for all the carriers). This is done until $\mathbf{E_{tot}}$ reaches $\mathbf{E_{max}}$ or $\mathbf{B_{min}}$ is fullfilled.

An interesting property of this greedy algorithm is that if $\mathbf{E_{max}=8}$ (no constraint on the transmit power), then the resulting bit allocation ($\mathbf{b_v}$ vector) is the one that achieves the desired bit rate with the minimum transmitted power. Conversely if $\mathbf{B_{min}=8}$ (no constraint on the required bit rate), then the final power allocation ($\mathbf{E_v}$ vector) is the one that achieves the highest bit rate. Both criteria can be utilized with the same algorithm. On the contrary, it is not so interesting the lack of computational efficiency of this program. This will be evaluated later on.

3.4. Chow's algorithm

This algorithm originally developed for DMT in ADSL systems is described in the famous paper from [Cho95]. In some literature, it is presented as the first sub-optimal solution to the bitloading problem in multicarrier systems with benefits concerning implementation issues.

Only the strategy of minimizing the transmit power for a given bit rate (criterion 1) is used to generate the transmitter configuration ($\mathbf{E_v}$**, bvhat**).

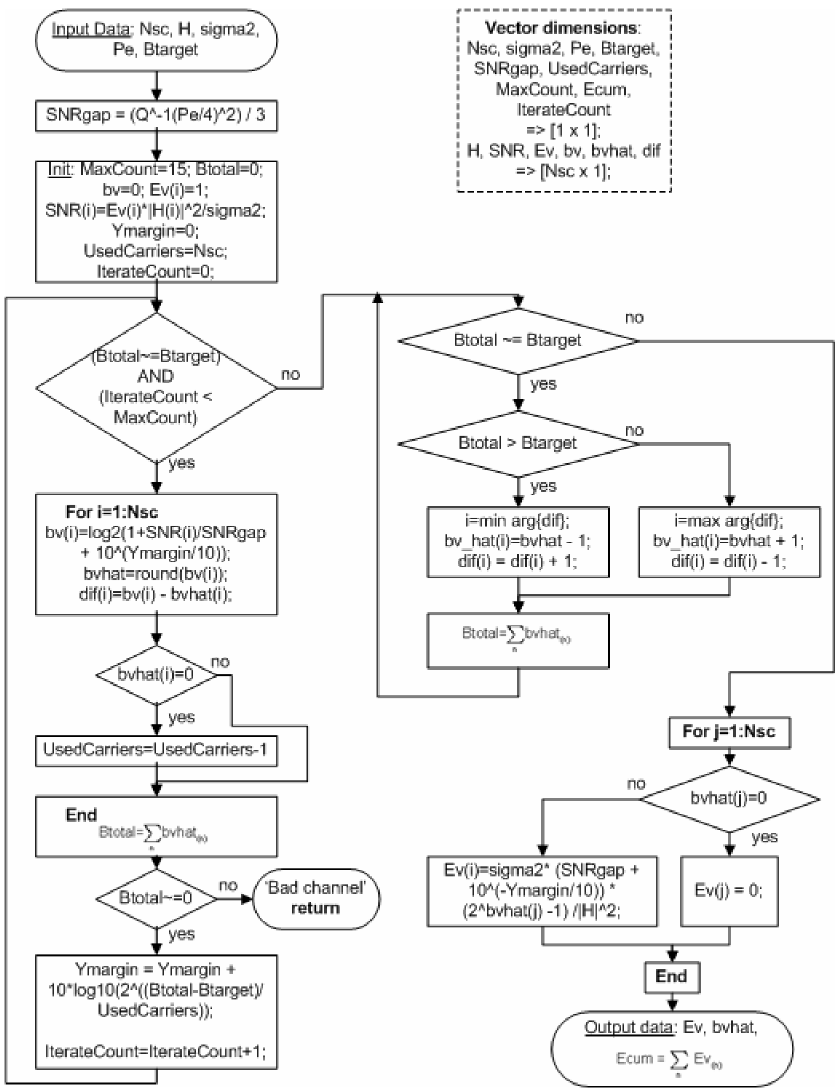


Fig. 3.4 Chow´s bitloading algorithm

Again, the "gap approximation" is used to relate the bit rate or spectral efficiency in QAM to capacity. Also the concept of "noise margin", already explained, is used in the proposed method (**Y_{margin}**).

When checking the input parameters to the program, the reader will notice that a slight change has been introduced. E_{target} in the original paper is not present as an input since it is desired that the algorithm produce the power allocation for a given bit rate rate (**B_{target}**). Hence, the 11th step in the original algorithm is not implemented and the final noise margin is deduced when computing the final power allocation. In this way, we can make fair comparisons with other bitloading algorithms.

The program receives the same parameters as the previous one (with the aforementioned exception) and computes first the **SNRgap** and the received **SNR** per sub-channel. The **Y_{margin}** is set to zero.

Then the bit allocation (**$b_v$hat**) is initially computed in one step using eq.(1) and (2) from [Cho95]. If any carrier receives no bits, that carrier is discarded in future steps (**UsedCarriers** is monitored).

All is repeated again to get a new bit distribution (after udating **Y_{margin}**) if that bit allocation does not match the desired bit rate or if the code has already performed **MaxCount** iterations. In the latter case the convergence is forced by adding or substracting iteratively bits to subcarrirers where the computed bit allocation (**b_v**) differs most from the rounded final value (**$b_v$hat**). Finally the power allocation is computed (**E_v**).

Fig. 3.4 shows the flow diagram of the algorithm.

3.5. Czylwik's algorithm

This algorithm was presented by its author in [Czy96]. There, two new algorithms were presented (named "modulator A" and "modulator B") and the second one was chosen to be implemented and verified since it is more general and allows the minimization of the transmit power for a given bit rate (**B_{target}**).

What is novel in the author's idea is not to take into account the SNR gap and to use a more accurate mapping between a certain sub-channel **SNR** and the achievable spectral efficiency by that carrier in that condition.

In this way, the author assumes Gray coding, QAM constellations, and a symbol error probability of $Pe=10^{-5}$ to get the following linear approximation to distribute bits to each subcarrier:

$$C'_{vQAM} \approx P_1.SNR_{v\,[dB]} + P_2 \qquad bits \tag{3.4}$$

With **P1**=0.31 and **P2**=-2.077

This new approach (related to the SNR tables described in section 2.3.2) gives better results in the overall adaptation, as it will be shown. But still, we can do it better if we try to find a more accurate mapping between the SNR and the bandwidth efficiency by using more precise (or exact if possible) QAM BER curves. In this sense we generalized the current method by computing new **P1** and **P2** coefficients via the **f(P_e)** function according to two different sources:

1- By using the QAM constellations and **P1** and **P2** coefficients proposed in [Czy96] directly. (**P1**=0.31 and **P2**=-2.077, for a $P_s=10^{-5}$).
2- By using the R-QAM constellations and exact BER curves described in section 2.3.2. (**P1**=0.32 and **P2**=-2.261, for a $P_b=10^{-5}$, derived from equation (2.15)).

The current basic algorithm (Fig. 3.5) receives the same input parameters than the previous methods and it first selects **P1** and **P2** as explained before. Then it computes an initial bit allocation **m** that it is adjusted to the desired bit rate **B_{target}** as in Chow's algorithm. The next step is to compute the required power for each sub-carrier **E_v** which is minimized by reallocating bits from some carriers to others where the power decrease is maximum (for the bits that are substracted) and the power increase is minimum (for the bits that are added) respectively.

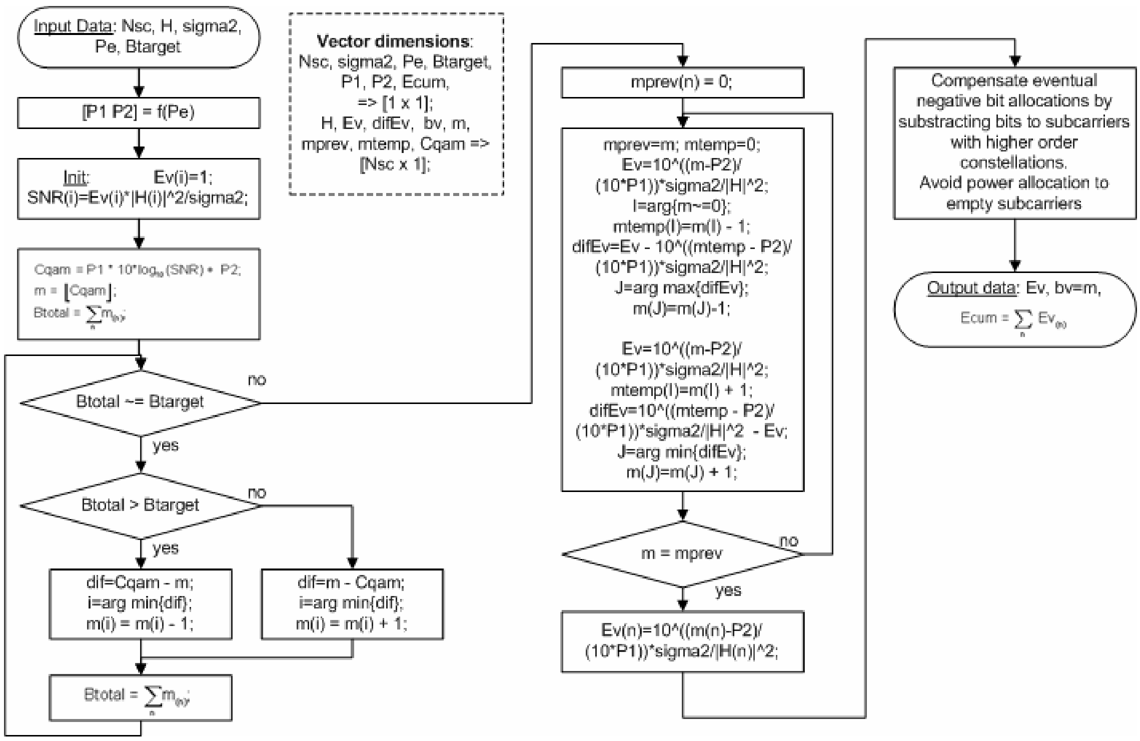


Fig.3.5 Czylwik's bitloading algorithm

It is possible that the algorithm gives negative bit allocations and allocates power to carriers with zero bits. Then, in the last step, this is solved by truncating to zero the forbidden bit and power allocations and by substracting (compensating) the negative bit allocations to the carriers with larger constellations.

3.6. Campello's algorithm

Campello proposed in his paper [Cam99] an optimal and efficient algorithm to implement in practice. Since it uses the gap approximation, its results are at most the same as the Hughes-Hartogs' method. However, the way of doing the adaptation is quite different with lower number of operations (faster implementation).

As before, the optimization criterion is to distribute the minimum transmit power among the sub-carriers for a given bit rate.

The algorithm receives the usual input parameters and computes the **SNR gap**, the channel gain to noise ratio **g_orig** and **g** (its normalized version). Then, the **k** coefficients (a discrete version of **g**) are grouped into **L** groups. Each group will receive the same bit allocation.

With the groups already created ($\mathbf{M_k}$) the program finds $\mathbf{i_B}$ thru an iterative process, which is a parameter that helps to obtain a fast and approximate bit allocation ($\mathbf{b_v} = [\mathbf{k} + \mathbf{i_B}]_0^{\mathbf{bmax}}$). $\mathbf{b_{max}}$ is the maximum allowed number of bits per carrier.

Once this bit distribution is obtained, then the remaining bits to reach $\mathbf{B_{target}}$ are added to the sub-channels that require smallest energy increments (see the BestIncrement algorithm in [Cam99] as an option to do this). Fig. 3.6 shows the flow diagram of this bitloading method.

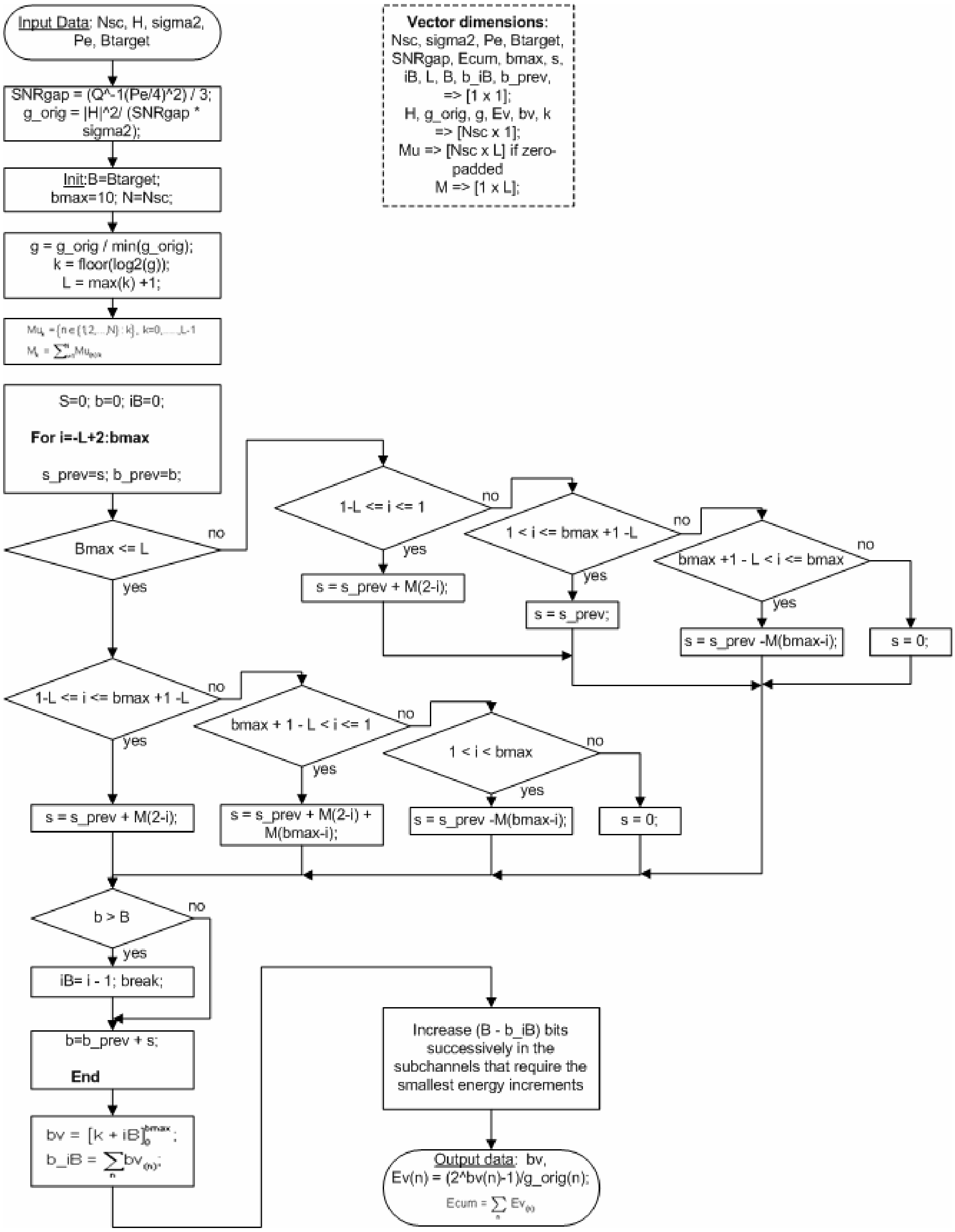


Fig.3.6. Campello's bitloading algorithm

3.7. Yu's algorithm

When trying to maximize the bit rate for a given transmit power a quite good sub-optimal solution is the so-called "constant power water-filling". This strategy greatly simplifies the transmitter and receiver design. It means that the same mean power is allocated to all the active carriers in the OFDM system. Yu [Yu01] derived a performance bound for the constant-power water-filling and proposed a very simple power allocation algorithm. Basically he observed that the critical task in approximate water-filling algorithms is to ensure that low SNR subchannels are allocated the correct amount of power. In particular, those subchannels that would receive zero power in exact water-filling, should not receive a positive power in sub-optimal algorithms, for otherwise, the power is almost wasted. As long as this is fulfilled, the performance is often close to optimal.

The algorithm receives the channel data and the available Power (**E_{target}**) and sorts the channel coefficients. Then it finds with a loop, the index (**m_{star}**) in the ordered vector. All the carriers from 1 to **m_{star}** will be assigned the power value S0. The rest of the carriers are assigned zero power.

Finally the power allocation vector (**E_v**) is re-ordered like the channel coefficients before the sorting process. This power allocation in the original paper nearly achieves the capacity of the frequency-selective channel (no QAM bitrates are considered, and the SNR gap is assumed equal to 1). Fig. 3.7 illustrates the proposed method.

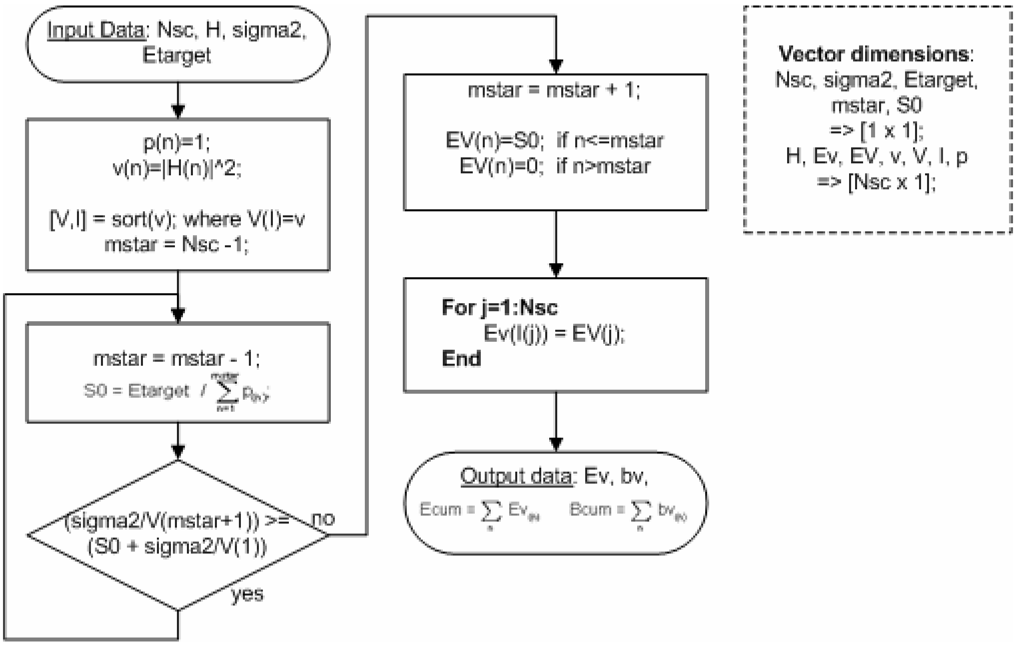


Fig.3.7. Yu's power allocation algorithm

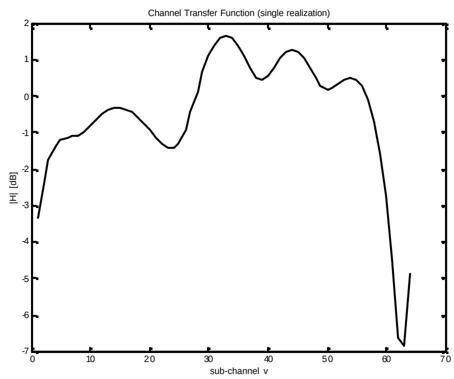
3.8. Evaluation of the presented algorithms

The described algorithms were implemented in Matlab, and an OFDM link over Hiperlan/2 channels was assumed (64 carriers and model-A channels [ETS98] were used). The path-loss in the channel model was not included since it does not alter the frequency-selectivity of the channel realizations (it only re-scales the amount of total used power). As the main interest was placed on the power and bit allocation, all the frequency bins were used and no pilots were considered. The channel state information (CSI) was assumed to be perfect and instantaneous. This means that before transmitting, the transmitter knows each sub-channel's SNR at the receiver end.

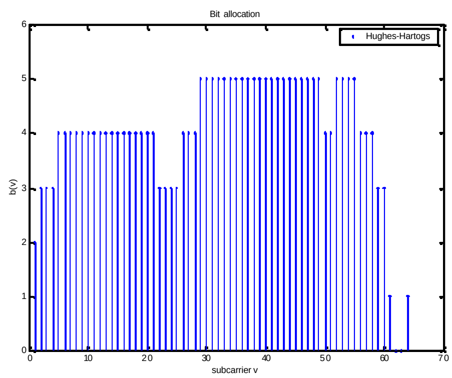
Power-Minimization methods: in this first experiment, all the algorithms solving the power-minimization problem were tried for the same channel and noise conditions:

The required bit rate was $B_{target}=256$ bits per OFDM symbol.
The noise variance of each sub-carrier was $s^2=10^{-4}$ (white broadband noise assumed).
The required error probability was $P_e=10^{-5}$.
Maximum number of bits per QAM subcarrier $b_{max}=10$.
Uncoded system (integer QAM constellations with $b \in \{0,1,2\ldots,b_{max}\}$ bits per symbol)
No power mask was predefined.

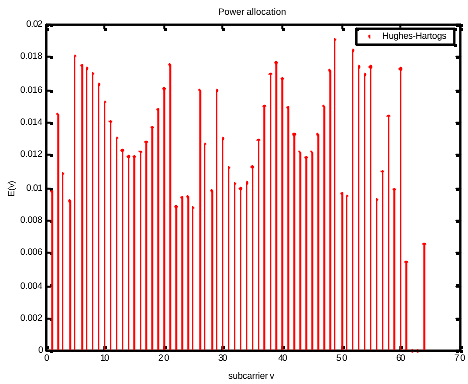
Fig. 3.8 displays one channel realization and the bit and power allocations that the different algorithms computed for this CSI.



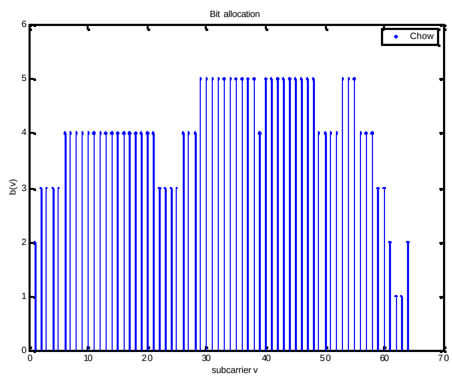
a



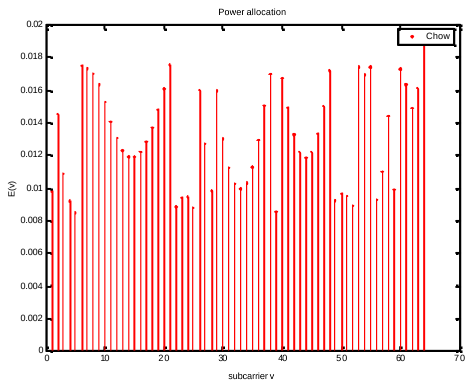
b



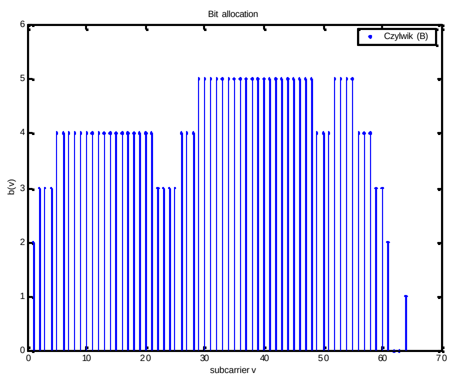
c



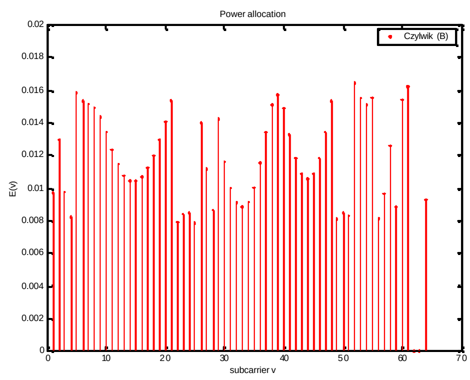
d



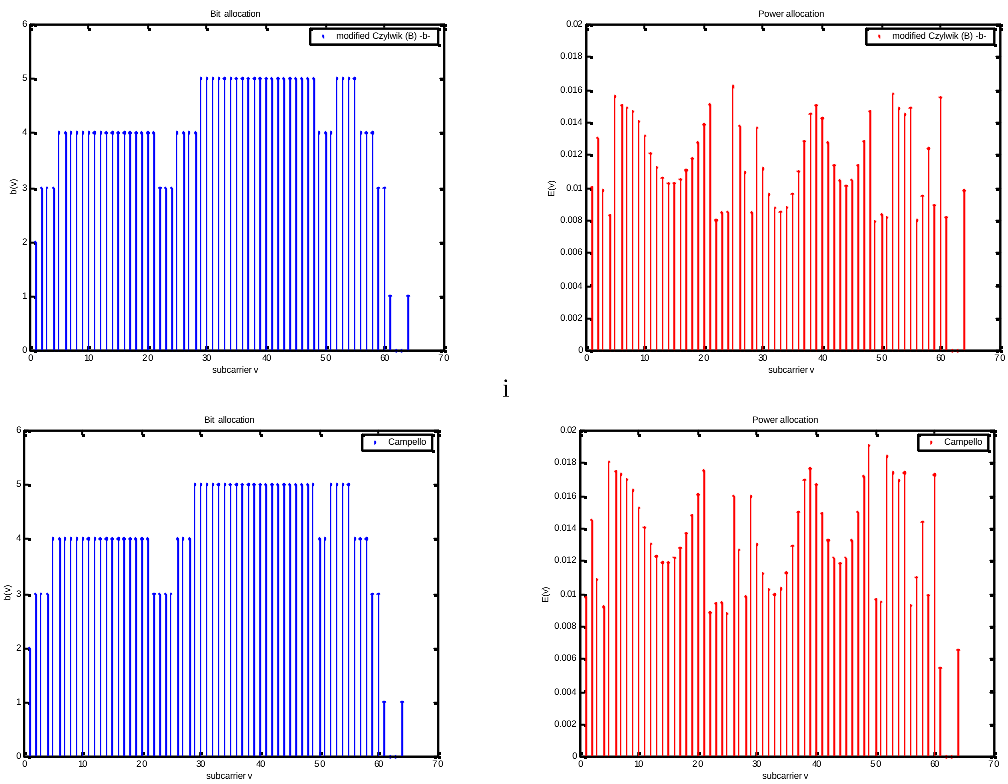
e



f



g



h i

j k

Fig. 3.8. Power and bit allocation for a given channel realization(different algorithms)

From Fig. 3.8 (*f* thru *i*), Czylwik's algorithm refers to the original algorithm presented in [Czy96] while the modified version of it uses more accurate coefficients **P1** and **P2** for the mapping between SNR and the achievable spectral bandwidth according to section 2.3.2. Notice some differences between both methods in subchannels 25 and 61. Besides, the second method is designed upon exact BER values (not symbol error rates) which make it better in terms of energy minimization as it will be shown.

It can be seen from the plots that Campello's algorithm gives the same bit and power allocation than Hughes-Hartog's (Fig. 3.8.*a* vs. Fig. 3.8.*j* , and Fig. 3.8.*b* vs. Fig. 3.8.*k*.). This is an interesting result (originally claimed by its author), since it is the optimal solution when using discrete constellation sizes and the gap approximation.

Chow's algorithm produced a bit different resource allocation (Fig. 3.8.*d* and *e*). For instance 2 bits were allocated to carriers 62 and 63 where no other method placed even one bit over these bad subchannels. Hence, Chow's power allocation has to be large over these carriers to transmit these bits.

After 50 trials on different channel realizations it was the modified version of Czylwik's algorithm that always gave the lowest transmit power, while Chow's algorithm gave the highest one (around 0.53dB between them). Table 3.1 shows typical results after the bitloading process from the described algorithms.

Table.3.1. required energy per OFDM symbol constrained to a required bit rate (different algorithms)

	Hughes-Hartogs	Chow	Czylwik	Czylwik (modified)	Campello
Total energy / OFDM symbol [dB]	2.2707	2.3541	1.9281	1.8204	2.2707

Table 3.2. Bitloading Algorithm's Processing Times

	Hughes-Hartogs	Chow	Czylwik	Czylwik (modified)	Campello
Processing Time [s]	0.0120	0.0031	0.0165	0.0172	N/A

It can be concluded that better performance (in terms of minimum used energy per symbol) of Czylwik's algorithms is based on the more accurate representation of the BER as a function of the SNR for the different used QAM schemes. Although the gap approximation is a very clever way of relating QAM performance to the capacity of the channel, it introduces errors that an exact SNR table can avoid.

To evaluate the complexity (i.e. implementation issues and processing times), we compared our Matlab implementations on the same computer platform to observe the relative speed of the different methods. Then Table 3.2 was produced. To avoid time-resolution errors of the computer clock each algorithm was measured after, at least, 50 running times for a given channel realization and then the individual processing time was taken as an average. Again, the experiment was done over 50 independent channels (i.e 50 independent time measurements).

We decided to exclude the Campello's algorithm from this evaluation since we didn't implement in our code the 4th step proposed in [Cam99] which seems to be the key of the algorithm's efficiency and low complexity. (It requires some understanding of advanced sorting methods). Instead we used a classic greedy method for this step that lead to times in the order of Czylwik's method.

Chow's method proved to be the fastest one of all. Huges-Hartogs' and both versions of Czylwik's had comparable velocities. This last result is reasonable since both are greedy methods that allocate one bit at a time.

Bit-Rate-Maximization methods: here, only the Hughes-Hartogs and Yu's algorithms were tried for the same channel and noise conditions:

The available energy per symbol to distribute was $E_{max} = 0.1$
The noise variance of each sub-carrier was $s^2=10^{-4}$ (white broadband noise assumed).
The required error probability was $P_e=10^{-5}$.
Maximum number of bits per QAM subcarrier $b_{max}=10$.
Uncoded system (integer QAM constellations with $b \in \{0,1,2\ldots,b_{max}\}$ bits per symbol)
No power mask was predefined.

To make the comparison possible, it was necessary to work with a SNRgap = 1 in the Hughes-Hartogs algorithm in order to make both of them reach the channel capacity[3]. Then, both power allocations were observed as well as their relative processing speeds.

Figure 3.9 displays one single channel realization and the power allocations generated by both algorithms. The constant power distribution from Yu's method is a highly attractive feature for a practical implementation since it decreases considerably the signalling information thru the feedback channel in the adaptive communication system. Only the subchannels that will not be used during the following periods need to be informed to the transmitter (or receiver). And this amount of information is much lower than the one that describes a detailed subcarrier-per-subcarrier optimal power-allocation.

Table 3.3 displays the results from both methods after computing 50 power allocations for 50 independent channel realizations. The mean used energy per symbol and mean number of allocated bits (for SNRgap = 1) are to be compared.

The integer bit loading in Hughes-Hartogs' algorithm makes that not all of the available energy per symbol be used. This does not happen with Yu's method (the full available power is distributed among the subchannels) as it can be noticed.

[3] In fact, Hughes-Hartog's method does not reach exactly the channel capacity since it can only allocate integer number of bits per sub-channel. Neither does Yu's algorithm since it is a sub-optimal method.

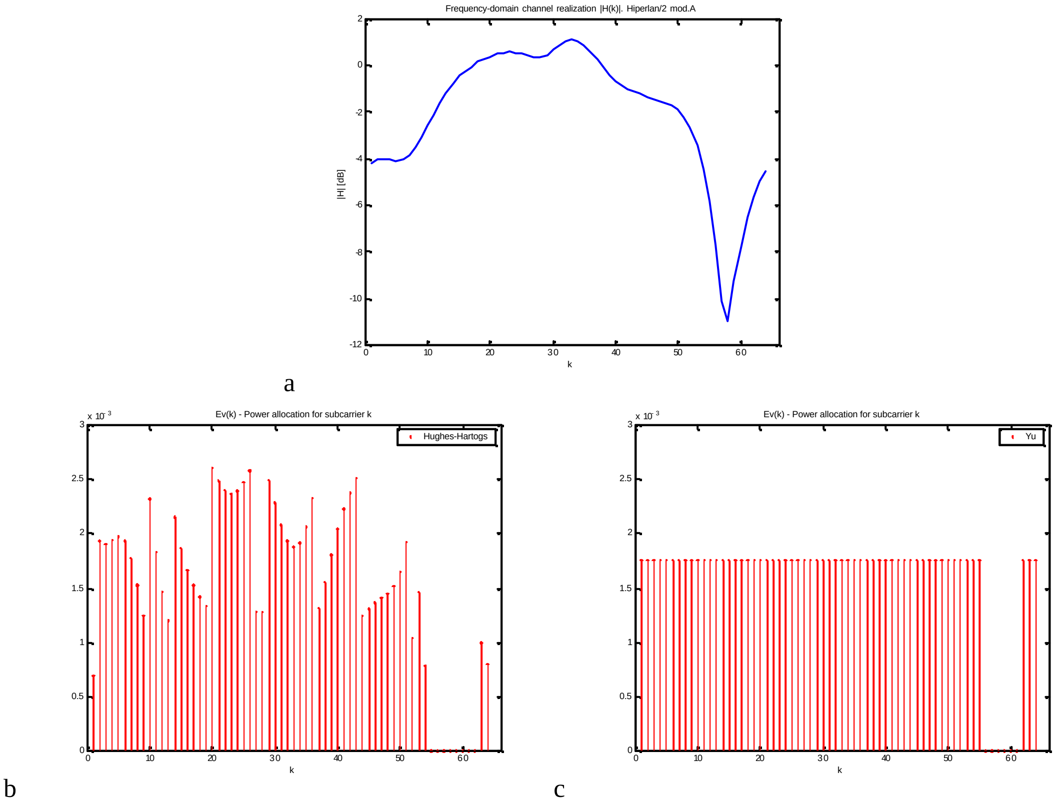


Fig.3.9 Power allocation for a given channel realization (two algorithms)

When looking at the relative speeds of both algorithms, Yu's method shows a significant advantage over Hughes-Hartog's. Its inherent simplicity and the fact that it does not allocate bits one by one in a greedy manner, make it faster and with much lower complexity. Table 3.4 compares both average processing times after 50 power allocations (in each power allocation, Yu's method had to be done 200 times to avoid resolution errors in the time measurements).

From both tables it can be concluded that the constant power adaptation is an excellent and cost-effective solution to maximize the bit rate in an MCM system.

Table.3.3 Mean Allocated Rates constrained to a limited transmitter energy per OFDM symbol (Emax= -10dB)

	Hughes-Hartogs	Yu
Total Rate / OFDM symbol [bits]	220.56	221.98
Used energy / OFDM symbol [dB]	-10.03	-10

Table.3.4 Mean processing times of the Bit-Rate-Maximization algorithms

	Hughes-Hartogs	Yu
Processing Time[s]	0.0103	0.0004

3.9. More about single-user bitloading algorithms

The presented algorithms represent only the basic ideas from the large research that has been carried out on this topic during the last decade. There is always a search towards the optimal adaptation at the expense of the minimal possible complexity (and maximal convergence speed). Most of the algorithms seen in the literature used the minimization of the transmit power or the maximization of the bit rate as the optimization criteria. But there are also proposals that tried to minimize the BER for a given transmit power and bit rate [Fis96]. It was shown before that the best adaptation could be obtained when using an accurate representation of the BER as a function of the received SNR per sub-channel. In this way, many authors like [Lev01] presented and/or patented many algorithms which obtained the best performance with reasonable complexity since look-up tables are used to obtain the bit and power allocations. One step ahead in reducing the complexity (and the necessary signalling overhead) is to work with clusters of carriers instead of with each of them individually. In [Grü01] there is an interesting proposal of adaptation based on a grid of modulation schemes for the different values of estimated SNR per sub-channel. Krongold, Ramchandran and Jones developed and optimal method [Kro00] by using a Lagrange multiplier and a fast bisection method to solve the problem numerically.

Finally, two more papers enhance the features of the classical Hughes-Hartogs' bitloading method: in [Son00], the authors propose a bit removal-strategy instead of a bit-filling one. With it they obtained faster convergence times for DSL applications. Recently, Baccarelli and Biagi [Bac04] presented a bitloading method that takes into account a constraint on the maximum transmit power per carrier. This algorithm is regarded as a generalization of the Hughes-Hartogs' well-known method.

4. Resource Allocation for the Multi-User Scenario

4.1. System Description

A multi-user OFDM system is the one where the orthogonal subcarriers are distributed among the users that communicate with a base station. In this kind of systems, each subchannel is allocated to only one user. As the transfer function is generally different for each user, it may happen that when some subchannels are in deep fade for one user, they are fine for other users. Fig. 4.1 provides the discrete-time channel model for an Orthogonal Frequency Division Multiple Access (OFDMA) scheme with U users and N subcarriers. In this typical uplink configuration, the receiver demodulates all the users' signals along the N subchannels. Each user is only allowed to transmit using certain subcarriers depending on the channel conditions and the user's resources or constraints. Thus, the subcarrier distribution is a key issue that should be adapted to the channel state. Once the users have their subchannels assigned, a bitloading algorithm determines the best bit and power allocation on their subchannels for each user. As it can be inferred, the multi-user resource-allocation problem increases the complexity of the optimization problem to solve when compared to the single-user case.

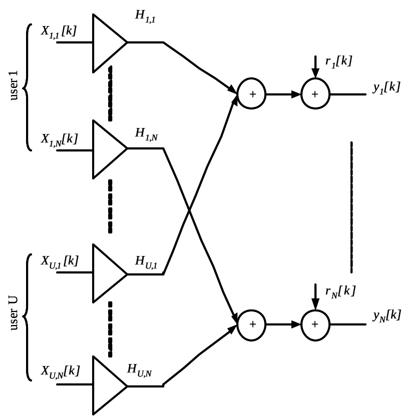


Fig. 4.1 Channel model for a multi-user OFDM uplink

$X_{u,n}$ represents the complex symbol that has been allocated to the user u and onto sub-carrier n (mapped according to a bit and power allocation algorithm). $H_{u,n}$ are the channel coefficients (i.i.d. Rayleigh distributed and generated according to section 2.4), r_n is the noise sequence affecting the sub-carrier n (with variance $s_n^2 = E[|r_n[k]|^2]$) and Y_n is the received symbol from the sub-channel n.

4.2. Optimization Criteria and Applications

The resource-allocation problem in multi-user OFDM, or how to distribute the subcarriers and how to allocate bits and power onto them, can be defined in many different ways according to the system requirements. Basically, there are two main optimization objectives that are pursued by most of the designs and that are usually combined with many secondary constraints that depend on the specific application or used hardware. These two different approaches, which are treated along the following sections, can be summarized as:

a. Power-minimization based resource-allocation: a system design that focuses on the reduction of the transmitter power for a given bit rate. Basically, the users need to transmit a certain bit rate at a given BER and the resource-allocation algorithms distribute subcarriers, bit and powers in a way that the transmitter power is minimized.

b. Capacity-maximization based resource allocation: a system design where the optimization goal is the maximization of the overall bit rate (or aggregated data rate, or sum capacity). The constraints are placed on the transmitter power that can be regarded per user (for the uplink) or for all the users together (for the downlink).

These broad design guidelines are usually implemented jointly with additional constraints as limits imposed on maximum power per subchannel (power spectrum masks). Moreover, some times it is desired that the subcarrier distribution be done in groups (not individual subchannels are assigned to the users). It is also possible that the system designer require as few subcarriers as possible to fulfill the joint power and bit-rate constraints. Once the problem is formulated, the algorithm that solves it must also meet some complexity requirements according to the hardware where it will be implemented and the update rate that the particular channel and terminal mobility demand. Lawrey's paper [Law99] about multi-user OFDM is a very good reference to introduce the interested reader into the basic aspects of this topic.

4.3. Power-Minimization Based Algorithms

4.3.1. Pfletschinger's Algorithm

This algorithm was presented in [Pfl02] and it was designed based on the two-steps idea described in [Yin00] for another resource allocation problem. The algorithm solves the following optimization problem for N subcarriers and U users:

$$\begin{aligned} &\min \sum_{n=1}^{N}\sum_{u=1}^{U} p_{n,u} \\ &s.t. \\ &\sum_{n=1}^{N} b_{n,u} \geq b_{\min}(u) \\ &\sum_{n=1}^{N} p_{n,u} \leq p_{\max}(u) \end{aligned} \tag{4.1}$$

Here, the overall transmitted power (sum of all the individual powers per user and subcarrier $p_{n,u}$) is minimized given that the available transmitter power per user $p_{max}(u)$ be not exceeded and the minimum number of bits that each user needs to transmit, $b_{min}(u)$, be effectively allocated. This problem description is typical in the uplink of a wireless system.

The algorithm proposed by Pfletschinger et al, as it is done in many other resource allocation methods, searches for an optimal distribution of the subcarriers among the users and then it performs individual bitloadings for each user using the assigned set of subchannels. To complete this first part, the algorithm divides the subcarrier allocation into two steps, based on the following principles:

- The number of subcarriers for one user mainly depends on its desired minimal bit rate $b_{min}(u)$ and on his *mean* CNR which is defined as

$$\overline{T_u} = \frac{1}{N}\sum_{n=1}^{N} T_{n,u} \tag{4.2}$$

- The selection of which subchannel is assigned to a user depends on the particular CNR value:

$$T_{n,u} = \frac{\left|H_{n,u}\right|^2}{\Gamma_u s_n} \tag{4.3}$$

First, estimation about how many subcarriers each user receives is done according the users' mean CNR, the desired bitrate and the users' maximum transmitter powers. In the second step it is determined which particular subcarrier is given to each user.

Subcarrier distribution - FIRST STEP

Each user is assigned k_u subcarriers such that $b_{min}(u)$ can be reached with the available transmitter power $p_{max}(u)$. Hence,

$$p_{tot}(u) = k_u \overline{T}_u^{-1}\left(2^{b_{\min}(u)/k_u} - 1\right) \leq p_{\max}(u) \ . \tag{4.4}$$

The first value that k_u takes is computed as if the largest constellation size could be applied to all the subcarriers (b_{max} is the maximum number of bits that can be allocated to any subcarrier):

$$k_u = \left\lceil b_{\min}(u) \big/ b_{\max} \right\rceil . \tag{4.5}$$

This value is usually lower than the final one, so new subcarriers are assigned to the users until the $p_{max}(u)$ is not exceeded. At this stage, if there are subcarriers left, the used powers are lowered by a small step, and the procedure repeats until no subcarriers remain. Fig 4.2 displays the pseudo-code of this first block of the algorithm.

$k_u = \lceil b_{\min}(u)/b_{\max} \rceil;$
$p_{tot}(u) = k_u \overline{T}_u^{-1}\left(2^{b_{\min}(u)/k_u} - 1\right); \forall u$
while $\sum_u k_u < N$
  for $u := 1,\ldots,U$
    while $p_{tot}(u) > p_{\max}(u)$
      $k_u = k_u + 1;$
      $p_{tot}(u) = k_u \overline{T}_u^{-1}\left(2^{b_{\min}(u)/k_u} - 1\right);$
    end while
  end for
  if $\sum_u k_u < N$
    $p_{\max}(u) = (1-e)\, p_{\max}(u); \forall u$
  end if
end while
while $\sum_u k_u > N$
  $p_{new}(u) = (k_u - 1)\overline{T}_u^{-1}\left(2^{b_{\min}(u)/(k_u - 1)} - 1\right); \forall u$
  $u' = \arg\min_u \{p_{new}(u) - p_{tot}(u)\};$
  $k_{u'} = k_{u'} - 1;$
  $p_{tot}(u') = p_{new}(u');$
end while

Fig. 4.2 First step of the subcarrier distribution process

Subcarrier distribution - SECOND STEP

Once each user has been assigned a number of subchannels, the users choose alternatingly the subcarrier with the best CNR until there is no subchannel left. As the users usually have unequal number of subcarriers, the order in which the users choose their subcarriers is important. This order is controlled by a procedure based on priorities. A reference priority $pr_0(u)$ is defined as:

$$pr_0(u) = \frac{k_u}{N} \tag{4.6}$$

After user u has taken one subcarrier, k_u is decremented by one and the actual priority of user u is computed as:

$$pr(u) = \frac{k_u}{\sum_{u=1}^{U} k_u}, \quad u = 1, \dots, U \tag{4.7}$$

The algorithm works like this: the user with the most subcarriers begins, then after each step the user with the greatest difference between reference and actual priority chooses its best carrier from the pool. **A** is the subcarrier allocation matrix with $A_{n,u}$ = 1 if subchannel n is assigned to user u, and 0 otherwise. Fig. 4.3 details this part of the algorithm.

$\mathbf{A} = \mathbf{0};$

$pr_0(u) = k_u / N \; ; \forall u$

$U_{set} = \left\{ u \middle| u = \arg\max_{u'=1\dots U} \{k_{u'}\} \right\};$

for $u \in U_{set}$

$\quad n_1 = \arg\min_{n \in M_{set}} \{T_{n,u}^{-1}\}$, **with** $M_{set} = \left\{ n \middle| \sum_{u'=1}^{U} A_{n,u'} = 0 \right\}$

$\quad k_u = k_u - 1;$

$\quad A_{n_1,u} = 1;$

end for

while $\sum_{\mathbf{u}} k_u > 0$

$\quad pr(u) = k_u \Big/ \sum_{u'} k_{u'} \; ; \forall u$

$\quad U_{set} = \{ u | u = \arg\max \{ pr(u') - pr_0(u') \} \};$

$\quad$ **for** $u \in U_{set}$

$\quad\quad n_1 = \arg\min_{n \in M_{set}} \{T_{n,u}^{-1}\}$, **with** $M_{set} = \left\{ n \middle| \sum_{u'=1}^{U} A_{n,u'} = 0 \right\}$

$\quad\quad k_u = k_u - 1;$

$\quad\quad A_{n_1,u} = 1;$

$\quad$ **end for**

end while

Fig. 4.3 Second step of the subcarrier distribution process

Single-user Bitloading

As it was described earlier, Pfletschinger's algorithm concludes by performing single-user bitloading operations over the set of subchannels assigned to each user. In this way, part of the optimization problem (4.1) is reduced to a simpler problem with a known optimal solution already described along Section 3. Then, we can summarize this procedure by defining a single-user bitloading function as

$$(\mathbf{b},\mathbf{p}) = \mathbf{bitload}\left(\mathbf{T}_{M_{set}}, \{0,1,\ldots,b_{\max}\}, b_{\min}, p_{\max}\right) \tag{4.8}$$

with M_{set} ={ M_{set} (1),…, M_{set} (M)} representing the set of dedicated subcarriers with cardinality M=|M_{set} |. $\mathbf{T}_{Mset}$ denotes the vector with the CNRs of the subcarriers that belong to user u, and $\mathbf{b}$=(b_1,…,b_M) and $\mathbf{p}$=(p_1,…,p_M) define the corresponding bit and power vectors.

4.3.2. Modified Pfletschinger's Algorithm

The algorithm proposed in this section introduces one modification respect to [Pfl02] described before. Basically, we perform the second step of the subcarrier distribution task following the procedure proposed by [Yin00] and adapted to the current optimization problem (4.1). The idea is that once the number of subcarriers k_u has been determined, the exact subcarrier assignment that minimizes the total power can be treated as an ordinary *Assignment Problem*[Ner93].

For our case, the assignment problem is presented as follows:

Given an N x N cost matrix $\mathbf{P}$ = [$p_{n,u}$], find an N x N allocation matrix $\mathbf{A}$ = [$a_{n,u}$] such that

$$P_{tot} = \sum_{u=1}^{N}\sum_{n=1}^{N} p_{n,u} a_{n,u} \tag{4.9}$$

is minimal. We let

$$p_{n,u} = T_{n,u}^{-1} k_u \left(2^{b_{\min}(u)/k_u} - 1\right) / k_u \tag{4.10}$$

and since U is usually lower than N, we split each user u into k_u virtual users in a way that each one of the N virtual users will be assigned one and only one subcarrier ($c_{n,u} \in \{0,1\}$). Then, by solving (4.9) for P_{tot}, we find a subcarrier allocation which minimizes the overall transmitter power when the number of subcarriers assigned to each user is given.

The so-called Hungarian algorithm [Khu55], which has been widely studied in the open literature, optimally solves the assignment problem.

As before, after the subchannels are distributed among the users, the power and bits allocated to each user can be allocated to these subcarriers locally. This is reduced to a single-user bitloading problem solved by (4.8).

4.3.3. Wong's Algorithm

The combined subcarrier, bit, and power allocation algorithm described here comes from [Won99]. This multi-user resource allocation method is known to be the best practical power-minimizer algorithm, i.e. the one that makes the system spend the least total power to transmit the desired user bitrates. This nearly optimal method solves the optimization problem described in (4.1). As this problem is a combinatorial optimization problem, Wong et al. relaxed the constraints of integer bits per subcarrier and no subcarrier sharing to allow the allocation coefficient $a_{u,n}$ is a real number within the interval [0,1]. This value represents the fraction of each subcarrier that each user takes. Also, the number of bits per subcarrier $b_{u,n}$ is now a real number within the interval [0, $a_{u,n}$.$b_{\max}$]. With these modifications, the new problem is the same as the original problem, but now the minimization of the cost function is done over a larger set. Besides, the objective function becomes convex over a convex set. Hence, standard convex optimization techniques can be used to solve (4.11).

$$P_T = \min_{\substack{b_{u,n} \in [0, a_{u,n} b_{\max}] \\ a_{u,n} \in [0,1]}} \sum_{u=1}^{U} \sum_{n=1}^{N} a_{u,n} T_{u,n}^{-1} \left(2^{\frac{b_{u,n}}{a_{u,n}}} - 1 \right)$$

$$\textbf{s.t.} \qquad \qquad (4.11)$$

$$b_{\min}(u) = \sum_{n=1}^{N} b_{u,n}, \quad \forall u \in \{1, \ldots, U\}$$

$$1 = \sum_{u=1}^{U} a_{u,n}, \qquad \forall n \in \{1, \ldots, N\}$$

As a consequence of this reformulation, we expect the minimum total power obtained in (4.11) to be a tight lower bound of the power obtained in the original problem. We use the gap approximation in the definition of the CNR ($T_{u,n}$) as it was done with the previous algorithms.

The solution proposed by Wong et al. is derived after obtaining the Lagrangian L from the constrained objective function and differentiating L with respect to $b_{u,n}$ and $a_{u,n}$ respectively. From here, the necessary conditions for the optimal solution are obtained and then it can be shown that, if $H_{u,n}(l_{q,u})$ are different for all k, then

$$a^*_{u',n} = 1, \; a^*_{u,n} = 0 \quad \forall u' \neq u \qquad (4.12)$$

where

$$u' = \arg\min_u \{H_{u,n}(l_{q,u})\} \qquad (4.13)$$

and

$$H_{u,n}(l) = \left[\frac{T_{u,n}.l_u}{\ln(2)} - T_{u,n}.l_u.\log_2\left(\frac{T_{u,n}.l_u}{\ln(2)}\right) - 1 \right] \frac{1}{T_{u,n}} \qquad (4.14)$$

With this, a fixed set of Lagrange multipliers l_u, $u \in \{1,\ldots,U\}$ can be used to determine u' for each n using (4.13). Although this leads to the optimal solution, the individual rate constraint $b_{\min}(u)$ may not be satisfied. To solve this, Wong et al. propose an iterative searching algorithm which is displayed in Fig. 4.4. Starting with some small values for all l_u, the iterative method increases one of the l_u until the data rate constraint for user u is fulfilled. Then, the process is repeated for the rest of the users, one at a time. The convergence of the algorithm is guaranteed as it can be shown in [Won99].

The results obtained from this resource allocation algorithm cannot be used immediately in our original problem (4.1). This is because the resulting $b_{u,n}$ may not be integer and within the available set of bits per subcarrier. Another mismatch may be a resulting $a_{u,n}$ within (0,1) indicating a time-sharing solution. And, simply quantizing $b^*_{u,n}$ and $a^*_{u,n}$ does not satisfy the individual rate constraints.

So, and in order to give a complete solution, Wong's method uses the algorithm described in Fig.4.4 to obtain the basic subcarrier allocation. Following, a single user bitloading algorithm -like (4.8)- is applied to each user on the allocated subcarriers. The short code displayed in Fig. 4.5 can be used to eliminate the time-shared subcarriers before single-user bitloading is applied to each user.

$l_u = \min_{n\in\{1,...,N\}} \{\ln(2)/T_{u,n}\}; \forall u \in \{1,...,U\}$ **// initial LM**

$b_{u,n} = \log_2(l_u.T_{u,n}/\ln(2));$

$H_{u,n} = \left[\frac{T_{u,n}.l_u}{\ln(2)} - T_{u,n}.l_u.\log_2\left(\frac{T_{u,n}.l_u}{\ln(2)}\right) - 1\right] \Big/ T_{u,n};$

$umn = \arg\min_{n\in\{1,...,N\}} \{H_{u,n}\}; \forall u \in \{1,...,U\}$

$a_{u,n} = 0; \quad a_{umn,n} = 1/|umn|;$ **// initial sc assignment (with sc sharing if necessary)**

$r_u = \sum_{n=1}^{N} a_{u,n}.b_{u,n}; \forall u \in \{1,...,U\}$ **// initial rates**

while $r_u < b_{\min}(u), \forall u \in \{1,...,U\}$

$l = \arg\max\{b_{\min}(u) - r_u\};$ **// user with largest rate deficiency**

lbl **for** $n1 = \{n | a_{l,n} \neq 1\};$ **// subcarriers that are not entirely assigned to user** l

$k(n1) = \arg\min_{k} \{H_{k,n1}\};$

find $l1(n1)$ **such that** $H_{l,n1} = H_{k(n1),n1} + \Delta;$ **// with** Δ **equal to some small value**

end for

$l_l = \min_{n1} \{l1(n1)\}; \quad m = \arg\min_{n1} \{l1(n1)\};$ **// new LM value and sc for user** l

$b_{l,n} = \log_2(l_l.T_{l,n}/\ln(2)); \forall n \in \{1,..., N\}$

$H_{l,n} = \left[\frac{T_{l,n}.l_l}{\ln(2)} - T_{l,n}.l_l.\log_2\left(\frac{T_{l,n}.l_l}{\ln(2)}\right) - 1\right] \Big/ T_{l,n}; \forall n \in \{1,..., N\}$

$a_{l,m} = 1; \quad a_{k,m} = 0; \forall k \neq l;$

$r_l = \sum_{n=1}^{N} a_{l,n}.b_{l,n};$ **// new rate for user** l

if $r_l \neq b_{\min}(l),$

if $r_l < b_{\min}(l)$

goto lbl

else

$r_l = r_l - b_{l,m};$ **// discard the contribution of the last added sc to the rate of user** l

if $r_l < b_{\min}(l),$

$a_{l,m} = (b_{\min}(l) - r_l)/b_{l,m};$ **// apply subcarrier sharing**

$b_{l,m} = b_{\min}(l) - r_l; \; a_{k(m),m} = 1 - a_{l,m};$

else

find l_l **such that** $\sum_{n=1}^{N} b_{l,n} = b_{\min}(u);$ **with** $b_{l,n} = \log_2(l_l.T_{l,n}/\ln(2))$ **// fine tune LM**

end if

end if

end if

$r_u = \sum_{n=1}^{N} a_{u,n}.b_{u,n}; \forall u \in \{1,...,U\};$

end while

Fig. 4.4 Pseudo-code of Wong's algorithm

$$
\begin{aligned}
&\textbf{for } n \in \{1,\ldots,N\},\\
&\quad a_{u,n} = 0; \forall u \in \{1,\ldots,U\}\\
&\quad u1 = \arg\max_{u \in \{1,\ldots,U\}} \{a_{u,n} \cdot b_{u,n}\};\\
&\quad a_{u1,n} = 1;\\
&\textbf{end}
\end{aligned}
$$

Fig. 4.5 Elimination of subcarrier sharing after Wong's method

4.3.4. A Spectrally Efficient Resource-Allocation Method

This algorithm was presented in [Ace05] and it proposes a new method for assigning subcarriers, constellation sizes and power to the users in an OFDMA system. The proposed scheme considers user-individual bit rate and power constraints, maximum constellation sizes and a spectral power density (PSD) mask. Although the algorithm aims to solve a different problem from (4.1) - we now minimize the total used bandwidth by reserving as many subcarriers as possible for future users - , we want to include it in this section since we found that only very few additional total transmit power is required in order to reserve a significant amount of subcarriers, even when compared to Wong's algorithm. The computational complexity of the algorithm is very low, which makes it suitable for practical implementation as we'll see in the simulations section.

There are various motivations to allocate only as few subcarriers as required in an OFDMA system. Subcarriers can be reserved for future users that might join the system or the free frequency bands can be made available to other systems, which might share this part of the spectrum without mutual interference. This latter approach is in line with recent developments like *spectrum pooling* [Weis04] and *spectrum agile radios* [Man04].

The proposed novel algorithm acts on the constraints of a minimum bit rate $b_{\min}(u)$ and a maximum available transmit power $p_{\max}(u)$ per user, as well as a maximum power $S_{\max}$ per subchannel, which corresponds to a spectral mask. The maximum number of bits that any subcarrier can convey is determined by $b_{\max}(u)$. Each subcarrier can be assigned to only one user. Denoting by $a_{n,u}$ the allocation coefficient, such that $a_{n,u} = 1$ if subcarrier n is assigned to user u and $a_{n,u} = 0$ otherwise, we can formulate the optimization problem as:

$$
\begin{aligned}
&\min \sum_{u=1}^{U} \sum_{n=1}^{N} a_{n,u}\\
&s.t.\\
&\sum_{n=1}^{N} b_{n,u} \geq b_{\min}(u)\\
&\sum_{n=1}^{N} p_{n,u} \leq p_{\max}(u)\\
&0 \leq p_{n,u} \leq S_{\max}\\
&\sum_{u=1}^{U} a_{n,u} \in \{0,1\}\\
&b_{n,u} \in \{0,1,\ldots,b_{\max}\}
\end{aligned}
\tag{4.15}
$$

The designed algorithm to solve (4.15) contains a single-user bitloading algorithm as a subfunction. Thus, part of the optimization problem is reduced to a simpler problem with a known solution. We use here (4.8) when single-user bitloading needs to be solved.

We define the mean CNR of user u as in (4.2) and sort the users in ascending order according to their mean CNR in the set U_{set}, which is a permutation of $\{1,2,\ldots,U\}$, such that

$$\overline{T}_{U_{set}(1)} \le \overline{T}_{U_{set}(2)} \le \ldots \le \overline{T}_{U_{set}(U)} \tag{4.16}$$

With this, the worst average channel appears first in U_{set}. Denoting by

$$F_{set}^{'} = \left\{ n : \sum_{u'=1}^{U} b_{n,u'} = 0 \right\} \tag{4.17}$$

the set of all not yet assigned subcarriers, we define the permutation p(.) such that the set F_{set}= p(F'_{set}) is ordered in descending order with respect to the CNR of user u (making the subcarrier with the highest CNR appear first in F_{set})

$$T_{F_{set}(1),u} \ge T_{F_{set}(2),u} \ge \ldots \tag{4.18}$$

To solve (4.15), we propose the algorithm depicted in Fig. 4.6 that can be summarized as

1. Sort all users according to their mean CNR in ascending order. The user with the worst channel will be processed first.
2. **for** all users
   - 2.1 Denote by F_{set} the set of all not yet assigned subcarriers. This pool is sorted such that the subcarriers with higher CNR appear first.
   - 2.2 Set m=1
     - 2.2.1 Apply single-user bitloading to the first m subcarriers of F_{set}
     - 2.2.2 If the bit rate, power and PSD constraints can be fulfilled with these m subcarriers, continue with step 2 (next user). If not, increment m and continue with step 2.2.1

$$
\begin{array}{l}
b_{n,u} = p_{n,u} = 0, \forall n,u \\
\mathit{for}\ u \in U_{set} \\
\quad F_{set}^{'} = \left\{ n : \sum_{u'=1}^{U} b_{n,u'} = 0 \right\} \\
\quad F_{set} = p\left(F_{set}^{'}\right) \\
\quad m = 1 \\
\quad \mathit{while} \\
\quad\quad M_{set} = \{F_{set}(1), \ldots, F_{set}(m)\} \\
\quad\quad (\mathbf{b^*}, \mathbf{p^*}) = \mathbf{bitload}\left(\mathbf{T}_{M_{set},u}, \{0,1,\ldots,b_{\max}\}, b_{\min}, p_{\max}\right) \\
\quad\quad \mathit{if}\ \sum_{m \in M_{set}} b_m^* = b_{\min}(u)\ \ AND\ \ p_m^* \le S_{\max}\ \forall m \in M_{set} \\
\quad\quad\quad b_{M_{set},u} = \mathbf{b^*} \\
\quad\quad\quad p_{M_{set},u} = \mathbf{p^*} \\
\quad\quad\quad \mathit{break} \\
\quad\quad \mathit{else} \\
\quad\quad\quad m = m + 1 \\
\quad\quad \mathit{end\ if} \\
\quad \mathit{end\ while} \\
\mathit{end\ for}
\end{array}
$$

Fig 4.6 Spectrally efficient resource-allocation algorithm

It should be noted that for the sake of a clear exposition, this algorithm does not include any conditions for error handling. It can of course happen, that for a given set of constraints no solution exists, e.g. if the desired bit rate cannot be reached with the given transmit power. Also, the number of subcarriers that are allocated to one user should be limited to a fraction of N to prevent that a user with few power (or a high path loss) tends to occupy many subcarriers. Depending on the application and the desired fairness conditions, such a user could be rejected or given a limited number of subcarriers. In any case, these conditions can be added easily to the described algorithm.

4.3.5. Simulation results

The previous algorithms were implemented in Matlab, and an OFDMA link (with the model described in Fig. 4.1) over Hiperlan/2 channels was assumed. Following [ETS98], model-A channels with $N = 64$ subcarriers were used. We focused on the subcarrier, power and bit allocation per user. Thus, all the frequency bins were used and no pilots were considered. The channel state information (CSI) was assumed to be perfect and instantaneous. This means that before transmitting, the transmitter knows each sub-channel's SNR at the receiver end.

We divided the simulations in two parts. The first one aimed to evaluate the algorithms presented in 4.3.1, 4.3.2 and 4.3.3 which solved the same basic optimization problem with differences in the set of constraints - see (4.1) and (4.11)-. Along the second part, we run some experiments to evaluate the algorithm described in 4.3.4. In both parts we also discuss about the obtained results.

A - Power-Minimization based algorithms:

Here, we tried the Pfletschinger, modified Pfletschinger and Wong's algorithms for the same channel and noise conditions:

The number of users sharing the OFDMA system ranged from 2 to 6
The required bit rate was $b_{\min}$=25 bits per user per OFDM symbol.
The noise variance of each sub-carrier was normalized to $s^2=1/N$ (white broadband noise assumed).
The required error probability was $P_e=10^{-5}$ (i.e the SNR gap = 6.95)
Maximum number of bits per QAM subcarrier b_{max}=6.
Uncoded system (integer QAM constellations with $b \in \{0,1,2\ldots,b_{\max}\}$)
No power mask was predefined.

To compare these three algorithms we observed the used total power per OFDM symbol that each algorithm allocated to all the users in order to allow the transmission of the required bit rates defined in the constraint set. Since the first two algorithms additionally consider user-individual power constraints, these individual power-budgets were chosen large enough, so that Wong's algorithm did not violate these constraints.

The implementation of the modified Pfletschinger's algorithm in Matlab included a function that performed the Hungarian algorithm. This function was taken from [Bor96].

We show in Fig. 4.7 and Fig 4.8 a particular set of channel realizations in a 4-users system and the corresponding subcarrier, power and bit allocations that each algorithm performed for the given scenario.

As it can be seen in these figures at first glance (which can be confirmed by running many resource allocations for many channel realizations), Wong's algorithm decreases the used power per subcarrier respect to the other two methods. Also, it uses lower constellation sizes when possible (recall that BPSK and QPSK are always more power-efficient than other high order constellation sizes). All this is done at the expense of increasing the occupied bandwidth (i.e. more subcarriers are distributed among the users).

Another interesting observation from these figures, which was also observed along many independent trials of the evaluated methods, is that the modified Pfletschinger's algorithm groups more the allocated subcarriers assigned to each user, when compared to the original Pfletschinger's algorithm. This gives better immunity to inter-carrier interference (ICI) problems.

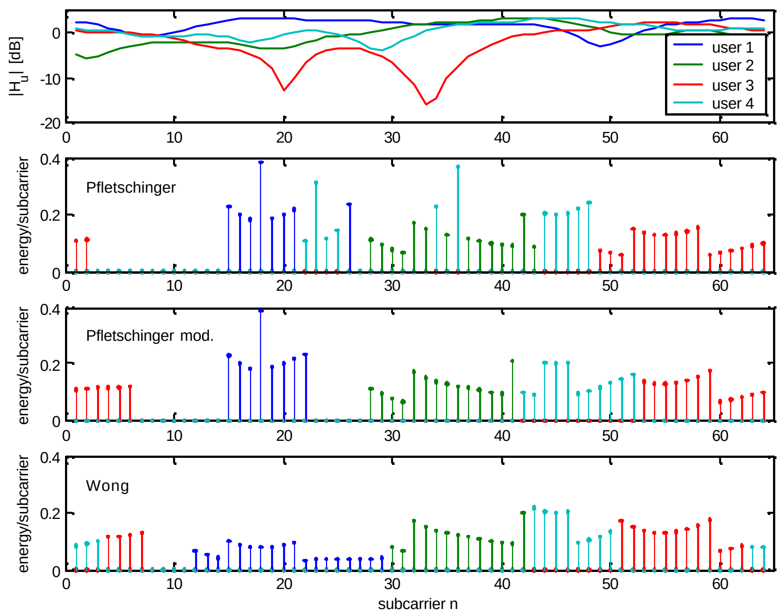


Fig. 4.7 Subcarrier and power allocation for a 4-users OFDMA system

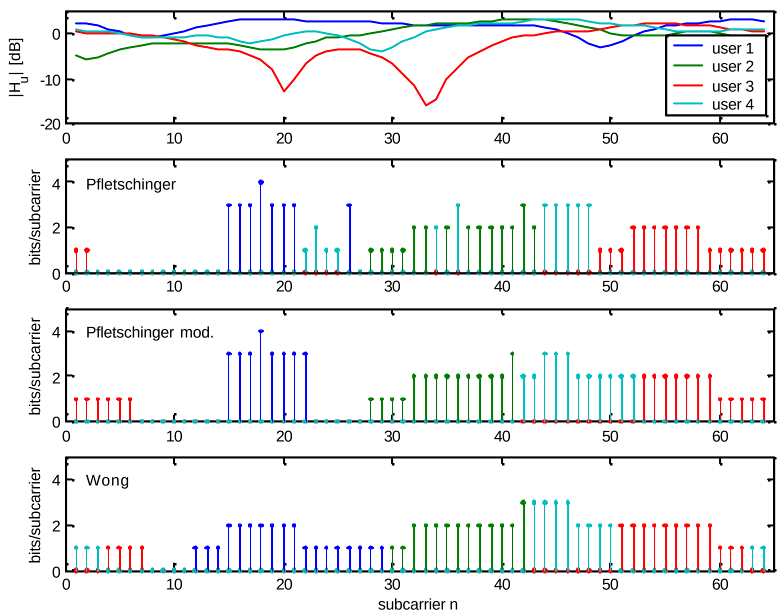


Fig. 4.8 Subcarrier and bit allocation for the same 4-users OFDMA system

In order to measure the algorithms' performance in terms of how well they achieve the power minimization objective and how fast each algorithm obtains a solution, we averaged results after 200 sets of independent channel realizations and different number of users. The results are displayed in Fig. 4.9 and Fig. 4.10 respectively.

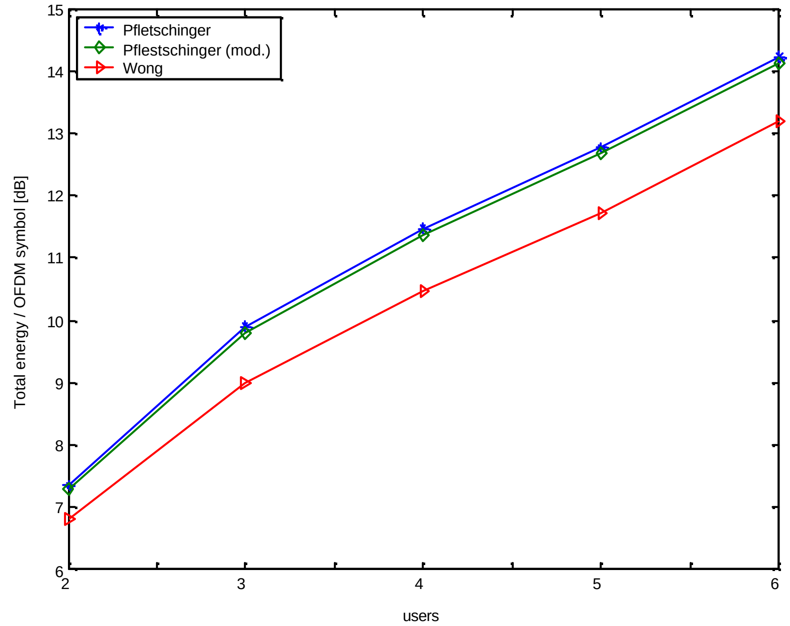


Fig. 4.9 Total used transmit power after the resource allocation within the OFDMA system

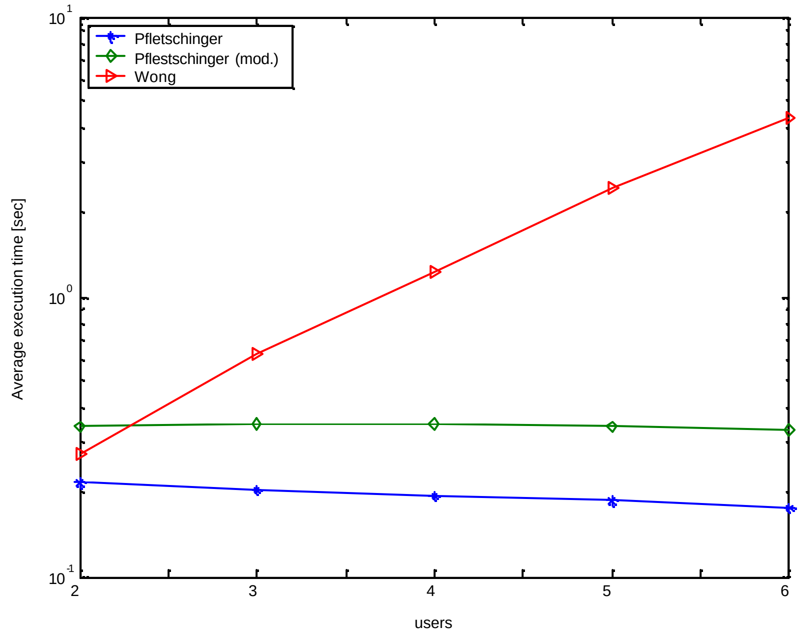


Fig. 4.10 Comparison of the mean execution times

It is clear from Fig. 4.9 that Wong's algorithm is the one that achieves the minimum total transmit power to allocate the desired user-bitrates. However, Wong's results are, at most, 1.06 dB lower than Pfletschinger's method for any number of users. The modified Pfletschinger's method including the Hungarian algorithm to solve the internal assignment problem has only a slight performance enhancement respect to the original method (0.1dB).

When we analyze the complexity and processing speed of the three algorithms, we can refer to Fig. 4.10. There, we observe the notorious increase of the mean execution time that Wong's algorithm has when add users to the system. This algorithm demands large computational resources which makes a practical implementation almost unthinkable by now. The other two algorithms have comparable execution times, being Pfletschinger's method better than its modified version with the Hungarian algorithm. In these two cases the execution time is nearly independent from the number of users (or even decreases with increasing number of users) due to the fact that there are always at most N subcarriers to distribute among the users.

B – Evaluation of the spectrally-efficient resource allocation method:

We evaluate the algorithm described in 4.3.4 in the same WLAN-like environment with N = 64 subcarriers. The constellation size is limited to 64-QAM and the channel has been generated according to the Hiperlan/2 model A [ETS98]. For all simulations, the noise was assumed to be white and normalized to zero dB, i.e. $\sigma_n^2 = 1/N$, and the SNR gap was set to $\Gamma = 4$, corresponding to an uncoded symbol error ratio of $P_s = 10^{-3}$.

As a reference for comparison with existing methods, we choose Wong's algorithm, which minimizes the total transmit power under user-individual bit rate constraints. Since the optimization criteria in (4.11) and (4.15) are distinct, a direct comparison of both algorithms is not possible. Instead, we use Wong's algorithm to calculate the bit and power allocation for a given set of bit rates. Based on this power allocation, we set the power constraints for the proposed algorithm as

$$p_{\max}(u) = k\sum_{n=1}^{N} p_{n,u}\,, \quad k > 1, \forall u \tag{4.19}$$

In other words, we apply 10 $\log_{10}(k)$ dB more power than Wong's algorithm would need. As a reward for the additional power we would expect a saving in bandwidth. This is confirmed by the simulation results in Fig. 4.11, where the average saving in bandwidth as a function of the additional transmitter power is depicted. The three scenarios of low, medium and high system load are defined by the total number of allocated bits to all the users {60, 120, 240} respectively. The results have been averaged over 200 channel realizations.

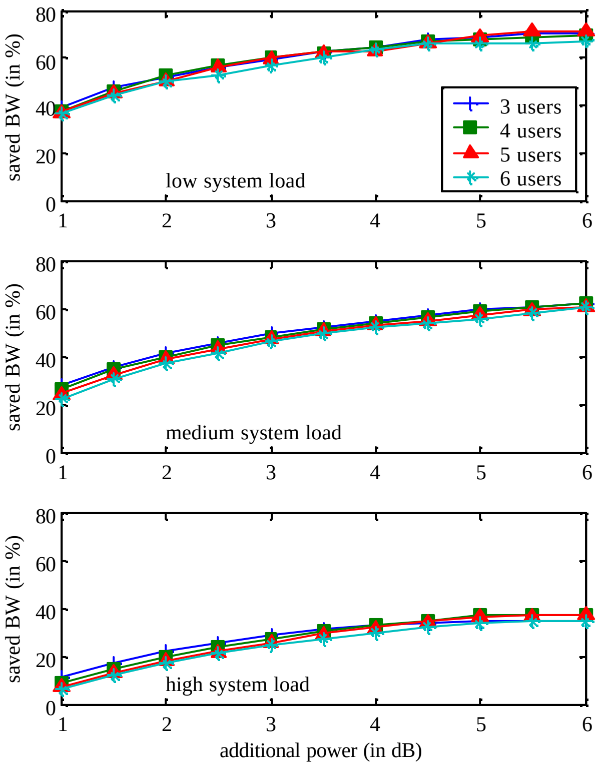


Fig. 4.11 Savings in bandwidth by applying additional transmit power for low, medium, and high system load

While the results are nearly independent from the number of users, the savings are significantly lower for high system load. This is intuitive since for this case, most subcarriers are already fully loaded, i.e. they operate at 64-QAM, and no subcarriers can be freed by re-allocating their bits to others. For low system load, and additional transmit power of just 1dB already saves ca. 20% of bandwidth and for 3dB more power, about half of the subcarriers can be left unused. Only for high system load, nearly all subcarriers have to be allocated.

Another possibility to compare the two algorithms is to set the spectral mask (the PSD constraint) to the maximum PSD that Wong's algorithm assigns: $S_{\max} = \max(p_{n,u})$. Considering the same three loading scenarios, Table 4.1 shows the additional allocated transmit power and the saved bandwidth. It is interesting to see that for low system load very little additional power is required while more than 20% of bandwidth is saved. For higher system load, the differences in power are still low and the spectral efficiency of the new method is always better.

These comparisons reflect the fundamental trade-off between bandwidth and power in digital communications: by increasing the transmit power less bandwidth is required for transmitting the same amount of bits and vice versa.

Fig. 4.12 shows the average execution times of the proposed method and Wong's algorithm. It is not surprising that second one is significantly slower since the aim of this algorithm is to derive a near-optimum solution rather than computational efficiency. However it is interesting to note that the proposed method has the nice property of hardly increasing or even reducing its execution time with the number of users for a constant system load. This is due to the fact that the embedded single-user bitloading algorithm is executed with the minimum number of subcarriers.

Table 4.1 Additional Transmit Power and Saved Bandwidth

	Low system load		High system load	
Number of users	Additional power	Saved bandwidth	Additional power	Saved bandwidth
3	0.5dB	19.5%	0.6dB	6.9%
4	0.6dB	21.6%	0.8dB	7.4%
5	0.6dB	24.7%	0.8dB	8.1%
6	0.6dB	24.9%	1.0dB	8.9%

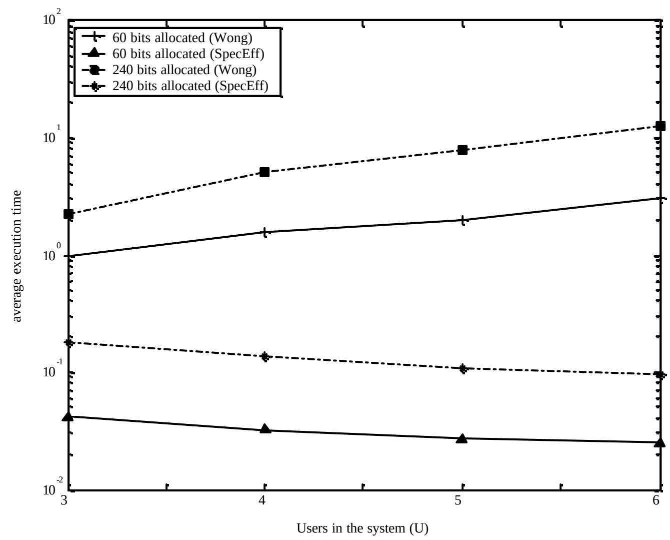


Fig. 4.12 Average execution times of Wong's vs. the spectrally efficient algorithm.

4.4. Capacity-Maximization Based Algorithms

4.4.1. The Multi-user Water-filling Theorem

The multi-user water-filling theorem was presented in [Che93] as a generalization of the well-known solution for the single-user AGN channels already described in section 2.3.1. In that paper, the authors use the idea of equivalent channels for the users of a multiple access scheme. These equivalent channels are equivalent to the original channels seen by each user, but with scaled versions of the respective water-filling diagrams. Each equivalent channel can be written as

$$\hat{H}(w) = \frac{H(w)}{\sqrt{l}} \tag{4.20}$$

which leads to adopt an equivalent transmit signal

$$\hat{s}(t) = \sqrt{l}\, s(t) \tag{4.21}$$

with the equivalent PSD

$$\hat{\Phi}_s(w) = l\,\Phi_s(w) \tag{4.22}$$

Then, the optimal PSD for the equivalent channel is the water-filling solution to:

$$\hat{\Phi}_s(w) = \left[1 - l\,T^{-1}(w)\right]^{+} \tag{4.23}$$

$$\hat{P}_s = l\,P_s = \frac{1}{p}\int_0^{\infty} \hat{\Phi}_s(w)\,dw \tag{4.24}$$

The capacities of both the equivalent and the original channels are naturally the same since the optimal PSD of the equivalent channel is a scaled version of the optimal PSD of the original channel. In order to extend the water-filling solution to U users, an appropriate scaling of the U water-filling diagrams is needed such that they can be combined to keep a single water level for all users. By choosing the multiplier λ such that the water level is fixed to one, the water-filling diagrams from many users can be combined into one diagram. As shown in Fig. 4.13 for a 2-users case, the bottom of the bowl is given by the user with the minimum equivalent CNR and the available spectrum is allocated to this corresponding user.

Provided that number of subchannels in an OFDMA system is large enough, we can use the water-filling theorem with a discrete frequency axis [Pfl03] if we define the equivalent channel coefficients and equivalent symbol energies per user as

$$\hat{H}_{u,n} = H_{u,n} \big/ \sqrt{l_u}\,, \qquad \hat{E}_{u,n} = l_u E_{u,n} \tag{4.25}$$

Then, the discrete water-filling theorem becomes

$$\hat{E}_{u,n} = \begin{cases} \left[1 - l_u T_{u,n}^{-1}\right]^{+} & \textbf{for } l_u T_{u,n}^{-1} \le l_l T_{l,n}^{-1},\ \forall l \ne u \\ 0 & \textbf{otherwise} \end{cases} \tag{4.26}$$

and the power constraint of user u given by

$$E_{tot}(u) = \sum_{n=1}^{N} E_{u,n} \le E_{\max}(u), \quad \forall u = 1,\ldots,U \tag{4.27}$$

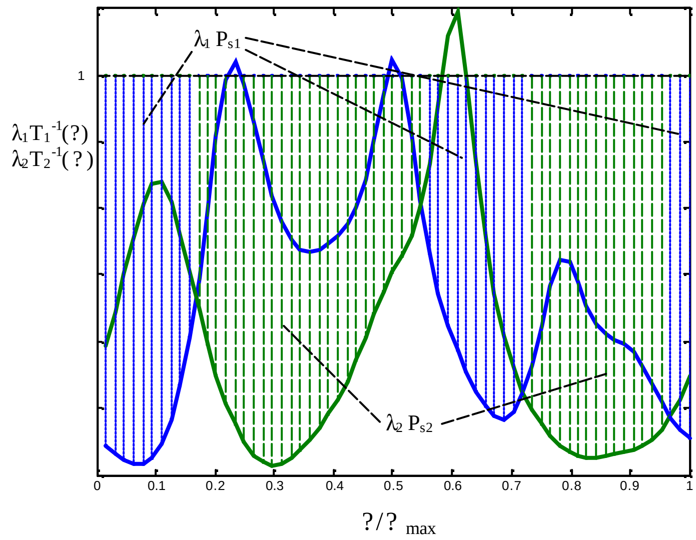


Fig. 4.13 Multi-user water-filling diagram for 2 users

4.4.2. Münz's Algorithm

The algorithm described here was presented by Münz et al in [Mün02] and also in [Pfl03], though we have made slight changes in it for the sake of a better convergence and a more stable performance. This algorithm generalized the two-user case solved by Diggavi in [Dig96].

The analytical formulation of the multi-user water-filling theorem tells us that a system of U equations must be solved after plugging (4.26) into (4.27) in order to compute the multipliers $\{\lambda_1,..., \lambda_U\}$. These multipliers are the unknowns and if the equation system is solved, the energy allocation is given by (4.26) and the subcarrier distribution can be found easily. But, the main problem is that the equation system is highly nonlinear and it is not easy to find efficient algorithms to obtain fast and accurate subcarrier, bit and power allocations.

Münz's algorithm was derived from the observation of (4.26) that shows that the equivalent transmit power $\hat{E}_{tot}(u)$ increases for decreasing λ_u while the equivalent available power $\hat{E}_{max}(u)$ decreases. For the values

$$\hat{l}_n = T_{u,n} \min_{l \neq u} \left\{ \hat{l}_l T_{l,n}^{-1} \right\}, \forall n \in \{1,..., N\}, \tag{4.28}$$

the equivalent transmit power $\hat{E}_{tot}(u)$ has a saltus because an additional subcarrier is assigned to user u for decreasing λ_u or a subcarrier is taken away for raising λ_u. In this algorithm, the multipliers λ_u are varied until (4.27) is fulfilled for all users with the best possible accuracy.

The subcarrier allocation matrix $\mathbf{A} = (a_{u,n})$ can be easily derived out of the values of $\{\lambda_1,..., \lambda_U\}$ and the channel to noise ratio coefficients $T_{u,n}$ as follows:

$$\mathbf{A} = \mathbf{g}(\underline{l}, \mathbf{T}) = \begin{cases} 1 & \textbf{for } l_u T_{u,n}^{-1} = \min\limits_{l \in \{1,\ldots,U\}} \left(l_l T_{l,n}^{-1} \right) AND\ l_u \leq T_{u,n} \\ 0 & \textbf{otherwise} \end{cases} \tag{4.29}$$

which is an improved definition of the subcarrier allocation matrix respect to [Münz02] that takes into account that there might be subcarries that are not allocated to any users.

The present method for multi-user water-filling shown in Fig. 4.14 starts with arbitrary multipliers that are varied for all the users (sorted according to their mean CNRs) until

$l_u = 1 \; \forall u \in \{1,\ldots,U\}$; $\quad \mathbf{A} = \mathbf{g}(\underline{l}, \mathbf{T})$; $count = 0$;
$U_{set} = \mathbf{sort}(\mathbf{mean}_n\{T_{u,n}\})$;
$\Delta\hat{E}(u) = \sum_{n=1}^{N} a_{u,n}\left[1 - l_u T_{u,n}^{-1}\right]^{+} - l_u E_{\max}(u)$;
while $\left(\Delta\hat{E}(u) \neq 0 \; \forall u\right)$ **and** $(count < MAXcount)$,
  for $u \in U_{set}$,
    $\Delta\hat{E}(u) = \sum_{n=1}^{N} a_{u,n}\left[1 - l_u T_{u,n}^{-1}\right]^{+} - l_u E_{\max}(u)$;
      if $\Delta\hat{E}(u) \neq 0$
        $\hat{l}_n = T_{u,n} \min_{l \neq u}\left\{\hat{l}_l T_{l,n}^{-1}\right\} \; \forall n \in \{1,\ldots,N\}$
        repeat
          $l_{OLD} = l_u$;
          if $\Delta\hat{E}(u) > 0$,
            $l_u = \min\left\{\hat{l}_n \middle| \hat{l}_n > l_{OLD}\right\}$;
            if $\min\left\{\hat{l}_n \middle| \hat{l}_n > l_{OLD}\right\}$
              $l_u = (1+e)\, l_{OLD}$;
            else
              $n_1 = \arg\min\left\{\hat{l}_n \middle| \hat{l}_n > l_{OLD}\right\}$;
              $a_{u,n_1} = 0$;
            end if
          else
            $l_u = \max\left\{\hat{l}_n \middle| \hat{l}_n < l_{OLD}\right\}$;
            if $\max\left\{\hat{l}_n \middle| \hat{l}_n < l_{OLD}\right\} = \{\ \}$,
              $l_u = (1-e)\, l_{OLD}$;
            else
              $n_1 = \arg\max\left\{\hat{l}_n \middle| \hat{l}_n < l_{OLD}\right\}$
              $a_{u,n_1} = 1$;
            end if
          end if
        until $\mathrm{sgn}\left(\Delta\hat{E}_{OLD}\right) \neq \mathrm{sgn}\left(\Delta\hat{E}(u)\right)$,
        $l_u = \left(l_{OLD}\Delta\hat{E}(u) - l_u \Delta\hat{E}_{OLD}\right) / \left(\Delta\hat{E}(u) - \Delta\hat{E}_{OLD}\right)$;
        $\mathbf{A} = \mathbf{g}(\underline{l}, \mathbf{T})$;
      end if
  end for
  $count := count + 1$;
end while

Fig. 4.14 Münz's algorithm for multi-user water-filling

$$\Delta\hat{E}(u) = \left(\sum_{n=1}^{N}\hat{E}_{u,n}\right) - l_u E_{\max}(u) \approx 0. \tag{4.30}$$

If ?$\hat{E}(u) > 0$ the energy assigned to user u is too big and then the corresponding multiplier λ_u must be increased and a subcarrier taken away from user u. In the case that ?$\hat{E}(u) < 0$, λ_u must be lowered and eventually and additional subcarrier is assigned to user u. In order to accelerate the search for the optimal multipliers, the saltuses in the equivalent transmit power (4.28) are computed and assigned to λ_u depending on the sign of ?$\hat{E}(u)$. The optimal value for λ_u lies between the two last values and is approximated linearly with

$$l_u = \left(l_{OLD}\Delta\hat{E}(u) - l_u\Delta\hat{E}_{OLD}\right) / \left(\Delta\hat{E}(u) - \Delta\hat{E}_{OLD}\right) \tag{4.31}$$

With the new values of the current multiplier the subcarrier allocation matrix **A** is determined and the algorithm continues with the following user. The overall process is repeated until (4.30) is fulfilled or a maximum number of iterations has been reached.

4.4.3. Yu's Iterative Water-Filling Algorithm

The optimum method to maximize the sum capacity for a Gaussian vector multiple-access channel has recently been described by Yu et al. [Yu04]. This method also applies for OFDMA and can be used to solve the following maximization problem (refer to [Tse98] for a complete and general analysis of the resource allocation problem in multi-access channels):

$$\begin{aligned} &\textbf{maximize} \quad \sum_{u=1}^{U}\sum_{n=1}^{N} b_{n,u} \\ &\textbf{s.t.} \\ &\qquad \sum_{n=1}^{N} p_{n,u} \le p_{\max}(u) \\ &\qquad \sum_{u=1}^{U} a_{n,u} \in \{0,1\} \\ &\qquad b_{n,u} \in B. \end{aligned} \tag{4.32}$$

As a general rule, the proposed algorithm allows each subcarrier to be assigned to only one user. This is denoted by the subcarrier allocation matrix $\mathbf{A} = (a_{u,n})$ already defined. We additionally consider that each user has a maximum power available $p_{\max}(u)$. This is the typical scenario that it is present when we desire to maximize the incoming traffic to the base station with limited spectrum resources and limited transmitter power per user. The aforementioned traffic is the aggregated data rate from all the users in such a way that only available bit rate (ABR) services are guaranteed. Additional buffers should be added at the transmitter and receiver end to support constant bit rate (CBR) services, but this consideration is out of the scope of the present work.

Yu's iterative water-filling algorithm is based on the idea that the optimal PSD of every user sharing the multiple-access channel can be obtained via an iterative process. At each step, each user water-fills all the subchannels with its available power while regarding the interference generated by all other users as additional noise. The iterative water-filling algorithm is more efficient than general convex programming routines [Boy04], since the algorithm decomposes the multi-user problem into a sequence of single-user problems, much easier to deal with.
The function that performs the single-user water-filling at each iteration is:

$$\mathbf{p}_u = \textbf{suwf}\left(\mathbf{T},\ p_{\max}(u)\right) \tag{4.33}$$

where $\mathbf{p}_u$ is the power allocation vector that user u is assigned along the N subchannels when the CNR is defined as

$$T_n = \left|H_{u,n}\right|^2 \Big/ \left(noise_n \cdot \Gamma\right); \quad \forall n \in \{1,\ldots,N\} \tag{4.34}$$

and

$$noise_n = \sum_{\substack{u'=1 \\ u' \neq u}}^{U} \left|H_{u',n}\right|^2 p_{u',n} + s^2; \quad \forall n \in \{1,\ldots,N\} \tag{4.35}$$

As usual, $H_{u,n}$ is the channel transfer function for user u and subchannel n. Γ is the SNR gap and s^2 the noise floor at every subchannel. The water-filling procedure is done according to Section 2.3.1.

In order to produce a practical resource-allocation algorithm based on the iterative water-filling idea (a pure information-theory approach), we have used Yu's method to obtain the optimal subcarrier allocation among users and then perform single-user bitloading for each user within the allocated subchannels using (4.8). The convergence of the practical algorithm is evaluated after each iteration by comparing the last two power allocations and checking that they are (almost) the same. Also, a maximum number of iterations *mxiter* is considered in the case that certain channel conditions do not allow a straightforward split of the subchannels between the users. However, as we will see later on, the convergence is fast (low number of iterations) most of the times. Figure 4.15 illustrates the described algorithm.

$\mathbf{p}_u = \mathbf{0}; \quad \mathbf{p\_old}_u = \mathbf{p}_u; \quad \forall u \in \{1,\ldots,U\}$

for $it \in \{1,\ldots,mxiter\}$,

  for $u \in \{1,\ldots,U\}$,

    $noise_n = \sum_{\substack{u'=1 \\ u' \neq u}}^{U} \left|H_{u',n}\right|^2 p_{u',n} + s^2; \quad \forall n \in \{1,\ldots,N\}$

    $T_n = \left|H_{u,n}\right|^2 \Big/ \left(noise_n \cdot \Gamma\right); \quad \forall n \in \{1,\ldots,N\}$

    $\mathbf{p}_u = \mathbf{suwf}\left(\mathbf{T},\ p_{\max}(u)\right);$

  end for

  $diffp = \left(\sum_{n=1}^{N}\sum_{u=1}^{U}\left|p_{u,n} - p\_old_{u,n}\right|^2\right) \Big/ \left(\sum_{n=1}^{N}\sum_{u=1}^{U}\left|p_{u,n}\right|^2\right);$

  if $diffp < e$,

    break;

  end if

  $\mathbf{p\_old}_u = \mathbf{p}_u; \quad \forall u \in \{1,\ldots,U\}$

end for

$a_{u,n} = 0; \qquad \forall u \in \{1,\ldots,U\}, \forall n \in \{1,\ldots,N\}$

for $n \in \{1,\ldots N\}$,

  $u' = \arg\max\{p_{u,n}\};$

  $a_{u',n} = 1;$

end for

Fig. 4.15 Practical Iterative Water-Filling Algorithm to obtain the subcarrier allocation

4.4.4. A Computationally Efficient Resource-Allocation Algorithm

Along the description of this novel method [Ace05b], we concentrate on the maximization of the aggregated throughput of all the users in an OFDMA system. The optimization problem that the current method solves has already been defined in (4.32).

Following the analysis done in [Cio91], the achievable bit rate per subchannel using QAM can be written as

$$b_{n,u} = \log_2\left(1 + E_{n,u} T_{n,u}\right). \tag{4.36}$$

If we disregard the power constraints, it is easy to see in (4.36) that the maximization of the aggregated data rate of U users can be obtained by simply allocating each subchannel to the user with the highest CNR. But the power constraints are to be considered and, when introduced, they make that the optimal subcarrier allocation be a quite complicated task. We have already reviewed Cheng and Verdú's work [Che93] and their method to solve the problem for a two-user case. It involved the computation of two scaling parameters (or multipliers) that modified the channels in such a way that the water-filling theorem on these "equivalent channels" could be combined into one diagram with common water level equal to one. From this latter diagram, the subcarrier allocation and the PSD for each user were obtained. This overall idea of the equivalent channels is very interesting since it involves the balance among how good each channel is and how much power each user has. However the practical implementation is not straightforward when dealing with many users. We may refer to Section 4.4.2 for a practical algorithm that computes the aforementioned multipliers.

What is clear from all the previous references (and also from [Yu02]), is that the key to solve our maximization problem is to find the optimal partition of the subchannels among the users. Once the subcarriers are assigned, the optimal bitloading for each user can be obtained by performing water-filling within the assigned band. The algorithm proposed here uses this strategy to solve the problem described in (4.32). First we find the subcarrier allocation among the users and then we perform U single-user bitloadings within the created subchannel partitions. The detailed description of this novel method named "XP" is presented in Fig. 4.16.

To begin with, we search for the values of the maximum subchannel CNR among all the users and the minimum user-available-power, both defined as T_{mx} and P_{mn} respectively. Then, and for each user, we compute its *mean* CNR, defined as in (4.2). Secondly, we obtain two normalization coefficients, *ct* and *cp*. The first one quantifies how much the maximum CNR is above the current $\overline{T_u}$. The latter one quantifies how much power has the current user compared to the weakest one. Following, we take the geometric mean *g* of both coefficients that it is used to re-scale the CNR of the current user *u*. Once this process is concluded for all the users, the modified CNRs $T'_{n,u}$ represents a proper balance between the goodness of the subchannels and the users' possibilities to use them in terms of transmit powers. Finally, each subchannel *n* is simply assigned to the user with highest $T'_{n,u}$.

To illustrate the above explanation about how the algorithm works, we show two symbolic cases with two users that have the same available power to face different channels. In Fig. 4.17 both users have almost the same mean CNR, and this value is not far from *Tmx*. Besides, the two users are not competing for the same subchannels (when user 1 has a good subcarrier, that subcarrier is bad for user 2). Hence, it makes no difference to perform the subcarrier allocation based either on the original or modified CNRs. But in Fig. 4.18, the situation is different. User 2 has a better mean CNR, with good opportunities over almost all the carriers. Meanwhile, from user-1's CNR we see that it has only reasonable opportunities near subchannels 9, 21, 31, and 54. Both users fight for some of these subchannels, claiming to be their best ones. The modified users' CNRs for this case (Fig.4.18 - lower) balance this situation to get a correct subcarrier allocation.

$$
\begin{array}{l}
Tmx = \max\{T_{n,u}\};\ \forall n = 1,\ldots,N,\ \forall u = 1,\ldots,U \\
Pmn = \min\{p_{\max}(u)\};\ \forall u = 1,\ldots,U \\
\textbf{for}\ \ u = 1,\ldots,U \\
\quad ct = Tmx / T_u; \\
\quad cp = p_{\max}(u) / Pmn; \\
\quad g = \sqrt{ct.cp};\ \ //\ \textbf{a geometric mean} \\
\quad T'_{n,u} = T_{n,u}.g; \\
\textbf{end for} \\
a_{n,u} = \mathbf{0};\ \ //\ \textbf{clear the allocation matrix} \\
\textbf{for}\ \ n = 1,\ldots,N \\
\quad u1 = \arg\max\{T'_{n,u}\};\ \forall u = 1,\ldots,U \\
\quad a_{n,u1} = 1; \\
\textbf{end for} \\
\textbf{for}\ \ u = 1,\ldots,U \\
\quad \mathrm{M} = \{n : a_{n,u} = 1\}; \\
\quad (b_{\mathrm{M},u}, p_{\mathrm{M},u}) = \textbf{bitload}(T_{\mathrm{M},u}, B, p_{\max}(u)); \\
\textbf{end for}
\end{array}
$$

Fig. 4.16 XP algorithm

Once the subchannels are distributed among the users, single-user bitloading is performed for each set of dedicated subcarriers using (4.8).

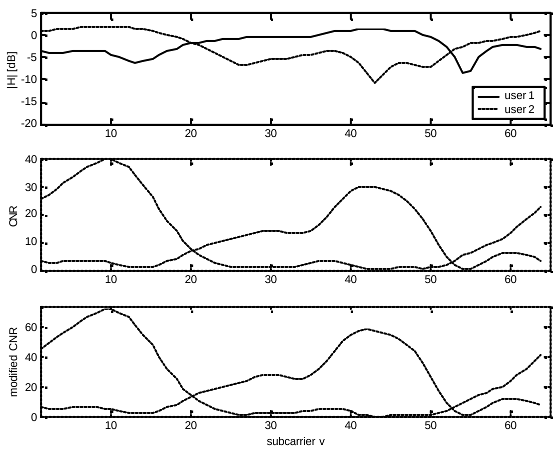


Fig. 4.17 CNRs and modified CNRs in a 2-user scenario (Case A)

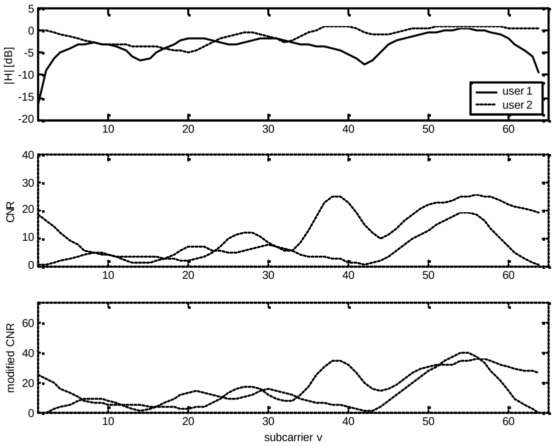


Fig. 4.18 CNRs and modified CNRs in a 2-user scenario (Case B)

It must be observed that the algorithm does not use iterative routines whose convergence (speed and precision) depend on particular channel or user conditions. Moreover, the subcarrier allocation procedure is done using few search and multiplication operations only.

4.4.5. Simulation Results

The algorithms described in 4.4.2, 4.4.3 and 4.4.4 were implemented in Matlab and, as we did in 4.3.5, an OFDMA uplink (with the model described in Fig. 4.1) over Hiperlan/2 channels [ETS98] was assumed (N = 64 subcarriers). All the frequency bins were used and no pilots were considered. The channel state information (CSI) was assumed to be perfect and instantaneous and signalled between the transmitter and the receiver via a separate channel.

The simulation results presented here show the performance of the above three algorithms in terms of how well and efficiently they maximize the aggregate throughput of all the users for the same channel and noise conditions. The following values were settled to define these system conditions:

The number of users sharing the OFDMA system ranged from 2 to 8
The available energy per user was fixed to E_{max}=100 (energy units per OFDM symbol).
The noise variance of each sub-carrier was normalized to $s^2=1/N$ (white broadband noise assumed).
Maximum number of bits per QAM subcarrier b_{max}=8.
Uncoded system (integer QAM constellations with $b \in \{0,1,2\ldots,b_{max}\}$)
No power mask was predefined.

In the following experiments we averaged results from the three algorithms after 200 independent channel realizations and observed:

a) the total number of bits allocated among all the users, or in other words the spectral efficiency per cell which is defined as the total number of allocated bits divided by N.
b) the execution times to get de corresponding subcarrier, bit and power allocations.

The first experiment was done with three different SNR-gap values (1.07dB, 4.19dB and 6.06dB corresponding to symbol error rates of 10^{-1}, 10^{-2} and 10^{-3} respectively). Figures 4.19 to 4.21 display the obtained spectral efficiencies per cell for increasing number of users.

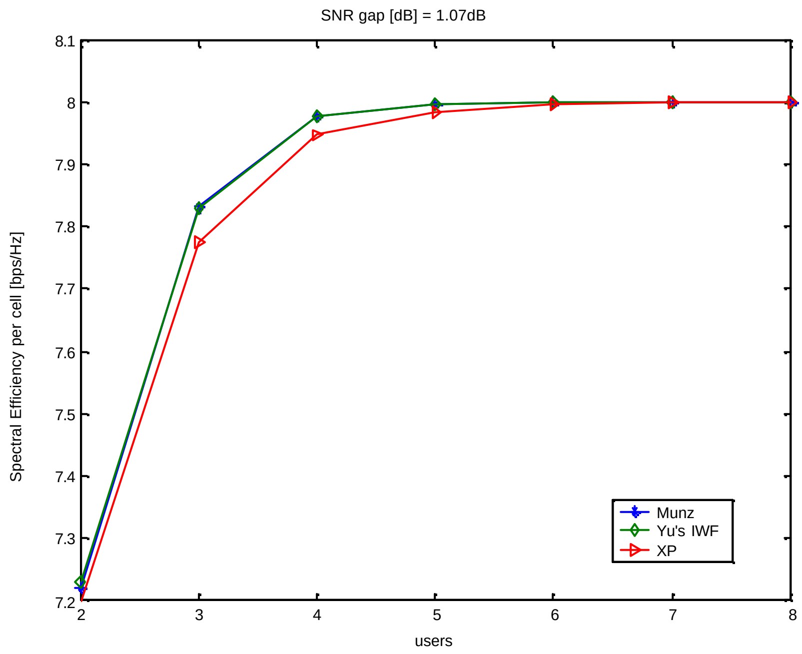


Fig. 4.19 Spectral efficiency per cell (G = 1.07 dB)

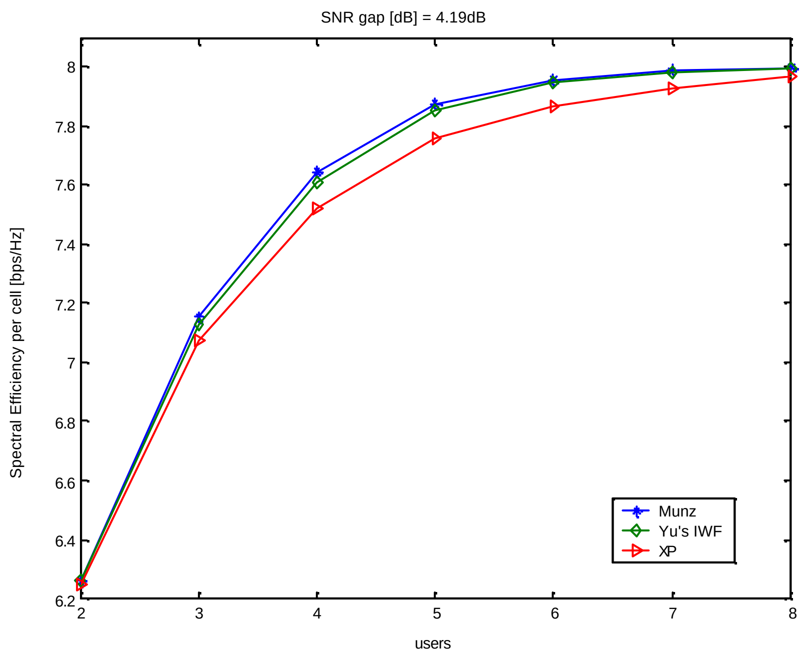


Fig. 4.20 Spectral efficiency per cell (G = 4.19 dB)

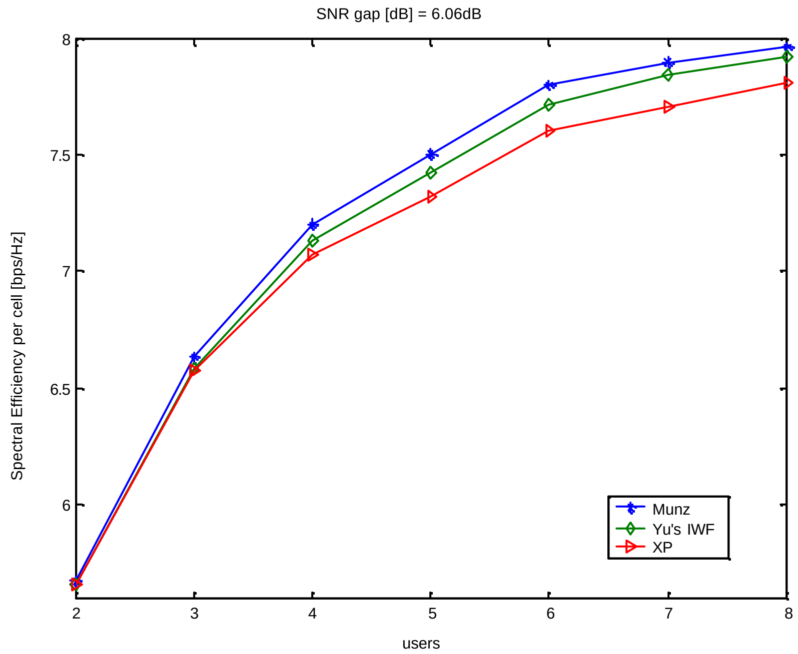


Fig. 4.21 Spectral efficiency per cell (G = 6.06 dB)

In all the three cases and for the three algorithms, the *multi-user diversity* effect can be seen. That is, there is a noticeable increase in the overall spectral efficiency as soon as the number of users increases.

We can see from Fig. 4.19 and 4.20 that both Münz and Yu's algorithms achieve (nearly) the same performance when they use a low SNR-gap value (the total number of allocated bits approaches the channel capacity). This is an expected result since both methods are derived from optimal information-theory solutions to the multi-user resource-allocation problem. However these two methods differ when a bigger gap value is introduced (Fig. 4.21).

The XP algorithm is always very close to the other two optimal methods; the obtained spectral efficiency is, at most, 0,19 bps/Hz away from the results given by any of the other two algorithms. This small difference makes the XP method truly interesting when we test its speed and discuss about its simple structure and fixed running time.

After the second experiment, we illustrate in Fig. 4.22 the average execution times of the three methods. The test was again performed with several sets of users and 200 channel realizations for each set. The XP algorithm is up to one order of magnitude faster than the Yu's algorithm and up to three orders of magnitude faster than Münz's algorithm.

Besides, the rate of increase of the processing time with the number of users is considerably lower for our method than for the other two algorithms. This is a nice feature if the system is to be loaded with many users, and if a real time update of the overall resource allocation is required. The fixed-running-time property of the XP algorithm can be verified by counting the number of operations that the algorithm processes for a given number of subcarriers and users. On the contrary, Yu and Münz's algorithms have no fixed running time, and its convergence speed depends mainly on the channel characteristics, e.g. two users with similar channel transfer functions and equal transmit powers make the algorithm iterate many times until a clear subcarrier allocation is produced.

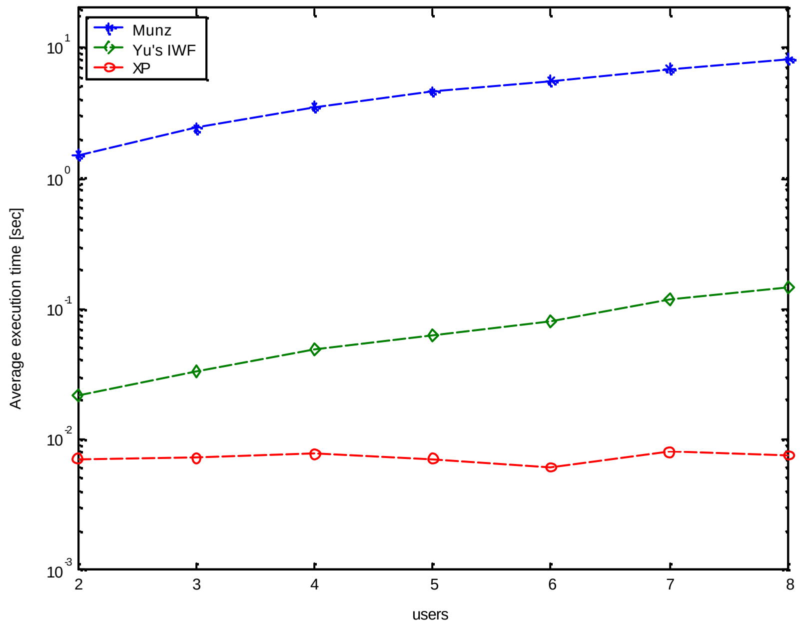


Fig. 4.22 Mean processing times of the three capacity-maximization algorithms

4.5. More about Multi-User OFDM

During the last years, the research activity related to resource-allocation algorithms for multi-user OFDM has been really extensive. Still, it is a topic that currently generates many scientific publications dedicated to solve a vast variety of related problems (OFDMA uplinks, OFDMA downlinks, both with overall power constraints, with user individual power constraints, or with additional constraints or different optimization approaches, etc, etc.). Along our project, it has been impossible to evaluate most of this previous research work available in the literature. However we want to mention here some well known papers as well as the references therein that have provided inspiration or extra knowledge to our work.

Sartenaer [Sar04] has deeply studied the resource allocation problem for multi-user multicarrier modulation applied to PLC (Power Line Communications) and proposed the maximization of the balanced capacity of the multi-user channels as an alternative to the sum capacity described in the theoretical background of the algorithms evaluated along this document.

Wong et al. [Won99b] proposed a real-time algorithm as an interesting alternative to the well known [Won99] evaluated in 4.3.3.

In [Wan03] its authors explore the impact of the multi-user diversity on adaptive OFDM systems, by both analysis and simulations.

Bakhtiari and Khalaj [Bak03] presents a novel power-minimization based algorithm that also evaluates the cumulative outage probability as a function of the distance to the transmitter.

García Armada [Gar01] proposed a CSMA-based algorithm to minimize the number of subcarriers used by all the users sharing the access to the same base station.

Finally, we also refer to [Rhe00] where both an optimal solution (using convex optimization) and a sub-optimal solution (low-complexity algorithm for practical implementation) are proposed to maximize the minimum of all users' throughput, assuming VBR (Variable Bit Rate) services for all the users in the cell.

5. Conclusions

Along this project, we have studied the resource-allocation problem for adaptive MCM systems like OFDM. We have reviewed the basic aspects of this kind of systems as well as several well-known bitloading algorithms for the single-user communication link. The channel and system models used for the simulations were also described.

All this background served as a starting point to study both power-minimization and capacity-maximization algorithms for the multi-user scenario (regarding the channel as a multiple-access one). In every case we evaluated how well and how fast each algorithm solved the corresponding assigned optimization problem.

The observation of the the key strengths and weaknesses of theses algorithms helped us to develop two novel resource-allocation methods. The first one of these two new algorithms saved as much bandwidth as possible at the expense of a small increase in the transmit power. The second novel method dealt with the capacity-maximization problem in OFDMA and it approached a very fast and sub-optimal solution from re-scaling the CNRs of each user. In this sense, the idea of modifying the CNRs with some transformation before taking decisions on how to split the subcarriers among the users is not widely deployed in the open literature and it could be the objective of future research work towards (nearly) optimal and very fast resource-allocation algorithms.

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