

# 64 × 64 spot-array generation based on freeform optics

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**Abstract:** A 64×64 spot-array generator using freeform surface is proposed. The Monte Carlo ray tracing simulation shows that the overall efficiency can be as high as 89% and a unique broadband performance can be obtained.

## 1. Introduction

A spot-array generator, which can transform the incident light beam to dot matrix pattern, is highly desired in many applications, such as parallel laser manufacturing and structured light illumination. Recently, diffractive optical elements (DOEs), which are optimally designed with micro-structures based on diffraction theory, are the most widely used laser spot-array generator. However, the overall efficiency of DOEs are only around 70% due to manufacturing difficulties and errors. In addition, the micro-structures of the DOEs are always designed for a specific wavelength and have strong dispersion effect, the overall efficiency and uniformity deteriorate seriously if the wavelength of incident light deviates from the design value. As a result, wavelength mismatched laser sources and non-coherent light sources such as LEDs are difficult to be applied with DOEs.

Freeform optical surface without rotational symmetry constraint is widely used for applications such as LED illumination and laser beam shaping. Unlike DOEs, freeform surface usually has smoothness constraint and is not suffered from strong dispersion effect due to diffraction. It has potential to achieve spot-array generation with high overall efficiency and wavelength independent using freeform surface. However, only few studies on spot-array generation using freeform surface have been reported. Si *et al.* [1] showed an 5 × 5 laser beam splitting system for parallel laser manufacturing application designed by ray mapping method [2], which is the most widely used method and contains two steps. Firstly, a ray mapping  $\phi : (x_s, y_s) \rightarrow (x_t, y_t)$  between the mesh grids of the source and target plane need to be built, which is the solution of the law of energy conversation:

$$I_s(x_s, y_s) = E_t(\phi(x_s, y_s)) \det(\nabla \phi(x_s, y_s)) \quad (1)$$

Where  $I_s$  and  $E_t$  are the radiance distributions on the source and target plane. Secondly, some methods based on Snell's law and smoothness constraint are used to construct the freeform surface. Solving ray mapping from the nonlinear second-order partial differential equation Eq.1 is of great difficulties. Some numerical methods such as finite difference method or least-squares method [3] have been developed. However, these methods are far slower and harder to converge on large scale problem, such as 2000 × 2000 mesh grids for building more spots beam splitting system.

In this paper, a freeform surface spot-array generator is proposed based on back-and-forth method (BFM) [4]. The Monte Carlo ray tracing simulation shows that this generator can create up to 64 × 64 spot-array with the total efficiency of 89% and has a unique broadband performance.

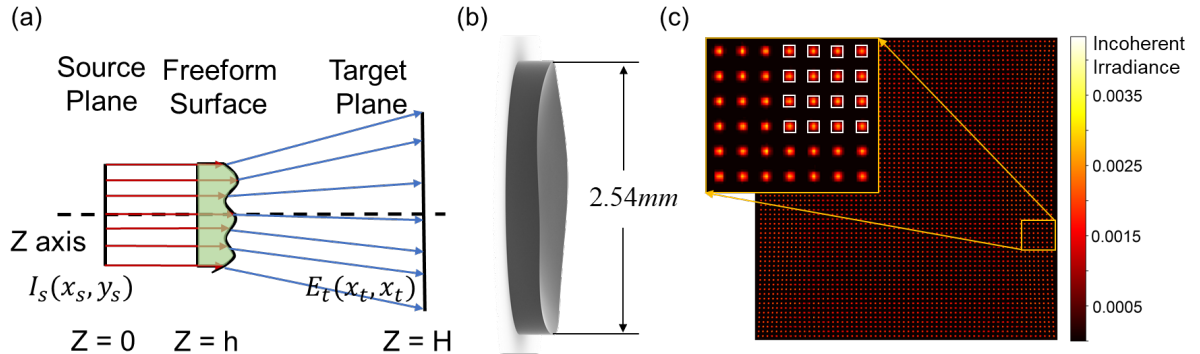


Fig. 1. (a) schematic of spot-array generator system using single freeform surface. (b) design result of the freeform surface. (c) output pattern of the Monte Carlo ray tracing simulation.

## 2. Freeform spot-array generation system design method

Fig.1 (a) shows a schematic of spot-array generation system using single freeform surface with collimated incident light source. Freeform surface redirects the incident ray following the Snell's law to generate the target radiance distribution which satisfies Eq.1. According to the theory of  $L^2$  optimal transport (LOT), the solution satisfying Eq.1 and ensuring the minimum overall grid points movement  $C = \sum [(x_i - x_j)^2 + (y_i - y_j)^2]$  exists and is unique. So that one could solve LOT problem to find the ray mapping instead of solving Eq.1 directly, which is so called optimal transport mapping method [2]. BFM is designed for solving the Kantorovich dual problem (DP) of optimal transport whose core idea is executing gradient ascent between two dual solution spaces of the DP back and forth. Compared with previous vanilla gradient ascent methods which operate only on one solution space, BFM converges far more rapidly and achieves approximately exponential convergence rate. In addition, optimized c-transform and gradient numerical algorithms are also employed here, resulting in only  $O(n \log(n))$  operations per iteration which is much faster than other methods for solving large scale problem.

In order to design and construct smooth freeform surface with BFM, continuous and positive spot-array radiance distribution  $E_t$  is required. Firstly, a summation of an array of gaussian distributed 2D patterns  $d(x_i, y_i)$  is employed to ensure continuous, which could be written as:

$$d(x_i, y_i) = \sum_i \sum_j \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x_i - p_{i,j})^2 + (y_i - p_{i,j})^2}{2\sigma^2}\right) \quad (2)$$

where the parameter  $\sigma$  represents the spot radius. Another parameter  $\alpha \in (0,1)$  in order to remove zero region is introduced to build the final pattern  $E_t(x_i, y_i) = (1 - \alpha)d(x_i, y_i) / \text{MAX}(d) + \alpha$ , which also introduces background noise to target distribution and reduces overall efficiency. These two parameters need to be optimized and to make a tradeoff between performance and manufacture difficulty.

Python program based on BFM and point-by-point integration algorithm [5] are realized to solve LOT and construct smooth surface model. In our simulation, working wavelength, spots number, separation angle and diameter of freeform surface are set as 940 nm,  $64 \times 64$ ,  $12^\circ \times 12^\circ$  and 25.4 mm, respectively. Freeform surface is located at  $h = 10$  mm, whose material is set as PMMA and the parameters  $\sigma$  and  $\alpha$  are chosen as 0.52 mm and 0.01. The target plane is located at  $H = 3$  m and the source and target distribution  $I_s$  and  $E_t$  are built on  $3001 \times 3001$  mesh grids. Design result and the output pattern of Monte Carlo ray tracing are shown in Fig.1 (b) and (c). Furthermore, different operating wavelengths are also considered and the ray tracing simulation results are shown in Table 1, which shows a superior broadband performance compared to DOEs.

Table 1. performances applied with different incident light source

Wavelength	940 nm (designed)	850 nm	1064 nm	920~960 nm (LED)
Overall Efficiency	89.32 %	89.28 %	89.30 %	87.32 %
Uniformity Error	10.81 %	10.86 %	10.83 %	12.87 %

The overall efficiency  $\eta$  and the uniformity error (UE) are calculated with  $\eta = \sum E_{i,j} / E_{out}$  and

$UE = \sqrt{(\sum (E_{i,j} - E_{obj})^2) / N^2} / E_{obj}$ , where  $E_{i,j}$  is the energy of one spot, which is integrated by the radiance within the design region (the white square shown in Fig.1 (c), whose length is  $4\sigma$ ) and  $E_{obj}$  is the ideal energy of one spot which is  $E_{out} / N^2$ .

## 3. Conclusion

A  $64 \times 64$  spot-array generator using freeform surface is proposed, which shows superior overall efficiency and comparable uniformity error with commercial DOE products. Moreover, the unique broadband performance demonstrates great potential for infrared spot-array illumination applications applied with high power LEDs.

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