ILP Formulation

Zoey Nielsen

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1 Content Covered

1. How do we give points to professors?
2. How do professors assign points?
3. What do edge weights mean?
4. How do we measure happiness of individual courses?
5. How do we evaluate the assignment?

2 Setup

Let $A = G \cup P$ be the set of applicants where P are the PhD and G are master's students and $G \cap P = \emptyset$. Let C be the set of courses and let r_c be the number of TAs a course c needs for all $c \in C$. Then, define our vertex set $V = A \cup C$ such that $A \cap C = \emptyset$. Let $\ell_c = r_c + k_c$ for some $k \in \mathbb{N}$ be the number of TA preferences a professor of course c provides. Then, construct our edge set E such that for all $e = (a, c)$ for some $a \in A$ and $c \in C$, applicant a applied for course c and the professor for course c listed a in their TA preferences.

3 Professor Bidding

Let $c \in C$ be a course that requires r_c TAs. Then, the professor associated with course c (which will henceforth just be referred to synonymously with c), will be required to provide ℓ_c TAs they would be willing to have as TAs for the course. Let this set be denoted as $A_c = \{a_{c1}, \ldots, a_{c\ell_c}\} \subseteq A$. Then, define $P_c : A_c \to \{0, \ldots, r_c\}$ such that $P_c(a_{ci}) = j$ if c has ranked applicant a_{ci} with value j.

Intuitively, this definition will allow a professor to distribute a ranking for some subset of applicants which is bounded by the total number of TAs they will ultimately be assigned. This bound is significant because it limits the variation within the ratings so that a professor can't skew their preferences in either extreme. Additionally, it is important to note that P_c is not bijective to more accurately capture the professor's preferences and how those points are distributed.

4 Edge Weights

Consider our graph G from above and define the weight function $w : E \to \{0, \dots, r_c\}$ as $w((a, c)) = P_c(a)$ for $(a, c) \in E$ such that $a \in A$ and $c \in C$. We now update our graph definition to be $G = (V, E, w)$.

5 Evaluation

To evaluate how satisfied a course assignment is, we will define the measure $\mathcal{P}_c : \mathscr{P}(A) \to [-1, 1]$ such that

$$\mathcal{P}_c(A') = n_c \sum_{a_i \in A'} (P_c(a_i) - B_c)$$

where $A' \in \mathscr{P}(A)$ such that $A' = \{a_1, \dots, a_n\}$. Additionally, we define B_c as the r_c^{th} value assigned to a TA applicant for some course c. Formally, if $A_c = (a_{c1}, \dots, a_{c\ell_c})$ as mentioned above is ordered and sorted in descending order, then $B_c = A_c[r_c]$. Then, we let $n_c = \frac{1}{|A'| \max_i \{|a_i - B_c|\}}$ to ensure that $\mathcal{P}_c(A') \in [-1, 1]$.

Definition of Normalization Constant

I think it is most advantageous to define it as $\frac{1}{|A'| \max_i \{|a_i - B|\}}$, but I was also wondering if $\frac{1}{r_c^2}$ would be better. The former tries to fit the satisfaction meaure to how they distributed their points but the latter would standardize that value across the different distribution (ie, someone rating lower scores in general would score lower and someone generally rating higher scores would score higher). I think this may flaw our results because if someone is rating lower anyway, there is not much we can really do to achieve a higher satisfaction. This could, however, be helpful in making it so we want to provide better TAs to professors who generally rank higher.

By shifting each rating by the respective baseline B_c, I aim to fit the distribution of ratings to a standard scale. Additionally, by using the normalization constant, it ensures that regardless of how a professor allocates their ratings or what values they are assigned, a standard measure can be used to compare satisfaction.

6 ILP Formulation

Let the set X be our decision variables. Note that $x_{a,c} \in X$ represents the edge $\{a, c\}$ and we define $x_{a,c} \in \{0, 1\}$ where $x_{a,c} = 1$ if applicant a is assigned to class c, otherwise, $x_{a,c} = 0$. Additionally, define the auxiliary variables $M, m \in X$ that will bound the minimum and maximum values. We then formulate the objective function as follows, using the definition of B_c and P_c as shown above:

$$\max \sum_{c \in C} (\mathcal{P}_c(I_c)) - (M - m)$$

where we modify $n_c = \frac{1}{r_c \max_i\{|a_i - B|\}}$ and I_c is the set of applicants adjacent to course c, subject to the constraints:

1. For all $p \in P$, let I_p denote the set of incident edges. Then, $\sum_{c \in I_p} x_{p,c} = 1$.
2. For all $a \in G$, let I_a denote the set of incident edges. Then, $\sum_{c \in I_a} x_{a,c} \leq 1$.
3. For all $c \in C$, let I_c denote the set of incident edges. Then, $\sum_{a \in I_c} a = r_c$.
4. For all $c \in C$, $(\mathcal{P}_c(I_c)) \leq M$.
5. For all $c \in C$, $(\mathcal{P}_c(I_c)) \geq m$.