Cohomology of Sheaves

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1 Homological Algebra

1.1 Exact categories

We consider a category with a **zero object** 0, that is, for every object *A* there is precisely one morphism $A \rightarrow 0$ and precisely one $0 \rightarrow A$.

A **zero morphism** $A \rightarrow B$ is a morphism which be factored $A \rightarrow 0 \rightarrow B$.

A **kernel**, ker(f) of a morphism $f: A \to B$ is a pair (ker(f), i), where $i: K \to A$ is a monomorphism with $f \circ i = 0$ and such that for any morphism $g: X \to A$ with $f \circ g = 0$ there is a commutative diagram

$$ker(f) \xrightarrow{\exists} X \downarrow g \downarrow 0$$

$$\downarrow g \downarrow g \downarrow 0$$

$$\downarrow g \downarrow g \downarrow A \rightarrow B$$

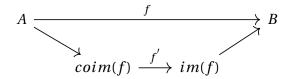
A **cokernel**, coker(f) of $f: A \to B$ is a pair (coker(f), p) where $p: B \to C$ is a epimorphism such that for any morphism $h: B \to Y$ with $h \circ f = 0$ there is a commutative diagram

$$coker(f) \xleftarrow{p} B \xleftarrow{f} A$$

We shall assume that every morphism has a kernel and cokernel.

We define the image im(f) of a morphism $f: A \rightarrow B$ to be the kernel of a cokernel.

We define the koimage coim(f) of a morphism $f:A\to B$ to be the corkernel of the kernel. Every morphism $f:A\to B$ has a **canonical factorization**



Definition 1.1.1. An **exact category** is a category with zero objects, kernels, cokernels, such that $coim(f) \xrightarrow{f'} im(f)$ is always an isomorphism.

In the remaining part of this section we shall work in an exact category.

Definition 1.1.2. A sequence of morphisms

$$\dots \xrightarrow{f^{n-2}} A^{n-1} \xrightarrow{f^{n-1}} A^n \xrightarrow{f^n} A^{n+1} \xrightarrow{f^{n+1}} \dots \tag{1.1.1}$$

is called **exact**, if $im(f^{n-1}) = ker(f^n) \forall n$.

Proposition 1.1.3. s