Amirkabir University of Technology
(Tehran Polytechnic)

Natural Language Processing

Lecture 5: LM Evaluation

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Outline

- **Entropy**
- Entropy and Linguistics
- Language Model Evaluation
- Parameter Tuning and Cross-validation

Entropy

- Entropy measures the amount of information in a RV
- Amount of information contained in a message (after removing all possible redundancy)
- number of bits that the message has after compression

Entropy

$$H(V) = E[-\log(p(V))]$$

$$H(V) = \sum_{w_i \in V} -p(w_i)\log\big(p(w_i)\big)$$

- Note: if you want the “unit” of the entropy to be “bit”, you have to use the log to the basis2

Example 1

- Reporting the result of rolling an 8-sided die
- Entropy:

$$H(X) = -\sum_{i=1}^{8} p(i)\ \ log(p(i))$$

$$H(X) = -\sum_{i=1}^{8} \frac{1}{8} \log\left(\frac{1}{8}\right) = -\log\left(\frac{1}{8}\right) = \log(8) = 3 \text{ bit}$$

The average length of the message needed to transmit an outcome of that variable using the optimal code

Example 1

- Reporting the result of rolling an 8-sided die
- The most efficient way is to simply encode the result as a 3 digit binary message:

1	2	3	4	5	6	7	8
001	010	011	100	101	110	111	000

Example 2

- Vocabulary with two words:

V = a; b

$$p(a) = x$$

$$p(b) = 1 - x$$

$$H = -x \log x - (1 - x) \log(1 - x)$$

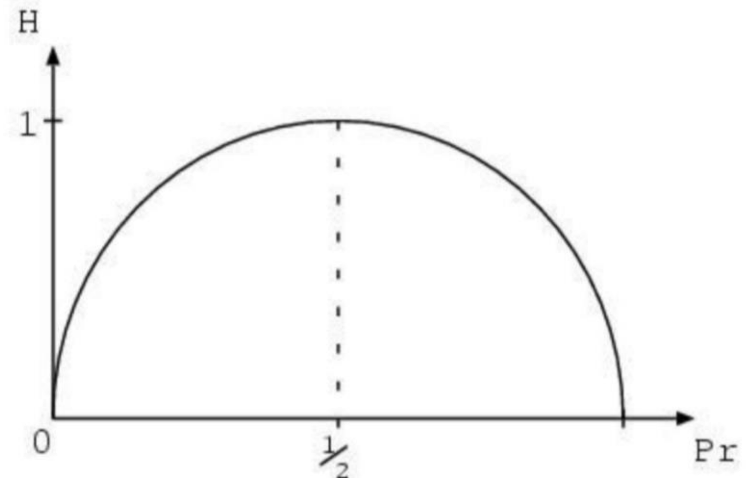
$$x = 0 \longrightarrow H = 0$$
$$x = 1 \longrightarrow H = 0$$

Example 2

- Vocabulary with two words:

V = a; b

$$H = -x \log x - (1 - x) \log(1 - x)$$



Example 3

- Vocabulary of W words w_i with uniform distribution $p(w_i) = \frac{1}{W}$

$$H = \sum_{i=1}^{W} -p(w_i)\ \ log(p(w_i)) = \sum_{i=1}^{W} -\frac{1}{W}\log(\frac{1}{W})$$

$$H = -W\frac{1}{W}\log(\frac{1}{W}) = -\log\left(\frac{1}{W}\right) = \log(W)$$

- Entropy for uniform distribution: log of the number of symbols

Joint Entropy

- The joint entropy of 2 $RV\ X;\ Y$ is the amount of the information needed on average to specify both their values:

$$H(x) = -\sum_{x \in X} \sum_{y \in Y} p(x,y) \log(p(x,y))$$

Conditional Entropy

- The conditional entropy of a $RV\ Y$ given another X, expresses how much extra information one still needs to supply on average to communicate Y given that the other party knows X

$$H(Y|X) = \sum_{x \in X} p(x) \mathrm{H(Y|X = x)}$$

$$H(Y|X) = -\sum_{x \in X} p(x) \sum_{y \in Y} p(y|x) \log(p(y|x))$$

$$H(Y|X) = -\sum_{x \in X} \sum_{y \in Y} p(x,y) \log(p(y|x)) = -E(\log(p(Y|X)))$$

Chain Rule

$$H(X|Y) = H(X) + H(Y|X)$$

$$H(X1, \ldots, Xn) = H(X1) + H(X2|X1) + \ldots + H(Xn|X1, \ldots, Xn-1)$$

Mutual Information

- $I(X, Y)$ is the mutual information between X and Y.
- The reduction of uncertainty of one RV due to knowing about the other, or the amount of information one RV contains about the other

$$H(X,Y) = H(X) + H(Y|X) = H(Y) + H(X|Y)$$

$$H(X) - H(X|Y) = H(Y) - H(Y|X) = I(X,Y)$$

Mutual Information

$$I(X,Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)$$

- $I(X,Y)$ is 0 only when X and Y are independent:

$$H(X|Y) = H(X)$$

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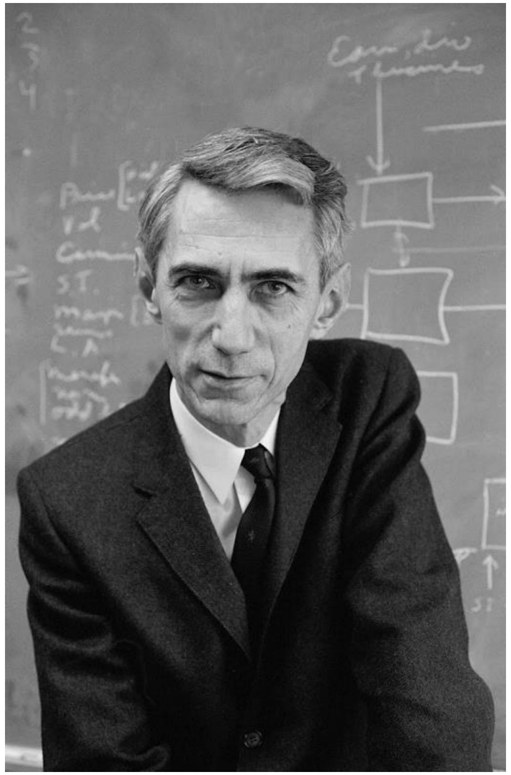
Shannon Game

- Shannon's Experiment to Calculate the Entropy of English



Claude Elwood Shannon
1916-2001
The Father Of Information Theory

http://www.math.ucsd.edu/~crypto/java/ENTROPY/

Complete the Sentence

- Th-r- -s -nly -n- w-y t- f-ll -n th- v-w-ls -n th-s s-nt-nc

Complete the Sentence

- Th-r- -s -nly -n- w-y t- f-ll -n th- v-w-ls -n th-s s-nt-nc

- There is only one way to fill in the vowels in this sentence

Entropy of a Language: Shannons Approach

- Show somebody the beginning of a text
- Ask him/her to guess the next letter
- Count the number of trials

Entropy and Linguistics

- Entropy is measure of uncertainty. The more we know about something the lower the entropy
- If a language model captures more of the structure of the language, then the entropy should be lower
- We can use entropy as a measure of the quality of our models

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Perplexity

- Definition:
  - Perplexity is a measurement of how well a probability distribution or probability model predicts a sample.

- The perplexity of a discrete probability distribution p is defined as

$$2^{H(p)} = 2^{-\sum_{w_i \in V} p(w_i)\log(p(w_i))}$$

Perplexity

- In natural language processing, perplexity is a way of evaluating language models.
- A language model is a probability distribution over entire sentences or texts

Branching Factor

- Branching factor is the number of possible words that can be used in each position of a text
  - Maximum branching factor for each language is V

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John eats an …

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Branching Factor

- A good language model should be able to
  - minimize this number
  - give a higher probability to the words that occur in real texts

Branching Factor

Can we give the same knowledge

to a computer to predict the next character?

Perplexity

$$P(S) = P(w1, w2, \dots, wn)$$

$$Perplexity(S) = P(w1, w2, \dots, wn)^{-\frac{1}{N}} = \sqrt[N]{\frac{1}{P(w1, w2, \dots, wn)}}$$

$$Perplexity(S) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(wi|w1,w2,\dots,wi-1)}}$$

Goal: giving higher probability to frequent texts

$\Longrightarrow$ minimizing the perplexity of the frequent texts

Evaluation

- The evaluation must give an indication of how well the learner will do when it is asked to make new predictions for data it has not already seen.
  - Dividing the corpus into two parts

  - Building a language model from the training set
  - Estimating the probability of the test set
  - Calculate the perplexity of the test set

Perplexity

- Maximum branching factor for each language is $|V|$

$$Perplexity(S) = \left(\prod_{i=1}^{N} P(wi|w1, w2, \ldots, w_{i-1})\right)^{-\frac{1}{N}}$$

Example: predicting next characters instead of next words ($|V| = 26$)

$$\frac{\_\_\_}{26} \quad \frac{\_\_\_}{26} \quad \frac{\_\_\_}{26} \quad \frac{\_\_\_}{26} \quad \frac{\_\_\_}{26}$$

$$Perplexity(S) = \left((1/26)^5\right)^{-\left(\frac{1}{5}\right)} = 26$$

Perplexity

- Wall Street Journal
  - Training set: 38 million word tokens
  - Test set: 1.5 million words

	Unigram	Bigram	Trigram
Perplexity	962	170	109

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Normal Evaluation

- Dividing the corpus into two parts

- Building a language model from the training set
- Estimating the probability of the test set
- Calculating the perplexity of the test set

Evaluation with Parameter Tuning

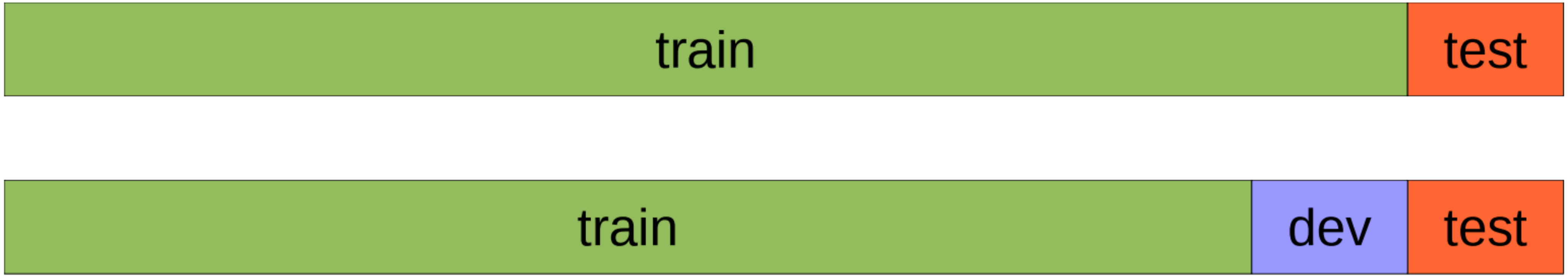
- Dividing the corpus into three parts

- Building a language model from the training set
- Calculating the perplexity of the development set with different parameter values
  - Also known as held-out data or validation set
- Choosing the best parameter value and use it to estimate the probability of the test set
- Calculating the perplexity of the test set

Motivation

- There is no guarantee that the chosen test set is representative enough to model our data
- Solution:
  - Assessing how the results of a statistical analysis will generalize to an independent data set
  - Performing multiple rounds of cross-validation using different partitions, and the validation results are averaged over the rounds

Cross-validation



- k-fold cross-validation
- Leave-one-out cross-validation

Further Reading

- Speech and Language Processing (3rd ed. draft)
  - Chapter 3