Started on	Thursday, 3 May 2018, 12:56 AM
State	Finished
Completed on	Thursday, 3 May 2018, 6:02 PM
Time taken	17 hours 5 mins
Grade	**3.50** out of 4.00 (**88**%)

Question **1**
Correct
Mark 0.50 out of 0.50

What is the clique number of a connected graph with 4 vertices and 5 edges?

Answer: 3 ✓

The only graph with 4 vertices and 6 edges is the 4-clique K_4, hence with one less edge the maximum clique size is 3.

Refer to lecture 6, slides 20,29

Question **2**
Correct
Mark 0.50 out of 0.50

Tick all graphs that have an Euler path.

Select one or more:

- [x] K_5 with one edge removed ✓
- [ ] The graph on slide 16 (lecture 6)
- [x] $K_{2,3}$ ✓
- [x] $K_{1,2,1}$ ✓

The graph on slide 16 (lecture 6)
 Not true. The graph has 4 vertices of odd degree, hence cannot have an Euler path.

$K_{2,3}$
 True. The graph has 2 vertices of degree 3 and 3 vertices of degree 2, hence must have an Euler path.

$K_{1,2,1}$
 True. The graph has 2 vertices of degree 3 and 2 vertices of degree 2, hence must have an Euler path.

K_5 with one edge removed
 True. The graph has 3 vertices of degree 4 and 2 vertices of degree 3, hence must have an Euler path.

Refer to lecture 6, slides 17,20

Question **3**
Correct
Mark 0.50 out of 0.50

What is the chromatic number of the graph on slide 16 (lecture 6)?

Answer: 3 ✓

The graph has a 3-clique, hence 3 colours are necessary. It is easy to find such a 3-colouring, hence $\chi(G) = 3$.

Refer to lecture 6, slides 20, 27-29

Question **4**
Correct
Mark 0.50 out of 0.50

Tick all graphs that have a Hamiltonian path.

Select one or more:

- [x] $K_{2,2,4}$ ✓
- [x] $K_{1,2,3}$ ✓
- [ ] $K_{1,4,1}$
- [x] $K_{1,1,3}$ ✓

$K_{1,2,3}$
 True. Start with one of the 3 vertices in the largest partition, then come back to this partition and finish with the three remaining vertices.

$K_{2,2,4}$
 True. If you start with one of the 4 vertices in the largest partition, it is easy to find a path that visits all vertices.

$K_{1,1,3}$
 True. Start with one of the 3 vertices in the largest partition, then come back to this partition and finish with the two remaining vertices.

$K_{1,4,1}$
 Not true. Even if you start with one of the 4 vertices in the largest partition, you can visit at most 3 of them before you have to revisit a vertex.

Refer to lecture 6, slides 20, 23-26

Question **5**
Correct
Mark 0.50 out of 0.50

Tick all statements that are true.

Select one or more:

- [x] When you remove two edges from K_6, you will never obtain a planar graph. ✓
- [ ] All graphs whose chromatic number is 4 are planar.
- [x] All graphs with 6 nodes and 8 edges are planar. ✓
- [ ] You can obtain a nonplanar graph by adding one edge to $K_{2,3}$.

When you remove two edges from K_6, you will never obtain a planar graph.
 True. The remaining graph will always contain $K_{3,3}$, which is nonplanar.

All graphs with 6 nodes and 8 edges are planar.
 True. K_5 requires 10 edges and $K_{3,3}$ 9 edges, hence there can be no nonplanar graph with only 8 edges.

You can obtain a nonplanar graph by adding one edge to $K_{2,3}$.
 Not true. $K_{2,3}$ has 5 vertices and 6 edges, so even with one more edge you cannot obtain K_5, which requires 10 edges.

All graphs whose chromatic number is 4 are planar.
 Not true. $K_{3,3}$ has chromatic number 2 but is not planar.

Refer to lecture 6, slides 35-40

Question **6**
Incorrect
Mark 0.00 out of 0.50

Let G be an undirected graph on 12 vertices with exactly two connected components. What is the maximum possible number of edges in G?

Answer: 91 ✗

Let the two connected components have n and m vertices respectively, with $n + m = 12$. The maximum number of edges is achieved by creating two complete graphs K_n and K_m with $n(n-1)/2 + m(m-1)/2$ edges overall. This number is maximal for $n = 11$, $m = 1$, which gives 55 edges.

Refer to lecture 6, slides 6-7, 20

Question **7**
Correct
Mark 0.50 out of 0.50

We would like to prove that $P(n)$ for all $n \geq 0$. Tick all conditions that imply this conclusion.

Select one or more:

- [x] $P(0)$ and $P(1)$ and $\forall n \geq 1\,(P(n) \Rightarrow P(2 \cdot n) \wedge P(2 \cdot n + 1))$ ✓
- [x] $P(1)$ and $\forall n \geq 0\,(P(n+1) \Rightarrow P(n) \wedge P(n+2))$ ✓
- [ ] $P(0)$ and $\forall n \geq 1\,(P(n-1) \Rightarrow P(n+1) \wedge P(n+2))$
- [ ] $P(0)$ and $P(1)$ and $\forall n \geq 1\,(P(n) \wedge P(n+1) \Rightarrow P(n+2))$

$P(1)$ and $\forall n \geq 0\,(P(n+1) \Rightarrow P(n) \wedge P(n+2))$
 True. All cases $n \geq 0$ are covered.

$P(0)$ and $P(1)$ and $\forall n \geq 1\,(P(n) \Rightarrow P(2 \cdot n) \wedge P(2 \cdot n + 1))$
 True. All cases $n \geq 0$ are covered.

$P(0)$ and $\forall n \geq 1\,(P(n-1) \Rightarrow P(n+1) \wedge P(n+2))$
 Not true. From $P(0)$ it follows that $P(2)$ and $P(3)$, but the case $n = 1$ is not covered.

$P(0)$ and $P(1)$ and $\forall n \geq 1\,(P(n) \wedge P(n+1) \Rightarrow P(n+2))$
 Not true. The ``first'' instance of the implication is $P(1) \wedge P(2) \Rightarrow P(3)$, but $P(2)$ is not given.

Refer to lecture 7, slides 17-28

Question **8**
Correct
Mark 0.50 out of 0.50

Suppose $f, g : \{a, b\}^* \longrightarrow \{a, b\}^*$ are recursively defined as follows:

- $f(\lambda) = b$
- $g(\lambda) = a$
- $f(a\omega) = f(\omega)g(\omega)$
- $f(b\omega) = g(\omega)f(\omega)$
- $g(a\omega) = g(b\omega) = f(\omega)$

What is $f(bab)$?

Answer: ababb ✓

$f(bab) = g(ab)f(ab) = f(b)f(b)g(b) = g(\lambda)f(\lambda)g(\lambda)f(\lambda)f(\lambda) = ababb$

For the basic definition of the concatenation operation on words, refer to lecture 1, slide 38