Started on	Sunday, 11 March 2018, 11:30 PM
State	Finished
Completed on	Thursday, 15 March 2018, 11:10 AM
Time taken	3 days 11 hours
Grade	**3.50** out of 4.00 (**88**%)

Question **1**
Correct
Mark 0.50 out of 0.50

How many numbers in the interval [1343, 9294] are divisible by 7 or 6 (or both)?

Answer: 2272

Numbers in [1343,9294] divisible by 7: floor(9294/7) - floor((1343-1)/7) = 1327 - 191 = 1136
Numbers in [1343,9294] divisible by 6: floor(9294/6) - floor((1343-1)/6) = 1549 - 223 = 1326
We need to subtract those that are counted twice:
Numbers in [1343,9294] divisible by 42: floor(9294/42) - floor((1343-1)/42) = 221 - 31 = 190
Answer: 1136 + 1326 - 190 = 2272

Refer to lecture 1, slide 9

Question **2**
Correct
Mark 0.50 out of 0.50

Which of the following is always true for sets A and B?

Select one or more:
- [x] $A \cap B \in Pow(A)$
- [ ] $B \subseteq Pow(B)$
- [ ] $|A \cap B| < |A \cup B|$
- [x] $A \subseteq (A \cap B) \cup (A \setminus B)$

$B \subseteq Pow(B)$
Not always true. Counterexample: if $B = \{1\}$ then $Pow(B) = \{\emptyset, \{1\}\}$. But $1 \notin \{\emptyset, \{1\}\}$, hence $B \nsubseteq Pow(B)$.
$A \cap B \in Pow(A)$
Always true. All elements in $A \cap B$ must be in A, hence $A \cap B \subseteq A$, which implies $A \cap B \in Pow(A)$.
$|A \cap B| < |A \cup B|$
Not always true. Counterexample: if $A = B$ then $A \cap B = A \cup B$, hence $|A \cap B| = |A \cup B|$.
$A \subseteq (A \cap B) \cup (A \setminus B)$
Always true. In fact, $A = (A \cap B) \cup (A \setminus B)$ since every element in A is either also in B or not in B.

Refer to lecture 1, slides 20-21 & 30-31

Question **3**
Incorrect
Mark 0.00 out of 0.50

Let $\Sigma = \{a, b, c\}$ and $\Psi = \{b, c, d, e\}$. How many words are in the set $\Sigma^2 \cup \Psi^{\leq 3}$?

Answer: 47

There are $4^0 + 4^1 + 4^2 + 4^3 = 85$ words of length ≤ 3, including the empty word, over alphabet Ψ.
There are $3^2 = 9$ words of length 2 over alphabet Σ. But 4 of these 2-letter words are also in $\Psi^{\leq 3}$, hence should not be counted twice.
The answer is 85 + 9 - 4 = 90.

Refer to lecture 1, slides 39-40

Question **4**
Correct
Mark 0.50 out of 0.50

Which of the following is always true for functions $f : S \longrightarrow T$ and $g : T \longrightarrow S$?

Select one or more:
- [x] $Dom(f) = Codom(g)$
- [ ] $Dom(g) \subseteq Im(f)$
- [x] $Im(g \circ f) \subseteq Dom(f)$
- [x] $Dom(f \circ g) = T$

$Dom(f \circ g) = T$
Always true. $Dom(f \circ g) = Dom(g) = T$
$Dom(g) \subseteq Im(f)$
Not always true. Counterexample: f maps every $s \in S$ to the same element $t_0 \in T$. Then $Im(f) = \{t_0\}$, which is not a subset of T if T has more than one element.
$Dom(f) = Codom(g)$
Always true. $Dom(f) = S = Codom(g)$
$Im(g \circ f) \subseteq Dom(f)$
Always true. $Im(g \circ f) \subseteq Codom(g \circ f) = Codom(g) = S = Dom(f)$

Refer to Lecture 1 slidesslides 41 & 46

Question **5**
Correct
Mark 0.50 out of 0.50

Consider the following propositions:
e - the paper tray is empty
p - the printer is printing
r - the printer is ready
Tick the two formulas that are logically equivalent to the following two statements:
- If the printer is not printing, then it is not ready or the paper tray is empty.
- When the paper tray is not empty, the printer is ready, provided it is not printing.

Select one or more:
- [x] $e \vee p \vee \neg r$
- [ ] $r \vee \neg p \vee \neg e$
- [x] $p \vee e \vee r$
- [ ] $\neg r \vee \neg e \vee \neg p$

If the printer is not printing, then it is not ready or the paper tray is empty.
Logical formalisation: $\neg p \Rightarrow \neg r \vee e$
By the laws on slides 21/22 (week 2), this is equivalent to $p \vee \neg r \vee e$
When the paper tray is not empty, the printer is ready, provided it is not printing.
Logical formalisation: $\neg p \Rightarrow (\neg e \Rightarrow r)$
By the laws on slides 21/22 (week 2), this is equivalent to $p \vee e \vee r$

Question **6**
Correct
Mark 0.50 out of 0.50

Consider three propositions A, B, C. Under how many of the 8 possible truth assignments is the following formula true ?
$B \wedge (\neg C \Rightarrow (A \vee \neg B))$

Answer: 3

There are 3 truth assignments under which the formula is true:
(1) A=F, B=T, C=T
(2) A=T, B=T, C=F
(3) A=T, B=T, C=T

Refer to lecture 2, slides 11 & 14 & 28-29

Question **7**
Correct
Mark 0.50 out of 0.50

Tick all the formulas that are logically entailed by:
$P \wedge (Q \vee \neg R)$

Select one or more:
- [x] $\neg R \Rightarrow P$
- [x] $R \Rightarrow Q$
- [ ] $\neg P$
- [ ] Q
- [ ] $\neg R$

There are 3 truth assignments under which $P \wedge (Q \vee \neg R)$ is true:
(1) P=T, Q=F, R=F
(2) P=T, Q=T, R=F
(3) P=T, Q=T, R=T
$\neg P$
Not entailed. $\neg P$ is false in all three truth assignments from above.
Q
Not entailed. Q is false in truth assignment (1) from above.
$\neg R$
Not entailed. R is true in truth assignment (3) from above.
$R \Rightarrow Q$
Entailed. In all of the assignments from above in which R is true, Q is true as well.
$\neg R \Rightarrow P$
Entailed. In all of the assignments from above in which R is false, P is true.

Refer to lecture 2, slides 32-37

Question **8**
Correct
Mark 0.50 out of 0.50

Solve Problem Set Week 2, Exercise 4 and tick all the statements that are correct.

Select one or more:
- [x] There are two liars and one truar.
- [x] Joan or Shane cannot both be truars.
- [x] One possibility is that Joan and Peter lie while Shane says the truth.

If Peter would tell the truth, then Joan and Shane are liars. But then Joan would have told the truth about Shane. This is a contradiction.
Since Peter lies, either Joan or Shane must be a truar. They cannot both be truars, because this would contradict Joan saying that Shane was a liar.
Hence, Peter is a liar and either Joan or Shane are truars but not both.
Therefore, all the statements are correct.