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Grade	**3.13** out of 4.00 (**78**%)

Question **1**
Partially correct
Mark 0.25 out of 0.50

Which of the following statements are always true for a Boolean algebra over a set T and all elements $x, y \in T$?

Select one or more:

- $\overline{x} + (\overline{y} \cdot x) = x + \overline{y}$
- $\overline{y} \cdot (x + \overline{y}) = x \cdot \overline{y}$
- $x + (\overline{y} \cdot \overline{x}) = x + \overline{y}$
- $y + (x + \overline{y}) = x + y + (\overline{x} \cdot \overline{y})$

$x + (\overline{y} \cdot \overline{x}) = x + \overline{y}$
Always true. $x + (\overline{y} \cdot \overline{x}) = (x + \overline{y}) \cdot (x + \overline{x}) = (x + \overline{y}) \cdot 1 = x + \overline{y}$

$y + (x + \overline{y}) = x + y + (\overline{x} \cdot \overline{y})$
Always true. $y + (x + \overline{y}) = x + (y + \overline{y}) = x + 1 = 1$. Likewise, $x + y + (\overline{x} \cdot \overline{y}) = (x + y + \overline{x}) \cdot (x + y + \overline{y}) = 1 \cdot 1 = 1$.

$\overline{x} + (\overline{y} \cdot x) = x + \overline{y}$
Not always true. Counterexample: Consider the standard Boolean algebra over $\mathbb{B} = \{0, 1\}$ with $x = y = 1$. Then $\overline{1} + (\overline{1} \cdot 1) = 0 + 0 = 0 \neq 1 = 1 + \overline{1}$.

$\overline{y} \cdot (x + \overline{y}) = x \cdot \overline{y}$
Not always true. Counterexample: Consider the standard Boolean algebra over $\mathbb{B} = \{0, 1\}$ with $x = y = 0$. Then $\overline{0} \cdot (0 + \overline{0}) = 1 \cdot 1 = 1 \neq 0 = 0 \cdot \overline{0}$.

Refer to lecture 2-3, slides 57,62

Question **2**
Partially correct
Mark 0.38 out of 0.50

Consider the following Boolean expression: $\phi \;=\; \overline{(p + \overline{r}) \cdot (\overline{p} + r)} + \overline{p} \cdot (\overline{q} + r) \cdot q$

Tick all formulas that are a DNF of ϕ.

Select one or more:

- $(\overline{p} + \overline{r}) \cdot (p + r)$
- $\overline{p}\, q\, r + \overline{p}\, \overline{q}\, r + p\, \overline{r}$
- $p\, q + \overline{p}\, \overline{q}\, \overline{r} + \overline{p}\, \overline{q} r$
- $p\, q + \overline{p}\, \overline{q}$
- $\overline{p}\, r + p\, \overline{r}$
- $(p + \overline{q}) \cdot (\overline{p} + q)$

$\phi = \overline{(p + \overline{r}) \cdot (\overline{p} + r)} + \overline{p} \cdot (\overline{q} + r) \cdot q = \overline{(p + \overline{r})} + \overline{(\overline{p} + r)} + \overline{p}qr + 0 = \overline{p}r + p\overline{r}$. This is a minimal DNF. The only other option that is both a DNF and equivalent to ϕ is $\overline{p}qr + \overline{p}\overline{q}r + p\overline{r}$.

Refer to lecture 2-3, slides 61,67

Question **3**
Correct
Mark 0.50 out of 0.50

Consider the following Boolean function f over variables v, w, x, y:

$f(v, w, x, y) \;=\; v\, \overline{w}\, x + v\, \overline{w}\, \overline{y} + \overline{w}\, \overline{x}\, y + \overline{v}\, \overline{w}\, \overline{x}\, \overline{y} + \overline{v}\, w\, y + \overline{v}\, w\, x\, \overline{y}$.

How many clauses does a minimal DNF of f have? Use a Karnaugh map to answer this question.

Select one:

- 3
- 6
- 5
- 7
- 4

A minimal DNF equivalent to the given function is: $v\, \overline{w} + \overline{w}\, \overline{x} + \overline{v}\, w\, y + \overline{v}\, w\, x$. This DNF has 4 clauses. You can verify that this DNF and f are the same functions by drawing their Karnaugh maps, which are identical and have the same 9 cells marked.

Refer to lecture 2-3, slides 72-76

Question **4**
Incorrect
Mark 0.00 out of 0.50

Consider the function:

$f : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R} \times \mathbb{R}$ given by $f :\; (x, y) \mapsto (2x - 4y, 4y - 2x)$.

Select one:

- f is both 1-1 and onto
- f is 1-1 but not onto
- f is not 1-1 but onto
- f is neither 1-1 nor onto

An example that shows that f is not 1-1 is: $f(0, 0) = f(2, 1)$.

The pair (u, v) resulting from applying f always satisfies $u + v = 0$. So $(1, 0)$, for example, will not be in the image of f, which means that the function is not onto either.

Refer to lecture 4, slides 3-4

Question **5**
Correct
Mark 0.50 out of 0.50

Let $\Sigma = \{0, 1\}$ and consider the functions $f, g :\; \Sigma^* \longrightarrow \Sigma^*$ given by
- $f(\omega) = \omega\omega$
- $g(\omega) = \omega 10$

What is $(f \circ g)(01)$?

Answer: 01100110

$(f \circ g)(01) = f(g(01)) = f(0110) = 01100110$

Refer to lecture 1, slide 38 lecture 4, slide 3

Question **6**
Correct
Mark 0.50 out of 0.50

Consider $f :\; \mathbb{Z} \longrightarrow \mathbb{Z}$ given by $f(x) = 2x - 1$.

What is the inverse image of $\{0, 1, 2\}$, that is, what is $f^{\leftarrow}(\{0, 1, 2\})$?

Select one:

- $\{2\}$
- $\{0, 1\}$
- $\{1, 3, 5\}$
- $\{0, 2, 4\}$
- $\{1, 2\}$
- $\{1\}$

Neither 0 nor 2 can be the result of $2x - 1$ for any $x \in \mathbb{Z}$. Hence, $f^{\leftarrow}(\{0, 1, 2\}) = f^{\leftarrow}(\{1\}) = \{1\}$.

Refer to lecture 4, slides 16-20

Question **7**
Correct
Mark 0.50 out of 0.50

Consider the 1×3 matrices $\mathbf{A} = [\,6 \;\; 8.8 \;\; 3\,]$ and $\mathbf{B} = [\,-9 \;\; 7 \;\; -2\,]$ along with the 3×1 matrix, displayed horizontally, $\mathbf{C} = [\,-1.6 \;\; -3 \;\; 6\,]$.

Compute the value of the inner product $(\mathbf{A} + \mathbf{B}) \cdot \mathbf{C}$.

Answer: -36.6

$\mathbf{A} + \mathbf{B} = [\,-3 \;\; 15.8 \;\; 1\,]$.
$(\mathbf{A} + \mathbf{B}) \cdot \mathbf{C} = -3 \cdot -1.6 + 15.8 \cdot -3 + 1 \cdot 6 = -36.6$.

Refer to lecture 4, slides 16, 18, 19

Question **8**
Correct
Mark 0.50 out of 0.50

Consider the relation $\mathcal{R} \subseteq \mathbb{Z} \times \mathbb{Z}$ given by: $(x, y) \in \mathcal{R}$ iff $2 \mid (x - y)$.

Tick all of the properties that $\mathcal{R}$ has.

Select one or more:

- reflexivity
- transitivity
- antisymmetry
- symmetry
- antireflexivity

It is easy to see that if $x = y$ then you obtain an even number. Hence, $(x, x) \in \mathcal{R}$ for all integers x, which is why the relation is reflexive.
The relation cannot be anti-reflexive since, for example, $(0, 0) \in \mathcal{R}$.
Observe that $(x, y) \in \mathcal{R}$ if, and only if, x and y are of the same "parity", that is, if they are both even or both odd.
It follows that the relation is symmetric: For all $x, y \in \mathbb{Z}$, if $(x, y) \in \mathcal{R}$ then x and y are of the same parity, hence $(y, x) \in \mathcal{R}$.
The relation is not anti-symmetric since, for example, $(0, 2) \in \mathcal{R}$ and $(2, 0) \in \mathcal{R}$ but $0 \neq 2$.
The relation is transitive: For all $x, y, z \in \mathbb{Z}$, if $(x, y) \in \mathcal{R}$ and $(y, z) \in \mathcal{R}$ then x and y are of the same parity and y and z are of the same parity, hence x and z must be of the same parity too, which implies $(x, z) \in \mathcal{R}$.

Refer to lecture 4, slides 34-37