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Grade	**2.50** out of 4.00 (**63**%)

Question **1**
Partially correct
Mark 0.50 out of 1.00

Tick all relations that are equivalence relations.

Select one or more:

- [ ] Real number r is related to real number s iff $|r - s| \leq 1$.
- [ ] Let $\Sigma = \{a, b\}$. Word $\nu \in \Sigma^*$ is related to word $\omega \in \Sigma^*$ iff $\nu = \omega\chi$ for some word $\chi \in \Sigma^*$.
- [ ] Propositional formula ϕ is related to propositional formula ψ iff $\phi \models \psi$ and $\models \psi \Rightarrow \phi$.
- [x] Pair of integers (i, j) is related to pair of integers (m, n) iff $i + j = m - n$ (mod 2). ✓

Pair of integers (i, j) is related to pair of integers (m, n) iff $i + j = m - n$ (mod 2).

True. $i + j$ is even iff i and j are of the same parity, likewise $m - n$ is even iff m and n are of the same parity. Hence, all pairs with numbers of the same parity are related to each other, and all pairs with numbers of different parity are related to each other.

Let $\Sigma = \{a, b\}$. Word $\nu \in \Sigma^*$ is related to word $\omega \in \Sigma^*$ iff $\nu = \omega\chi$ for some word $\chi \in \Sigma^*$.

Not true. Counterexample: ab is related to a but a is not related to ab.

Real number r is related to real number s iff $|r - s| \leq 1$.

Not true. Counterexample: 3.1 and 2.2 are related, 2.2 and 1.3 are related, but 3.1 and 1.3 are not related.

Propositional formula ϕ is related to propositional formula ψ iff $\phi \models \psi$ and $\models \psi \Rightarrow \phi$.

True. $\phi \models \psi$ is equivalent to $\models \phi \Rightarrow \psi$. Hence, ϕ and ψ are related iff $\models \phi \Leftrightarrow \psi$, that is, iff ϕ and ψ are logically equivalent.

Refer to lecture 4, slide 52

Question **2**
Incorrect
Mark 0.00 out of 1.00

Consider the following equivalence relation $\mathcal{R}$ on $\{0, 1, 2, 3, 4, 5\}$:
$(m, n) \in \mathcal{R}$ iff $2m = 2n$ (mod 6)
How many equivalence classes does $\mathcal{R}$ have?

Answer: 12 ✗

$2 \cdot 0 \bmod 6 = 2 \cdot 3 \bmod 6 = 0$. $2 \cdot 1 \bmod 6 = 2 \cdot 4 \bmod 6 = 2$. $2 \cdot 2 \bmod 6 = 5 \cdot 2 \bmod 6 = 4$. Hence, there are 3 equivalence classes: $\{0, 3\}$, $\{1, 4\}$ and $\{2, 5\}$.

Refer to lecture 4, slides 52-53, 60

Question **3**
Correct
Mark 1.00 out of 1.00

Tick all relations that are partial orders.

Select one or more:

- [x] Natural number m is related to natural number n iff $\text{gcd}(m, n) = n$. ✓
- [x] Integer m is related to integer n iff $m^3 \leq n^3$. ✓
- [ ] Natural number m is related to natural number n iff $n^3 > m^2 + 1$.
- [ ] Natural number m is related to natural number n iff m is a prime divisor of n.

Integer m is related to integer n iff $m^3 \leq n^3$.

True. $m^3 \leq n^3$ if, and only if, $m \leq n$, which is reflexive, antisymmetric and transitive.

Natural number m is related to natural number n iff $\text{gcd}(m, n) = n$.

True. $\text{gcd}(m, n) = n$ if, and only if, n is a divsor of m, which is reflexive, antisymmetric and transitive.

Natural number m is related to natural number n iff m is a prime divisor of n.

Not true. Counterexample: 4 is not a prime divisor of 4, hence the relation is not reflexive.

Natural number m is related to natural number n iff $n^3 > m^2 + 1$.

Not true. Counterexample: $1^3 \not> 1^2 + 1$, hence the relation is not reflexive.

Refer to lecture 4, slide 67

Question **4**
Correct
Mark 1.00 out of 1.00

Consider the lattice $(\{1, 2, 3, 5, 6, 10, 15, 30\}, \preceq)$ with $x \preceq y$ iff $x \mid y$.
Compute $\text{glb}(\{6, 10, 30\}) + \text{lub}(\{2, 3, 5\})$.

Answer: 32 ✓

$\text{glb}(\{6, 10, 30\}) = 2$ and $\text{lub}(\{2, 3, 5\}) = 30$.

Refer to lecture 4, slides 67-69