WAVE EQUATION

(IN 2D) (VIBRATION OF A STRING)

$$\partial_t^2 \phi = c^2 \left[\partial_x^2 \phi + \partial_y^2 \phi\right]$$

$\Rightarrow T^{-2} = c^2 L^{-2}$

$\Rightarrow c^2 = \left(\frac{L}{T}\right)^2$

$\Rightarrow c$ VELOCITY OF THE WAVE

VIBRATION OF A MEMBRANE WITH FIXED BORDERS

$$\phi(t,0,y) = \phi(t,L_x,y) = \phi(t,x,0) = \phi(t,x,L_y) = 0$$

SEP. VAR.

$$\phi = T(t)\,X(x)\,Y(y)$$

$$\Rightarrow c^2\left(\frac{\partial_x^2 X}{X} + \frac{\partial_y^2 Y}{Y}\right) = k = \frac{\partial_t^2 T}{T}$$

$$\Rightarrow \boxed{\partial_t^2 T = kT}$$

$$\boxed{\frac{\partial_x^2 X}{X} = \frac{k}{c^2} - \frac{\partial_y^2 Y}{Y} \equiv k_x}$$

$$\frac{\partial_y^2 Y}{Y} = \frac{k}{c^2} - k_x \equiv k_y$$

AS BEFORE $\Rightarrow$

$$k_x = -\frac{m_x^2 \pi^2}{L_x^2} \qquad k_y = -\frac{m_y^2 \pi^2}{L_x^2}$$

$\Rightarrow$ CALL THIS C_x,

$C_x = -w_x^2$?

$$S(x,y) = \sum_{m_x, m_y} A'_{m_x m_y} \sin\left(\frac{m_x \pi}{L_x} x\right) \sin\left(\frac{m_y \pi}{L_y} y\right)$$

$$= \sum_{n,m} A_{n,m} S_n^{L_x}(x) S_m^{L_y}(y)$$

$$S_n^{L_x}(x) \equiv \sqrt{\frac{2}{L_x}} \sin\left(\frac{n \pi}{L_x} x\right)$$

IF WE ASSUME THAT

$$f(x,y) = \sum_{n,m} A_n B_m S_n(x) S_m(y)$$

$\Rightarrow$

$$\int_0^{L_x} dx \int_0^{L_y} dy \; f(x,y) S_n(x) S_m(y) =$$

$$\iint \sum_{\ell n} A_\ell B_n S_\ell(x) S_n(y) S_m(x) S_m(y) = \sum_{\ell n} \delta_{\ell m} \delta_{n m} A_\ell B_n = A_m B_m (= A_{mm}$$

IN DIRAC NOTATION $(\Rightarrow)$ WE WRITE

$$\sum_{n,m} |n,m\rangle \langle n,m| = \mathbb{1}$$

$\Rightarrow$

$$|f\rangle = \sum_{n,m} |n,m\rangle \langle n,m|f\rangle$$

Our initial condition will thus be

$$\phi(0,x,y) = f(x,y) \qquad \partial_t \phi(0,x,y) = g(x,y)$$

$T(t)$:

$$\partial_t^2 T = kT \qquad K = (k_x + k_y)\, c^2$$

$$= \left(-\frac{n_x^2 \pi^2}{L_x^2} - \frac{n_y^2 \pi^2}{L_y^2} \right) c^2 \equiv - W_{n_x,n_y}^2$$

$$\Rightarrow \partial_t^2 T = -W^2 T$$

if $L_x = L_y = L$

$$W = \frac{|n| c \pi}{L}$$

$$\Rightarrow T(t) = A e^{i \omega_{n_x,n_y} t} + B e^{-i \omega_{n_x,n_y} t}$$

if we ask T real

$$\Rightarrow T(t) = A \sin(\omega_{n,m} t) + B \cos(\omega_{n,m} t)$$

$\Rightarrow$

$$\phi(t,x,y) = \sum_{n,m} A_{n,m} \sin(W_{n,m} t)\, S_n^{L_x}(x)\, S_m^{L_y}(y) + B_{n,m} \cos(W_{n,m} t)\, S_n^{L_x}(x)\, S_m^{L_y}(y)$$

$$W_{n,m} = \left(\frac{n_x^2 \pi^2}{L_x^2} + \frac{n_y^2 \pi^2}{L_y^2} \right)^{\frac{1}{2}} c$$

$$\phi(0,x,y) \equiv f(x,y)$$

$$= \sum_{n,m} B_{n,m} S_n^{L_x}(x) S_m^{L_y}(y) \Rightarrow B_{n,m} = \iint f(x,y) S_n^{L_x}(x) S_m^{L_y}(y)$$

$$\partial_t \phi(0,x,y) \equiv g(x,y)$$

$$= \sum_{n,m} W_{nm} A_{n,m} S_n^{L_x}(x) S_m^{L_y}(x) \Rightarrow A_{n,m} = \left[\iint g(x,y) S_n^{L_x}(x) S_m^{L_y}(y)\right] / W_{n,m}$$

(or $\partial_t^2 \phi = c^2 \nabla^2 \phi$)

$$\partial_t^2 \phi = c^2 \partial_x^2 \phi$$

c is usually used for the speed of light.

If $t' = ct$ $\Rightarrow \frac{\partial}{\partial t'} = \frac{\partial}{\partial t}\frac{\partial t}{\partial t'} = \frac{\partial}{\partial t}\frac{1}{c}$

$$\Rightarrow \partial_t^2 = c^2 \partial_{t'}^2$$

$$\Rightarrow \partial_{t'}^2 \phi = \partial_x^2 \phi$$

"$c = 1$"

In relativity theory we use the same unit for length and time (light year) so that c disappears from eqs.