Harbour crane analysis

Figure 2 shows the model of the harbour crane depicted in Figure 1.

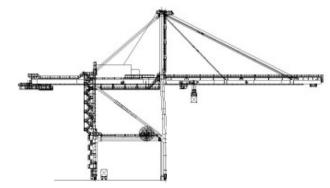


Figure 1: harbour crane

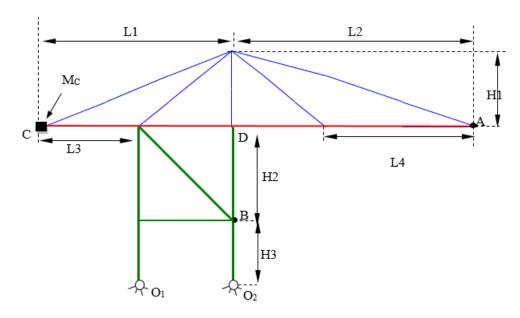


Figure 2: harbour crane model

Structural damping is to be accounted for according to the proportional damping assumption, $[C]=\alpha[M]+\beta[K]$, with values α and β assigned in the input data table.

The students are asked to:

- 1) Define a finite element model valid in the frequency range $0 \div 10 \; \text{Hz}$
- 2) Compute the system's natural frequencies and related modes of vibration in the frequency range 0÷10 Hz
- 3) Compute the following frequency response functions (FRF) in the frequency range $0 \div 10$ Hz with step 0.01 Hz:
 - a. Input: vertical force at point A; output: vertical displacement of point A;
 - b. Input: vertical force at point A; output: horizontal displacement of point B;
 - c. Input: vertical force at point A; output: vertical component of the constraint force in the hinge O₂;
 - d. Input: vertical force at point C; output: vertical component of the constraint force in the hinge O₂.

- 4) Compute the frequency response functions (FRF) in the frequency range 0 ÷ 10 Hz, step 0.01 Hz for a horizontal distributed load (magnitude 1 N/m) applied on the right leg of the crane between points B and D. Outputs:
 - a. horizontal displacement of point A;
 - b. vertical displacement of point A;
 - c. vertical displacement of point C.
- 5) Assume a winch is placed at A and moves up and down a mass M_A according to a periodic time history $y_{rel}(t)$:

$$y_{rel}(t) = \sum_{i=1,3} A_i \cos\left(i\frac{2\pi}{T}t + \varphi_i\right)$$

which represents the periodic vertical relative displacement of the load with respect to point A (positive if upwards directed). Compute the corresponding time history of the absolute vertical displacement of point A.

6) Modify the structure in order to reduce at least by 50 % the maximum value of the constraint force computed at point 3, without increasing the total mass of the crane more than 5 % of its original value. Any change of the material and/or of the constraints (by adding or moving them) is not allowed. In case of change of one or more beams sections, both the inertial and the stiffness parameters m, EA, EJ have to be computed taking into account a real geometry of the new sections and the physical properties of steel material.

Input data:

Main horizontal red beam: linear mass (kg/m)	m ₁	312
Main horizontal red beam: axial stiffness (N)	EA ₁	8.2 E9
Main horizontal red beam: bending stiffness (Nm²)	EJ ₁	1.40 E9
Vertical, diagonal and secondary horizontal green beams: linear mass (kg/m)	m ₂	200
Vertical, diagonal and secondary horizontal green beams: axial stiffness (N)	EA ₂	5.4 E9
Vertical, diagonal and secondary horizontal green beams: bending stiffness (Nm²)	EJ ₂	4.5 E8
Blue beams: linear mass (kg/m)	m ₃	90.0
Blue beams: axial stiffness (N)	EA ₃	2.4E9
Blue beams: bending stiffness (Nm²)	EJ ₃	2.0 E8
Point mass in C (kg)	Mc	2000
Moment of inertia of the point mass in C (kgm²)	Jc	320
L ₁ length (m)	L ₁	30
L ₂ length (m)	L ₂	39
L ₃ length (m)	L ₃	15
L ₄ length (m)	L ₄	24
H ₁ height (m)	H ₁	12
H ₂ height (m)	H ₂	15
H ₃ height (m)	H ₃	10
Moving load M _A (kg)	MA	800
Point 5, amplitude of the first harmonic (m)	A ₁	0.25
Point 5, amplitude of the second harmonic (m)	A ₂	0.25
Point 5, amplitude of the third harmonic (m)	A ₃	0.15
Point 5, phase of the first harmonic (rad)	Φ1	0
Point 5, phase of the second harmonic (rad)	φ2	π
Point 5, phase of the third harmonic (rad)	φ3	π
Point 5: period (s)	T	1.2
Coefficient for the structural damping evaluation	α	0.1
Coefficient for the structural damping evaluation	β	2.0e-4