Ritardi nei sistemi di controllo

Fall 2020

Project no. 1

(submission deadline: 18.12.2020, 11:00am)

Background

Consider the discrete system

$$\begin{cases} x[t+1] = Ax[t] + B_w w[t] + B_u u[t] \\ y[t] = Cx[t] + D_w w[t], \end{cases} \tag{1}$$

where $x[t] \in \mathbb{R}^n$ is a state vector, $u[t] \in \mathbb{R}^{m_u}$ is a measured input signal, $w[t] \in \mathbb{R}^{m_w}$ is an unmeasured input signal, and $y[t] \in \mathbb{R}^p$ is the measured output. The H_2 (steady-state Kalman) filter for this process, reconstructing its state, is

$$\hat{x}[t+1] = A\hat{x}[t] + B_u u[t] - L(y[t] - C\hat{x}[t]) = (A + LC)\hat{x}[t] + B_u u[t] - Ly[t], \tag{2}$$

where $L = -(B_w D_w' + AYC')(D_w D_w' + CYC')^{-1}$ and $Y = Y' \geq 0$ is the stabilizing solution to the DARE

$$Y = AYA' + B_w B_w' - (B_w D_w' + AYC')(D_w D_w' + CYC')^{-1}(D_w B_w' + CYA'), \tag{3}$$

i.e. such that $A + LC$ is Schur. This solution yields the optimal estimation error $\epsilon[t] := x[t] - \hat{x}[t]$ under a unit-variance white w. It is known that the stabilizing solution of (3) exists if (C, A) is detectable and the matrix $\left[\begin{smallmatrix} A - e^{j\theta} I & B_w \\ C & D_w \end{smallmatrix}\right]$ has full row rank for all $\theta \in [-\pi, \pi]$ (i.e. the realization (A, B_w, C, D_w) has no invariant zeros on the unit circle). Thus, we assume hereafter that these conditions hold true. We also assume, for simplicity, that $B_w D_w' = 0$, meaning that $w = \left[\begin{smallmatrix} w_x \\ w_y \end{smallmatrix}\right]$ contains two components, a process disturbance w_x and measurement noise w_y, which are uncorrelated.

The problem

Consider now the system

$$\begin{cases} x[t+1] = Ax[t] + B_w w[t] + B_u u[t-\tau] \\ y[t] = Cx[t] + D_w w[t], \end{cases}$$

where $\tau \in \mathbb{N}$ is the delay. We know that if u is perfectly measurable, then the reduced-order observer $\hat{x}[t+1] = A\hat{x}[t] + B_u u[t-\tau] - L(y[t] - C\hat{x}[t])$ does the trick. But we assume that

- the control input is measured as $u_{\mathrm{m}}[t] = u[t] + D_u n_u[t]$, where n_u is a unit-variance white process.

The system dynamics read then

$$\begin{cases} x[t+1] = Ax[t] + B_w w[t] - B_u D_u n_u[t-\tau] + B_u u_{\mathrm{m}}[t-\tau] \\ y[t] = Cx[t] + D_w w[t] \end{cases}$$

So, assuming that the delay-free system (1) satisfies the assumptions guaranteeing the solvability of the H_2 estimation problem for (1), do the following.

1. Find the state vector x_τ of this system and write its state and measurement equations in form (1), with new unmeasured input signal w_τ, which should be white unit-variance signal, a new measured input signal, and corresponding matrices A_τ, $B_{w,\tau}$, $B_{u,\tau}$, C_τ, and $D_{w,\tau}$.

2. Under what conditions on D_u the existence conditions for the corresponded DARE are satisfied for this system (assuming they are satisfied for (1))?

3. Find the stabilizing solution to the corresponding DARE. Its elements should require only one n-dimensional DARE, all other parts of the solution should have closed-form expressions. What are eigenvalues of $A_\tau + LC_\tau$?
Hint: First, solve it numerically (Matlab's `idare`) and use the gained insight for your educated guess on the structure of Y.

4. Derive the delay counterpart of (2). Try to have as transparent and simple structure as possible. Under what conditions on D_u it is a reduced-order observer?