Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

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| **Course Number:** | 843 |
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| **Instructor:** | Dr. Guanghui (Richard) Wang |

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| *Assignment/Lab Number:* | HW Assighnement 4 |
| *Assignment/Lab Title:* | Problem Set 4 |

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| *Submission Date:* | November 20th 2023 |
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| Student LAST Name | Student FIRST Name | Student Number | Section | Signature* |
|---|---|---|---|---|
| Rahim | Farzaan | 501038746 | 01 | Farzaan Rahim |
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# Problem 1

## Part 1

Canonical form of the plane at infinity

$$\boxed{\pi = (0,0,0,1)^T}$$

## Part 2

$$\pi'_\infty = H_A^{-T}\pi_\infty = \begin{bmatrix} A^{-T} & 0 \\ -A_t & 1 \end{bmatrix}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \pi_\infty$$

## Part 3

A plane as defined by 3 points can be expressed as:

$$\begin{bmatrix} \pi_1^T \\ \pi_2^T \\ \pi_3^T \end{bmatrix} X = 0$$

Undergoing the transform of the 3D points:

$$X' = HX$$

$$\pi' = H^{-T}\pi$$

Problem 2

## Part 1

General form :

$$K = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

$\alpha_x$ = focal length of x-axis $= f m_x$

$\alpha_y$ = focal length of y axis $= f m_y$

$s$ = the skew

$x_0, y_0$ = principal points.

## Part 2

Projection matrix

$$P = [M \mid p_4]$$

The internal and external parameters can be recovered by

$$P = KR[I \mid -C] = M[I \mid M^{-1} p_4]$$

$K$ = internal parameter

$R, t$ = external parameter

## Part 3

The image of the point at infinity can be represented by

$$D = (d^T, 0)^T$$

$$x = PD = [M \mid p_4] D = Md$$

## Problem 3

### Part 1

$PC = 0$

or

$X = \det([p_2, p_3, p_4])$

$Z = \det([p_1, p_2, p_4])$

$Y = -\det([p_1, p_3, p_4])$

$T = -\det([p_1, p_2, p_3])$

### Part 2

$$s_x V_x = P X_w = [p_1, p_2, p_3, p_4] [1, 0, 0, 0]^T = p_1$$

$$s_y V_y = P Y_w = [p_1, p_2, p_3, p_4] [0, 1, 0, 0]^T = p_2$$

$$s_z V_z = P Z_w = [p_1, p_2, p_3, p_4] [0, 0, 1, 0]^T = p_3$$

### Part 3

Projection plan = $[x, y, 0]$

$$\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} p_1^T \\ p_2^T \\ p_3^T \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# Problem 4

## Part 1

3D Points

$$x = [p_1 \quad p_2 \quad p_3 \quad p_4] \begin{pmatrix} X \\ Y \\ 0 \\ 1 \end{pmatrix}$$

2D Points

$$x = [p_1 \quad p_2 \quad p_4] \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

$$[p_1 \quad p_2 \quad p_3 \quad p_4] \begin{pmatrix} X \\ Y \\ 0 \\ 1 \end{pmatrix} = [p_1 \quad p_2 \quad p_4] \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

$$P = K[R|t]$$

$$H = K[r_1, r_2, t]$$

## Part 2

Back projection of a real line in 3D

$$\pi = P^T l$$

x will line of the line if

$$x^T l = 0$$

real point on the real line in 3D if and only if

$$X^T P^T l = 0$$

therefore

$P^T l$ represents the plane. 2D point $x$ lies on this point if and only if

$$\pi = P^T l$$

## Part 3

$$x = K[I|0]X$$

$$x' = K'[I|0]X$$

$$x' = K' K^{-1}(K[I|0]X) = K' K^{-1} x$$

$$x' = Hx$$

$$H = K' K^{-1}$$

## Problem 5

### Part 1

Real points on the line of infinity:

$P = KR[I | -\tilde{C}]$

point in the image

$x = PX_\infty = KR[I | -\tilde{C}]\begin{pmatrix} d \\ 0 \end{pmatrix} = KRd$

Map observed point from line of infinity $x = Hd$

$H = KR$

absolute conic

$w = (KK^T)^{-1}$

### Part 2

The angle between two rays is

$$\cos(\theta) = \frac{x_1^T w x_2}{\sqrt{x_1^T w x_1}\sqrt{x^T w x_2}}$$

if 2 rays are orthogonal

$x_1^T w x_2 = 0$

### Part 3

$h_1^T w h_2 = 0$

$h_1^T w h_1 = h_2^T w h_2$

### Part 4

The restraints for the square pixel assumption for the image of absolute conic is

$W_{12} = W_{21} = 0$

and

$W_{11} = W_{22}$

Part 2:

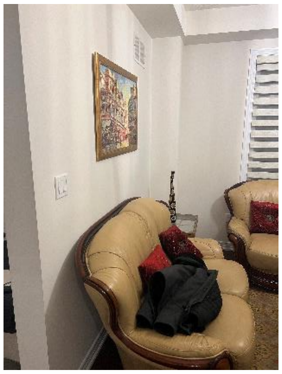
SIFT (Scale Invariant Feature Transform) is a method for obtaining distinct and invariant features from photos. It can be used for a variety of computer vision tasks, such as image matching and object detection. The SIFT method consists of several stages:

Scale-space extrema detection: In this step, potential interest points in the image that are both orientation- and scale-invariant are sought after. This is achieved by identifying regions where intensity changes noticeably using a difference-of-Gaussian function.

Key point localization: A comprehensive model is fitted to each possible point of interest to ascertain its exact location and scale. The selection of key points is based on stability metrics.

Orientation assignment: Every key point is given one or more orientations based on the local image gradient directions. This step ensures that the image rotation won't affect it.

Key point descriptor: Each key point’s surrounding local image gradients are measured at the selected scale. These gradients are transformed into a form of representation that allows significant variations in illumination and local shape distortion. This descriptor captures the special characteristics of the key point.



For image matching, SIFT features are extracted from a set of reference photos and stored in a database. Euclidean distance is used to compare each feature individually to the features in the database whenever a new image is displayed. To find candidate matching features, their feature vectors are compared for similarity. This process allows for the reliable matching of different perspectives of an object or scene.

When it comes to image stitching, SIFT features can be used to align and combine multiple images into a panoramic view. The key point descriptors between the images are matched, and the transformation parameters (such as translation and rotation) are estimated, to properly align the images. By using the matched key points to blend the overlapping areas of the images, a seamless panoramic image is created.

When all is said and done, SIFT features provide resistance to image rotation, scale, affine distortion, noise, and changes in lighting. Precise matching against a large feature database is possible due to their extreme distinctiveness. Standard hardware can achieve real-time performance because SIFT features are computed efficiently.

My camera could be rotated horizontally, and the algorithm recognized this and stitched the images appropriately. There are two black "circles" on top and bottom because of the camera's rigidly horizontal movement. If I wanted to eliminate this blank space, I could simply take pictures above and below the horizontal while maintaining an overlap.