Problem 2

Ideal point : $(x, y, 0)$

2D transformation 3x3 matrix : $\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$

Apply transformation : $[x \ y \ 0]$

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} h_{11} \cdot x + h_{12} \cdot y + h_{13} \cdot 0 \\ h_{21} \cdot x + h_{22} \cdot y + h_{23} \cdot 0 \\ h_{31} \cdot x + h_{32} \, y + h_{33} \cdot 0 \end{bmatrix} = \begin{bmatrix} h_{11} \cdot x + h_{12} \cdot y \\ h_{21} \cdot x + h_{22} \cdot y \\ h_{31} \cdot x + h_{32} \, y \end{bmatrix}$$

line in homogeneous coordinates ; $l^T \cdot x = 0$

$l'^T \cdot x' = 0$

$l'^T \cdot H_x = 0$

To ensure that $l^T \cdot x = 0$

$l' = H^{-T} \cdot l$

verify : $C' = H^{-T} \cdot C \cdot H^{-1}$

Conic in homogeneous coordinates : $x^T \cdot C_x = 0$

$x'^T \cdot C' \cdot x' = 0$

$(H_x)^T \cdot C' \, H_x = 0$

$C' = H^{-T} \cdot C \cdot H^{-1}$