Problem 1

Part 1

Canonical form of the plane at infinity

$$\boxed{\pi = (0,0,0,1)^T}$$

Part 2

$$\pi'_\infty = H_A^{-T}\pi_\infty = \begin{bmatrix} A^{-T} & 0 \\ -At & 1 \end{bmatrix}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \pi_\infty$$

Part 3

A plane as defined by 3 points can be expressed as:

$$\begin{bmatrix} \pi_1^T \\ \pi_2^T \\ \pi_3^T \end{bmatrix} X = 0$$

Undergoing the transform of the 3D points:

$$X' = HX$$

$$\pi' = H^{-T}\pi$$

Problem 2

Part 1

General form :

$$K = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

α_x = focal length of x-axis = $f m_x$

α_y = focal length of y axis = $f m_y$

s = the skew

x_0, y_0 = principal points.

Part 2

Projection matrix

$$P = [M \mid p_4]$$

The internal and external parameters can be recovered by

$$P = KR[I \mid -C] = M[I \mid M^{-1} p_4]$$

K = internal parameter

R, t = external parameter

Part 3

The image of the point at infinity can be represented by

$$D = (d^T, 0)^T$$

$$x = PD = [M \mid p_4] D = Md$$

Problem 3

Part 1

$PC = 0$

or

$X = \det([p_2, p_3, p_4])$

$Z = \det([p_1, p_2, p_4])$

$Y = -\det([p_1, p_3, p_4])$

$T = -\det([p_1, p_2, p_3])$

Part 2

$$s_x V_x = P X_w = [p_1, p_2, p_3, p_4]\,[1, 0, 0, 0]^T = p_1$$

$$s_y V_y = P Y_w = [p_1, p_2, p_3, p_4]\,[0, 1, 0, 0]^T = p_2$$

$$s_z V_z = P Z_w = [p_1, p_2, p_3, p_4]\,[0, 0, 1, 0]^T = p_3$$

Part 3

Projection plan = $[x, y, 0]$

$$\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} p_1^T \\ p_2^T \\ p_3^T \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Problem 4

Part 1

3D Points

$$x = [p_1 \quad p_2 \quad p_3 \quad p_4] \begin{pmatrix} X \\ Y \\ 0 \\ 1 \end{pmatrix}$$

2D Points

$$x = [p_1 \quad p_2 \quad p_4] \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

$$[p_1 \quad p_2 \quad p_3 \quad p_4] \begin{pmatrix} X \\ Y \\ 0 \\ 1 \end{pmatrix} = [p_1 \quad p_2 \quad p_4] \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

$P = K[R|t]$

$H = K[r_1, r_2, t]$

Part 2

Back projection of a real line in 3D

$\pi = P^T l$

x will line of the line if

$x^T l = 0$

real point on the real line in 3D if and only if

$X^T P^T l = 0$

therefore

$P^T L$ represents the plane. 2D point X lies on this point if and only if

$\pi = P^T l$

Part 3

$x = K[I|0]X$

$x' = K'[I|0]X$

$x' = K'K^{-1}(K[I|0]X) = K'K^{-1}x$

$x' = Hx$

$H = K'K^{-1}$

Problem 5

Part 1

Real points on the line of infinity:

$P = KR[I | -\tilde{C}]$

point in the image

$x = PX_\infty = KR[I | -\tilde{C}] \begin{pmatrix} d \\ 0 \end{pmatrix} = KRd$

Map observed point from line of infinity $x = Hd$

$H = KR$

absolute conic

$w = (KK^T)^{-1}$

Part 2

The angle between two rays is

$$\cos(\theta) = \frac{x_1^T w x_2}{\sqrt{x_1^T w x_1} \sqrt{x^T w x_2}}$$

if 2 rays are orthogonal

$x_1^T w x_2 = 0$

Part 3

$h_1^T w h_2 = 0$

$h_1^T w h_1 = h_2^T w h_2$

Part 4

The restraints for the square pixel assumption for the image of absolute conic is

$w_{12} = w_{21} = 0$

and

$w_{11} = w_{22}$