Problem 3

Projective

general form = $$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

degree of freedom = in 2D it has 8 degree of freedom since the matrix has 9 parameters but is defined up to a scalar multiple

invariant properties = preserves collinearity and cross ratio of points but do not preserve angles, lengths or parallelism

Affine Transformations

general form = $$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

degree of freedom = in 2D has 6 degrees of freedom, 2 for translation, 1 for rotation, 1 for scalling and 2 for shearing

invariant Properties = Preserves points, straight lines and planes and also preserves parallellism and ratios of segments on parrallel lines

Similarity Transformation

general form = $$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s\cos\theta & -s\sin\theta & t_x \\ s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

degree of Freedom = in 2D has 4 degrees of freedom 1 for uniform scalling, 1 for rotation an 2 for translation

invariant properties = Preserve Shape of geometrical figures. They maintain the proportionality of lengths and angles. They enlarge or reduce the figures without changing the shape.

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} ax + by \\ dx + ey \\ 0 \end{bmatrix}$$

w' is still $= 0$

Line at infinity = $l_1 x + l_2 y = 0$ $\therefore l_{3u} = 0$

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \qquad A^{-T} = \begin{bmatrix} a & d \\ b & e \end{bmatrix} = \begin{bmatrix} a & b \\ d & e \end{bmatrix}^{-1}$$

$$A^{-T} l = \begin{bmatrix} a & b \\ d & e \end{bmatrix}^{-1} \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} = \begin{bmatrix} l'_1 \\ l'_2 \end{bmatrix}$$