Department of Electrical,
Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

Course Title:	CPS
Course Number:	843
Semester/Year (e.g.F2016)	F2023

Instructor:	Dr. Guanghui (Richard) Wang

Assignment/Lab Number:	HW Assighnment 3
Assignment/Lab Title:	Problem Set 3

Submission Date:	November 6th 2023
Due Date:	November 6th 2023

Student LAST Name	Student FIRST Name	Student Number	Section	Signature*
Rahim	Farzaan	501038746	01	Farzaan Rahim

*By signing above you attest that you have contributed to this written lab report and confirm that all work you have contributed to this lab report is your own work. Any suspicion of copying or plagiarism in this work will result in an investigation of Academic Misconduct and may result in a “0” on the work, an “F” in the course, or possibly more severe penalties, as well as a Disciplinary Notice on your academic record under the Student Code of Academic Conduct, which can be found online at: http://www.ryerson.ca/senate/current/pol60.pdf

Problem 1

Part 1

1 $y = -0.5x + 2 \rightarrow 0.5x + y - 2z = 0$

0 $3x + 6y = 5 \rightarrow 3x + 6y - 5z = 0$

$L_1 = (0.5, 1, -2)$

$L_2 = (3, 6, -5)$

$$L_1 \times L_2 = \begin{array}{c} \begin{array}{ccc} i & j & k \end{array} \\ \begin{bmatrix} 0.5 & 1 & -2 \\ 3 & 6 & -5 \end{bmatrix} \end{array}$$

$$\Delta = \left(1(-5) - (-2)(6)\right)i + \left[\left((-2)(3) - (0.5)(-5)\right)\right]j + \left((0.5)(6) - (1)(3)\right)$$

$$(-5 + 12)i + [-6 + 2.5]j + [3 - 3]k$$

$$7i - 3.5j + 0k$$

$$(7, 3.5, 0)$$

* no intersection since $z = 0$

Part2

ideal point $= [x, y, 0]$ line at infinity $w = 0$

$[7, 3.5, 0]$ since z or $w = 0$ it lies on the line of infinity

Part3

in homogeneous form

$$(0.5x + y - 2)(3x + 6y - 5) = 0$$

$$(0.5x)(3x) + (0.5x)(6y) + (0.5x)(-5) + (y)(3x) + (y)(6y) + (y)(-5) + (-2)(3x) + (-2)(6y) + (-2)(-5) = 0$$

$$1.5x^2 + 3xy - 2.5x + 3xy + 6y^2 - 5y - 6x - 12y + 10 = 0$$

$$1.5x^2 + 6xy - 8.5x + 6y^2 - 17y + 10 = 0$$

homogeneous form

$$1.5(x/w)^2 + 6(x/w)(y/w) + 6(y/w)^2 - 8.5(x/w) - 17(y/w) + 10 = 0$$

multiply by w^2

$$1.5x^2 + 6xy + 6y^2 - 8.5xw - 17yw + 10w^2 = 0$$

Matrix form

$$C = \begin{bmatrix} A & B & D \\ B & C & E \\ D & E & F \end{bmatrix} = \begin{bmatrix} 1.5 & 3 & -4.25 \\ 3 & 6 & -8.5 \\ -4.25 & -8.5 & 10 \end{bmatrix}$$

$$\begin{bmatrix} x & y & w \end{bmatrix} \begin{bmatrix} 1.5 & 3 & -4.25 \\ 3 & 6 & -8.5 \\ -4.25 & -8.5 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix} = 0$$

Problem 2

Ideal point : $(x, y, 0)$

2D transformation 3x3 matrix : $\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$

Apply transformation : $[x \; y \; 0]$

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} h_{11} \cdot x + h_{12} \cdot y + h_{13} \cdot 0 \\ h_{21} \cdot x + h_{22} \cdot y + h_{23} \cdot 0 \\ h_{31} \cdot x + h_{32}\, y + h_{33} \cdot 0 \end{bmatrix} = \begin{bmatrix} h_{11} \cdot x + h_{12} \cdot y \\ h_{21} \cdot x + h_{22} \cdot y \\ h_{31} \cdot x + h_{32}\, y \end{bmatrix}$$

line in homogeneous coordinates ; $l^T \cdot x = 0$

$l'^T \cdot x' = 0$

$l'^T \cdot H_x = 0$

To ensure that $l^T \cdot x = 0$

$l' = H^{-T} \cdot l$

verify : $C' = H^{-T} \cdot C \cdot H^{-1}$

Conic in homogeneous coordinates : $x^T \cdot C_x = 0$

$x'^T \cdot C' \cdot x' = 0$

$(H_x)^T \cdot C' \, H_x = 0$

$C' = H^{-T} \cdot C \cdot H^{-1}$

Problem 3

Projective

general form = $$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

degree of freedom = in 2D it has 8 degree of freedom since the matrix has 9 parameters but is defined up to a scalar multiple

invariant properties = preserves collinearity and cross ratio of points but do not preserve angles, lengths or parallelism

Affine Transformations

general form = $$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

degree of freedom = in 2D has 6 degrees of freedom, 2 for translation, 1 for rotation, 1 for scalling and 2 for shearing

invariant Properties = Preserves points, straight lines and planes and also preserves parallellism and ratios of segments on parrallel lines

Similarity Trarsformation

general form = $$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s\cos\theta & -s\sin\theta & t_x \\ s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

degree of freedom = in 2D has 4 degrees of freedom 1 for uniform scalling, 1 for rotation an 2 for translation

invariant properties = Preserve Shape of geometrical figures. They maintain the proportionality of lengths and angles. They enlarge or reduce the figures without changing the shape.

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} ax + by \\ dx + ey \\ 0 \end{bmatrix}$$

w' is still $= 0$

Line at infinity = $l_1 x + l_2 y = 0$ $\therefore l_{3\infty} = 0$

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \qquad A^{-T} = \begin{bmatrix} a & d \\ b & e \end{bmatrix} = \begin{bmatrix} a & b \\ d & e \end{bmatrix}^{-1}$$

$$A^{-T} l = \begin{bmatrix} a & b \\ d & e \end{bmatrix}^{-1} \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} = \begin{bmatrix} l'_1 \\ l'_2 \end{bmatrix}$$

Problem 4

for circular points: $I = [1, i, 0]^T$ $\quad J = [1, -i, 0]^T$

for a circle: $x^2 + y^2 + Ax + By + C = 0$

Intersection with I

$1^2 + i^2 + A + Bi + C = 0$

$1 - 1 + A + Bi + C = 0$

Real: $A + C = 0$

imaginary: $B = 0$

Intersection with B

$1^2 + (-i)^2 + A - Bi + C = 0$

$1 - 1 + A - Bi + C = 0$

real: $A + C = 0$

imaginary: $B = 0$

* Every circle intersects the infinite line at the circular points

Under point transformation: $x' = H_s \cdot x$

$$C_\infty^{*'} = H_s C_\infty^* H_s^T = C_\infty^*$$

$$\Rightarrow I^T l_\infty = J^T l_\infty = 0$$

$$C_\infty^* l_\infty = (IJ^T + JI^T) l_\infty = I(J^T l_\infty) + J(I^T l_\infty) = 0$$

For 2 orthogonal lines:

$l = (l_1, l_2, l_3)^T$ $\quad m = (m_1, m_2, m_3)^T$

Projective Space: $\cos\theta = \dfrac{l^T C_\infty^* m}{\sqrt{(l^T C_\infty^* l)(m^T C_\infty^* m)}}$

lines l and m are orthogonal if $l^T C_\infty^* m = 0$

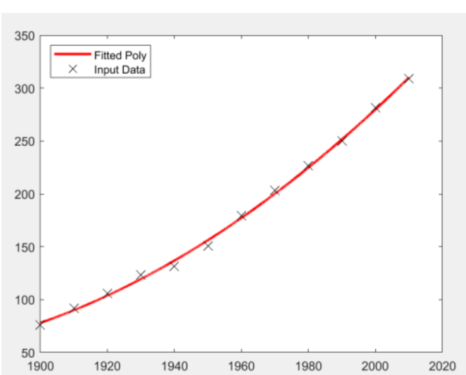
Part 1 (problem 5):

```
data=[ 1900 75.995;
       1910 91.972;
       1920 105.711;
       1930 123.203;
       1940 131.669;
       1950 150.697;
       1960 179.323;
       1970 203.212;
       1980 226.505;
       1990 249.633;
       2000 281.422;
       2010 308.748;];

g=[data(:,1)];
tMeasured=[data(:,2)];
a=polyfit(g, tMeasured, 2);

gModel=min(g):0.01:max(g);
tModelled=polyval(a, gModel);

fittedDates = [2020];
fittedY = polyval(a, fittedDates);
disp(fittedY);
```

Code 1

```
fittedDates = [2020];
fittedY = polyval(a, fittedDates);
disp(fittedY);

figure
plot(gModel,tModelled,'r-','LineWidth',2);
hold on
plot(g,tMeasured,'kx','MarkerSize',10)
legend('Fitted Poly','Input Data','Location','NorthWest');
```

Code 2



Result 1

According to the result of the data, after plotting it we can predict the 2020 population to be 350 by looking at the graph and 342 by looking at the table.

Part 2:

The MLESAC approach is a dependable estimator for determining image geometry. The two main steps are non-linear minimization and robust estimation.

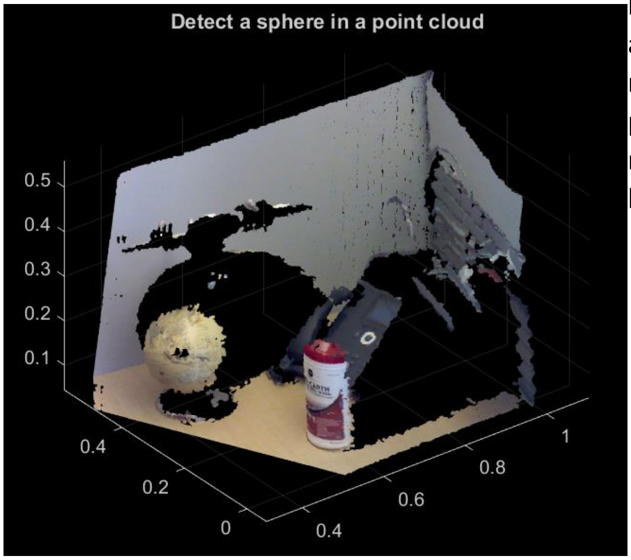
To provide potential solutions during the robust estimating phase, MLESAC randomly chooses a small selection of correspondences from the given data. Then, each solution is evaluated using a cost function that quantifies the probability that the correspondences will agree with the estimated link. Unlike traditional RANSAC, MLESAC uses the log probability of the solution as the support measure. This allows for a more accurate assessment of the solution's quality by taking the distribution of outliers into consideration.

Once a robust estimate is obtained, the estimated relation is revised using the non-linear minimization phase. An adequate parametrization is used to place the necessary limits on the matrix members. The minimization is done using a gradient descent approach, such the one described in Gill and Murray. A ceiling on error levels is a useful way to deal with outliers during the reduction process, until their parameters alter to the point where they could be classified as inliers.

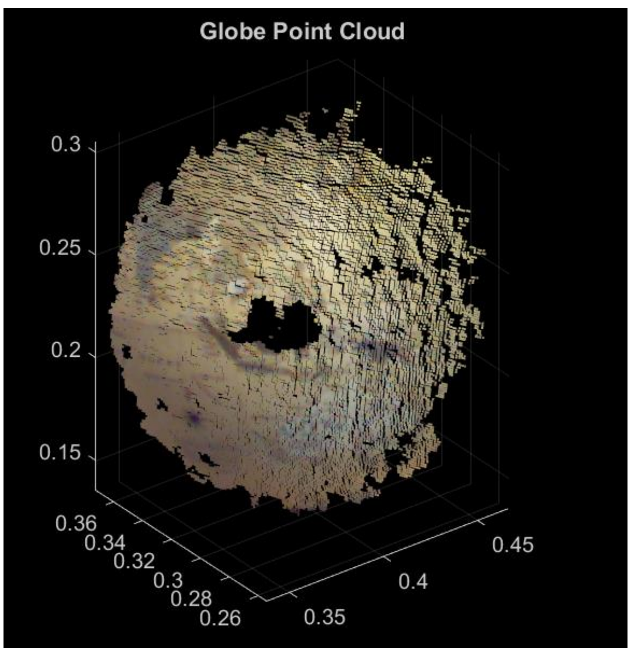
Experiments on both synthetic and real images have shown how effective the MLESAC approach is at estimating image geometry. It produces superior results and provides a consistent parametrization that enforces the necessary constraints on the matrix members compared to other methods such as RANSAC.

```
load ("object3d.mat");
figure
pcshow(ptCloud)
title ("Detect a sphere in a point cloud")
maxDistance = 0.01;
roi = [-inf, 0.5; 0.2,0.4;0.1,inf];
sampleIndicies = findPointsInROI(ptCloud,roi);
[model,inlierIndicies] = pcfitsphere(ptCloud,maxDistance,SampleIndices=sampleIndicies);
globe = select(ptCloud,inlierIndicies);
figure
pcshow(globe)
title("Globe Point Cloud")
```

Code 1



Original Picture



Globe Point Cloud (sphere extracted)

Part 3:

For the project the group I have enrolled in is Group 105. It is with 2 other peers of mine; we have decided to go with the idea of tracking objects revolving around self driving cars. The progress we have made in the group so far relates to settling on an idea, we have also been able to do some research work and most importantly we have planned for our next steps which is how we will be moving forward with this idea and making it happen.