Problem 4

for circular points: $I = [1, i, 0]^T$ $\quad J = [1, -i, 0]^T$

for a circle: $x^2 + y^2 + Ax + By + C = 0$

Intersection with I

$1^2 + i^2 + A + Bi + C = 0$

$1 - 1 + A + Bi + C = 0$

Real: $A + C = 0$

imaginary: $B = 0$

Intersection with B

$1^2 + (-i)^2 + A - Bi + C = 0$

$1 - 1 + A - Bi + C = 0$

real: $A + C = 0$

imaginary: $B = 0$

* Every circle intersects the infinite line at the circular points

Under point transformation: $x' = H_S \cdot X$

$$C_\infty^{*'} = H_S C_\infty^* H_S^T = C_\infty^*$$

$$\Rightarrow I^T l_\infty = J^T l_\infty = 0$$

$$C_\infty^* l_\infty = (IJ^T + JI^T) l_\infty = I(J^T l_\infty) + J(I^T l_\infty) = 0$$

For 2 orthogonal lines:

$l = (l_1, l_2, l_3)^T$ $\quad m = (m_1, m_2, m_3)^T$

Projective Space: $\cos\theta = \dfrac{l^T C_\infty^* m}{\sqrt{(l^T C_\infty^* l)(m^T C_\infty^* m)}}$

lines l and m are orthogonal if $l^T C_\infty^* m = 0$