Price Response Curves and Proxies

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1 Introduction

When trying to fit price-response curves it's necessary to have some variation in the prices offered else there is only one point which does not define a curve. The response data will be considered to be of one of two kinds: with and without zero response values. These considerations give rise to the following methods.

2 Data

Response data is modelled as quadruples of segment, time, price and response i.e. of the form (s_i, t_i, p_i, r_i) where $s_i \in \{1, \ldots, n_s\}$, $t_i \in (-\infty, 0]$ and $p_i, r_i \in [0, \infty)$. r_i will be restricted to $(0, \infty)$ if zeroes are not present. Data is filtered according to a given price range, $[P_{min}, P_{max}]$.

3 Method

A set of n_t time windows are established, $(T_1, \ldots, T_{n_t})$ each of the form $[t_i^{min}, t_i^{max}]$, presumed to be of comparable length. These will determine which responses are used. A vector of time interval weights $(\tau_1, \ldots, \tau_{n_t})$ will determine the significance of each window. A vector of segment weights $(\sigma_1, \ldots, \sigma_{n_s})$ will determine the significance of each of the n_s segments available in the response data. To denote the exposure size of each segment in a given time window requires a matrix of non-negative real numbers $(m_t^s)_{s,t}$.

First compute a combined weighting matrix $(\alpha_t^s)_{s,t}$ by:

$$\alpha_t^s = \frac{\sigma_s \cdot \tau_t}{m_t^s}$$

Using this the response space can be weighted by ω_t^s using.

$$\omega_t^s = \sum_{\{j | t \in T_j\}} \alpha_j^s$$

The expected total response is:

$$E(\Sigma r) = \frac{\sum_i r_i \cdot \omega_{t_i}^{s_i}}{\sum_{s,t} \alpha_t^s}$$

The expected total price (revenue) is:

$$E(\Sigma p) = \frac{\sum_i p_i \cdot \omega_{t_i}^{s_i}}{\sum_{s,t} \alpha_t^s}$$

The expected number of responses:

$$E(N) = \frac{\sum_i 1 \cdot \omega_{t_i}^{s_i}}{\sum_{s,t} \alpha_t^s}$$

The expected individual response is:

$$E(r) = \frac{E(\Sigma r)}{E(N)}$$

The expected individual price is:

$$E(p) = \frac{E(\Sigma p)}{E(N)}$$

It is important to note that these are only valid in the price range. However, by looking at the whole datasetthe expected market size is:

$$E(M) = \frac{\sum_i 1 \cdot \omega_{t_i}^{s_i}}{\sum_{s,t} \alpha_t^s}$$

This leads to an expected projected total response (volume):

$$E(V) = E(M) \cdot E(r)$$

Here the m_t^s must need only be correct in relative terms [???].

For a finite population and responses restricted to $\{0, 1\}$ this represents a binomial model with $E(r)$ being the probability of acceptance for an individual.

4 Missing zeros

If zero responses are missing then some form of exposure distribution is required. This says that in any given time window how prices were offered as a distribution over all possible prices. Let $Q_t^s(p)$ be the cumulative probability of an offer being made for segment s in time window t below price p. For the given price range $[P_{min}, P_{max}]$ define $q_t^s = Q_t^s(P_{max}) - Q_t^s(P_{min})$. Here the m_t^s must need to be correct in absolute terms since it yields the number of zero points added at an average level.

Then, some of the numerators change in the following ways:

$$\sum_i r_i \cdot \omega_{t_i}^{s_i} \to \sum_i r_i \cdot \omega_{t_i}^{s_i}$$

$$\sum_i p_i \cdot \omega_{t_i}^{s_i} \to \sum_{s,t} \left[\int_{P_{min}}^{P_{max}} p \cdot [Q_t^s(p+dp) - Q_t^s(p)]\, dp \right] \cdot m_t^s \cdot q_t^s \cdot \alpha_t^s$$

$$\sum_i 1 \cdot \omega_{t_i}^{s_i} \to \sum_{s,t} m_t^s \cdot q_t^s \cdot \alpha_t^s$$

Looking at the market size across all prices reduces the q_t^s to 1 yielding:

$$E(M) = \frac{\sum_{s,t} m_t^s \cdot \alpha_t^s}{\sum_{s,t} \alpha_t^s}$$

5 Curve fitting

To actually produce a curve that will determine total or individual responses over a fixed time horizon requires taking some sample points. For a quadratic three points are required. By selecting three consecutive price ranges and calculating the average in each will suffice. The price to which these are assigned will be the expected price in each of the three ranges. The bounds of the ranges whould be calculated such that the combined weight of the responses is equal across all three.

6 Examples

Consider taking 4 consecutive weeks to today $T_1 = [-28, -21]$, $T_2 = [-21, -14]$, $T_3 = [-14, -7]$ and $T_4 = [-7, 0]$. Price ranges might be $[85, 95]$, $[95, 105]$ and $[105, 115]$ and $\alpha_t^s = 1$. This defines 12 buckets of data. Summing the responses in each bucket and then averaging across the 4 weeks for each price range will be $E(\Sigma r)$. This is the projection for the next 7 days in terms of sales volume.

Other variations are:

- Include a time decay in the weighting function to favour more recent data.
- Determine the price ranges to equalise the number of sample points in each range.
- Take non-consecutive weeks such as the first week of the previous 4 years.

Next consider three sales channels A, B and C with sizes 1, 2, 3 respectively (note only relative values matter in this example). Using the previous example there will be 12 buckets for each of the three channels. Assuming an equal weighting across each, which is realistic for the same product, the sums are computed as before. However, now each one is scaled back by divdiing through by m_t^s to make them all of comparable size. Multiply through by m^A which is the expected market size over the projected time horizion. Averages are taken over time giving the expected volume of sales for three price ranges through channel A.