CSD2250 Linear Algebra Week 5 Homework

10th June 2022

You are given until 17th of June 2022, 2359 HRS to submit this homework.

Question 1 (Orthogonality/Orthonormality)

Let

$$\boldsymbol{v} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} \quad \text{and} \quad \boldsymbol{w} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}.$$

(a) Check that $\boldsymbol{v}$ and $\boldsymbol{w}$ are orthogonal.

(b) Using $\boldsymbol{v}$ and $\boldsymbol{w}$, determine a pair of orthonormal vectors.

Question 2

Determine the values of k for which

$$\boldsymbol{v} = \begin{bmatrix} k \\ k \\ 1 \end{bmatrix} \quad \text{and} \quad \boldsymbol{w} = \begin{bmatrix} k \\ 5 \\ 6 \end{bmatrix}$$

are orthogonal.

Question 3 (Projection onto a line)

Let

$$\boldsymbol{a} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad \boldsymbol{b} = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}.$$

(a) Find the projection $\boldsymbol{p}$ of $\boldsymbol{b}$ onto the line l through $\boldsymbol{a}$.

(b) Determine the shortest distance from the point $\boldsymbol{b}$ to the line l.

Question 4 (Invertibility of A^TA)

Let

$$\boldsymbol{a}_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \qquad \boldsymbol{a}_2 = \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix},$$

and A be the 3×2 matrix with $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ as its columns (from left to right).

(a) Verify that the vectors $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ are linearly independent.

(b) Verify that A^TA is invertible.

Question 5 (Projection onto a subspace)

Let $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, and A be defined as in Question 4. Set $\boldsymbol{b}$ as the vector

$$\boldsymbol{b} = \begin{bmatrix} 3 \\ 1 \\ 11 \end{bmatrix}.$$

Furthermore, let S be the subspace spanned by the vectors $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$.

(a) Verify that $\boldsymbol{b} \notin C(A)$, by showing that $A\boldsymbol{x} = \boldsymbol{b}$ has no solution.

(b) Find the projection $\boldsymbol{p}$ of $\boldsymbol{b}$ onto S.

(c) Determine the shortest distance from $\boldsymbol{b}$ to the subspace S.