CSD2250 Linear Algebra Week 4 Homework

3rd June 2022

You are given until 10th of June 2022, 2359 HRS to submit this homework.

Question 1 (Linear Independence)

Check that the columns of the following matrix

$$A = \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix}$$

are linearly independent by showing that the only solution to $A\boldsymbol{x} = \mathbf{0}$ is $\boldsymbol{x} = \mathbf{0}$.

Question 2

Determine if the sets of vectors below are linearly independent or dependent.

(a) $B_1 = \{(1, -1, 0, 0), (1, 0, -1, 0), (1, 0, 0, -1)\}$.

(b) $B_2 = \{(1, -1, 0, 0), (1, 0, -1, 0), (1, 0, 0, -1), (0, 1, -1, 0)\}$.

(c) $B_3 = \{(2, 0, 0, 0), (3, 6, 0, 0), (4, 7, 0, 0), (1, 0, 9, 2)\}$.

(d) $B_4 = \{(2, 0, 0, 4), (3, 6, 0, 6), (4, 7, 1, 8), (0, -1, 2, 3)\}$.

(e) $B_5 = \{(2, 0, 0, 4), (3, 6, 0, 6), (4, 7, 1, 8), (0, -1, 2, 3), (1, -1, 0, 0)\}$.

Question 3 (Basis)

For each of the sets of vectors in Question 2, are they bases for $\mathbb{R}^4$? Explain your answer.

Question 4 (Finding bases for $C(A)$ and $C(A^T)$)

Let A be the matrix

$$\begin{bmatrix} -1 & 2 & 4 & 3 \\ 4 & 2 & 3 & 9 \end{bmatrix}.$$

(a) Find a basis for the column space $C(A)$.

(b) Find a basis for the row space $C(A^T)$.

(c) Previously, if asked to describe the column space of A, we would write

$$C(A) = \left\{ a \begin{bmatrix} -1 \\ 4 \end{bmatrix} + b \begin{bmatrix} 2 \\ 2 \end{bmatrix} + c \begin{bmatrix} 4 \\ 3 \end{bmatrix} + d \begin{bmatrix} 3 \\ 9 \end{bmatrix} : a, b, c, d \in \mathbb{R} \right\}.$$

Now, it should be clear to you that some of these columns in this description are redundant. Using the basis for $C(A)$ we found in part (a), describe $C(A)$.

Question 5

Using the matrix A found in Question 4,

(a) Find a basis for the nullspace $N(A)$.

(b) **Using the information you obtained in this question and in Question 4**, find a basis for $\mathbb{R}^4$.