CSD2250 Linear Algebra Week 2 Homework

20th May 2022

You are given until 27th of May 2022, 2359 HRS to submit this homework.

Question 1 (Matrix multiplication and transpose)

Let A be a 3×4 matrix where $A_{ij} = 2i - 3j$, and B be a 4×3 matrix where

$$B_{ij} = \begin{cases} -1 & \text{if } i + j \text{ is odd} \\ 1 & \text{if } i + j \text{ is even,} \end{cases}$$

$$\text{and } C = \begin{bmatrix} 5 & -1 \\ 9 & 1 \\ 2 & 0 \end{bmatrix}.$$

Compute the following, whenever possible:

(a) CA,

(b) AB,

(c) BC,

(d) C^TC,

(e) CC^T.

Question 2 (Finding inverse through Gauss-Jordan)

Let

$$A = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 3 & 1 \\ 0 & 1 & 3 \end{bmatrix}.$$

Apply the Gauss-Jordan algorithm to compute A^{-1}. You are expected to explain every step of the process clearly.

Question 3

Let A be the matrix in Question 2. Using the A^{-1} you have computed in Question 2, solve the system $A\boldsymbol{x} = \boldsymbol{b}$ where

$$\boldsymbol{b} = \begin{bmatrix} 8 \\ 16 \\ 24 \end{bmatrix}.$$

Question 4 (Determinants and invertibility)

Let

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 3 & 0 \\ 4 & 2 & 0 \end{bmatrix}.$$

Without using cofactor expansion, explain why $\det(A) = 0$.

Question 5

Using cofactor expansion, by expanding along a row or column of your choice, compute $\det(A)$ where

$$A = \begin{bmatrix} 4 & 0 & -3 & 1 \\ 5 & 1 & 0 & -8 \\ 2 & -5 & 9 & -1 \\ 0 & 3 & -1 & 0 \end{bmatrix}.$$

(a) Is A invertible? Explain.

(b) If A is invertible in part (a), without explicitly computing A^{-1}, determine $\det(A^{-1})$. If A is not invertible, you may ignore this part.