

CSD2250 Linear Algebra Week 1 Homework

13th May 2022

You are given until 20th of May 2022, 2359 HRS to submit this homework.

Question 1 (Column form)

Let

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}.$$

Compute $A \cdot \mathbf{b}$ by first expressing it in terms of the column form.

Question 2 (Method of Elimination)

Solve the following linear system by the method of elimination and back-substitution.

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 8 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - 7x_2 + 4x_3 &= 10 \end{aligned}$$

Question 3 (Elimination matrices)

- (a) Write down the elimination matrix E_{21} which subtracts 5 times of row 1 from row 2 of a 3 by 3 matrix A .
- (b) Write down the elimination matrix E'_{21} which *adds* 5 times of row 1 to row 2 of a 3 by 3 matrix A .
- (c) Show that $E_{21} \cdot E'_{21} = I_3$ **and** $E'_{21} \cdot E_{21} = I_3$, where I_3 is the 3 by 3 identity matrix.

Question 4 (Permutation matrices)

- (a) Write down the permutation matrix P_{31} which swaps row 1 with row 3 of a 3 by 3 matrix A .
- (b) Show that $P_{31} \cdot P_{31} = I_3$, where I_3 is the 3 by 3 identity matrix.

Question 5 (Augmented matrices)

- (a) Write down the augmented matrix form $[A \ \mathbf{b}]$ for the system in Question 2.
- (b) Let

$$E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}.$$

Compute $E_{31} \cdot [A \ \mathbf{b}]$.