

CSD2250 Linear Algebra Week 3 Homework

27th May 2022

You are given until 3rd of June 2022, 2359 HRS to submit this homework.

Question 1 (Subspaces)

For (a) and (b), show that these sets are subspaces of \mathbb{R}^3 .

(a) $\{(a, b, c) : 4a = 3b\}$

(b) $\{(x, y - 1, z) : x + y - 2z = 1\}$.

For (c) and (d), show that these sets are subspaces of $M_2(\mathbb{R})$.

(c) $\left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : b = 0 \right\}$

(d) $\left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a + b - c - d = 0 \right\}.$

Question 2

In this question, we demonstrate a technique to show that a subset S of a vector space V is **not** a subspace of V .

- (a) Write down examples of 2 by 2 matrices A_1 and A_2 such that $\det(A_1) = \det(A_2) = 1$ **but** $\det(A_1 + A_2) \neq 1$.
- (b) Use (a) to show that

$$S = \{A \in M_2(\mathbb{R}) : \det(A) = 1\}$$

is not a subspace of $M_2(\mathbb{R})$.

Question 3 (Column spaces)

Let A be the matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 5 \end{bmatrix}.$$

- (a) What is $C(A)$ a subspace of?
- (b) Describe $C(A)$. Is it a line or a plane?

Question 4 (Nullspaces)

Let A is the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 4 & 8 & 12 \\ 3 & 6 & 7 & 13 \end{bmatrix}.$$

- (a) Find the special solutions of A using the algorithm found in the lecture notes. How many of such special solutions are there?
- (b) How many pivots/pivot columns of this matrix are there?
- (c) Using the special solutions you found in (a), describe $N(A)$.

Question 5

Repeat the same exercises (a) to (c) in Question 4, but using the following matrix

$$A = \begin{bmatrix} 4 & 6 & 8 & 10 \\ 1 & 3 & 0 & 5 \\ 1 & 1 & 3 & 3 \end{bmatrix}.$$

For both matrices in Question 4 and 5, what do you observe about the number of pivots/pivot columns + number of special solutions?