The mean inside diameter of a sample of 250 washers produced by a machine is 5.05 *mm* (millimeters) and the standard deviation is 0.05 *mm* (millimeters). The purpose for which these washers are intended allows a maximum tolerance in the diameter of 4.95 *mm* to 5.10 *mm*, otherwise the washers are considered defective. Determine the percentage of defective washers produced by the machine assuming the diameters are normally distributed.

Solutions

First Method (Using Table and for attempt in exam)

Let $X =$ Inside Diameter of Washers

$$X \sim N(5.05, 0.05^2)$$

$$\mu = 5.05$$

$$\sigma = 0.05$$

Probability of Maximum Tolerance Limit in the diameter

(i.e., Minimum inside diameter of a washer should be 4.95 millimeters and maximum inside diameter of a washer should be 5.10 millimeters)

$$P(4.95 \leq X \leq 5.10)$$

Standardizing X with variable Z with zero mean and unit variance. i.e.,

$$P\left(\frac{X-\mu}{\sigma} < Z < \frac{X-\mu}{\sigma}\right)$$

$$P(4.95 \leq X \leq 5.10) = P\left(\frac{4.95-5.05}{0.05} < Z < \frac{5.10-5.05}{0.05}\right)$$

$$= P(-2 \leq Z \leq 1)$$

$$= P(Z \leq 1) - P(Z \leq -2)$$

$$\because P(a \leq Z \leq b) = P(Z \leq b) - P(Z \leq a)$$

$$= P(Z \leq 1) - [1 - P(Z < 2)]$$

$$\because P(Z \leq -a) = 1 - P(Z \leq a)$$

$$P(4.95 \leq X \leq 5.10) = \Phi(1) - 1 + \Phi(2)$$

Z	**0.00**	0.01	0.02	0.03
0.8	**0.78814**	0.79103	0.79389	0.79673
0.9	**0.81594**	0.81859	0.82121	0.82381
1.0	**0.84134**	**0.84375**	**0.84614**	**0.84849**
1.1	**0.86433**	0.86650	0.86864	0.87076

Z	**0.00**	0.01	0.02	0.03
1.8	**0.96407**	0.96485	0.96562	0.96638
1.9	**0.97128**	0.97193	0.97257	0.97320
2.0	**0.97725**	**0.97778**	**0.97831**	**0.97882**
2.1	**0.98214**	0.98257	0.98300	0.98341

$$P(4.95 \leq X \leq 5.10) = 0.84134 - 1 + 0.97725 = 0.81859$$

The washers will be considered defective if the inside diameter of the washers lies outside the tolerance limits. i.e.

$$1 - P(4.95 \leq X \leq 5.10) = 1 - 0.81859$$

$$1 - P(4.95 \leq X \leq 5.10) = 0.18141 \text{ or } 18.141\%$$

Second Method (Using CASIO)

Find the probability of washers being defective.

The tolerance range is given as follows

$$P(4.95 \leq X \leq 5.10)$$

The probability of washers being defective is as follows

$$1 - \text{Tolerance Range}$$

$$= 1 - P(4.95 \leq X \leq 5.10)$$

$$= 1 - P\left(\frac{4.95 - \mu}{\sigma} \leq Z \leq \frac{5.10 - \mu}{\sigma}\right)$$

$$= 1 - P\left(\frac{4.95 - 5.05}{0.05} \leq Z \leq \frac{5.10 - 5.05}{0.05}\right)$$

$$= 1 - P(-2 \leq Z \leq 1)$$

$$= 1 - [P(Z \leq 1) - P(Z \leq -2)]$$

$$\because \text{using CASIO}$$

$$= 1 - [P(1) - P(-2)]$$

$$= 1 - (0.84134 - 0.0228) = 0.18146 \text{ or } 18.146\%$$

Third Method (Using Definite Integrals)

1. Direct Method

Tolerance Limit

$$P(4.95 \leq X \leq 5.10) = \int_{4.95}^{5.10} \frac{1}{(0.05)\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-5.05}{0.05}\right)^2} dx \approx 0.8186$$

Probability of washers being defective

$$1 - \text{Tolerance Limit}$$

$$= 1 - P(4.95 \leq X \leq 5.10) = 1 - 0.8186$$

$$1 - P(4.95 \leq X \leq 5.10) = 0.1814 \text{ or } 18.14\%$$

2. First, Transform Tolerance Limit into standard normal distribution as already attempted in first method i.e.,

$$P(4.95 \leq X \leq 5.10) = P(-2 \leq Z \leq 1)$$

$$P(-2 \leq Z \leq 1) = \int_{-2}^{1} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz \approx 0.8186$$

Probability of washers being defective

$$1 - \text{Tolerance Limit}$$

$$= 1 - P(4.95 \leq X \leq 5.10) = 1 - P(-2 \leq Z \leq 1)$$

$$1 - P(4.95 \leq X \leq 5.10) = 1 - 0.8186$$

$$1 - P(4.95 \leq X \leq 5.10) = 0.1814 \text{ or } 18.14\%$$