**What is Expected Value actually?**

$$E(\textbf{something}) = \frac{\sum \textbf{something}}{n}$$

**something** is the variable or a random variable which contains observations / values / numbers.

$n$ is the number of observations in variable **something**

e.g.

$$E(\boldsymbol{X_i}) = \frac{\sum_{i=1}^{n} \boldsymbol{X_i}}{n}$$

$$E(\boldsymbol{X_i^2}) = \frac{\sum_{i=1}^{n} \boldsymbol{X_i^2}}{n}$$

$$E(\boldsymbol{BMW_i}) = \frac{\sum_{i=1}^{n} \boldsymbol{BMW_i}}{n}$$

And so on.

So, it is clear that **Expected Value** is nothing but the **Average / Arithmetic Mean**.

For group data let’s recap the arithmetic mean for group data.

In group data, we have either a data set in a variable called $X_i$ along with its corresponding frequencies or classes in which we obtain mid – points by taking arithmetic mean of lower-class limit and upper-class limit.

# Discrete Grouped Data

| Persons having Number of Children ($X_i$) | No. of Persons (Frequency) ($f_i$) | Relative Frequency $\left(\frac{f_i}{\sum_{i=1}^{n} f_i}\right)$ | ($X_i$) x (Relative Frequency) |
|---|---|---|---|
| 1 | 6 | $\frac{6}{83}$ | $1 \times \frac{6}{83} = \frac{6}{83}$ |
| 2 | 10 | $\frac{10}{83}$ | $2 \times \frac{10}{83} = \frac{20}{83}$ |
| 3 | 22 | $\frac{22}{83}$ | $3 \times \frac{22}{83} = \frac{66}{83}$ |
| 4 | 18 | $\frac{18}{83}$ | $4 \times \frac{18}{83} = \frac{72}{83}$ |
| 5 | 15 | $\frac{15}{83}$ | $5 \times \frac{15}{83} = \frac{75}{83}$ |
| 6 | 9 | $\frac{9}{83}$ | $6 \times \frac{9}{83} = \frac{54}{83}$ |
| 7 | 3 | $\frac{3}{83}$ | $7 \times \frac{3}{83} = \frac{21}{83}$ |
| **Total / Sum** | **83** | **1** | $\mathbf{\frac{314}{83}}$ |

$$\bar{X} = \sum_{i=1}^{n} X_i(i - \text{th Relative Frequency}) = E(X_i) = \mathbf{\frac{314}{83}}$$

# Traditional Method for finding Arithmetic Mean of Grouped Data

| Persons having Number of Children $(X_i)$ | No. of Persons (Frequency) $(f_i)$ | $(X_i)$ x (Frequency) $f_i X_i$ |
|---|---|---|
| 1 | 6 | $1 \times 6 = 6$ |
| 2 | 10 | $2 \times 10 = 20$ |
| 3 | 22 | $3 \times 22 = 66$ |
| 4 | 18 | $4 \times 18 = 72$ |
| 5 | 15 | $5 \times 15 = 75$ |
| 6 | 9 | $6 \times 9 = 54$ |
| 7 | 3 | $7 \times 3 = 21$ |
| **Total / Sum** | **83** | **314** |

$$\bar{X} = \frac{\sum_{i=1}^{n} f_i X_i}{\sum_{i=1}^{n} f_i} = \frac{\mathbf{314}}{\mathbf{83}}$$

## Mathematically

$$\bar{X} = \frac{\sum_{i=1}^{n} f_i X_i}{\sum_{i=1}^{n} f_i}$$

$$= \frac{f_1 X_1 + f_2 X_2 + f_3 X_3 + \cdots + f_n X_n}{\sum_{i=1}^{n} f_i}$$

$$= \frac{f_1 X_1}{\sum_{i=1}^{n} f_i} + \frac{f_2 X_2}{\sum_{i=1}^{n} f_i} + \frac{f_3 X_3}{\sum_{i=1}^{n} f_i} + \cdots + \frac{f_n X_n}{\sum_{i=1}^{n} f_i}$$

$$= X_1\left(\frac{f_1}{\sum_{i=1}^{n} f_i}\right) + X_2\left(\frac{f_2}{\sum_{i=1}^{n} f_i}\right) + X_3\left(\frac{f_3}{\sum_{i=1}^{n} f_i}\right) + \cdots + X_n\left(\frac{f_n}{\sum_{i=1}^{n} f_i}\right)$$

$$= X_1(\text{First Relative Frequency}) + X_2(\text{Second Relative Frequency}) + X_3(\text{Third Relative Frequency}) + \cdots + X_n(\text{n} - \text{th Relative Frequency})$$

$$\bar{X} = \sum_{i=1}^{n} X_i(i - \text{th Relative Frequency}) = E(X_i)$$

In sampling distribution, the word Relative Frequency replaced as *p.d.f.* or *p.m.f.* i.e., $P(x)$ or $f(x)$ etc.

## The sampling distribution of $\overline{x}$

| $\overline{x}$ | $f_i$ | $P(\overline{x})$ (Equivalent to Relative Frequency) | $\overline{x}P(\overline{x})$ |
|---|---|---|---|
| 2 | 2 | $\frac{2}{14}$ | $2 \times \frac{2}{14} = \frac{4}{14}$ |
| 2.5 | 3 | $\frac{3}{14}$ | $2.5 \times \frac{3}{14} = \frac{7.5}{14}$ |
| 3 | 4 | $\frac{4}{14}$ | $3 \times \frac{4}{14} = \frac{12}{14}$ |
| 3.5 | 3 | $\frac{3}{14}$ | $3.5 \times \frac{3}{14} = \frac{10.5}{14}$ |
| 4 | 2 | $\frac{2}{14}$ | $4 \times \frac{2}{14} = \frac{8}{14}$ |
| **Total / Summation** | **14** | **1** | $\mathbf{\frac{42}{14}}$ |

$$E(\bar{x}) = \sum_{i=1}^{n} \bar{x}_i P(\bar{x}_i) = \frac{42}{14} = \overline{(\bar{x})}$$

## Let's see the same question with traditional method

| $\overline{\boldsymbol{x}}_{\boldsymbol{i}}$ | $\boldsymbol{f}_{\boldsymbol{i}}$ | $\boldsymbol{f}_{\boldsymbol{i}}\overline{\boldsymbol{x}}_{\boldsymbol{i}}$ |
|---|---|---|
| 2 | 2 | $2 \times 2 = 4$ |
| 2.5 | 3 | $2.5 \times 3 = 7.5$ |
| 3 | 4 | $3 \times 4 = 12$ |
| 3.5 | 3 | $3.5 \times 3 = 10.5$ |
| 4 | 2 | $4 \times 2 = 8$ |
| **Total / Summation** | **14** | **42** |

$$\overline{(\bar{x})} = \frac{\sum_{i=1}^{n} f_i \bar{x}_i}{\sum_{i=1}^{n} f_i} = \frac{\mathbf{42}}{\mathbf{14}} = E(\bar{x}_i)$$