

Standard Error: The Standard Deviation of a sampling distribution is known as Standard Error.

Standard Error of sample mean (\bar{x})

Case 1: Population N is very large and sample size n is small, (less than 5% of the population).

$$S.E(\bar{x}) = \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Case 2: Population N is finite, sampling without replacement or sample size is large i.e., (5% or greater than 5% of the population).

$$S.E(\bar{x}) = \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}}$$

Note: the factor

$$\frac{N-n}{N-1}$$

Known as **Finite Population Correction (FPC)** and uses for adjusting the variability in large samples.

Standard Error of sample proportion (p)

Case 1: Population N is very large and sample size n is small, (less than 5% of the population).

$$S.E(p) = \sigma_p = \sqrt{\frac{p(1-p)}{n}}$$

Case 2: Population N is finite, sampling without replacement or sample size is large i.e., (5% or greater than 5% of the population).

$$S.E(p) = \sigma_p = \sqrt{\frac{p(1-p)}{n}} \times \sqrt{\frac{N-n}{N-1}}$$

Confidence Intervals

$$\bar{x} - \sigma_{\bar{x}} \times z_{1-\frac{\alpha}{2}} < \mu < \bar{x} + \sigma_{\bar{x}} \times z_{1-\frac{\alpha}{2}} \quad (\sigma \text{ known})$$

$$p - \sigma_p \times z_{1-\frac{\alpha}{2}} < \pi < p + \sigma_p \times z_{1-\frac{\alpha}{2}} \quad (\text{Proportion})$$

$$\bar{x} - \frac{\hat{S}}{\sqrt{n}} \times z_{1-\frac{\alpha}{2}} < \mu < \bar{x} + \frac{\hat{S}}{\sqrt{n}} \times z_{1-\frac{\alpha}{2}} \quad (\sigma \text{ unknown})$$

$$\bar{x} - \frac{\hat{S}}{\sqrt{n}} \times t_{v, (1-\frac{\alpha}{2})} < \mu < \bar{x} + \frac{\hat{S}}{\sqrt{n}} \times t_{v, (1-\frac{\alpha}{2})} \quad (v = d.f = n - 1)$$

Z – Distribution (Standard Normal Distribution) and student’s – t distribution

| Sample Size (n) | Population Standard Deviation (σ) | Sample statistic \bar{x} | Sampling Distribution |
|-------------------------------------|--|--|------------------------------|
| $n \geq 30$ | Known | Known | Z |
| $n < 30$ | Known | Known | Z |
| $n \geq 30$ | Unknown | Known | Z |
| $n < 30$ | Unknown | Known | T |

1. Sometimes, \bar{x} unknown but data set is provided, then we calculate \bar{x} by using the formula.

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

2. When σ unknown and $n \geq 30$, either \hat{S} is provided already or we have to calculate \hat{S} by the following formulas.

$$\hat{S} = \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2 - \left(\frac{1}{n} \sum_{i=1}^n x_i \right)^2}$$

OR

$$\hat{S} = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

3. When σ unknown and $n < 30$, then, either \hat{s} is provided already or we have to calculate \hat{s} by the following formulas.

$$\hat{s} = \sqrt{\frac{1}{n-1} \left[\sum_{i=1}^n x_i^2 - \frac{1}{n} \left(\sum_{i=1}^n x_i \right)^2 \right]}$$

OR

$$\hat{s} = \sqrt{\frac{1}{n-1} \left[\sum_{i=1}^n (x_i - \bar{x})^2 \right]}$$