Page 266 - Question 12 (a) (ii)

Find

$$\hat{y} = a + bx + cx^2$$

Taking 1980 as origin

Year (t)	**Profit** (y)	$x = t - 1980$	x^2	x^3	x^4	xy	x^2y
1977	88	-3	9	-27	81	-264	792
1978	101	-2	4	-8	16	-202	404
1979	105	-1	1	-1	1	-105	105
1980	91	0	0	0	0	0	0
1981	113	1	1	1	1	113	113
1982	120	2	4	8	16	240	480
1983	132	3	9	27	81	396	1188
	750	**0**	**28**	**0**	**196**	**178**	**3082**

Taking 1977 as origin

Year (t)	**Profit** (y)	$x = t - 1977$	x^2	x^3	x^4	xy	x^2y
1977	88	0	0	0	0	0	0
1978	101	1	1	1	1	101	101
1979	105	2	4	8	16	210	420
1980	91	3	9	27	81	273	819
1981	113	4	16	64	256	452	1808
1982	120	5	25	125	625	600	3000
1983	132	6	36	216	1296	792	4752
	750	**21**	**91**	**441**	**2275**	**2428**	**10900**

For origin 1980

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} n & \sum x & \sum x^2 \\ \sum x & \sum x^2 & \sum x^3 \\ \sum x^2 & \sum x^3 & \sum x^4 \end{bmatrix}^{-1} \begin{bmatrix} \sum y \\ \sum xy \\ \sum x^2 y \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 7 & 0 & 28 \\ 0 & 28 & 0 \\ 28 & 0 & 196 \end{bmatrix}^{-1} \begin{bmatrix} 750 \\ 178 \\ 3082 \end{bmatrix} = \begin{bmatrix} 103.238 \\ 6.357 \\ 0.976 \end{bmatrix}$$

For origin 1977

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} n & \sum x & \sum x^2 \\ \sum x & \sum x^2 & \sum x^3 \\ \sum x^2 & \sum x^3 & \sum x^4 \end{bmatrix}^{-1} \begin{bmatrix} \sum y \\ \sum xy \\ \sum x^2 y \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 7 & 21 & 91 \\ 21 & 91 & 441 \\ 91 & 441 & 2275 \end{bmatrix}^{-1} \begin{bmatrix} 750 \\ 2428 \\ 10900 \end{bmatrix} = \begin{bmatrix} 92.952 \\ 0.5 \\ 0.976 \end{bmatrix}$$

It is possible that we can find a, b and c for origin 1977 without solving it, using the equation of the origin 1980.

Second degree equation from the origin 1980 is given as follows

$$\hat{y} = 103.238 + 6.357x + 0.976x^2$$

Because 1977 is 3 units backward from 1980, so we replace x with $(x - 3)$.

$$\hat{y} = 103.238 + 6.357(x - 3) + 0.976(x - 3)^2$$

$$\hat{y} = 103.238 + 6.357x - 19.071 + 0.976(x^2 + 9 - 6x)$$

$$\hat{y} = 103.238 - 19.071 + 9 \times 0.976 + 6.357x - 6x(0.976) + 0.976x^2$$

$$\hat{y} = 92.951 + 6.357x - 5.856x + 0.976x^2$$

$$\hat{y} = 92.951 + 0.501x + 0.976x^2$$