Data taken from 50 Ramen Shops in Kyoto, Japan

Prices of Ramen Bowl

Lower Class Limit	**Upper Class Limit**	**Frequency**	**Mid-Point (x_i)**	$\boldsymbol{f_i x_i}$
500	600	4	550	2200
600	700	13	650	8450
700	800	18	750	13500
800	900	12	850	10200
900	1000	3	950	2850
		50		**37200**

Arithmetic Mean:

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{37200}{50} = 744$$

$(\boldsymbol{x_i - \bar{x}})$	$(\boldsymbol{x_i - \bar{x}})^2$	$(\boldsymbol{x_i - \bar{x}})^3$	$(\boldsymbol{x_i - \bar{x}})^4$
-194	37636	-7301384	1416468496
-94	8836	-830584	78074896
6	36	216	1296
106	11236	1191016	126247696
206	42436	8741816	1800814096

$\boldsymbol{f_i(x_i - \bar{x})}$	$\boldsymbol{f_i(x_i - \bar{x})^2}$	$\boldsymbol{f_i(x_i - \bar{x})^3}$	$\boldsymbol{f_i(x_i - \bar{x})^4}$
-776	150544	-29205536	5665873984
-1222	114868	-10797592	1014973648
108	648	3888	23328
1272	134832	14292192	1514972352
618	127308	26225448	5402442288
0	**528200**	**518400**	**13598285600**

First four moments about mean.

$$m_1 = \frac{\sum_{i=1}^{n} f_i(x_i - \bar{x})}{\sum_{i=n}^{n} f_i} = \frac{0}{50} = 0$$

$$m_2 = \frac{\sum_{i=1}^{n} f_i(x_i - \bar{x})^2}{\sum_{i=n}^{n} f_i} = \frac{528200}{50} = 10564$$

$$m_3 = \frac{\sum_{i=1}^{n} f_i(x_i - \bar{x})^3}{\sum_{i=n}^{n} f_i} = \frac{518400}{50} = 10368$$

$$m_4 = \frac{\sum_{i=1}^{n} f_i(x_i - \bar{x})^4}{\sum_{i=n}^{n} f_i} = \frac{13598285600}{50} = 271965712$$

Alternative Method

Lower Class Limit	**Upper Class Limit**	**Frequency**	**Mid-Point (x_i)**	$f_i x_i$
500	600	4	550	2200
600	700	13	650	8450
700	800	18	750	13500
800	900	12	850	10200
900	1000	3	950	2850
		50		**37200**

x_i^2	x_i^3	x_i^4
302500	166375000	91506250000
422500	274625000	178506250000
562500	421875000	316406250000
722500	614125000	522006250000
902500	857375000	814506250000

$f_i x_i^2$	$f_i x_i^3$	$f_i x_i^4$
1210000	665500000	366025000000
5492500	3570125000	2320581250000
10125000	7593750000	5695312500000
8670000	7369500000	6264075000000
2707500	2572125000	2443518750000
28205000	**21771000000**	**17089512500000**

Moments about Zero

$$m_1' = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{37200}{50} = 744$$

$$m_2' = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} = \frac{28205000}{50} = 564100$$

$$m_3' = \frac{\sum_{i=1}^{n} f_i x_i^3}{\sum_{i=1}^{n} f_i} = \frac{21771000000}{50} = 435420000$$

$$m_4' = \frac{\sum_{i=1}^{n} f_i x_i^4}{\sum_{i=1}^{n} f_i} = \frac{17089512500000}{50} = 341790250000$$

Moments about Mean

$$m_1 = m_1' - m_1' = 0$$

$$m_2 = m_2' - (m_1')^2$$

$$m_3 = m_3' - 3m_2' m_1' + 2(m_1')^3$$

$$m_4 = m_4' - 4m_3' m_1' + 6m_2'(m_1')^2 - 3(m_1')^4$$

Calculations:

$$m_1 = m_1' - m_1' = 0$$

$$m_1 = 744 - 744 = 0$$

$$m_2 = m_2' - (m_1')^2$$

$$m_2 = 564100 - (744)^2 = 10564$$

$$m_3 = m_3' - 3m_2'm_1' + 2(m_1')^3$$

$$m_3 = 435420000 - 3(564100)(744) + 2(744)^3 = 10368$$

$$m_4 = m_4' - 4m_3'm_1' + 6m_2'(m_1')^2 - 3(m_1')^4$$

$$m_4 = 341790250000 - 4(435420000)(744) + 6(564100)(744)^2 - 3(744)^4$$

$$m_4 = 271965712$$

Beta Ratios

$$b_1 = \frac{(m_3)^2}{(m_2)^3}$$

$$b_2 = \frac{m_4}{(m_2)^2}$$

Skewness

$$\sqrt{b_1} = \frac{m_3}{(m_2)^{\frac{3}{2}}} = \frac{m_3}{(S.D)^3} = \frac{10368}{(10564)^{\frac{3}{2}}} = 0.0095489$$

If $\boldsymbol{\sqrt{b_1} < 0}$, the distribution is **negatively skewed**.

If $\boldsymbol{\sqrt{b_1} = 0}$, the distribution is **symmetric**.

If $\boldsymbol{\sqrt{b_1} > 0}$, the distribution is **positively skewed**.

Kurtosis

$$b_2 = \frac{m_4}{(m_2)^2} = \frac{271965712}{(10564)^2} = 2.4370103$$

If $\boldsymbol{b_2 < 3}$, the distribution is **platykurtic**.

If $\boldsymbol{b_2 = 3}$, the distribution is **mesokurtic**.

If $\boldsymbol{b_2 > 3}$, the distribution is **leptokurtic**.

Ungrouped Data

Shop	**Price ¥ (x_i)**	x_i^2	x_i^3	x_i^4
1	700	490000	343000000	240100000000
2	850	722500	614125000	522006250000
3	600	360000	216000000	129600000000
4	650	422500	274625000	178506250000
5	980	960400	941192000	922368160000
6	750	562500	421875000	316406250000
7	500	250000	125000000	62500000000
8	890	792100	704969000	627422410000
9	880	774400	681472000	599695360000
10	700	490000	343000000	240100000000
11	890	792100	704969000	627422410000
12	720	518400	373248000	268738560000
13	680	462400	314432000	213813760000
14	650	422500	274625000	178506250000
15	790	624100	493039000	389500810000
16	670	448900	300763000	201511210000
17	680	462400	314432000	213813760000
18	900	810000	729000000	656100000000
19	880	774400	681472000	599695360000
20	720	518400	373248000	268738560000
21	850	722500	614125000	522006250000
22	700	490000	343000000	240100000000

Shop	Price ¥ (x_i)	x_i^2	x_i^3	x_i^4
23	780	608400	474552000	370150560000
24	850	722500	614125000	522006250000
25	750	562500	421875000	316406250000
26	780	608400	474552000	370150560000
27	590	348100	205379000	121173610000
28	650	422500	274625000	178506250000
29	580	336400	195112000	113164960000
30	750	562500	421875000	316406250000
31	800	640000	512000000	409600000000
32	550	302500	166375000	91506250000
33	750	562500	421875000	316406250000
34	700	490000	343000000	240100000000
35	600	360000	216000000	129600000000
36	800	640000	512000000	409600000000
37	800	640000	512000000	409600000000
38	880	774400	681472000	599695360000
39	790	624100	493039000	389500810000
40	790	624100	493039000	389500810000
41	780	608400	474552000	370150560000
42	600	360000	216000000	129600000000
43	670	448900	300763000	201511210000
44	680	462400	314432000	213813760000
45	650	422500	274625000	178506250000

Shop	Price ¥ (x_i)	x_i^2	x_i^3	x_i^4
46	890	792100	704969000	627422410000
47	930	864900	804357000	748052010000
48	650	422500	274625000	178506250000
49	777	603729	469097433	364488705441
50	700	490000	343000000	240100000000
Total	**37147**	**28174829**	**21790926433**	**17163876935441**

Moments about Zero

$$m_1' = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{37147}{50} = 742.94$$

$$m_2' = \frac{\sum_{i=1}^{n} x_i^2}{n} = \frac{28174829}{50} = 563496.58$$

$$m_3' = \frac{\sum_{i=1}^{n} x_i^3}{n} = \frac{21790926433}{50} = 435818528.66$$

$$m_4' = \frac{\sum_{i=1}^{n} x_i^4}{n} = \frac{17163876935441}{50} = 343277538708.82$$

Moments about Mean

$$m_1 = m_1' - m_1' = 0$$

$$m_2 = m_2' - (m_1')^2$$

$$m_3 = m_3' - 3m_2'm_1' + 2(m_1')^3$$

$$m_4 = m_4' - 4m_3'm_1' + 6m_2'(m_1')^2 - 3(m_1')^4$$

Calculations:

$$m_1 = m_1' - m_1' = 0$$

$$m_1 = 742.94 - 742.94 = 0$$

$$m_2 = m_2' - (m_1')^2$$

$$m_2 = 563496.58 - (742.94)^2 = 11536.7364$$

$$m_3 = m_3' - 3m_2'm_1' + 2(m_1')^3$$

$$m_3 = 435818528.66 - 3(563496.58)(742.94) + 2(742.94)^3 = 32173.63277$$

$$m_4 = m_4' - 4m_3'm_1' + 6m_2'(m_1')^2 - 3(m_1')^4$$

$$m_4 = 343277538708.82 - 4(435818528.66)(742.94) + 6(563496.58)(742.94)^2 - 3(742.94)^4$$

$$m_4 = 315366132.97803$$

Alternative Method

Shop	Price ¥ (x_i)	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$(x_i - \bar{x})^3$	$(x_i - \bar{x})^4$
1	700	-42.94	1843.8436	-79174.64418	3399759.2213
2	850	107.06	11461.8436	1227104.976	131373858.7109
3	600	-142.94	20431.8436	-2920527.724	417460232.8949
4	650	-92.94	8637.8436	-802801.1842	74612342.0581
5	980	237.06	56197.4436	13322165.98	3158152667.1752
6	750	7.06	49.8436	351.895816	2484.3845
7	500	-242.94	59019.8436	-14338280.8	3483341938.5685
8	890	147.06	21626.6436	3180414.208	467711713.4014
9	880	137.06	18785.4436	2574732.9	352892891.2488
10	700	-42.94	1843.8436	-79174.64418	3399759.2213
11	890	147.06	21626.6436	3180414.208	467711713.4014
12	720	-22.94	526.2436	-12072.02818	276932.3265
13	680	-62.94	3961.4436	-249333.2602	15693035.3960
14	650	-92.94	8637.8436	-802801.1842	74612342.0581
15	790	47.06	2214.6436	104221.1278	4904646.2750
16	670	-72.94	5320.2436	-388058.5682	28304991.9633
17	680	-62.94	3961.4436	-249333.2602	15693035.3960
18	900	157.06	24667.8436	3874331.516	608502507.8741
19	880	137.06	18785.4436	2574732.9	352892891.2488
20	720	-22.94	526.2436	-12072.02818	276932.3265
21	850	107.06	11461.8436	1227104.976	131373858.7109
22	700	-42.94	1843.8436	-79174.64418	3399759.2213

Shop	Price ¥ (x_i)	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$(x_i - \bar{x})^3$	$(x_i - \bar{x})^4$
23	780	37.06	1373.4436	50899.81982	1886347.3224
24	850	107.06	11461.8436	1227104.976	131373858.7109
25	750	7.06	49.8436	351.895816	2484.3845
26	780	37.06	1373.4436	50899.81982	1886347.3224
27	590	-152.94	23390.6436	-3577365.032	547122208.0222
28	650	-92.94	8637.8436	-802801.1842	74612342.0581
29	580	-162.94	26549.4436	-4325966.34	704872955.4696
30	750	7.06	49.8436	351.895816	2484.3845
31	800	57.06	3255.8436	185778.4358	10600517.5477
32	550	-192.94	37225.8436	-7182354.264	1385763431.7317
33	750	7.06	49.8436	351.895816	2484.3845
34	700	-42.94	1843.8436	-79174.64418	3399759.2213
35	600	-142.94	20431.8436	-2920527.724	417460232.8949
36	800	57.06	3255.8436	185778.4358	10600517.5477
37	800	57.06	3255.8436	185778.4358	10600517.5477
38	880	137.06	18785.4436	2574732.9	352892891.2488
39	790	47.06	2214.6436	104221.1278	4904646.2750
40	790	47.06	2214.6436	104221.1278	4904646.2750
41	780	37.06	1373.4436	50899.81982	1886347.3224
42	600	-142.94	20431.8436	-2920527.724	417460232.8949
43	670	-72.94	5320.2436	-388058.5682	28304991.9633
44	680	-62.94	3961.4436	-249333.2602	15693035.3960
45	650	-92.94	8637.8436	-802801.1842	74612342.0581

Shop	Price ¥ (x_i)	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$(x_i - \bar{x})^3$	$(x_i - \bar{x})^4$
46	890	147.06	21626.6436	3180414.208	467711713.4014
47	930	187.06	34991.4436	6545499.44	1224401125.2120
48	650	-92.94	8637.8436	-802801.1842	74612342.0581
49	777	34.06	1160.0836	39512.44742	1345793.9590
50	700	-42.94	1843.8436	-79174.64418	3399759.2213
Total	**37147**	**0**	**576836.82**	**1608681.6384**	**15768306648.9172**

First four moments about mean.

$$m_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})}{n} = \frac{0}{50} = 0$$

$$m_2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n} = \frac{576836.82}{50} = 11536.7364$$

$$m_3 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^3}{n} = \frac{1608681.6384}{50} = 32173.63277$$

$$m_4 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^4}{n} = \frac{15768306648.9172}{50} = 315366132.97835$$

Beta Ratios

$$b_1 = \frac{(m_3)^2}{(m_2)^3}$$

$$b_2 = \frac{m_4}{(m_2)^2}$$

Skewness

$$\sqrt{b_1} = \frac{m_3}{(m_2)^{\frac{3}{2}}} = \frac{m_3}{(S.D)^3} = \frac{32173.63277}{(11536.7364)^{\frac{3}{2}}} = 0.025964242$$

If $\sqrt{\boldsymbol{b_1}} < \mathbf{0}$, the distribution is **negatively skewed**.

If $\sqrt{\boldsymbol{b_1}} = \mathbf{0}$, the distribution is **symmetric**.

If $\sqrt{\boldsymbol{b_1}} > \mathbf{0}$, the distribution is **positively skewed**.

Kurtosis

$$b_2 = \frac{m_4}{(m_2)^2} = \frac{315366132.97803}{(11536.7364)^2} = 2.369458537$$

If $\boldsymbol{b_2} < \mathbf{3}$, the distribution is **platykurtic**.

If $\boldsymbol{b_2} = \mathbf{3}$, the distribution is **mesokurtic**.

If $\boldsymbol{b_2} > \mathbf{3}$, the distribution is **leptokurtic**.

Data taken from 50 Ramen Shops in Kyoto, Japan

Prices of Ramen Bowl

Let’s take an arbitrary value from Mid-Point column $a = 750$

As we already know that class interval is $h = 100$

So, we can change the origin and scale of data as follows

$$u_i = \frac{x_i - a}{h} = \frac{x_i - 750}{100}$$

Lower Class Limit	**Upper Class Limit**	**Frequency**	**Mid-Point (x_i)**	$\boldsymbol{u_i = \frac{x_i - 750}{100}}$
500	600	4	550	-2
600	700	13	650	-1
700	800	18	750	0
800	900	12	850	1
900	1000	3	950	2
		50		

$\boldsymbol{u_i}$	$\boldsymbol{u_i^2}$	$\boldsymbol{u_i^3}$	$\boldsymbol{u_i^4}$
-2	4	-8	16
-1	1	-1	1
0	0	0	0
1	1	1	1
2	4	8	16

f_iu_i	$f_iu_i^2$	$f_iu_i^3$	$f_iu_i^4$
-8	16	-32	64
-13	13	-13	13
0	0	0	0
12	12	12	12
6	12	24	48
-3	**53**	**-9**	**137**

Moments about Zero (After Changing Scale of a group data with equal class interval h)

$$m_r' = \frac{\sum_{i=1}^{n} f_i u_i^r}{\sum_{i=1}^{n} f_i} \times (h)^r$$

$$m_1' = \frac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f_i} \times h = \frac{-3}{50} \times 100 = -6$$

$$m_2' = \frac{\sum_{i=1}^{n} f_i u_i^2}{\sum_{i=1}^{n} f_i} \times h^2 = \frac{53}{50} \times (100)^2 = 10600$$

$$m_3' = \frac{\sum_{i=1}^{n} f_i u_i^3}{\sum_{i=1}^{n} f_i} \times h^3 = \frac{-9}{50} \times (100)^3 = -180000$$

$$m_4' = \frac{\sum_{i=1}^{n} f_i u_i^4}{\sum_{i=1}^{n} f_i} \times h^4 = \frac{137}{50} \times (100)^4 = 274000000$$

Moments about Mean

$$m_1 = m_1' - m_1' = 0$$

$$m_2 = m_2' - (m_1')^2$$

$$m_3 = m_3' - 3m_2'm_1' + 2(m_1')^3$$

$$m_4 = m_4' - 4m_3'm_1' + 6m_2'(m_1')^2 - 3(m_1')^4$$

Calculations:

$$m_1 = m_1' - m_1' = 0$$

$$m_1 = -6 - (-6) = 0$$

$$m_2 = m_2' - (m_1')^2$$

$$m_2 = 10600 - (-6)^2 = 10564$$

$$m_3 = m_3' - 3m_2'm_1' + 2(m_1')^3$$

$$m_3 = -180000 - 3(10600)(-6) + 2(-6)^3 = 10368$$

$$m_4 = m_4' - 4m_3'm_1' + 6m_2'(m_1')^2 - 3(m_1')^4$$

$$m_4 = 274000000 - 4(-180000)(-6) + 6(10600)(-6)^2 - 3(-6)^4$$

$$m_4 = 271965712$$