1. Prove that r**egression coefficients** are independent of **origin** but NOT independent of **scale**.
2. Prove that co-efficient of correlation is independent of **BOTH** origin and scale.

| **Given Data** | | **Data after Changing the Origin** | | **Data after Changing the Scale** | |
|---|---|---|---|---|---|
| $\boldsymbol{X}$ | $\boldsymbol{Y}$ | $\boldsymbol{U = X - 70}$ | $\boldsymbol{V = Y - 120}$ | $\boldsymbol{A = \frac{X}{4}}$ | $\boldsymbol{B = \frac{Y}{5}}$ |
| 78 | 125 | 8 | 5 | 19.5 | 25 |
| 89 | 137 | 19 | 17 | 22.25 | 27.4 |
| 97 | 156 | 27 | 36 | 24.25 | 31.2 |
| 69 | 112 | -1 | -8 | 17.25 | 22.4 |
| 59 | 107 | -11 | -13 | 14.75 | 21.4 |
| 79 | 136 | 9 | 16 | 19.75 | 27.2 |
| 68 | 123 | -2 | 3 | 17 | 24.6 |
| 61 | 104 | -9 | -16 | 15.25 | 20.8 |
| **600** | **1000** | **40** | **40** | **150** | **200** |

| Data | | First Method | | | Deviation Method | | |
|---|---|---|---|---|---|---|---|
| $\boldsymbol{U}$ | $\boldsymbol{V}$ | $\boldsymbol{UV}$ | $\boldsymbol{U^2}$ | $\boldsymbol{V^2}$ | $(\boldsymbol{U}-\bar{\boldsymbol{U}})(\boldsymbol{V}-\bar{\boldsymbol{V}})$ | $(\boldsymbol{U}-\bar{\boldsymbol{U}})^{\boldsymbol{2}}$ | $(\boldsymbol{V}-\bar{\boldsymbol{V}})^{\boldsymbol{2}}$ |
| 8 | 5 | 40 | 64 | 25 | 0 | 9 | 0 |
| 19 | 17 | 323 | 361 | 289 | 168 | 196 | 144 |
| 27 | 36 | 972 | 729 | 1296 | 682 | 484 | 961 |
| -1 | -8 | 8 | 1 | 64 | 78 | 36 | 169 |
| -11 | -13 | 143 | 121 | 169 | 288 | 256 | 324 |
| 9 | 16 | 144 | 81 | 256 | 44 | 16 | 121 |
| -2 | 3 | -6 | 4 | 9 | 14 | 49 | 4 |
| -9 | -16 | 144 | 81 | 256 | 294 | 196 | 441 |
| **40** | **40** | **1768** | **1442** | **2364** | **1568** | **1242** | **2164** |

# Changing of Origin

Regression Coefficient for V on U

$$b_{VU} = \frac{n\sum UV - \sum U \sum V}{n\sum U^2 - (\sum U)^2}$$

$$b_{VU} = \frac{\sum(U - \bar{U})(V - \bar{V})}{\sum(U - \bar{U})^2}$$

Regression Coefficient for U on V

$$b_{UV} = \frac{n\sum UV - \sum U \sum V}{n\sum V^2 - (\sum V)^2}$$

$$b_{UV} = \frac{\sum(U - \bar{U})(V - \bar{V})}{\sum(V - \bar{V})^2}$$

Co-efficient of correlation

$$r_{UV} = \frac{n\sum UV - \sum U \sum V}{\sqrt{[n\sum U^2 - (\sum U)^2][n\sum V^2 - (\sum V)^2]}}$$

$$r_{UV} = \frac{\sum(U - \bar{U})(V - \bar{V})}{\sqrt{[\sum(U - \bar{U})^2][\sum(V - \bar{V})^2]}}$$

$$r_{UV} = \pm\sqrt{b_{UV} \times b_{VU}}$$

| Data | | First Method | | | Deviation Method | | |
|---|---|---|---|---|---|---|---|
| $\boldsymbol{A}$ | $\boldsymbol{B}$ | $\boldsymbol{AB}$ | $\boldsymbol{A^2}$ | $\boldsymbol{B^2}$ | $(\boldsymbol{A}-\overline{\boldsymbol{A}})(\boldsymbol{B}-\overline{\boldsymbol{B}})$ | $(\boldsymbol{A}-\overline{\boldsymbol{A}})^2$ | $(\boldsymbol{B}-\overline{\boldsymbol{B}})^2$ |
| 19.5 | 25 | 487.5 | 380.25 | 625 | 0 | 0.5625 | 0 |
| 22.25 | 27.4 | 609.65 | 495.0625 | 751 | 8.4 | 12.25 | 5.76 |
| 24.25 | 31.2 | 756.6 | 588.0625 | 973 | 34.1 | 30.25 | 38.44 |
| 17.25 | 22.4 | 386.4 | 297.5625 | 502 | 3.9 | 2.25 | 6.76 |
| 14.75 | 21.4 | 315.65 | 217.5625 | 458 | 14.4 | 16 | 12.96 |
| 19.75 | 27.2 | 537.2 | 390.0625 | 740 | 2.2 | 1 | 4.84 |
| 17 | 24.6 | 418.2 | 289 | 605 | 0.7 | 3.0625 | 0.16 |
| 15.25 | 20.8 | 317.2 | 232.5625 | 433 | 14.7 | 12.25 | 17.64 |
| **150** | **200** | **3828.4** | **2890.125** | **5086.56** | **78.4** | **77.625** | **86.56** |

# Changing of Scale

Regression Coefficient for B on A

$$b_{BA} = \frac{n\sum AB - \sum A \sum B}{n\sum A^2 - (\sum A)^2}$$

$$b_{BA} = \frac{\sum(A - \bar{A})(B - \bar{B})}{\sum(A - \bar{A})^2}$$

Regression Coefficient for A on B

$$b_{AB} = \frac{n\sum AB - \sum A \sum B}{n\sum B^2 - (\sum B)^2}$$

$$b_{AB} = \frac{\sum(A - \bar{A})(B - \bar{B})}{\sum(B - \bar{B})^2}$$

Co-efficient of correlation

$$r_{AB} = \frac{n\sum AB - \sum A \sum B}{\sqrt{[n\sum A^2 - (\sum A)^2][n\sum B^2 - (\sum B)^2]}}$$

$$r_{AB} = \frac{\sum(A - \bar{A})(B - \bar{B})}{\sqrt{[\sum(A - \bar{A})^2][\sum(B - \bar{B})^2]}}$$

$$r_{AB} = \pm\sqrt{b_{AB} \times b_{BA}}$$