# QB Metrics Documentation

1. **System Metrics**
   (a) **Number of electrons** $\eta$
   (b) **Number of natural orbitals** $N_{\text{nao}}$
   (c) **Number of qubits** $n$
   (d) **Log FCI Size** $\log_{10}\left(\binom{N}{N_\uparrow}\binom{N}{N_\downarrow}\right)$

2. **One-norm**

$$\lambda(H) = \sum_{ij} |h_{ij}^{(1)}| + \frac{1}{2}\sum_{\ell=1}^{L} |\lambda_\ell| \left(\sum_{pq} |g_{pq}^{(\ell)}|\right)^2 \tag{1}$$

3. **Double Factorization Metrics**
   (a) **Rank** $L$
   (b) **Eigenvalues** $\{\lambda_\ell\}_{\ell=1}^{L}$

4. **Hypergraph Metrics**
   Let $G_{Interaction}(H) = (V, E)$ where $V = [n]$ for an $n$-qubit Hamiltonian $H$ where the edge set contains hyperedges $e_i = (i_1, ..., i_{k(i)}) \in E$ where $i_1, ..., i_{k(i)} \in \{X, Y, Z\}$ are all those non-identity Pauli string terms. The graph has edge weights $w(e) = h_e$ where $h_e$ is the coefficient of Pauli string $e \in E$ where $H = \sum_{e\in E} h_e P_e$. We take statistics (max, min, mean, std. dev.) on edge order (Pauli weight), vertex degree, and edge weights.
   (a) **Number of Pauli Strings**

$$|E| = \left|\left\{P : |h_P| > 0, H = \sum_P h_P P\right\}\right| \tag{2}$$

   (b) **Edge Order**

$$\text{ord}(e_i) = k(i) \tag{3}$$

   (c) **Vertex Degree**

$$\deg(v) = |\{v \in e : e \in E\}| \tag{4}$$

5. **Other Graphs**
   (a) **Frustration Graph** $G_{\text{frus}}$
   (b) **Fermionic Graph** $G_F = (V = [N], E = E_F)$ comes from which terms appear in the one-body terms of the Hamiltonian $\sum_{ij} h_{ij}^{(1)} \hat{a}_i^\dagger \hat{a}_j$, two orbital are connected by an edge $e = (i, j) \in E_F$ if $|h_{ij}^{(1)}| > 0$