Chapter 5

n-Person Games

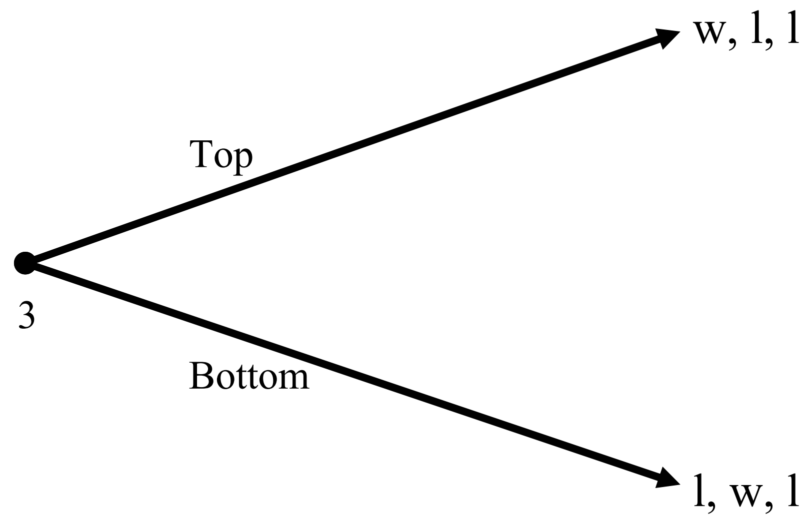
in

Normal Form

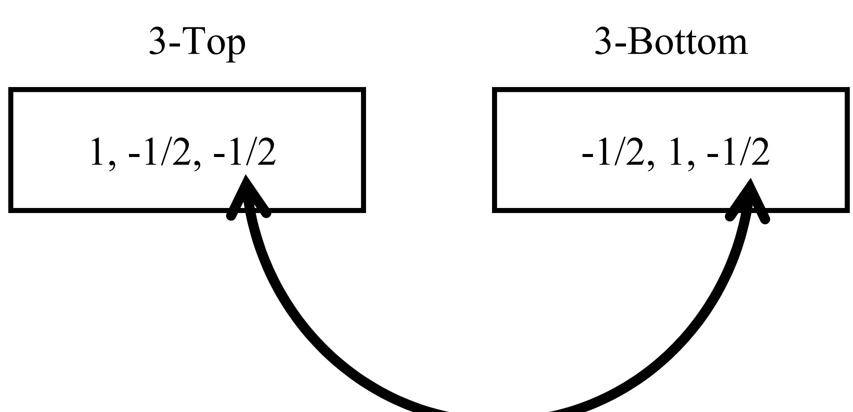
Fundamental Differences with 3 Players: the Spoilers

- Counterexamples
- The theorem for games like Chess does not generalize
- The solution theorem for 0-sum, 2-player games does not generalize
- A player playing the spoiler

Indeterminate three-person game



Multiple solutions, 3-person, zero-sum game



Competitive Advantage and Market Niche with 3 Players

- The row-column matrix representation for 3 players
- Games where no player plays the spoiler
- Pure and mixed strategy equilibria for 3-player games

Competitive Advantage, three firms

Firm 3 - New Technology

Firm 1 \ Firm 2	New Technology	Stay Put
New Technology	0, 0, 0	a/2, -a, a/2
Stay Put	-a, a/2, a/2	-a/2, -a/2, a

Firm 3 - Stay Put

Firm 1 \ Firm 2	New Technology	Stay Put
New Technology	a/2, a/2, -a	a, -a/2, -a/2
Stay Put	-a/2, a, -a/2	0, 0, 0

Competitive Advantage, three firms: Strategy for Firm 1

Firm 3 - New Technology

Firm 1 \ Firm 2	New Technology	Stay Put
New Technology	**0**, 0, 0	**a/2**, -a, a/2
Stay Put	-a, a/2, a/2	-a/2, -a/2, a

Firm 3 - Stay Put

Firm 1 \ Firm 2	New Technology	Stay Put
New Technology	**a/2**, a/2, -a	**a**, -a/2, -a/2
Stay Put	-a/2, a, -a/2	0, 0, 0

Competitive Advantage, three firms: Strategy for Firm 2

Firm 3 - New Technology

Firm 1 \ Firm 2	New Technology	Stay Put
New Technology	0, **0**, 0	a/2, -a, a/2
Stay Put	-a, **a/2**, a/2	-a/2, -a/2, a

Firm 3 - Stay Put

Firm 1 \ Firm 2	New Technology	Stay Put
New Technology	a/2, **a/2**, -a	a, -a/2, -a/2
Stay Put	-a/2, **a**, -a/2	0, 0, 0

Competitive Advantage, three firms: Strategy for Firm 3

Firm 3 - New Technology

Firm 1 \ Firm 2	New Technology	Stay Put
New Technology	0, 0, 0	a/2, -a, a/2
Stay Put	-a, a/2, a/2	-a/2, -a/2, a

Firm 3 - Stay Put

Firm 1 \ Firm 2	New Technology	Stay Put
New Technology	a/2, a/2, -a	a, -a/2, -a/2
Stay Put	-a/2, a, -a/2	0, 0, 0

Competitive Advantage, three firms:The Nash equilibrium

Firm 3 - New Technology

Firm 1 \ Firm 2	New Technology	Stay Put
New Technology	0, 0, 0 ★	a/2, -a, a/2
Stay Put	-a, a/2, a/2	-a/2, -a/2, a

Firm 3 - Stay Put

Firm 1 \ Firm 2	New Technology	Stay Put
New Technology	a/2, a/2, -a	a, -a/2, -a/2
Stay Put	-a/2, a, -a/2	0, 0, 0

Market Niche for three firms

Firm 3 - Enter

Firm 1 \ Firm 2	Enter	Stay Out
Enter	-50, -50, -50	-50, 0, -50
Stay Out	0, -50, -50	0, 0, 100

Firm 3 - Stay Out

Firm 1 \ Firm 2	Enter	Stay Out
Enter	-50, -50, 0	100, 0, 0
Stay Out	0, 100, 0	0, 0, 0

Market Niche, three firms: Strategy for Firm 1

Firm 3 - Enter

Firm 1 \ Firm 2	Enter	Stay Out
Enter	-50, -50, -50	-50, 0, -50
Stay Out	<u>0</u>, -50, -50	<u>0</u>, 0, 100

Firm 3 - Stay Out

Firm 1 \ Firm 2	Enter	Stay Out
Enter	-50, -50, 0	<u>100</u>, 0, 0
Stay Out	<u>0</u>, 100, 0	0, 0, 0

Market Niche, three firms: Strategy for Firm 2

Firm 3 - Enter

Firm 1 \ Firm 2	Enter	Stay Out
Enter	-50, -50, -50	-50, **0**, -50
Stay Out	0, -50, -50	0, **0**, 100

Firm 3 - Stay Out

Firm 1 \ Firm 2	Enter	Stay Out
Enter	-50, -50, 0	100, **0**, 0
Stay Out	0, **100**, 0	0, 0, 0

Market Niche, three firms: Strategy for Firm 3

Firm 3 - Enter

Firm 1 \ Firm 2	Enter	Stay Out
Enter	-50, -50, -50	-50, 0, -50
Stay Out	0, -50, -50	0, 0, **100**

Firm 3 - Stay Out

Firm 1 \ Firm 2	Enter	Stay Out
Enter	-50, -50, **0**	100, 0, **0**
Stay Out	0, 100, **0**	0, 0, 0

Market Niche, three firms: Three pure strategy equilibria

Firm 3 - Enter

Firm 1 \ Firm 2	Enter	Stay Out
Enter	-50, -50, -50	-50, 0, -50
Stay Out	0, -50, -50	0, 0, 100

Firm 3 - Stay Out

Firm 1 \ Firm 2	Enter	Stay Out
Enter	-50, -50, 0	100, 0, 0
Stay Out	0, 100, 0	0, 0, 0

Mixed Strategy equilibrium in Market Niche with 3 players

From the standpoint of the market, the distribution of number of firms in the market niche, according to mixed strategy equilibria is as follows:

- *p*(3 firms enter) = .08
- *p*(2 firms enter) = .31
- *p*(1 firm enters) = .42
- *p*(No firm enters)= .19

3-Player Versions of Coordination, Deal-Making, and Advertising

- Video System Coordination with 3 firms
- Let's Make a Deal with 3 firms
- Cigarette Advertising on Television with 3 firms

Video System Coordination, three firms: The payoff matrices

Firm 3 - Beta

Firm 1 \ Firm 2	Beta	VHS
Beta	1, 1, 1	0, 0, 0
VHS	0, 0, 0	0, 0, 0

Firm 3 - VHS

Firm 1 \ Firm 2	Beta	VHS
Beta	0, 0, 0	0, 0, 0
VHS	0, 0, 0	1, 1, 1

Video System Coordination, three firms: Strategy for Firm 1

Firm 3 - Beta

Firm 1 \ Firm 2	Beta	VHS
Beta	**1**, 1, 1	**0**, 0, 0
VHS	0, 0, 0	**0**, 0, 0

Firm 3 - VHS

Firm 1 \ Firm 2	Beta	VHS
Beta	**0**, 0, 0	0, 0, 0
VHS	**0**, 0, 0	**1**, 1, 1

Video System Coordination, three firms: Strategy for Firm 2

Firm 3 - Beta

Firm 1 \ Firm 2	Beta	VHS
Beta	1, **1**, 1	0, 0, 0
VHS	0, **0**, 0	0, **0**, 0

Firm 3 - VHS

Firm 1 \ Firm 2	Beta	VHS
Beta	0, **0**, 0	0, **0**, 0
VHS	0, 0, 0	1, **1**, 1

Video System Coordination, three firms: Strategy for Firm 3

Firm 3 - Beta

Firm 1 \ Firm 2	Beta	VHS
Beta	1, 1, 1	0, 0, 0
VHS	0, 0, 0	0, 0, 0

Firm 3 - VHS

Firm 1 \ Firm 2	Beta	VHS
Beta	0, 0, 0	0, 0, 0
VHS	0, 0, 0	1, 1, 1

Video System Coordination, three firms: Two pure strategy equilibria

Firm 3 - Beta

Firm 1 \ Firm 2	Beta	VHS
Beta	1, 1, 1	0, 0, 0
VHS	0, 0, 0	0, 0, 0

Firm 3 - VHS

Firm 1 \ Firm 2	Beta	VHS
Beta	0, 0, 0	0, 0, 0
VHS	0, 0, 0	1, 1, 1

Let’s make a deal, three players: Payoffs in millions of dollars

Player 3 - Yes

Player 1 \ Player 2	Yes	No
Yes	5, 5, 5	0, 0, 0
No	0, 0, 0	0, 0, 0

Player 3 - No

Player 1 \ Player 2	Yes	No
Yes	0, 0, 0	0, 0, 0
No	0, 0, 0	0, 0, 0

Let’s make a deal, three players: Strategy for player 1

Player 3 - Yes

Player 1 \ Player 2	Yes	No
Yes	5, 5, 5	0, 0, 0
No	0, 0, 0	0, 0, 0

Player 3 - No

Player 1 \ Player 2	Yes	No
Yes	0, 0, 0	0, 0, 0
No	0, 0, 0	0, 0, 0

Let’s make a deal, three players: Strategy for player 2

Player 3 - Yes

Player 1 \ Player 2	Yes	No
Yes	5, **5**, 5	0, 0, 0
No	0, **0**, 0	0, **0**, 0

Player 3 - No

Player 1 \ Player 2	Yes	No
Yes	0, **0**, 0	0, **0**, 0
No	0, **0**, 0	0, **0**, 0

Let’s make a deal, three players: Strategy for player 3

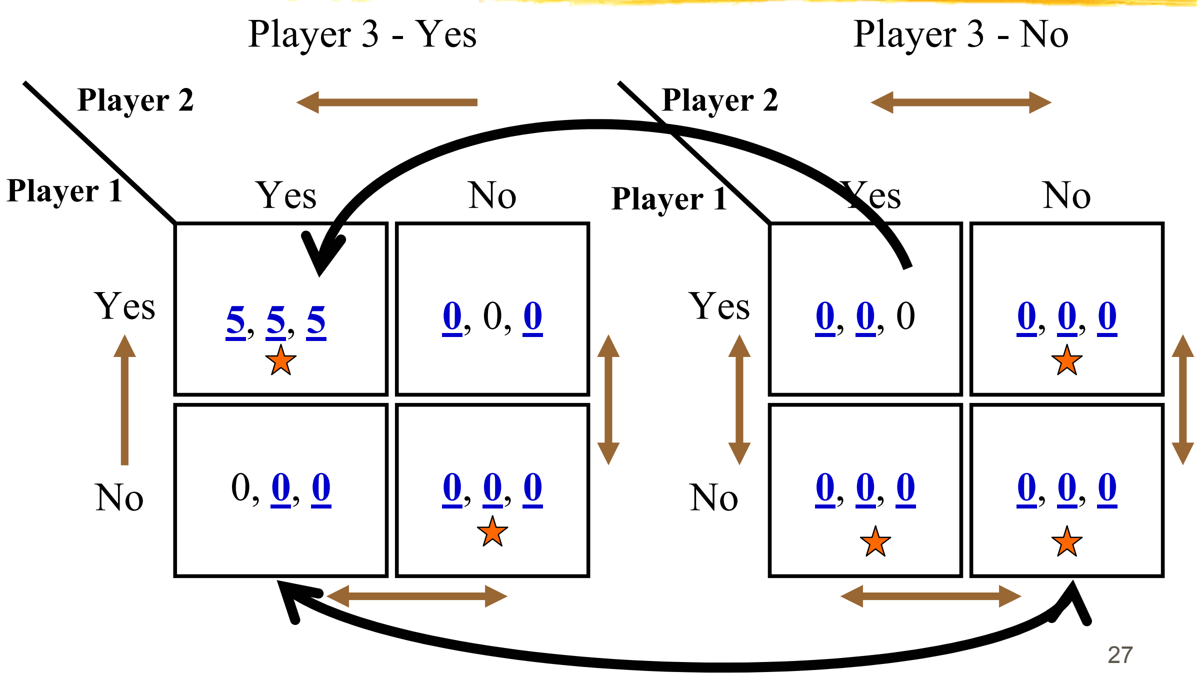
Player 3 - Yes

Player 1 \ Player 2	Yes	No
Yes	5, 5, **5**	0, 0, **0**
No	0, 0, **0**	0, 0, **0**

Player 3 - No

Player 1 \ Player 2	Yes	No
Yes	0, 0, 0	0, 0, **0**
No	0, 0, **0**	0, 0, **0**

Let's make a deal, three players: Five pure strategy equilibria



Stonewalling Watergate

- Watergate as a 3-person Prisoner's Dilemma
- Strictly dominant strategies and uniqueness of equilibrium
- Equilibria which are bad for the players

Stonewalling Watergate: D = Dean, E = Ehrlichman, H = Halderman

H - Stonewall

D \ **E**	Stonewall	Talk
Stonewall	-3, -3, -3	-5, -2, -2
Talk	-2, -5, -5	-2, -2, -5

H - Talk

D \ **E**	Stonewall	Talk
Stonewall	-5, -5, -2	-5, -2, -2
Talk	-2, -5, -2	-4, -4, -4

Stonewalling Watergate: Strategy for Dean

H - Stonewall

D \ **E**	Stonewall	Talk
Stonewall	-3, -3, -3	-5, -2, -5
Talk	**-2**, -5, -5	**-2**, -2, -5

H - Talk

D \ **E**	Stonewall	Talk
Stonewall	-5, -5, -2	-5, -2, -2
Talk	**-2**, -5, -2	**-4**, -4, -4

Stonewalling Watergate: Strategy for Ehrlichman

H - Stonewall

D \ E	Stonewall	Talk
Stonewall	-3, -3, -3	-5, **-2**, -5
Talk	-2, -5, -5	-2, **-2**, -5

H - Talk

D \ E	Stonewall	Talk
Stonewall	-5, -5, -2	-5, **-2**, -2
Talk	-2, -5, -2	-4, **-4**, -4

Stonewalling Watergate: Strategy for Halderman

H - Stonewall

D \ E	Stonewall	Talk
Stonewall	-3, -3, -3	-5, -2, -5
Talk	-2, -5, -5	-2, -2, -5

H - Talk

D \ E	Stonewall	Talk
Stonewall	-5, -5, **-2**	-5, -2, **-2**
Talk	-2, -5, **-2**	-4, -4, **-4**

Stonewalling Watergate: The Nash equilibrium

H - Stonewall

D \ **E**	Stonewall	Talk
Stonewall	-3, -3, -3	-5, -2, -5
Talk	-2, -5, -5	-2, -2, -5

H - Talk

D \ **E**	Stonewall	Talk
Stonewall	-5, -5, -2	-5, -2, -2
Talk	-2, -5, -2	-4, -4, -4 ★

Symmetry and Games with Many Players

- A compact notation for utility functions
- A generalized symmetry sufficient condition
- A symmetric game may have asymmetric equilibria

Solving Symmetric Games with Many Strategies

- A test for when a game is symmetric
- Symmetry makes games easier to solve
- Solving a game of common interest by exploiting the symmetry of the game
  - The Nash Demand Game

The Nash demand game: the payoff matrix

Player 1 \ Player 2	\$2	\$1	\$0
\$2	0, 0	0,0	2, 0
\$1	0, 0	1, 1	1, 0
\$0	0, 2	0, 1	0, 0

The Nash demand game: player 1's strategy

Player 1 \ Player 2	$2	$1	$0
$2	0, 0	0,0	2, 0
$1	0, 0	1, 1	1, 0
$0	0, 2	0, 1	0, 0

The Nash demand game: player 2's strategy

Player 1 \ Player 2	$2	$1	$0
$2	0, 0	0,0	2, 0
$1	0, 0	1, 1	1, 0
$0	0, 2	0, 1	0, 0

The Nash demand game: Nash equilibrium

Player 1 \ Player 2	\$2	\$1	\$0
\$2	0, 0 ☆	0,0	2, 0 ☆
\$1	0, 0	1, 1 ☆	1, 0
\$0	0, 2 ☆	0, 1	0, 0

Stag Hunt

- Game requiring cooperation for efficient outcome
- Adding third player leads to qualitatively different outcome
  - additional Nash equilibria
  - asymmetric outcomes
  - possibility of free riding

Stag Hunt, two hunters: The payoff matrix

Hunter 1 \ Hunter 2	hunt big	hunt small
hunt big	3, 3	0, 1
hunt small	1, 0	1, 1

Stag Hunt, two hunters: Strategy for hunter 1

Hunter 1 \ Hunter 2	hunt big	hunt small
hunt big	_3_, 3	0, 1
hunt small	1, 0	_1_, 1

Stag Hunt, two hunters: Strategy for hunter 2

Hunter 1 \ Hunter 2	hunt big	hunt small
hunt big	3, 3	0, 1
hunt small	1, 0	1, 1

Stag Hunt, two hunters: The equilibrium

Hunter 1 \ Hunter 2	hunt big	hunt small
hunt big	3, 3 ★	0, 1
hunt small	1, 0	1, 1 ★

Stag Hunt, three hunters: The payoff matrix

hunter 3: hunt big

hunter 1 \ hunter 2	hunt big	hunt small
hunt big	3, 3, 3	3, 5, 3
hunt small	5, 3, 3	1, 1, 0

hunter 3: hunt small

hunter 1 \ hunter 2	hunt big	hunt small
hunt big	3, 3, 5	0, 1, 1
hunt small	1, 0, 1	1, 1, 1

Stag Hunt, three hunters: Strategy for hunter 1

hunter 3: hunt big

hunter 1 \ hunter 2	hunt big	hunt small
hunt big	3, 3, 3	3, 5, 3
hunt small	5, 3, 3	1, 1, 0

hunter 3: hunt small

hunter 1 \ hunter 2	hunt big	hunt small
hunt big	3, 3, 5	0, 1, 1
hunt small	1, 0, 1	1, 1, 1

Stag Hunt, three hunters: Strategy for hunter 2

hunter 3: hunt big

hunter 1 \ hunter 2	hunt big	hunt small
hunt big	3, 3, 3	3, 5, 3
hunt small	5, 3, 3	1, 1, 0

hunter 3: hunt small

hunter 1 \ hunter 2	hunt big	hunt small
hunt big	3, 3, 5	0, 1, 1
hunt small	1, 0, 1	1, 1, 1

Stag Hunt, three hunters: Strategy for hunter 3

hunter 3: hunt big

hunter 1 \ hunter 2	hunt big	hunt small
hunt big	3, 3, 3	3, 5, 3
hunt small	5, 3, 3	1, 1, 0

hunter 3: hunt small

hunter 1 \ hunter 2	hunt big	hunt small
hunt big	3, 3, 5	0, 1, 1
hunt small	1, 0, 1	1, 1, 1

Stag Hunt, three hunters: Nash equilibria

hunter 3: hunt big

hunter 1 \ hunter 2	hunt big	hunt small
hunt big	3, 3, 3	3, 5, 3 ★
hunt small	5, 3, 3 ★	1, 1, 0

hunter 3: hunt small

hunter 1 \ hunter 2	hunt big	hunt small
hunt big	3, 3, 5 ★	0, 1, 1
hunt small	1, 0, 1	1, 1, 1 ★

The Tragedy of the Commons

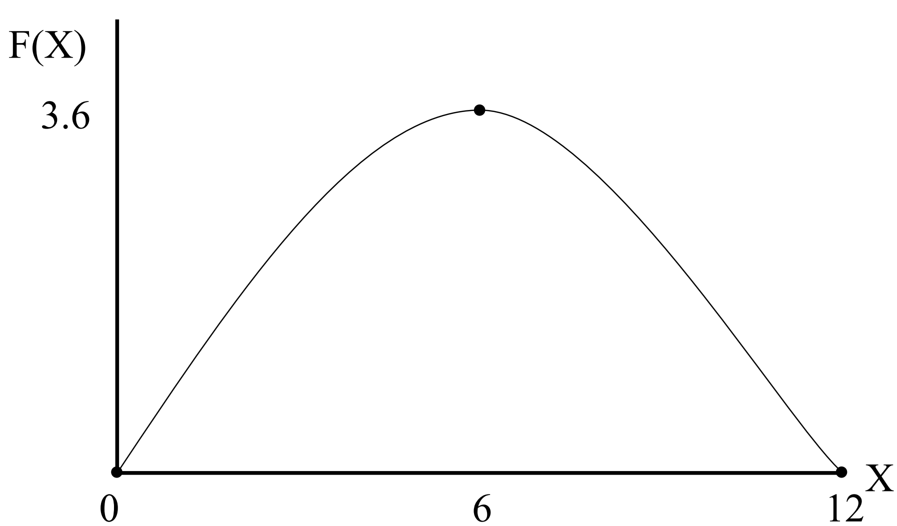
- Games played on a commons
- The equilibrium of such a game has a tragic outcome
- Externalities
- First Welfare Theorem
- The case of the Geysers of Northern California

Tragedy of the Commons: score sheet

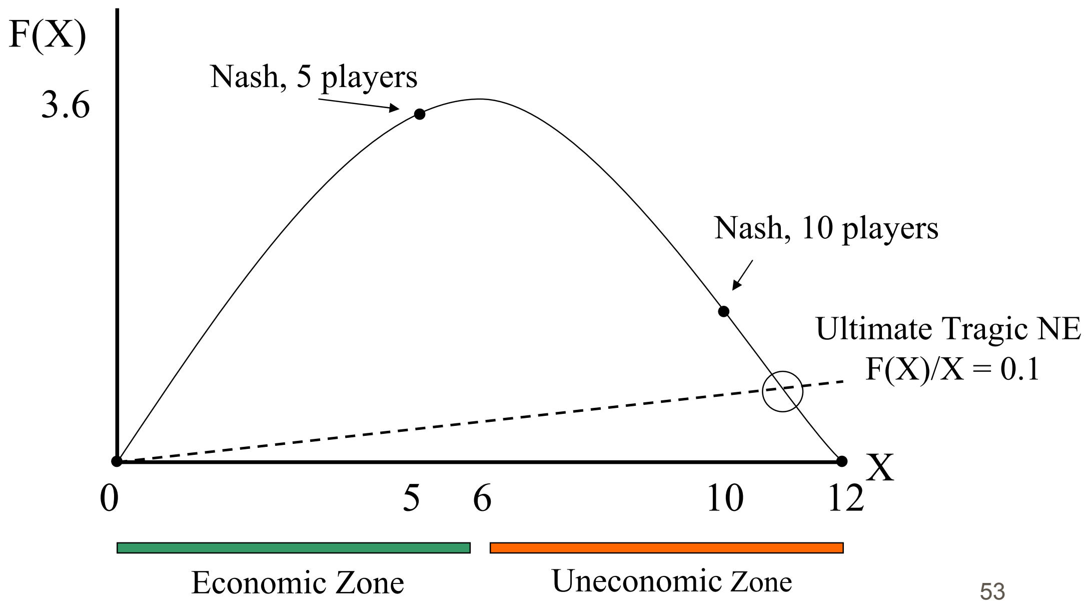
Payoff = $5(10 - x_i) + x_i (23 – 0.25\ \Sigma x_i)$

	strategy (x_i)	payoff
1		
2		
3		
4		

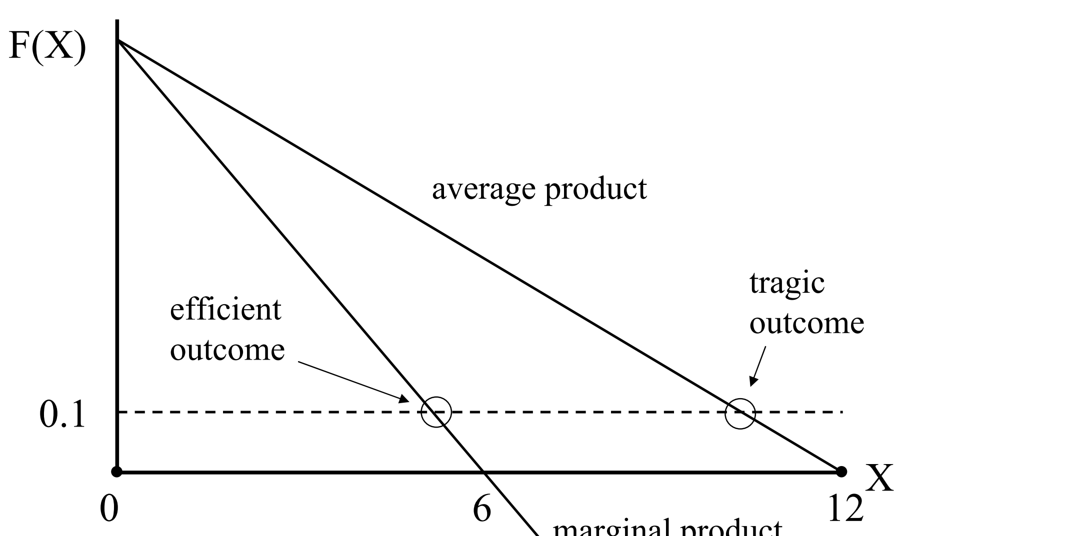
Tragedy of the Commons: Commons production function



Tragedy of the Commons



Tragedy of the Commons



Appendix. Tragedy of the Commons in the Laboratory

- Playing a game in a behavior laboratory
- Tragic outcomes of a game played on a commons in a laboratory
- Unexplained phenomena