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Ejercicios

1.
$$F(t)=\begin{cases}2 & -1\le t<0\\ 3 & 0<t\le 1\end{cases}$$

$$a_0=\int_{-1}^{0}2\,dt+\int_{0}^{1}3\,dt=2t\Big|_{-1}^{0}+3t\Big|_{0}^{1}=2+3=5$$

$$a_n=\int_{-1}^{1}F(t)\cos n\pi t=\int_{-1}^{0}2\cos n\pi t+\int_{0}^{1}3\cos n\pi t$$

$$=\frac{2\,\text{sen}\,n\pi t}{n\pi}\Big|_{-1}^{0}+\frac{3\,\text{sen}\,n\pi t}{n\pi}\Big|_{0}^{1}$$

$$=0$$

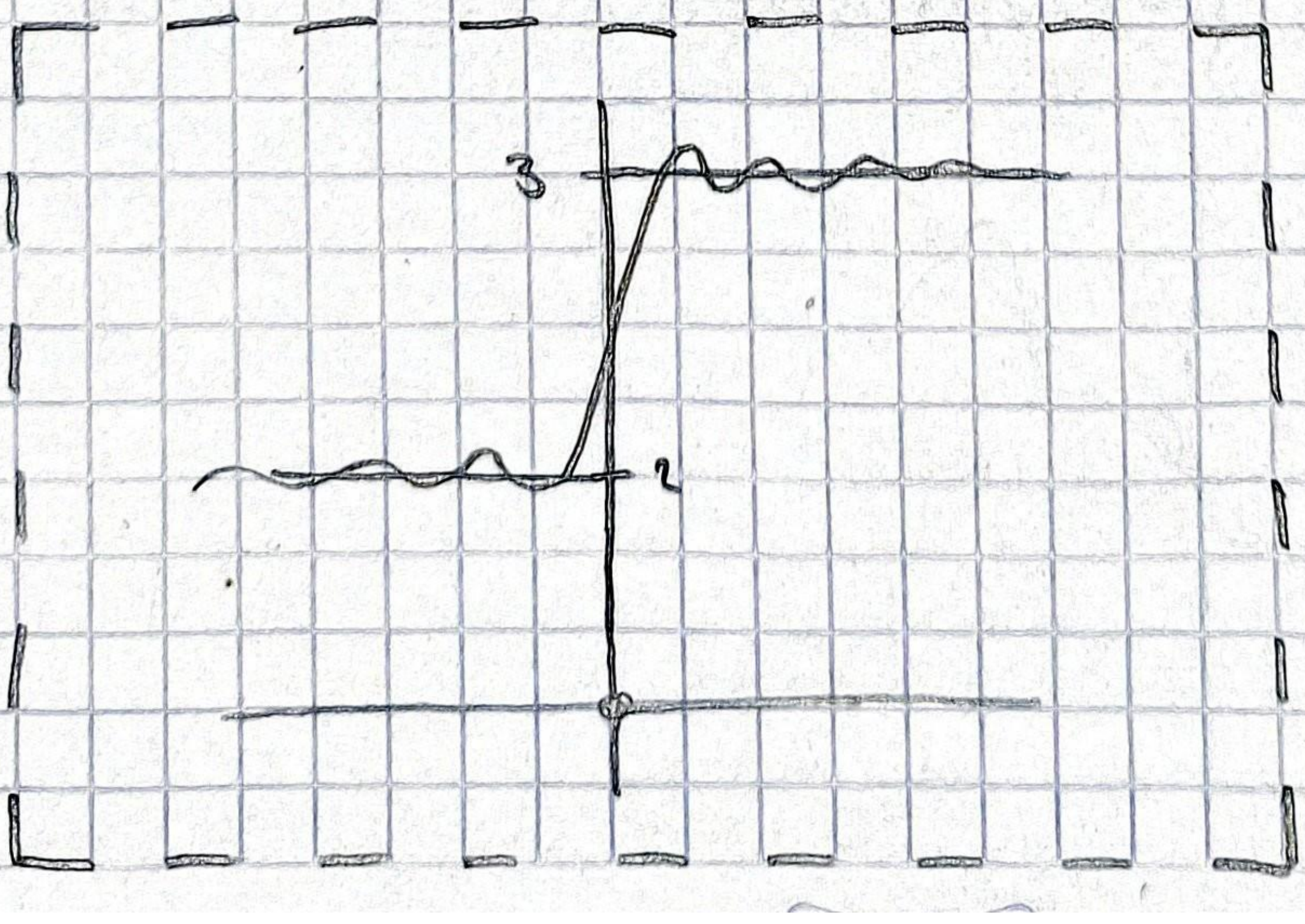
$$b_n=\int F(t)\,\text{sen}\,n\pi t=\int_{-1}^{0}2\,\text{sen}\,n\pi t+\int_{0}^{1}3\,\text{sen}\,n\pi t$$

$$=-\frac{2\cos n\pi t}{n\pi}\Big|_{-1}^{0}-\frac{3\cos n\pi t}{n\pi}\Big|_{0}^{1}$$

$$=\frac{-2}{n\pi}+\frac{2(-1)^n}{n\pi}+\frac{3}{n\pi}-\frac{3(-1)^n}{n\pi}$$

$$=\frac{1}{n\pi}\left(1-(-1)^n\right)$$

$$F(t)=\frac{5}{2}+\sum_{n=1}^{\infty}\frac{1}{n\pi}\left(1-(-1)^n\right)\text{sen}\,n\pi t$$



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2) $f(t) = t+1 \quad -\pi < t < \pi$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} t+1\, dt = \frac{1}{\pi}\left(\frac{t^2}{2} + t \Big|_{-\pi}^{\pi}\right) = \frac{1}{\pi}\left(\frac{\pi^2}{2} + \pi - \frac{\pi^2}{2} + \pi\right)$$

$$= 2$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt = \frac{1}{\pi}\int_{-\pi}^{\pi}(t+1)\cos nt$$

$u = t+1 \quad dv = \cos nt$

$du = 1 \quad v = \frac{\operatorname{sen} nt}{n}$

$$= \frac{1}{\pi}\left(\frac{(t+1)\operatorname{sen} nt}{n} + \frac{\cos nt}{n^2}\Big|_{-\pi}^{\pi}\right)$$

$$= \frac{1}{\pi}\left(\frac{\cos n\pi}{n^2} - \frac{\cos(-n\pi)}{n^2}\right)$$

$$= 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\, dt = \frac{1}{\pi}\int_{-\pi}^{\pi}(t+1)\operatorname{sen} nt$$

$u = (t+1) \quad dv = \operatorname{sen} nt$

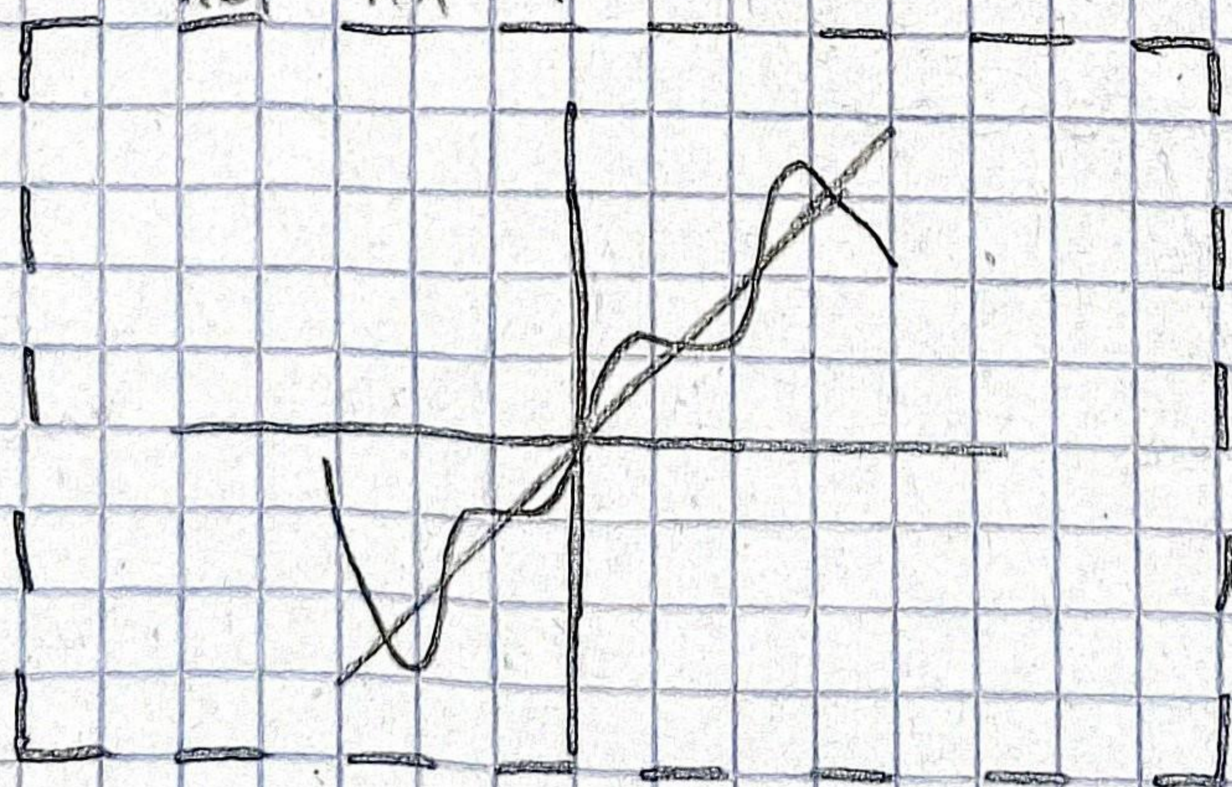
$du = 1 \quad v = -\frac{\cos nt}{n}$

$$= \frac{1}{\pi}\left(-\frac{(t+1)\cos nt}{n} + \frac{\operatorname{sen} nt}{n^2}\Big|_{-\pi}^{\pi}\right)$$

$$= \frac{1}{\pi}\left(-\frac{(\pi+1)\cos n\pi}{n} + \frac{(-\pi+1)\cos -n\pi}{n}\right)$$

$$= \frac{(-1)^n}{\pi n}\left(-(\pi+1) + (-\pi+1)\right)$$

$$f(t) = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{\pi n}(-2\pi)\operatorname{sen} nt$$



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3.-
$$f(t)=\begin{cases}0 & -\pi/2 \le t<0\\ \operatorname{sen} t & 0\le t\le \pi/2\end{cases}$$

$$a_0=\frac{2}{\pi}\int_0^{\pi/2}\operatorname{sen} t\,dt=\frac{2}{\pi}\left(-\cos t\right)\Big|_0^{\pi/2}=\frac{-2\cos \pi/2}{\pi}+\frac{2\cos 0}{\pi}$$
$$=\frac{2}{\pi}$$

$$a_n=\frac{2}{\pi}\int_0^{\pi/2}(\operatorname{sen} t)\cos\frac{n\pi}{\pi/2}t=\frac{2}{\pi}\int_0^{\pi/2}(\operatorname{sen} t)\cos 2nt$$
$$=\frac{2}{\pi}\int_0^{\pi/2}\frac{1}{2}\left(\operatorname{sen}(t-2nt)+\operatorname{sen}(t+2nt)\right)$$
$$=\frac{1}{\pi}\int_0^{\pi/2}\operatorname{sen}(t-2nt)\,dt+\frac{1}{\pi}\int_0^{\pi/2}\operatorname{sen}(t+2nt)\,dt$$
$$=\frac{1}{\pi}\left(\frac{-\cos(t-2nt)}{(1-2n)}-\frac{\cos(t+2nt)}{(1+2n)}\Big|_0^{\pi/2}\right)$$
$$=\frac{-\cos(\pi/2-2n\pi/2)}{(1-2n)\pi}-\frac{\cos(\pi/2+2n\pi/2)}{(1+2n)\pi}+\frac{1}{(1+2n)\pi}+\frac{1}{(1+2n)\pi}$$
$$=\frac{1-\cos(\pi/2-2n\pi/2)}{(1-2n)\pi}+\frac{1-\cos(\pi/2+2n\pi/2)}{(1+2n)\pi}$$

$$b_n=\frac{2}{\pi}\int_0^{\pi/2}(\operatorname{sen} t)(\operatorname{sen} 2nt)\,dt=\frac{2}{\pi}\int_0^{\pi/2}\frac{1}{2}\left(\cos(t-2nt)-\cos(t+2nt)\right)dt$$
$$=\frac{1}{\pi}\left(\frac{\operatorname{sen}(t-2nt)}{(1-2n)}-\frac{\operatorname{sen}(t+2nt)}{(1+2n)}\right)\Big|_0^{\pi/2}$$
$$=\frac{\operatorname{sen}(\pi/2-2n\pi/2)}{(1-2n)\pi}-\frac{\operatorname{sen}(\pi/2+2n\pi/2)}{(1+2n)\pi}$$

$$F(t)=\frac{1}{\pi}+\sum_{n=1}^{\infty}\left(\frac{1-\cos(\pi/2-2n\pi/2)}{(1-2n)\pi}+\frac{\cos(\pi/2+2n\pi/2)}{(1-2n)\pi}\right)\cos 2nt$$
$$+\sum_{n=1}^{\infty}\left(\frac{\operatorname{sen}(\pi/2-2n\pi/2)}{(1-2n)\pi}-\frac{\operatorname{sen}(\pi/2+2n\pi/2)}{(1+2n)\pi}\right)\operatorname{sen} 2nt$$

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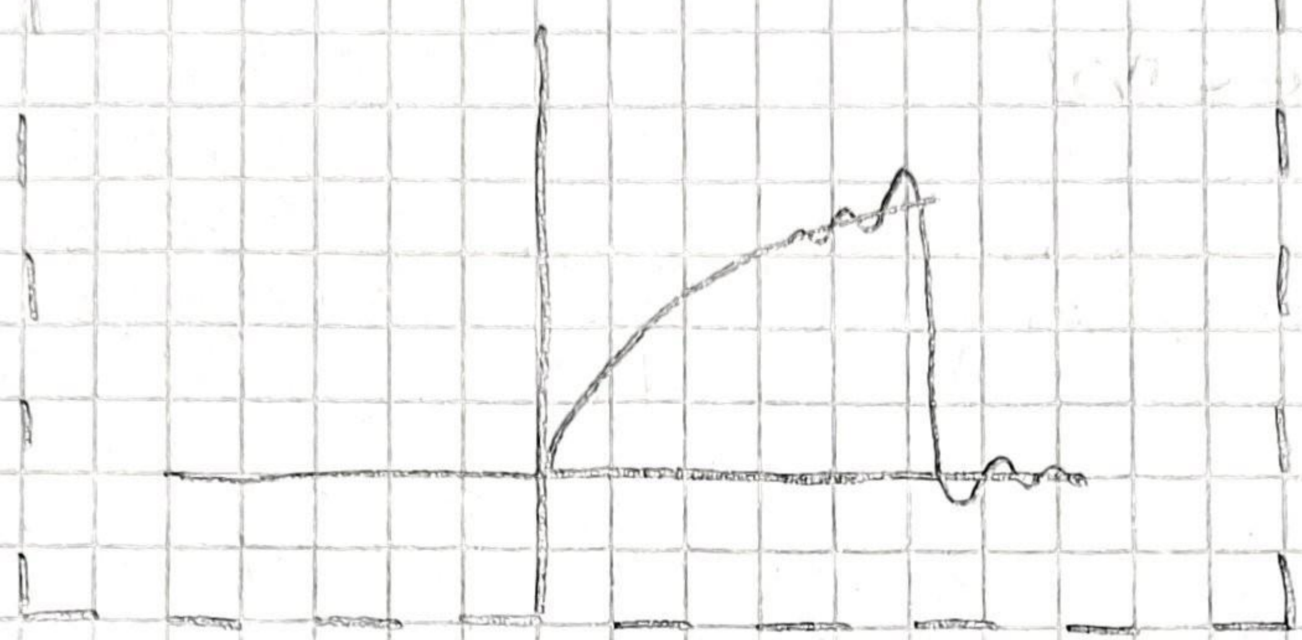
Gráfica



4.- $f(t) = \begin{cases} 0 & -\pi < t < 0 \\ e^t & 0 < t < \pi \end{cases}$

$$a_0 = \frac{1}{\pi}\int_0^\pi e^t\,dt = \frac{1}{\pi} e^t \Big|_0^\pi = \frac{e^\pi}{\pi} - \frac{1}{\pi}$$

$$a_n = \frac{1}{\pi}\int_0^\pi f(t)\cos nt = \frac{1}{\pi}\left(\frac{e^t(\cos nt + n\,\mathrm{sen}\,nt)}{(1+n^2)}\right|_0^\pi$$

$$= \frac{1}{\pi}\left(\frac{e^\pi(-1)^n}{(1+n^2)} - \frac{1}{(1+n^2)}\right)$$

$$= \frac{1}{(1+n^2)\pi}\left(e^\pi(-1)^n - 1\right)$$

$$b_n = \frac{1}{\pi}\int_0^\pi f(t)\,\mathrm{sen}\,nt = \frac{1}{\pi}\int_0^\pi e^t\,\mathrm{sen}\,nt = \frac{1}{\pi}\left(\frac{e^t(\mathrm{sen}\,nt - n\cos nt)}{(1+n^2)}\right)\Big|_0^\pi$$

$$= \frac{1}{\pi}\left(\frac{e^\pi(-n(-1)^n)}{(1+n^2)} + \frac{n}{(1+n^2)}\right)$$

$$= \frac{e^\pi(-n(-1)^n)}{(1+n^2)\pi} + \frac{n}{(1+n^2)\pi}$$

$$f(t) = \frac{1}{2}\left(\frac{e^\pi}{\pi} - \frac{1}{\pi}\right) + \sum_{n=1}^{\infty}\left(\frac{1}{(1+n^2)\pi}\left(e^\pi(-1)^n - 1\right)\right)\cos nt$$
$$+ \sum_{n=1}^{\infty}\left(\frac{1}{(1+n^2)\pi}\left(e^\pi(-n(-1)^n) + n\right)\right)\mathrm{sen}\,nt$$

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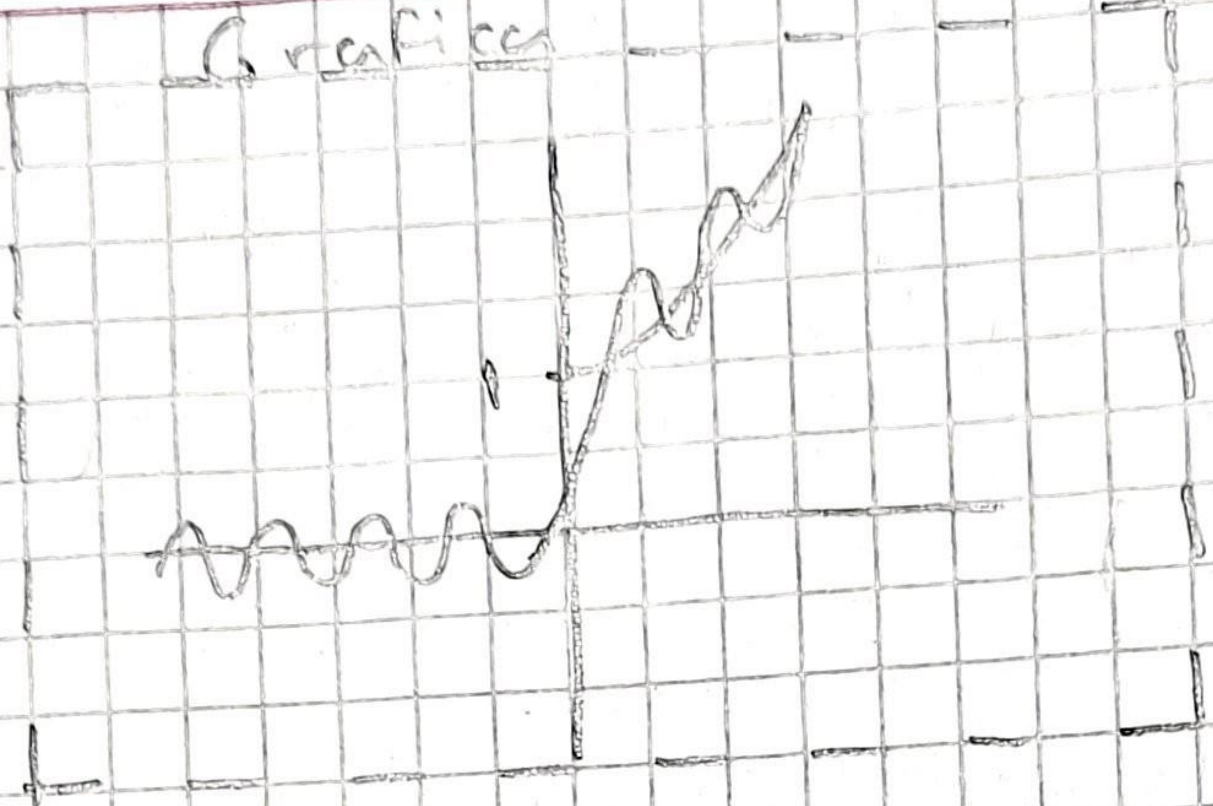

5.- $f(t) = \begin{cases} 0 & -2<t<0 \\ t & 0<t<1 \\ t-2 & 1<t<2 \end{cases}$

$$a_0 = \frac{3}{4}\left(\int_0^1 t\,dt + \int_1^2 (t-2)\,dt\right) = \frac{3}{4}\left(\frac{t^2}{2}\Big|_0^1 + \left(\frac{t^2}{2} - 2t\right)\Big|_1^2\right)$$

$$= \frac{3}{8} - \frac{3}{8}$$

$$= 0$$

$$a_n = \frac{3}{4}\left(\int_0^1 t\cos\frac{3n\pi t}{4} + \int_1^2 (t-2)\cos\frac{3n\pi t}{4}\right)$$

$$= \frac{3}{4}\left(\frac{4t\,\mathrm{sen}\,3n\pi t}{3n\pi} - \frac{4}{3n\pi}\left(\frac{-\cos 3n\pi t}{3n\pi}\right)\right) +$$
$$\frac{3}{4}\left(\frac{4(t-2)\,\mathrm{sen}\,3n\pi t}{3n\pi} - \frac{4}{3n\pi}\left(\frac{-\cos 3n\pi t}{3n\pi}\right)\right)$$

$$= \left(\frac{t\,\mathrm{sen}\,3n\pi t}{n\pi} + \frac{\cos 3n\pi t}{3(n\pi)^2}\right)\Big|_0^1 + \left(\frac{(t-2)\,\mathrm{sen}\,3n\pi t}{n\pi} + \frac{\cos 3n\pi t}{3(n\pi)^2}\right)\Big|_1^2$$

$$= \frac{(-1)^n}{3(n\pi)^2} - \frac{1}{3(n\pi)^2} + \frac{(-1)^n}{3(n\pi)^2} - \frac{(-1)^n}{3(n\pi)^2}$$

$$= \frac{1}{3(n\pi)^2}\left((-1)^n - 1\right)$$

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$$b_n = \frac{3}{4}\int_0^1 t \operatorname{sen} \frac{3n\pi t}{4} + \frac{3}{4}\int_1^2 (t-1) \operatorname{sen} \frac{3n\pi t}{4}$$

$$= \frac{3}{4}\left(\frac{-4t\cos 3n\pi t}{3n\pi} + \frac{4\operatorname{sen} 3n\pi t}{(3n\pi)^2} \right) + \frac{3}{4}\left(\frac{-4(t-2)\cos 3n\pi t}{3n\pi} + \frac{4\operatorname{sen} 3n\pi t}{(3n\pi)^2} \right)$$

$$= \left(\frac{-t\cos 3n\pi t}{n\pi} + \frac{\operatorname{sen} 3n\pi t}{3(n\pi)^2} \right)\Big|_0^1 + \left(\frac{-(t-2)\cos 3n\pi t}{n\pi} + \frac{\operatorname{sen} 3n\pi t}{3(n\pi)^2} \right)\Big|_1^2$$

$$= \frac{-(-1)^n}{n\pi} - \frac{(-1)^n}{n\pi} = \frac{-2(-1)^n}{n\pi}$$

$$F(t) = \frac{3}{8} + \sum_{n=1}^{\infty} \left(\frac{1}{3(n\pi)^2}\left((-1)^n - 1\right) \right) \cos \frac{3n\pi t}{4} + \sum_{n=1}^{\infty} \left(-\frac{2(-1)^n}{n\pi} \right) \operatorname{sen} \frac{3n\pi t}{4}$$