***Linear Programming Background:***

**Mathematical Programming:** a process for determining the optimal allocation of limited resources to meet certain goals or objectives.

**Dictionary definition of model:** a model is an idealized representation of a real-life system.

**Model Types (\*Indicates focus of Mathematical Programming):**

1. **Iconic Model (Scale):** physical model scaled up or down
2. **Analog Model:** one property is substituted for another (e.g., clock)
3. **Symbolic Model (Math):** system behavior is represented by mathematical symbols and functions *[can be optimized]\**
4. **Simulation Model:** computer program that attempts to replace system behavior

**Key Tradeoff of Mathematical Programming:**

*Tractability versus Validity*

Tractability: how well model lends itself to solution and analysis

Validity: how well the output is applicable to the real system

**Generic Math Program:**

**Objective Function**: f(x)

(maximize or minimize)

**Constraints**:

(subject to) g1(x) b1

g2(x) b2

g3(x) b3

Where the box can be ≤, ≥, <, >, =, or ≠

Constraints are typically technological and resource restrictions. May also include relationships between variables.

**Variables:** x = (x1, x2, …)T

**A linear program (LP) is a math program in which all functions [f(x), gi(x)] are linear.**

We will typically only use decision variables in this class; however, large programs have intermediate variables – which are derived variables (from the decision variables) to keep track of problem characteristics, or accounting variables. Also, in real-world programs there are typically many objective functions, rather than just one (multiple objective programming).

**Key Assumptions of Linear Programming:**

1. Deterministic: data is certain (that is, there are no probability distributions)

2. Proportionality: e.g., if 1 unit of a product requires 5 units of a resource; then 2 units of the product require 10 units of the resource. Thus, no economies of scale (i.e., no setup costs or learning curves)

3. Additivity: e.g., the total cost is the sum of the individual costs (i.e., no chemical reactions)

4. Divisibility: the decision variables are continuous (can take on fractional units) [not integers]

\*Note, assumption #4 is the most significant assumption.

**George Dantzig:**

George Dantzig is affectionately known as the “Father of Linear Programming.”

He developed the simplex algorithm to solve linear programs as part of Project SCOOP (Scientific Computation of Optimal Programs) while he worked for the Air Force. His work on linear programming was released/published in 1947 (it had been kept secret during World War II) and his algorithm was used for military operations planning. In that time, the “computer” was the digital computer (i.e., punchcards). Eventually his algorithm and applications expanded, to include (not an exhaustive list), product mix problems, production scheduling, inventory control, manpower planning, capital budgeting, and transportation/distribution problems. George Dantzig showed the use of his algorithm by solving a 70 by 70 assignment problem (by-hand and on a computer). (Note, a 70 x 70 assignment problem has 4900 possible solutions if doing an exhaustive search.)

His applications of linear programming for the Air Force not only included aircraft scheduling, but also finding the correct nutritional mix (i.e., a complicated product mix problem) for airmen, distribution of goods, planning of goods and people, etc.

George Dantzig was in graduate school (Cal. Berkeley’s PhD program) when World War II started. He took a leave of absence, and was an Air Force employee (civilian) for the Air Force’s Office of Statistical Control (Chief of the Combat Analysis Branch for USAF Headquarters Statistical Control and as Mathematical Advisor for USAF Headquarters). He completed his Ph.D. and stayed on with the Air Force for a few years before joining the RAND Corporation as a researcher. He eventually went on to academia.

Air Force Award: 1944, War Department Exceptional Civilian Service Medal

A George Dantzig quote about his initial Air Force appointment:

*“My office collected data about sorties flown, bombs dropped, aircraft lost... I also helped other divisions of the Air Staff prepare plans called "programs". ... everything was planned in greatest detail: all the nuts and bolts, the procurement of airplanes, the detailed manufacture of everything. There were hundreds of thousands of different kinds of material goods and perhaps fifty thousand specialties of people. My office collected data about the air combat such as the number of sorties flown, the tons of bombs dropped, attrition rates. I also became a skilled expert on doing planning by hand techniques.”*

Note, it is called the simplex method because it is named for a “simplex” in mathematical terms. (I’m simplifying this a bit), a simplex is putting k points into a k+1 dimensional space. For example, 1 point would have a simplex that is a line segment; whereas, 2 points would have a simplex that is a triangle; and 3 points a tetrahedron. The reason this is important to the algorithm is because an optimal solution (if one exists) will be at the corner points of the simplex; thus, you only need to evaluate/consider the corner points.